COHERENT BEAM BREAK-UP DUE TO SPACE CHARGE

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Abstract

A mechanism is described which leads to a beam break-up in storage rings and which yields limiting currents in good agreement with some experiments made with the A.C.O. storage ring.

## 1. Introduction

In electron and positron storage rings the maximum currents are limited by space charge effects. The limit is usually expressed by the Q-shift, i.e. the change of the betatron wave number, which was first investigated in[1] and which was experimentally found to be approximately 0.03. But there is no general theory from which this value can be computed with sufficient accuracy. In the following investigation it is shown that at certain Q-shifts the closed orbits become unstable.

## 2. Description of the mechanism

To investigate the stability of the closed orbit, we assume that one of the two beams has a virtuel transverse displacement in the interaction point. Then the other beam does not pass through the center of the first beam and sees a transverse force. It is obvious that this force causes a closed orbit distortion. Now the question arises, in what direction the second beam is displaced at the interaction point. If the force is attractive, as between electron and positron beams, the second beam is not necessarily shifted toward the first beam. Depending on the betatron wave number Q, it may as well be moved in the opposite direction. To show this, we use the formula for the closed orbit distortion[2].

$$z(s) = \frac{\sqrt{\beta(s)}}{2E\sin(\pi Q)} \int_s^{s+C} F(\ell)\sqrt{\beta(\ell)}\cos(\psi(\ell)-\psi(s)-\pi Q)\,d\ell \qquad (1)$$

with

$\beta(s)$ = amplitude function

$\psi(s)$ = phase of the betatron oscillation

$E$ = particle energy

$F(s)$ = disturbing force

$C$ = circumference

If the force is given by the displacement $z_1$ of the first beam and if the bunch length is small as compared to the betatron wave length, one obtains for the displacement $z_2$ of the second beam at the interaction point:

$$z_2 = 2\pi D\cot(\pi Q) z_1 \qquad (2)$$

with

$$D = \frac{r_e N_B \beta_z}{2\pi\gamma\sigma_z(\sigma_x+\sigma_z)}$$

$r_e$ = electron radius

$N_B$ = number of particles per bunch

$\sigma_{x,z}$ = standard deviations for beam dimensions

For Q not too close to an integer, D is the shift of the betatron wave number per interaction region

$$D = \Delta Q \ . \qquad (3)$$

Eq.(2) shows that the second beam is moved in the opposite direction if $\cot(\pi Q)$ is negative.

It is evident that, inversly, the displaced second beam leads to a closed orbit distortion of the first beam which, in the interaction region, is given by

$$z_1 = 2\pi D\cot(\pi Q) z_2 \qquad (4)$$

If this last $z_1$ is larger than that $z_1$ which was assumed to produce the displacement $z_2$, then the closed orbit distortion will increase and is unstable. With $z_2 = -z_1$, the limit for stability can then be written in thr form:

$$D = -\frac{1}{2\pi}\tan(\pi Q) \qquad (5)$$

In Eq.(5), Q as well as $\beta$ are the disturbed values which contain the action of the space charge as a rigid thin lens.

## 3. General formulation of the limiting Q-shift

The last result can be generalized. One can consider more than one interaction point and gets several different modes for an unstable closed orbit. Also, one can consider those oscillations which yield a diplacement of opposite sign after one revolution. If the absolute value of the displacement is increased after one revolution, then the two beams will oscillate with growing amplitude. Finally one can consider all modes which lead on an integral or half-integral stopband.

A convenient way for computing the limiting currents is the following. If $M_o$ [2] is the transfer matrix for one of p equal sections between p interaction points, we get for the coherent motion of the first beam $(z_1, z_1')$ the relation:

$$\begin{pmatrix} z_1 \\ z_1' \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & 0 \\ -4\pi D/\beta & 1 \end{pmatrix} M_o \begin{pmatrix} z \\ z' \end{pmatrix}_n$$

$$+4\pi D/\beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} z_{2,n+1} \qquad (6)$$

For $z_2 = -z_1$ one gets

$$\begin{pmatrix} z_1 \\ z_1' \end{pmatrix}_{n+1} = M \begin{pmatrix} z_1 \\ z_1' \end{pmatrix}_N \qquad (7)$$

with

$$M = \begin{pmatrix} 1 & 0 \\ -8\pi D/\beta & 1 \end{pmatrix} M_o$$

The limit for stability is now given by the condition that the eigenvalues $\lambda^p$ of the distorted revolution matrix $M^p$ are ± 1. This condition yields the limits

$$D = \frac{\cos(2\pi Q/p) - \cos(\pi(2m+1/2 \pm 1/2)/p)}{4\pi \sin(2\pi Q/p)}^{+} \qquad (8)$$

with

$$m = 0, \pm 1, \pm 2, \ldots$$

In Eq.(8) Q and β are the undisturbed values. Fig.1 shows the limits for 1, 2, 4 and 6 interaction points.

## 4. Comparison with measurements

These limits can in fact be observed in the A.C.O. storage ring. In this ring the betatron wave numbers are 2.845 and 0.845 for $Q_x$ and $Q_z$, respectively. The limiting Q-shift according to Eq. (8) is 0.020 for 2 as well as for 4 interaction points. The experimental values[3] are 0.028 for 2 and 0.019 for 4 interaction points, independent of energy.

Comparing these values, one has to keep in mind that the bunch dimensions were measured in the magnets and then transferred to the interaction point by calculation using undisturbed β-functions. If, in this computation, one takes into account the change of the β-function due to the space charge (for instance, $\Delta\beta_z/\beta_z$ at the interaction point is + 23% for 2 and - 3% for 4 interaction points) one gets 0.020 for 2 and 0.018 for 4 interaction points, which appears to be in good agreement with the theoretical value.

Applying this formalism to the Adone storage ring, one finds that the measured currents are much smaller than the limits computed here. Therefore, it seems that at least two competing mechanisms limit the stored currents.

+ For m = 0 one obtains the two limits

$$D = -\frac{1}{4\pi} \tan(\pi Q/p)$$

$$D = \frac{1}{4\pi} \cot(\pi Q/p) \;.$$

These modes have the property that the absolute value of λ becomes larger than 1 with increasing D, which leads to a growing oscillation within one section.

References:

1. F. Amman, D. Ritson, Report LNF-61/38(1961)
2. E.D. Courant, H.S. Snyder, Ann. of Phys. 3, 1-48 (1958)
3. Groupe de l'Anneau de Collisions d'Orsay (private communication)

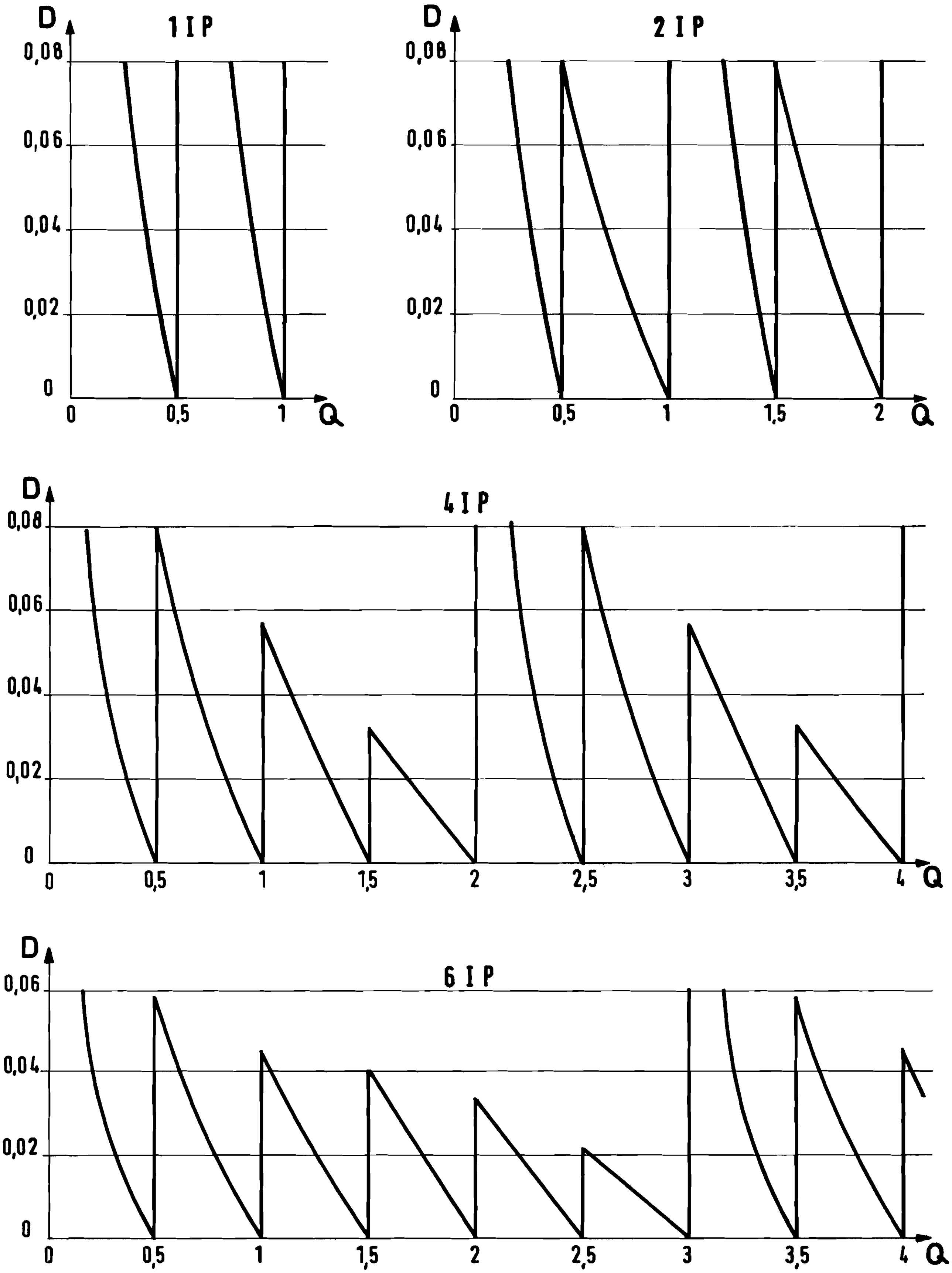


Fig. 1. Dependence of the Q-shift on Q.