# SOME PROBLEMS OF CREATION OF COLLECTIVE ACCELERATORS

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Abstract

The question of formation of the magnetic field necessary for extraction of the electron-ion ring from the setup, in which it is created, to the accelerating system of the collective accelerator, by means of the "computer + display" system is considered. The requirements for tolerances on the magnetic field of the initial region of the accelerating system are obtained. The effect of accumulation of ions on the ring formation is considered. The problems concerning the electron ring creation in a static magnetic field necessary for the collective accelerator are discussed.

## 1. Introduction

The results of the theoretical investigations and calculations on some problems connected with creation of collective accelerators are presented in this paper. The selection of material is determined, on the one hand, by the subject at which the authors are working. On the other hand, we think the given problems to be of importance when designing collective accelerators.

## 2. Formation of the Magnetic Field by Means of the "Display System"

At present the electron ring is created, as is known, in the symmetry plane of the pulsed weak-focusing of the ADHESATOR (compressor) magnetic field[1]. After finishing the process of compression and injection of ions, the field symmetry is violated and the ring is extracted from the ADHESATOR to the constant magnetic field of the accelerating section.

The configuration of the coils, creating a required field, is selected by using the "computer + display" system. The work is carried out by the following scheme. The magnetic fields of the coils and of the accelerating section are calculated by a computer. The coil parameters and the diagrams of the summary field are seen on the display screen. With the help of a light pen the parameters of the coils are changed and are sent to be recalculated. The field diagram is analysed. In this manner the system of the coils, creating the required magnetic field, has been selected comparatively fast (Fig.1).

The magnetic field induced in the walls of the ADHESATOR chamber may be taken into account. The vertex electric field E in the conductive walls is calculated from the Fredholm integral equation of the second kind[2]:

$$E(Q)+i\lambda\sigma\int_D E(M)\sqrt{\frac{r_M}{r_Q}}f(k)\,dS_M=-i\lambda\int_{D_{ex}} j_{ex}(M)\sqrt{\frac{r_M}{r_Q}}f(k)\,dS_M \qquad (1)$$

where $\lambda=\frac{\omega\mu_0}{2\pi}$ ; Q, M are the observation and integration points, respectively; $r_Q$, $r_M$ are the radial coordinates of the points; $\omega$ is the circular frequency of currents in the coils; $\mu_0=4\pi\times10^{-7}\,\Omega$ sec/m ; $\sigma$ is the conductivity of the chamber walls of the ADHESATOR; D, $D_{ex}$ are the cross-sections of the conductive walls of the chamber and of the ring coils in the plane $\theta$=const ( $\theta$ is the azimuth coordinate).

$$f(k)=\left(\frac{2}{k}-k\right)K(k)-\frac{2}{k}E(k) \qquad (2)$$

E(k), K(k) are the complete elliptic integrals of the first and second kind of modulus k where

$$k^2=\frac{4r_Q r_M}{(r_Q+r_M)^2+(z_Q-z_M)^2} \qquad (3)$$

$z_Q$,$z_M$ are the longitudinal coordinates of points Q and M; $j_{ex}$ is the current density in the coils (system of loops). In formula (1) E and j are the complex amplitudes of the corresponding values. Having found induced currents in the walls $j=\sigma E$ , it is possible to determine the magnetic fields created by them. A quasi-stationary approximation was used in (1).

## 3. Magnetic Field Tolerances of the Initial Section of the Accelerating System

Let us consider tolerances on the value of the magnetic field of the initial section of the accelerating system where the bunch motion is not yet relativistic. The acceleration of the bunch under the effect of the magnetic field component $B_r$ leads to ion oscillations in the bunch with the frequency $\omega=\sqrt{2\pi e^2 n/M}$ where n is the electron density in the ring, M is the ion mass. The ion deflection from the bunch centre[3] at the moment t is

$$x(t)=-\frac{e}{m\omega}\int_0^t B_r(t)\sin\omega(t-t')\,dt' \qquad (4)$$

if the deflection and the velocity at the initial moment are equal to zero.

Here m is the electron mass increased by its rotation. Time in (4) is a function of the coordinate of the bunch centre $t=t(X_e)$ which is determined by the motion law of this centre relative to the stationary origin $X_e=X_e(t)$, i.e. t is an inverse function of $X_e=X_e(t)$. Accidental perturbations of the magnetic field increase the deflection of ions from the centre. Their contribution to the mean square deflection is equal to

$$\overline{\Delta x^2} = \frac{e^2 \Delta \overline{B_r^2}}{m^2 \omega^2} \int_0^{x_e}\int_0^{x_e} \frac{dx_1\, dx_2}{v(x_1) v(x_2)}\, g(|x_1-x_2|) \times \\ \times \sin\omega[t(x_e)-t(x_1)] \sin\omega[t(x_e)-t(x_2)] \tag{5}$$

where $v = \frac{dx_e}{dt}$, g is the correlation function. If the correlation length $\ell \ll \frac{v}{\omega}$, then $g=\ell\delta(x)$. In this case

$$\overline{\Delta x^2} = \frac{e^2 \Delta \bar{B}_r^2 \ell t}{2m^2 \omega^2 \bar{v}} \left(1 - \frac{\sin 2\omega t}{2\omega t}\right) \tag{6}$$

where $\bar{v}$ is the mean value of the bunch velocity. For the acceleration time much longer than the period of the ion oscillation

$$\overline{\Delta x^2} = \frac{e^2 \Delta \bar{B}_r^2 \ell t}{2m^2 \omega^2 \bar{v}} \tag{7}$$

The tolerance value will be found supposing for estimates $\sqrt{\overline{\Delta x^2}} \lessapprox a$ where a is a minor radius of the ring.

$$\sqrt{\Delta \bar{B}_r^2} \lessapprox m\bar{v}\sqrt{\frac{2N_e}{\pi MRL\ell}} \tag{8}$$

where L is the acceleration length, R is a major radius of the ring.

For example, for a number of electrons in the bunch $N_e=5\times10^{12}$, L=200 cm, R=5 cm, $\ell$=2cm, $v=10^{10}$ cm/sec, $m=40m_o$ ($m_o$ is the electron rest mass).

$\sqrt{\overline{B_r^2}}$ = 12 G (for protons). Correspondingly $\Delta(\frac{\partial B_r}{\partial z})$ =5 G/cm. The effect of single perturbations of the magnetic field on ions can be estimated according to ref.[4]. If the length of a single pertubation is equal to $\ell_o$ ($\ell_o \ll \frac{v}{\omega}$) then the ratio of the tolerance on $\Delta B_r$ to the tolerance (8) for accidental perturbations is $\sqrt{\ell L/2\ell_0^2}$. The formulae presented are used for estimating tolerances for the magnetic field of the accelerator and for adjustment of the magnetic fields of the ADHESATOR and the accelerating system. In the case of relativistic motion of the ring these formulae should be generalised.

## 4. Accumulation of Ions in the Electron Ring

The possibility of acceleration of heavy ions by the collective method has been shown in refs.[5,6]. The subject of accumulation of ions in the electron ring has been discussed in the same refs. and in [7]. Accumulation of ions is supposed to occur at the end of the ring compression, and their effect on the ring dimensions may be neglected. In this paper the effect of ions on the formation process of the electron-ion bunch is taken into account. The necessity for this arises, for example, when the gas, ions of which are accumulated in the bunch, is present in the ADHESATOR chamber for all the time of the ring compression under pressures higher than $10^{-7}$ torr or at a sufficiently powerful pulsed gas injection. Accumulation of a significant number of ions can markedly act upon the ring cross-section dimensions and lead to the crossing of the resonance value $\nu_r=1$ of the radial frequency of betatron oscillations.

Thus, in the equations permitting to find the number of accumulated ions[6], a decrease of the ring volume V should be taken into account:

$$\frac{dN_k}{dt} = \lambda_{k-1}\frac{N_{k-1}}{V} - \lambda_k \frac{N_k}{V} \tag{9}$$

where V≠ const; $N_k$ is the number of ions with the charge ke, $\lambda_k = N_e c\sigma_k$, $\sigma_k$ is the cross-section of ion ionization.

The system of equations (9) has been solved by the method of successive approximations together with the system of equations for semi-axes of the ring elliptic cross-section $\bar{b}$ (along axis z) and $\bar{g}$ (along axis r) from ref.[1] The contribution of ions is additionally taken into consideration in expressions for Coulomb shifts of frequencies.

1) Let us consider the accumulation of ions of the gas medium in which the ring compression takes place. The initial parameters of the ring are the following: $\gamma_{\perp 0}=4$ ($\gamma_\perp = (1-\beta_\theta^2)^{-1/2}$ is the transverse relative electron energy); major radius $R_o$=38 cm, $\bar{g}_o=g_oR_o$=1.5 cm, $\bar{b}_o=b_oR_o$=0.75cm, $N_e=5\times10^{12}$, field index $n_o$=0.5. The final values are $R_f$=6cm, $n_f$=0.06. The compression time is $2\times10^4$sec.

For helium under pressure $2\times10^{-7}$ torr, the final parameters are the following: $\bar{g}_f$=1.81 mm, $\bar{b}_f$=1.33 mm, $\nu_r^2$ =0.986, $N_1=3.2\times10^{10}$, $N_2=1.4\times10^{10}$. The increasing of gas pressure results in the crossing of the resonance $\nu_r$ =1. The corresponding maximum allowable pressure for nitrogen is equal to $2\times10^{-8}$ torr. In this case $\bar{g}_f$=1.81 mm, $\bar{b}_f$=1.36 mm, $N_1=9.14\times 10^9$, $N_2=1.07\times10^{10}$, $N_3=5.36\times10^9$,

$N_4=1.06\times10^9$, the number of ions with the charge larger 4 is insignificant. At $N_e=10^{13}$ we practically have the same results. Calculations for the helium medium have been used for determining the number of $\alpha$-particles trapped in the compressed electron ring in ref.[8)]

2) Let us consider the accumulation of ions of heavy atoms for the ring with $N_e=10^{14}$ when the compression of the ring with the initial parameters $R_o=35$cm, $\gamma_{\perp 0}=7$ and $\bar{b}_o=\bar{g}_o=1.5$ cm takes place in a high vacuum (the accumulation of ions from the residual gas in the process of compression may be neglected) up to the final values $R_f=5$ cm, $\bar{g}_f=0.84$ mm, $\bar{b}_f=2.4$ mm. After that, the magnetic field of the ADHESATOR keeps constant, and the ring is intersected by a flux of neutral atoms for time T. T is much shorter than the accumulation time (accumulation time is a period of time from the beginning of gas injection till the beginning of ring acceleration). In this case at the end of accumulation in the ring there are mainly ions with a large charge. The process of the accumulation of ions with a large charge is considerably accelerated when taking into account the adiabatic decrease of the dimensions $\bar{b}$ and $\bar{g}$ of the ring transverse cross-section in the process of accumulation. The decrease is caused by the fact that ions compensate a defocusing effect of the self-field of the electron ring.

Numbers of ions with different charges for N, Ar, Xe, Hg under pressures $10^{-5}$, $10^{-6}$, $10^{-7}$ torr in the gas jet have been determined by the method described. A more appropriate pressure is $10^{-6}$ torr. Calculations show that under this pressure and $T=10^{-5}$ sec to $15\times10^{-5}$ sec, it is possible to accumulate a necessary number of nitrogen ions with charge 7 (Fig.2), xenon (Fig.3) and argon ions with charge 8÷10 and mercury ions with charge 9÷11 in the ring.

It should be noted that while accumulating multicharge ions it is impossible to avoid the crossing of the resonance $\nu_r=1$. The effect of the crossing of this resonance on the formation of the electron-ion ring was not taken into account in the calculations presented.

## 5. On the Subject of the Static ADHESATOR (Compressor)

In refs.[9÷11)] different variants of the static compressor (ADHESATOR) have been proposed. The idea of the static ADHESATOR in itself is alluring. However, there are some difficulties which should be overcome when designing such a method of the electron-ion bunch creation.

One of these difficulties consists in a provision of conditions for longitudinal (along axis z) transportation and focusing of the ring.

The static ADHESATOR, proposed in ref.[9)], imposes very stringent requirements upon angular and energetic beam spread (external z-focusing is absent). In ref.[11)] a dynamic potential well is supposed to be used for transportation and external focusing, but such a compressor is not substantially a static one. In the original proposal[10)] external linear z-focusing is also absent[x)].

We consider the variant of the static ADHESATOR in which longitudinal focusing is provided by the interaction of the ring with an external metal tube cut along its sides[12)]. Transportation and compression of the ring on the major radius occur automatically (the latter takes place in ref.[10)] as well).

Let the laws for changing the radius and the longitudinal momentum of a thin electron ring along the axis of the static ADHESATOR L in length be given:

$$\frac{R}{R_0}=\eta(z),\quad \frac{p_z}{p_{\theta_0}}=\xi(z) \qquad (10)$$

where $R_o$, $p_{\theta_0}$ are the initial radius of the ring and the initial azimuth momentum equal to the total one, respectively. Assume that $\eta(0)=1$, $\eta(L)=1/k$, k is the coefficient of compression, $\xi(0)=\xi(L)=0$, $\xi<1$. Then a necessary magnetic field on the surface $r=R(z)$ takes the form:

$$B_z(z)=B_0\frac{\sqrt{1-\xi^2(z)}}{\eta(z)},\ B_r=B_0R_0\frac{\xi(z)}{\sqrt{1-\xi^2(z)}}\frac{d\xi}{dz} \qquad (11)$$

Define also

$$\bar{n}(z)=\frac{\ln(B_z/B_0)}{\ln(R/R_0)}=\frac{\ln[1-\xi^2(z)]}{2\ln\eta(z)}-1 \qquad (12)$$

At $\eta(z)<1$ one obtains $\bar{n}(z)>-1$.

In particular, the known cases follow from 10-12):

1) At $1-\xi^2=\frac{1}{\eta^2}$ $\bar{n}=-2$; $\frac{R}{R_0}=\left(\frac{B_0}{B_z}\right)^{1/2}$ is the acceleration ( $d\xi/dz>0$ ) in the magnetic field decreasing in z-direction and almost homogeneous in r-direction.

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x) In the magnetic field adiabatically changing along z, longitudinal focusing in the bunch relative to its moving centre is absent.

2) $\eta = 1$, then $\bar{n} \to \infty$, $R=R_o$=const, $B_z = \langle B_z \rangle /2 + const$ where $\langle B_z \rangle$ is the mean field in the circle of the radius R. This is an acceleration in the decreasing field at constant R.

3) $\xi \equiv 0$, $\eta \neq 1$, then $\bar{n}=-1$, $B_r=0$, $R/R_0 = B_0/B_z$. This correponds to the case considered in ref.[11].

The frequencies of betatron oscillations at a sufficiently slow change of parameters along z are

$$\nu_r^2 = 1 - \frac{2r_0 N_e \gamma_\perp}{\pi R(\gamma_\parallel^2 - 1)} \left[ T_r + \frac{R^2}{\gamma_\perp^2 \bar{g}(\bar{g} + \bar{b})} \right] \approx 1$$

$$\nu_z^2 = \frac{2r_0 N_e \gamma_\perp}{\pi R(\gamma_\perp^2 - 1)} \left[ T_z - \frac{R^2}{\gamma_\perp^2 \bar{b}(\bar{g} + \bar{b})} \right] \qquad (13)$$

Here $r_o = 2.8\times10^{-13}$cm, $\gamma_\perp = \gamma/\gamma_\parallel$ where $\gamma$ is the total relative energy of electrons in the ring.
$\gamma_\parallel = (1 - \dot{z}^2/c^2)^{-1/2}$ is the longitudinal relative energy of electrons of the ring; $T_r$ =1.33, $T_z$=1.68 are the values determined by image forces and depending on geometric parameters.

The adiabatic change of the amplitudes of betatron oscillations is the following:

$$\bar{b} \approx \left(\frac{R}{\nu_z}\right)^{1/2}, \ \bar{g} \approx \left(\frac{R}{\nu_r}\right)^{1/2} \approx R^{1/2} \qquad (14)$$

By the end of compression $\nu_z \approx R^{-1/2}, \bar{b} \approx R^{3/4}$.

The monotony of the decrease of $\bar{b}$ and $\bar{g}$ is provided if $\bar{n} > -1$ and this value is not very close to 0. The magnetic field of the configuration specified on the surface R(z) is created by special coils with average radii $R_1(z)$ and $R_2(z)$, the ring moving between these coils: $R_1(z) > R(z) > R_2(z)$. The injection of ions into the ring takes place when admitting gas into the setup. Figure 4 presents the calculations at the following parameters:

$\gamma_{\perp 0} = \gamma$ =40, $N_e=10^{14}$, k=8, $\bar{g}_o$= 0.204cm, $\bar{b}_o$=0.78 cm (the initial parameters are denoted by index "o"). The phase area $\epsilon_0$ of the injected beam is taken to be equal to 40 cm mrad. The laws of changing R and the longitudinal momentum are chosen as follows:

$$\eta(z) = 1 - \left(1 - \frac{1}{k}\right)\frac{z}{L}, \ \xi(z) = \frac{\xi_{max}}{2}\left(1 - \cos 2\pi \frac{z}{L}\right) \qquad (15)$$

$$0 \le z \le L$$

Such an ADHESATOR permits to obtain the best field for accelerating ions, corresponding to the Coulomb field at the edge of the bunch, E ≈ 9 MV/cm.

The questions of injection and creation of a necessary configuration of the magnetic field should be particularly considered. It should be noted that the phase area of an injected beam could be larger than the presented one. This will decrease the Coulomb field due to the increasing of the ring cross-section. Estimates show that $E \approx \epsilon_0^{-1}$.

References


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## Figures

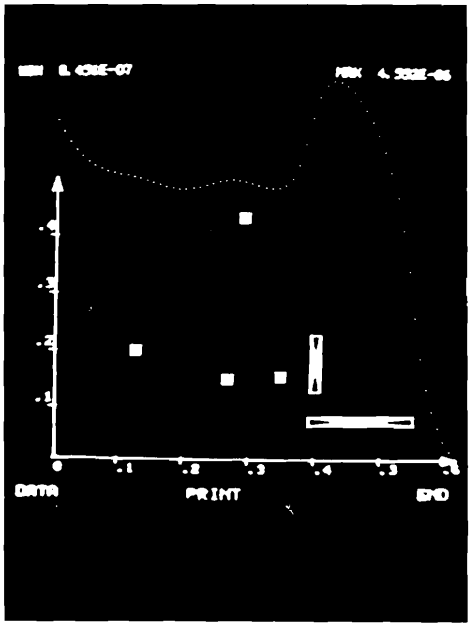


Fig. 1. Example of the distribution of the magnetic field of the Adhesator when the ring is extracted without taking into account the shielding by the chamber walls for specific positions of the coils with different currents in the "Display" program.

Horizontal axis - z(M).
Vertical axis - r(M).
Dotted curve - diagram $B_z$.
White squares and rectangles - loops and coils with current.

Maximum and minimum $B_z$ at the given interval z are shown above. The peak of the field distribution is cut off to take into account the shielding effect of the Adhesator wall.

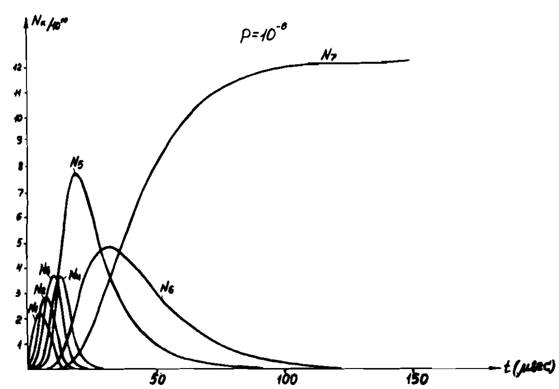


Fig. 2. Diagram of the accumulation of nitrogen ions in the ring.

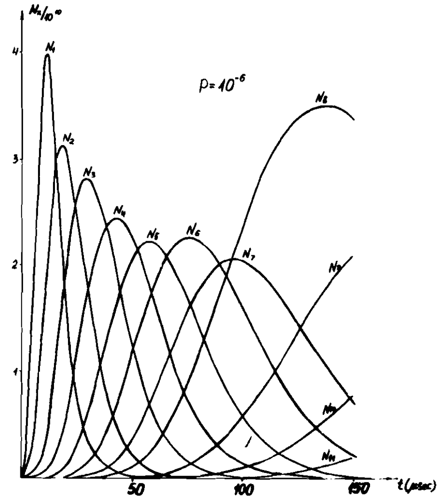


Fig. 3. Diagram of the accumulation of xenon ions in the ring.

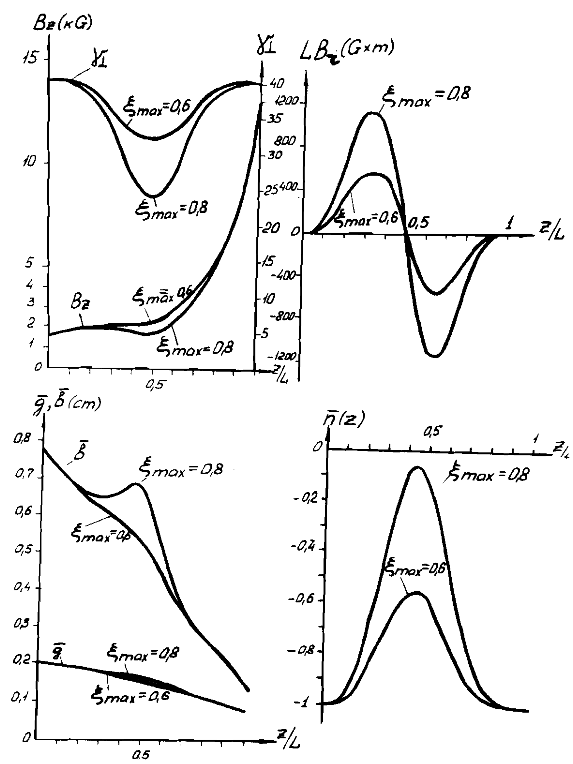


Fig. 4. Change of main physical parameters along the axis of the static adhesator.