THE TRANSVERSE STABILITY OF THE BUNCHED BEAM INTERACTING WITH THE LOW - Q - VALUE SYSTEM

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Experimental investigation of the beam coherent stability in storage rings has shown that there are the coherent effects in which the decrements and increments do not depend on the choice of the operating betatron and synchrotron frequencies of particle oscillations. This fact shows that these effects are due to the beam interaction with the low-Q-value systems in which the fields excited by the beam decrease during the time smaller than the period of particle revolution. The characteristic feature of such effects is their dependence not only on full charge of a bunch but on its length (and independence on the number of bunches). Earlier the authors of this report investigated the interaction of a bunched beam with the low-Q-value resonator and a matched transmission line[1, 2]. These works consider the stability of the main types of the beam collective betatron oscillations in the limit of short length of bunches. The main results of these works are the conclusions of the stability of a short beam interacting with a matched transmission lines[1] and the possibility of arising an instability due to the beam interaction with the low-Q-value resonator[2]. The dependence of betatron frequencies on energy in these works was not taken into account ($d\omega_z/dE_s = 0$). The importance of this dependence was pointed out by Pellegrini[3]. But Pellegrini's increments tend to zero when the machine chromatism vanishes:

$$\delta \sim \frac{d\ln\nu}{d\ln R_0}\,\ell_\beta$$

(ν - betatron wave number, $2\pi R$- orbit perimeter, ℓ_β- bunch length); it seems to us that it is connected with not quite correct (in electromagnetic relation) account of the fields excited by a beam.

In this paper we give some results of stability investigation of axial and axial-longitudinal collective excitations of finite length bunch when derivative $d\omega_z/dE_s$ is not zero.

Undisturbed motion of particles is described by

$$Z = a\left[f_z(\Theta)\, e^{i\left(\psi_z + \frac{d\omega_z}{d\omega_s}\varphi\right)} + C.C.\right];\ \dot{\psi}_z = \omega_z(E_s)$$

$$\Theta = \omega_s t + \varphi;\ \omega_s = \omega_0(E_s);\ \varphi = \varphi_0 \sin\psi_c;\ \dot{\psi}_c = \omega_c$$

$$\frac{d\omega_z}{d\omega_s} = \left(\frac{d\omega_z}{dE_s} \Big/ \frac{d\omega_s}{dE_s}\right)$$

Here, $f_z(\Theta)$- is Floke's periodic function satisfying the normalization condition

$$f_z \cdot \left(f_z'^{*} - i\nu_z f_z^{*}\right) - C.C. = -2i\ ;$$

Θ - generalized asimuth; φ - phase of longitudinal motion; E_s - equilibrium meaning of energy.

When the frequency shifts are small ($|\Delta\omega| << \omega_c$) due to the collective interaction the beam normal excitation near some stationary state (the distribution function F_0 of which does not depend on phases ψ_z, ψ_c) is described by function

$$F(a,\varphi_0,\psi_z,\psi_c,t) = F_0(a,\varphi_0) + \left[F_m(a,\varphi_0)\, e^{i(m_z\psi_z + m_c\psi_c) - i\omega t} + C.C.\right]$$

m_z, m_c are integer numbers which define multipolarity of excitation, ω - characteristic frequency which is to be found. The spectrum of eigen values ω are found with the help of Vlasov's equations for F function and electromagnetic field excited by beam. The solution for ω may be easily obtained in the case of δ function distribution in amplitudes of phase oscillations

$$F_0(a,\varphi_0) \sim \delta\left(\overline{\varphi_0^2} - \varphi_0^2\right)$$

An ideal exampleof low-Q-value system is matched line in which the time of disappearing fields is equal to a plate length (ℓ) divided by velocity of light (it is supposed that the filling is absent) and much less than the revolution period ($\ell/c << \frac{2\pi}{\omega_s}$). For matching at the ends the eigen-functions of a field free oscillations in such line can be written

$$\vec{A}_K(\vec{z},t) = \sqrt{cZ}\ \frac{\vec{\mathcal{E}}(\vec{z})}{U_0} \cdot \frac{e^{iK(R\Theta - ct)}}{(2\pi)^{3/2}}\ ;\quad U = \sqrt{cZ}\ \frac{U_a}{U_0}$$

where $\vec{\mathcal{E}} = -\vec{\nabla} U_a$ -electrostatic field which is produced by the potential U_0 between the plate and chamber walls, Z -wave impedance, c -velocity of light, $2\pi/K$ -the length of a field wave in line. Here we take into account the interaction only with the main wave (TEM) of a two-connected wave-guide concerning the bunches with lengthes(ℓ_β) more than the chamber transversal dimensions ($\ell_\perp$). $\ell_\beta > \ell_\perp$

The results given below were obtained for the case of arbitrary multipolarity when for $U(\vec{z}_\perp,\theta)$ the expansion in multipoles is true

$$U(\vec{z}_\perp,\theta) = \sum_{m_z=0}^{\infty} \sum_{m_r=0}^{\infty} \frac{Z^{m_z}}{m_z!} \cdot \frac{r^{m_r}}{m_r!} \cdot \left(\frac{\partial^{m_z\, m_r} U}{\partial Z^{m_z}\, \partial r^{m_r}}\bigg|_{\vec{z}_\perp = \vec{z}_{\perp s}}\right)$$

here m_z, m_r are the numbers of multipolarity in r and in Z degrees of freedom.

In this approximation ($a_r << \ell_\perp, a_z << \ell_\perp$) the expression for decrements is ($\delta = -\mathrm{Im}\,\omega$): a. short bunch ($\ell_\beta < \ell$) in ultrarelativistic case $\gamma^2 \ell_\beta > \ell$ ($\gamma = \frac{E_s}{mc^2}$ m is particle mass)

$$\delta = N\delta_0\left(\Phi_1 + \frac{4\ell_\beta}{\pi^2 \ell}\left[\Phi_2 - \frac{d\nu_z}{d\ln R}\cdot\Phi_3\right]\ \frac{1}{4m_c^2 - 1}\right) \quad (1)$$

where

$$\delta_0 = \frac{|m_z| r_0 c}{2\gamma}\left(\frac{1}{(|m_z|-1)!} \cdot \frac{\partial^{|m_z|} U}{\partial Z^{|m_z|}}\bigg|_{\vec{z}_\perp = \vec{z}_{\perp s}}\right)^2 \left(\frac{\ell}{2\pi R}\right) \langle a^{2|m_z|-2} \rangle$$

$r_0 = e^2/mc^2$ -classical particle radius, brackets < > mean averaging with F_0, and factors Φ_1 Φ_2 Φ_3 are

$$\Phi_1 = J^2_{m_c}\left(\frac{m_z \ell_B}{2}\frac{d\nu_z}{dR}\right)\cdot\frac{R}{\ell}\int_0^{\ell/R} d\theta \left|f_z(\theta)\right|^{2|m_z|-2}$$

$$\Phi_2 = \begin{cases} \frac{3|m_z|-2}{2|m_z|-2}\cdot\left|f_z(\theta=\ell/R)\right|^{2|m_z|-2} + \frac{|m_z|-2}{2|m_z|-2}\left|f_z(\theta=0)\right|^{2|m_z|-2}, & |m_z|>1 \\ 2+\ln\left(\frac{|f_z(\theta=\ell/R)|}{|f_z(\theta=0)|}\right), & |m_z|=1 \end{cases}$$

$$\Phi_3 = \left|f_z\left(\theta=\ell/R\right)\right|^{2|m_z|} + \left|f_z(\theta=0)\right|^{2|m_z|}$$

$\theta=\ell/R$ and $\theta=0$ the coordinates of the plate ends, $J_m(x)$ - Bessel's function of m order.

The formula (1) differs from (3) by the presence of terms proportional to Φ_1 and Φ_2 and by the fact the it is obtained for the excitations of arbitrary multipolarity of betatron oscillations.

The first term in (1) which is proportional to a plate length is connected with the energy radiation in a matched line due to the bunch coherent transversal motion. Note, that it is always positive and corresponds to the early obtained[1] decrement of a "fast damping". The second term which does not depend on a plate length corresponds to so called "head tail" effect. The sign of this term depends on the excitation type, sign and value $d\nu_z/d\ln R$. Its appearance is physically connected with the fact that at the ends of a plate where the electric field of the "main wave" has the longitudinal component the wave excitation (TEM) by the longitudinal motion is occured.

Note, that at $m_c \neq 0$ factor Φ_1 is proportional to a small value

$$\left(\frac{m_z \ell_B}{2}\,\frac{d\nu_z}{dR}\right)^{2m_c} << 1$$

and that is why synchro-betatron excitations may become unstable. The stability of excitations $m_c=0$ may be provided by the choice of the plate sufficient lengths.

Note, that the value of the "edge" terms is proportional to m_z-degree of modulus of Floke's function at the ends of line. This fact may be determinant for the choice of a location of plates in the machines with large beating Floke's function (the machines with low β - function).

Let us neglect Floke's beating $(f_z = 1/\sqrt{\nu_z})$ then the stability condition ($\delta > 0$) is

1. $m_c=0$ $\quad \frac{8\ell_B}{\pi^2\ell}\left(1-\frac{d\ln\nu_z}{d\ln R}\right) = \frac{8\ell_B}{\pi^2\ell}\;\frac{1}{\nu_z}\left(\frac{d\,\omega_z}{d\,\omega_0}\right)_s < 1$

2. $m_c \neq 0$ $\quad \left(\frac{d\,\omega_z}{d\,\omega_0}\right)_s > 0$

More general stability condition(for strong focusing) may be obtained from (1).

In conclusion we give the expression of decrement for more important type of oscillations when $m_z = 1$:

$$\delta = \frac{N z_0 c}{2\gamma}\left(\frac{\ell}{2\pi R}\right)\left(\frac{\partial U}{\partial z}\right)^2\left[J^2_{m_c}\left(\frac{\ell_B}{2}\,\frac{d\nu_z}{dR}\right) + \frac{4\ell_B}{\pi^2\ell}\left\{2 + \ln\frac{|f_z(\theta=\ell/R)|}{|f_z(\theta=0)|} - \frac{\partial\nu_z}{d\ln R}\left(|f_z(\theta=\ell/R)|^2 + |f_z(\theta=0)|^2\right)\right\}\frac{1}{4m_c^2-1}\right] \tag{2}$$

b. For long bunches $\ell_B >> m_c\ell$ (in ultra-relativistic approximation) the decrements for the case of azimuth-symmetric focusing are obtained:

$$\delta = \frac{\delta_0 N\ell}{\pi^2\ell_B}\left(1+2\frac{d\ln\nu}{d\ln R}\cdot\ln\frac{\ell_B}{(m_c+1)\ell}\right) \tag{3}$$

It is seen that instability may occur when the value $(d\ln\nu_z/d\ln R)$ is negative. Note, that the decrement in (3) is inversely proportional to the bunch length ℓ_B.

The functional dependence of threshold current (I_{th}) of instability on energy and chromatism $(d\ln\nu_z/d\ln R)$ may be easily obtained from the expressions (1,2,3) by equilizing $(-\delta)$ to the full decrement of damping $\Delta\nu$ (equal to the decrements owing to Landau's damping and radiation friction). For example, for the case when the bunch is longer than a plate ($\ell_B > \ell$)

$$N_{th} = \frac{\pi^2}{\delta_0}\;\frac{\Delta\nu\,\ell_B\,\gamma}{\left(2\cdot\ln\frac{\ell_B}{(m_c+1)\ell}\cdot\left|\frac{d\ln\nu_z}{d\ln R}\right|-1\right)}$$

That is in a qualitative agreement with the experimental results obtained on machines ACO[4] and ADONE[5].

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