NON-LINEAR BEAM-BEAM EFFECT

A. Jejcic, J. Le Duff

Laboratoire de l'Accélérateur Linéaire, Université de Paris-Sud, Centre d'Orsay, Orsay, France

Abstract

An analytical approach to the non-linear space charge effects in electron-positron colliding beam rings is presented. In particular, the distribution function of the betatron frequencies is computed in some special cases of interest, such as head-on collisions, uncomplete compensation with four beams.

Introduction

Many approaches have been tried to investigate beam-beam effect which limits current density in storage rings. The linear incoherent or coherent theories[1,2,3] lead to frequency shifts for betatron motion, but unfortunately such explanations do not account for the dependence of current limits on the operating point, which has been experimentally established. Another method has been developped, consisting in numerical computation of the betatron motion of a particle passing turn after turn through the non linear space charge field created by a bunch circulating in the opposite direction[4,5].

In the present work an analytical approach is made considering the beam-beam effect as a strong non-linear phenomena. More precisely one gives a method of calculating the resulting wave number distribution in any possible case. This can be also a starting point for a study of amplitude evolution.

Perturbation method

A circular beam with gaussian distributed charges creates in its own frame an electrostatic potential which can be written :

$$V^* = V_o^* + \frac{Q^*}{4\pi\varepsilon_o} \sum_{k=1}^{\infty} \frac{(-1)^k}{k.\ k!} \left[\frac{x^2 + y^2}{2\sigma^2}\right]^k \qquad (1)$$

where Q^* is the charge per unit lenght, and σ the standard deviation for the radial density distribution.

In the laboratory, with $\beta \approx 1$, the magnetic potential is purely longitudinal so that it can be added to the electrostatic one, and one can write :

$$V = 0 \qquad\qquad cA_y = 2\gamma V^*$$

which, refering to G. Leleux[6], leads to perturbating hamiltonian :

$$k_1 = -\frac{2e\ R^2}{p\ r_m^2} \frac{\gamma}{c} V^* \qquad (2)$$

expressed with the normalized variables

$$X = \frac{x}{r_m}, \qquad Z = \frac{z}{r_m}$$

(r_m = magnetic radius, R = mean radius) and with the independant variable θ ($ds = R\, d\theta$).

Let a and ϕ be the amplitude and phase of the usual Floquet solutions ; the method of variation of canonical constants[7] then gives the equations :

$$\frac{d\phi_{x,y}}{d\theta} = \frac{\partial k_1}{\partial a_{x,y}} \quad (3a) \qquad \frac{da_{x,y}}{d\theta} = -\frac{\partial k_1}{\partial \phi_{x,y}} \quad (3b)$$

describing the amplitude and phase evolution induced by the perturbating space charge forces.

Distribution function for the betatron frequencies

Introducing into the perturbating hamiltonian(2) the unperturbated Floquet solutions one obtains from equation (3a) :

$$\frac{d\phi_x}{d\theta} = -\frac{2e\ R^2\ \gamma\ Q^*}{4\pi\ \varepsilon_o\ c\ p\ r_m^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k.k!} \frac{r_m^{2k}}{a^{2k}}$$

$$\cdot \sum_{p=0}^{k} C_p^k\ 2^{2k}\ (k-p)\ a_z^p\ q_z^{2p} \cos^{2p}(\nu_z\theta + \phi_z) \qquad (4)$$

$$\cdot\ a_x^{k-p-1}\ q_x^{2(k-p)} \cos^{2(k-p)}(\nu_x\theta + \phi_x)$$

In an electron positron storage ring particles are gathered in short bunches, so one can assume that during interaction the transverse position of the particle remains unchanged. Then, in (4) the longitudinal charge density must be integrated over the interaction region. Moreover, taking into account the fact that the interaction regions are well defined, the Floquet factors can be expressed in terms of Twiss functions. Finally the wave number variation due to one interaction is given by :

$$\Delta\nu_x = \frac{\Delta\phi_x}{2\pi} = -\frac{N r_o \beta_{xi}}{4\pi\ \gamma\ \sigma^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k.\ k!} \frac{1}{2^{3k-2}}$$

$$\cdot \sum_{p=0}^{k} C_p^k\ (k-p) \left(\frac{\hat{z}_i}{\sigma}\right)^{2p} \left(\frac{\hat{x}_i}{\sigma}\right)^{2(k-p-1)} \qquad (5)$$

$$\left[\sum_{n=0}^{p-1} 2\ C_n^{2p} \cos 2\ (p-n)(\nu_z\theta + \phi_z) + C_p^{2p}\right]$$

$$\cdot \left[\sum_{m=0}^{k-p-1} 2\ C_m^{2(k-p)} \cos 2\ (k-p-m)(\nu_x\theta+\phi_x) + C_{k-p}^{2(k-p)} \right]$$

where $\hat{z}_i$ and $\hat{x}_i$ are the betatron amplitudes in the interaction region.

After a great number of interactions and assuming the operating point is out of non-linear resonances, the mean value for the wave number displacement becomes :

$$< \Delta\nu_x > = - \Delta\nu_{xo} \sum_{k=1}^{\infty} \frac{(-1)^k}{k.k!} \frac{1}{2^{3k-2}} \tag{6}$$

$$\cdot \sum_{p=0}^{k} (k-p)\ C_p^k\ C_p^{2p}\ C_{k-p}^{2(k-p)} \left(\frac{\hat{z}_i}{\sigma}\right)^{2p} \left(\frac{\hat{x}_i}{\sigma}\right)^{2(k-p-1)}$$

where $\Delta\nu_{xo}$ is often called the linear frequency shift. A similar formula is obtained for the vertical motion.

The same method applied to equation 3b would show no change in amplitude.

A numerical study of (6) has been done using a Monte Carlo method. Rayleigh distributed numbers representing the amplitude were generated and put into the formula (6) giving the wave number distribution represented on fig.1. It must be noticed that the series in (6) has a very low convergence, but numerical improvements were made to obtain rapidly the result with a good precision.

The present calculation can be also performed for a Gaussian beam with an elliptical cross section using the corresponding potential formula[8,9,10]. The flat beam case leads to an analytical formulation for the distribution function[11].

<u>Discussion about possible amplitude evolution</u>

Until now the non-linear resonances have been neglected, assuming the operating point was chosen properly. Nevertheless in the preceeding paragraph it is shown that when the current increases, the spread in betatron frequencies increases, so it may be possible to bring some particles on non-linear resonances whose strength still remains difficult to appreciate.

A non linear resonance means that there is a phase relation between two interaction points so that the average effect has not the same signification. To take this new effect into account one can write for the perturbation hamiltonian :

$$k_1(\theta) = k_1 \times \delta\ (\theta - q\ \Theta) \tag{7}$$

where $\Theta = \frac{2\pi}{M}$ (M being the number of interaction points per turn).

Furthermore from formulae (2) and (3) one can show that :

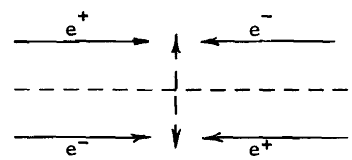
$$\frac{da}{d\theta} = -\ 2\ a\ \mathrm{tg}\left(\nu\theta+\phi(\theta)\right) \frac{d\phi}{d\theta} \tag{8}$$

Then it appears that non linearities will introduce a slow varying part for the phase evolution which also leads to a change in amplitude.

<u>Space charge compensation</u>

The idea developped by the ACO group to put in interaction four beams does not permit to expect a perfect space charge compensation, in particular due to the fact that dipole defects can remain in the machine.

Assuming the four beams have the same intensity, they will be separated from the machine axis by a quantity $\frac{\delta}{2}$, as shown on the figure.



It has been interesting to look to residual dispersion in wave numbers due to the unperfect compensation. Calculations similar to the head-on collisions case have been done by adding the potential due to the separated "companion beams". The results are represented on fig.2, where σ is the standard deviation of the distribution, δ_1 and δ_2 are respectively the radial and vertical separations of the beams.

<u>Conclusion</u>

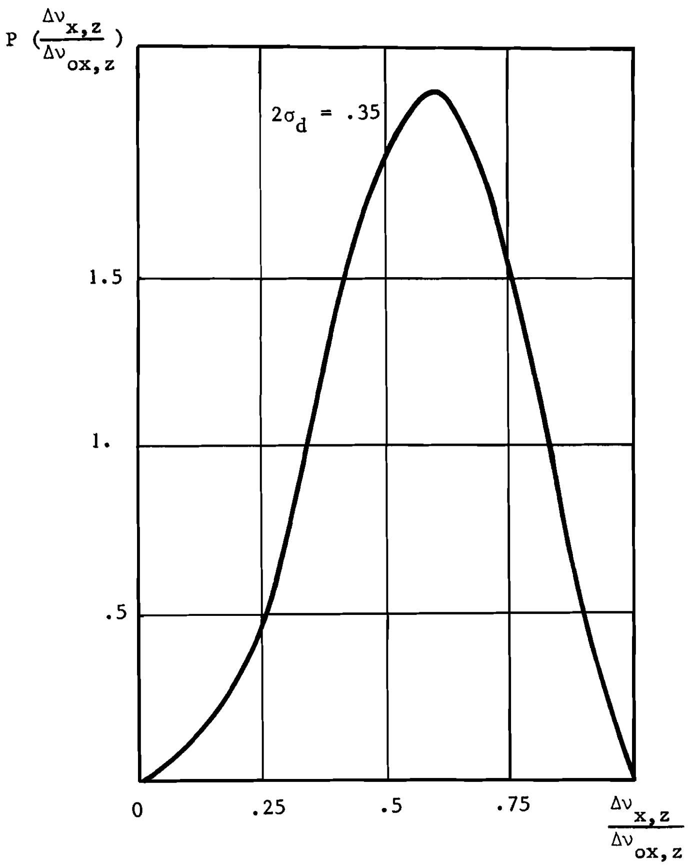
The present study does not permit to define any current limitation for the beam-beam effect as long as the effects of the non-linear resonances and amplitudes growth are not well understood.

Nevertheless it can help to give a better idea on these phenomena if one can later measure the distribution function.

The method can be also used in the case of beams crossing at an angle.

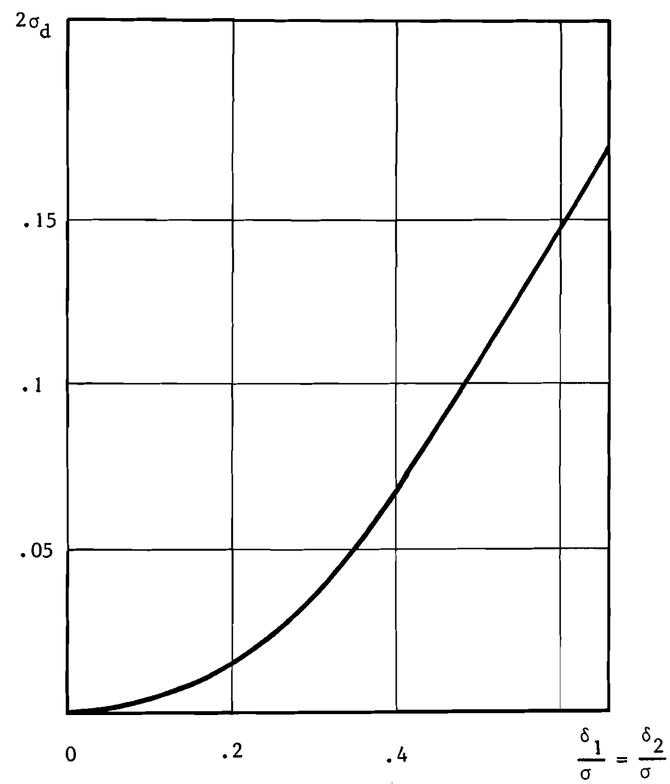
<u>Références</u>

1. F. Amman - D. Ritson "Int. Conf. on High Energy Acc." Brookhaven (1961), p. 471.
2. C. Pellegrini - A. Sessler, CERN 67-19 ISR (1967).
3. D. Potaux - X. Buffet, Note technique 19-70, Anneaux de Collisions, Acc. Lin. Orsay.
4. J.R. Rees, ISEPSR VI-7, Saclay (1966).
5. M. Bassetti, "Int. Conf. on High Energy Acc." Frascati (1965), p. 708.
6. G. Leleux, Soc. Alis 18, CEN - Saclay.
7. A. Schoch, CERN 57-21.
8. P.M. Lapostolle et al., CERN 68-35
9. D. Potaux - X. Buffet, RT/5-70, Anneaux de Collisions, Acc. Lin. Orsay.
10. J.E. Augustin, RT/3-70, Anneaux de Collisions, Acc. Lin. Orsay.
11. J.E. Augustin, RT/8-70, Anneaux de Collisions, Acc. Lin. Orsay.



Wave Number Distribution : Head-On Collision

Fig. 1



Wave Number Distribution Versus Orbit Separation

Fig. 2