ENHANCEMENT OF THE ν-SPREAD IN A PROTON BEAM BY THE INSERTION OF AN ELECTRON BEAM

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Abstract

For a circular beam with uniform density the transverse space charge forces are linear. The effect of the linear space charge is just a reduction $\Delta\nu$ of the transverse oscillation wave number. With nonuniform density distribution in addition to a reduction $\Delta\nu$ of the average ν-value, one expects the nonlinear space charge forces to produce a ν-spread $\delta\nu$. In many cases a ν-spread is desirable for supplying Landau damping to transverse instabilities.

The ν-spread can be enhanced by the insertion of a thin beam of particles of the opposite charge in the middle of the original beam. Because of the dependence of the space charge force on γ^{-2} we can expect a low-γ thin beam of opposite charge to produce a large ν-spread. The ν-spread in a nonuniform beam and its enhancement by the beam of opposite charge are computed in this paper.

A. NONLINEAR SPACE CHARGE FORCES IN PROTON BEAM

Let us consider an infinitely long straight beam of protons with uniform distribution in the longitudinal direction and Gaussian distribution in the transverse plane. For the special case that the beam has circular geometry the scalar potential $\Phi(x,y)$ is[1)]

$$\Phi(x,y) = \lambda_p e \sum_{k=1}^{\infty} \left(\frac{x^2+y^2}{\sigma_p^2}\right)^k \frac{(-1)^k}{k(k!)} \tag{1}$$

with

(x,y) = transverse Cartesian coordinates with origin on beam axis,
e = proton charge,
λ_p = number of protons per unit length,
σ_p = standard deviation of the Gaussian distribution.

Neglecting the end effect, we can consider (1) valid also for a bunched beam. In this case λ_p is the ratio of the total number of particles to the total length of the bunch.

The vector potential, $\vec{A}$, has no transverse components if the transverse motion of the protons is neglected. Thus it is given by

$$|\vec{A}| = A = \frac{v}{c}\Phi = \beta\Phi \tag{2}$$

where v is the beam velocity and c the speed of light.

From (1) and (2) we can calculate the electric and the magnetic fields and, then, the total space charge force F on a proton. Specifically, on the x and y axes we have

$$F_x = -e(1-\beta^2)\frac{\partial\Phi}{\partial x}, \qquad y = 0$$

$$F_y = -e(1-\beta^2)\frac{\partial\Phi}{\partial y}, \qquad x = 0$$

or with (1)

$$F_x\Big|_{y=0} = 2\frac{\lambda_p e^2}{\sigma_p\gamma^2} f\left(\frac{x}{\sigma_p}\right)$$

$$F_y\Big|_{x=0} = 2\frac{\lambda_p e^2}{\sigma_p\gamma^2} f\left(\frac{y}{\sigma_p}\right) \tag{3}$$

where $\gamma^{-2} = (1-\beta^2)$, and the function

$$f(u) = \frac{1-e^{-u^2}}{u} \tag{4}$$

is plotted in Fig. 1, for positive u.

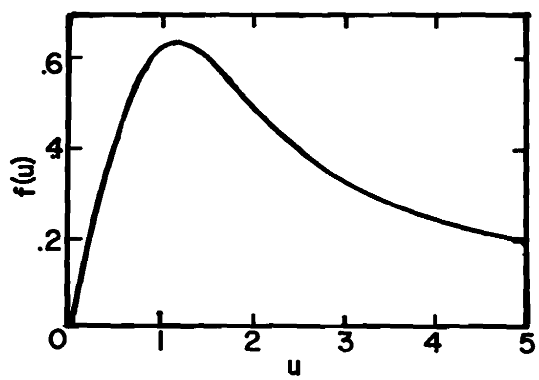

Fig. 1

The equation of a transverse motion (say, x) of a proton in the presence of this space charge force is, then

$$x'' + \nu_0^2 x - \frac{2\lambda_p e^2}{m_0\gamma^3\sigma_p\omega_0^2} f\left(\frac{x}{\sigma_p}\right) = 0 \tag{5}$$

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where a prime denotes derivative with respect to the angular longitudinal coordinate, θ, and

ω_0 = beam angular velocity,
ν_0 = betatron oscillation wave number due to the external guide field,
m_0 = proton rest mass.

Assuming the space charge term to be small compared to the $\nu_0{}^2$ term we take as an approximate solution of (5)

$$x(\theta) = a \sin \psi(\theta) \tag{6}$$

with

$$\psi(\theta) = \int \nu(\theta) d\theta,$$

$$\nu(\theta) = \nu_0 + \delta_p(a,\theta), \text{ and } |\delta_p(a,\theta)| \ll \nu_0 .$$

In the following we neglect the weak θ-dependence of the amplitude a. Introducing (6) in (5) and neglecting terms in ψ'' and $\delta_p{}^2$ we get

$$\delta_p(a,\theta) = - \frac{\lambda_p e^2}{m_0 \gamma^3 \sigma_p{}^2 \omega_0{}^2 \nu_0} \frac{f\left(\frac{a \sin \nu_0 \theta}{\sigma_p}\right)}{\frac{a \sin \nu_0}{\sigma_p}} .$$

The shift is, therefore,

$$\delta_p(a) = \frac{1}{2\pi} \int_0^{2\pi} \delta_p(a,\theta) d\theta = -\eta_p I\left(\frac{a}{\sigma_p}, 0\right) \tag{7}$$

with

$$\eta_p = \frac{\lambda_p r_0 R^2}{2\pi \gamma^3 \sigma_p{}^2 \beta^2 \nu_0{}^2} \tag{8}$$

and

$$I(w,p) = \int_0^{2\pi\nu_0} \frac{f(w \sin\theta + p) - f(p)}{w \sin\theta} d\theta \tag{9}$$

where

$$r_0 = \frac{e^2}{m_0 c^2} = \text{classical proton radius}$$

R = closed orbit radius.

The function $I(w,p)$ is plotted in Fig. 2 for $\nu_0 = 20.25$ which corresponds to the NAL Main Ring. $I(w,p)$ is an even function of p if ν_0 is an integer. For large values of ν_0, it is approximately an even function of p and has only a weak dependence on the fractional part of ν_0.

The full ν-spread in the proton beam due to space charge can be defined as

$$\Delta_p = \eta_p [I(0,0) - I(1,0)]. \tag{10}$$

For the NAL Main Ring, it is, from Fig. 2,

$$I(0,0) - I(1,0) = 25$$

and with the following numbers

$\lambda_p = 7 \times 10^8 \text{ cm}^{-1}$, $\beta = 1.0$
$R = 10^5$ cm, $\gamma = 10$
$\nu_0 = 20.25$, $\sigma_p = 0.5$ cm

we get

$\eta_p = 1.7 \times 10^{-3}$ $\Delta_p = 0.04$.

This ν-spread is only 40% of the minimum required for beam stabilization against coherent oscillations[2].

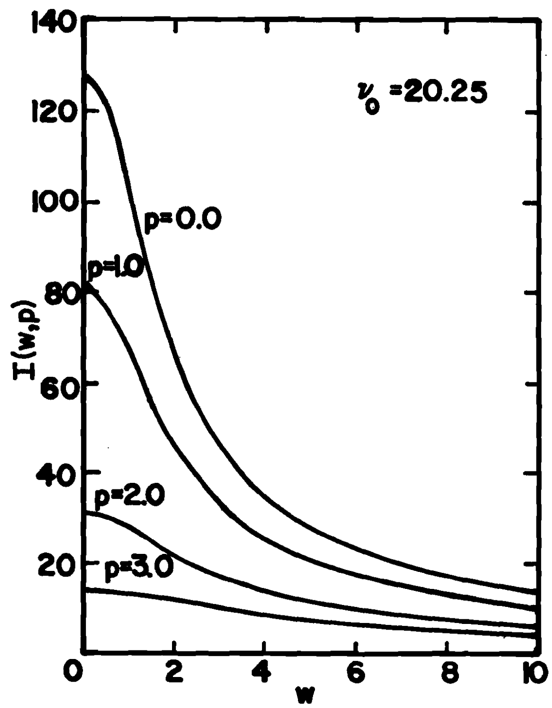

Fig. 2

B. EFFECT OF THE NONLINEAR SPACE CHARGE FORCE INDUCED BY AN ELECTRON BEAM

We insert a thin electron beam in the proton beam for a fraction ε of the accelerator circumference. Assuming the electron beam to be centered on the proton beam axis and having a Gaussian distribution with standard deviation σ_e, we have now the following ν-shift for a proton as a function of its amplitude a

$$\delta_e(a) = \eta_e I\left(\frac{a}{\sigma_e}, 0\right) \tag{11}$$

with

$$\eta_e = \frac{\lambda_e r_0 R^2}{2\pi\gamma\gamma_e^2\sigma_e^2\beta^2\nu_0^2}\,\varepsilon$$

$$= \varepsilon\,\frac{\lambda_e}{\lambda_p}\left(\frac{\gamma}{\gamma_e}\right)^2\left(\frac{\sigma_p}{\sigma_e}\right)^2\eta_p = \alpha\eta_p \qquad (12)$$

where

λ_e = number of electrons per unit length
γ_e = relativistic energy factor for electrons,

and we have taken the effect of the electron beam as being spread over one turn.

The ν-spread in the proton beam now is

$$\Delta_e = \eta_e\left[I(0,0) - I\left(\frac{\sigma_p}{\sigma_e},\ 0\right)\right] \qquad (13)$$

$$= \alpha K\left(\frac{\sigma_p}{\sigma_e}\right)\Delta_p$$

with

$$K(x) = \frac{I(0,0) - I(x,0)}{I(0,0) - I(1,0)}\ . \qquad (14)$$

The total ν-spread Δ_t of the proton beam is given by the algebraic sum of the partial spreads due to the electrons and the protons, namely, if $\Delta_e > \Delta_p$,

$$\Delta_t = \Delta_e - \Delta_p = (\alpha K - 1)\Delta p$$

from which

$$\alpha K = 1 + \frac{\Delta_t}{\Delta_p}. \qquad (15)$$

For the NAL Main Ring, we have $\Delta_p = 0.04$. To obtain the required $\Delta_t = 0.10$ we must have

$\alpha K = 3.5.$

Let us take $\sigma_p/\sigma_e = 10$. From Fig. 2 we get

$K = 4.25$ and $\alpha = 0.77.$

With $\varepsilon = 10^{-3}$ and an electron beam of very low energy ($\gamma_e \sim 1$) we obtain

$$\lambda_e = 0.077\ \lambda_p = 5.4 \times 10^7\ \text{cm}^{-1}$$

which, for low energies, corresponds to very modest current i_e of the electron beam.

C. EFFECT OF AN OFF-CENTER ELECTRON BEAM

Similar calculation with the electron beam displaced from the axis of the proton beam in the x-direction by an amount x_0 gives for the ν-shift

$$\delta_t(a) = -\eta_p\left\{I\left(\frac{a}{\sigma_p},\ 0\right) - \alpha I\left(\frac{a}{\sigma_e} - \frac{x_0}{\sigma_e}\right)\right\}. \qquad (16)$$

This equation shows that for certain values of α, σ_p/σ_e, and a it is possible to have $\delta_t = 0$.

Let us neglect the contribution of the protons on the right-hand side of Eq. (16). Again for the example of the NAL Main Ring, the minimum α required is plotted in Fig. 3 against the lateral displacement x_0/σ_e. Taking the same parameters as before we have

$$\lambda_e = 0.1\ \alpha\lambda_p = 7.0 \times 10^7\ \alpha\ \text{cm}^{-1}.$$

For a 5 keV electron beam, the equivalent current, i_e, is given on the right-hand side of Fig. 3.

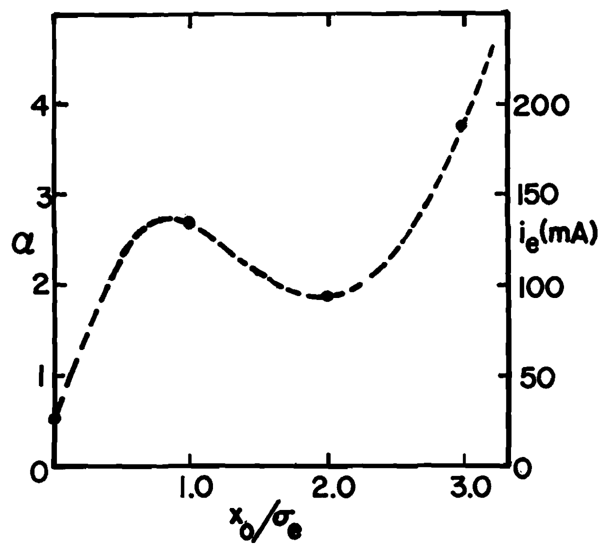

Fig. 3

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