E. Keil

CERN, Geneva, Switzerland

Abstract

At high beam currents, the beam lifetime in the ISR is shorter than that expected from nuclear and Coulomb scattering in the residual gas. Two possible explanations for this observation are put forward: intra-beam scattering of the protons of one beam on each other driving them into non-linear resonances, and a slow stochastic instability, called Arnold diffusion, which is excited by neutralised regions in the ISR beam.

1. Introduction

The best beam lifetimes in the ISR are compatible with nuclear scattering only for currents up to about 1 A. For higher currents, the beam lifetime decreases with increasing current. Furthermore, the beam lifetime depends on the degree of neutralisation of the beam: when clearing electrodes are switched off, the beam lifetime is reduced. In the following, we shall discuss possible explanations for these observations¹).

2. Intra-beam Scattering

Coulomb scattering of the protons of the same beam on each other was considered as a possible means of beam growth in the ISR by Pellegrini²⁾. He found that the fastest process is a change of momentum which has a diffusion constant for the relative momentum error given by:

$$a^{2} = \frac{d}{dt} < \left(\frac{\Delta p}{p}\right)^{2} > = \frac{\pi^{\frac{1}{2}} \operatorname{Nr}_{o} c F(\lambda_{e})}{\sqrt{\sqrt{x^{2} > \frac{1}{2}} \gamma^{3} \beta^{3}}}$$
(1)

Here r_0 is the classical proton radius, V is the volume of the beam, $\langle x'^2 \rangle_2^2$ is the r.m.s. betatron angle, and $F(\lambda_e)$ is a numerical factor related to the cut-off in the impact parameter; it can be taken as 65 for a wide range of values of λ_e . In the ISR, part of the beam size comes from betatron oscillations and part from the momentum spread. We take V to be the betatron volume, and assume that on average particles with a momentum difference corresponding to two horizontal betatron amplitudes N.

Within the lifetime of the ISR beam, the growth of the beam size due to diffusion in momentum is completely negligible². A stronger limitation arises when one considers intra-beam scattering in connection with non-linear resonances. It has been found necessary to run the ISR with a substantial spread of Q values across the stack¹. Consequently, there are always fairly strong non-linear resonances close to its upper and lower edges. We shall explain the observed beam lifetimes by intra-beam scattering into these resonances which we assume to be fatal in the sense that the lifetime of a proton within a resonance is short compared to the beam lifetime. Since the latter is some 10¹⁰ revolutions, this assumption seems well justified.

A stack which is bordered by two resonances a distance $\Delta p/p$ apart has an asymptotic lifetime given by:

$$\tau = (\Delta p/p)^2 / \pi^2 a^2$$
 (2)

A rectangular stack reaches this decay rate within a factor of two after about 0.05 lifetimes. The initial decay rate is higher or lower depending on whether the stack density at the edges is higher or lower than that of the sine-shaped equilibrium distribution.

For typical ISR parameters, the diffusion constant a^2 is of the order of $4 \times 10^{-11} s^{-1}$. The distances between resonances which would explain certain decay rates are shown in Table I.

Table I

Relation between decay rate \dot{I}/I and distance r between adjacent resonances

$$\frac{1}{1/1} \begin{bmatrix} \min^{-1} \\ 10^{-3} & 10^{-4} & 10^{-5} \end{bmatrix}$$

r [cm] 0.61 2.0 6.1

Since the distance between adjacent non-linear resonances is typically a few centimetres, decay rates of the order of 10⁻⁴ min⁻¹ might well be explained by intra-beam scattering. The best lifetimes observed in the ISR are between the nuclear scattering lifetime and the multiple scattering lifetime and are therefore inconclusive as far as intra-beam scattering is concerned. However, we observe that these good lifetimes only last for a few hours at best. This is much shorter than expected from the growth in beam size due to multiple scattering. We also observe rather large fluctuations of the beam lifetime at the same current which cannot be explained by variations in the vacuum pressure. They may be due to changes in the beam position relative to the resonances from one run to the next.

3. Arnold diffusion

This process was put forward by $Chirikov^3$ as a possible danger for the beam lifetime in proton storage rings due to beam-beam interaction. It requires coupling between horizontal and vertical betatron motion and the presence of many azimuthal harmonics in the non-linear force. It was not considered dangerous in the ISR because the large crossing angle causes the beam-beam force to be essentially vertical, therefore not introducing any coupling, and because of the number of superperiods in the ISR.

The required coupling and harmonic spectrum do exist when the ISR beam becomes neutralised over some distance. A reduction of the beam lifetime has been observed when the clearing electrodes are switched off over as little as one eighth of the ISR circumference¹).

The space-charge forces in a neutralised region are very much equivalent to the kicks assumed by Chirikov for the beam-beam force. Following his analysis, we have calculated the beam lifetime where Arnold diffusion becomes important as a function of the Q shift, ΔQ , which is produced by a single kick per revolution. The parameters entering into the calculation are the Q shift at which the particle motion becomes stochastic ΔQ_s and the coupling parameter β . The stochasticity limit ΔQ_s is known from electron storage rings⁴). The coupling parameter β is given by the ratio of the width of coupling resonances. The result of our calculation is shown in Fig. 1.

At a momentum of 15 GeV/c the neutralised Q shift is about

$$\Delta Q = 2 \times 10^{-7} L I \tag{3}$$

where L is the neutralised length, assuming complete neutralisation, and I is the beam current. It may be seen, that a drastic reduction in the beam lifetime sets in when the product L*I exceeds about 100, corresponding to one neutralised long straight section at 5 A beam current. There are several observations which lead us to believe that Arnold diffusion may be one of the mechanisms which limit the ISR beam lifetime at present. Large pressure rises have been observed in several regions. up to pressures where the clearing electrodes are known to become inefficient¹⁾⁵). Sometimes these regions coincide with enlarged portions of the ISR vacuum chamber in which clearing is difficult because they form longitudinal potential wells in which the electrons may be trapped longer than in normal straight sections with efficient clearing⁵⁾. However, conclusive experiments have not been made, because artificial neutralised regions would only appear in addition to those already present in the ISR and their combined effect would be very difficult to analyse.

Arnold diffusion itself only affects particles inside the stochastic layers which should occupy only a small fraction of phase space. However, phenomena like Coulomb scattering on the residual gas or intra-beam scattering can transport particles into the stochastic layers, thus causing Arnold diffusion to affect the whole beam.

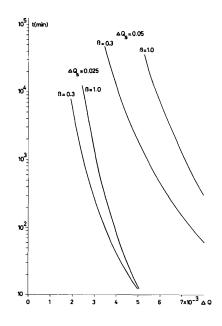


Fig. 1:Arnold diffusion lifetime as a function of
the strength of the non-linear kick ΔQ .
 ΔQ_s is the stochasticity limit and β is the
coupling parameter.

4. Conclusions

Two mechanisms have been presented which might explain the beam lifetime shortening observed in the ISR at high currents. At the time of this writing it is impossible to affirm that either of these mechanisms has definitely been seen in the ISR. Experiments to show their existence or absence are continuing.

References

- W. Schnell "Experimental Observation of Beam Behaviour in the ISR". Contribution to this Conference.
- C. Pellegrini, Frascati Report LNF-68/1, unpublished.
- B.V. Chirikov, Institute of Nuclear Physics, Novosibirsk. Preprint 267, Section 4.3., unpublished.
- 4) M. Sands, "The physics of Electron Storage Rings. An Introduction " SLAC-121 (1970).
- 5) B. Angerth, E. Fischer, O. Gröbner "The clearing fields of the ISR". Contribution to this Conference.