BEAM-BUNCH LENGTH MATCHING AT TRANSITION CROSSING

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Abstract

A multiparticle computer program was written to study the γ_t-jump and ϕ_s-switch schemes[1)] for matching beam-bunch length in crossing the transition energy when the longitudinal space-charge force is significant. This computation takes into account nonlinearities in the forces and can, therefore, yield information about the bunch stability.

The results show that all ϕ_s-switch schemes suffer from negative-mass instability shortly after transition. On the other hand, γ_t-jump schemes are stable and are effective for bunch-length matching.

A. THE COMPUTER PROGRAM

The phase motion of each individual particle is given by the difference equations

$$\begin{cases} \psi_{n+1}=\psi_n+2\pi h\dfrac{\gamma_{s,n}}{\gamma^2_{s,n}-1}\left(\dfrac{1}{\gamma^2_{t,n}}-\dfrac{1}{\gamma^2_{s,n}}\right)\left(\delta_n+\dfrac{\Delta_n}{2}\right) \\ \delta_{n+1}=\delta_n+\left(\gamma_{s,n+1}-\gamma_{s,n}\right)\left[\dfrac{\sin\left(\phi_{s,n+1}+\psi_{n+1}\right)}{\sin\phi_{s,n+1}}-1\right] \end{cases}$$
$$+\frac{1}{2}\left(\Delta_n+\frac{\gamma^2_{s,n+1}}{\gamma^2_{s,n}}\Delta_{n+1}\right) \qquad (1)$$

where

h = rf harmonic number
ϕ_s = synchronous rf phase
γ_s = synchronous energy in mc^2 units $=\left(1-\beta_s^2\right)^{-1/2}$
γ_t = transition energy in mc^2 units
ψ = deviation of particle rf phase from ϕ_s
δ = deviation of particle γ from γ_s
Δ = energy gain per turn due to space charge
n(in subscript) = revolution number.

For a long and thin cylindrical beam bunch of length ℓ and radius a inside a perfectly conducting thin cylindrical vacuum chamber of radius b ($b\ll\ell$) the space-charge term can be written as

$$\Delta_n = 2\pi h^2\frac{r_p}{R_s}\frac{g}{\gamma^2_{s,n}}\left(N\frac{df}{d\psi}\right)_n \qquad (2)$$

where

r_p = classical proton radius $= 1.53\times10^{-18}$ m
R_s = synchronous orbit radius
N = number of protons per bunch
$Nf(\psi)$ = number of protons per unit ψ
g = geometrical factor $= 1+2\ln(b/a)$.

For the rf force these difference equations accurately represent the case of one accelerating gap and give an adequately good approximation when there are many accelerating gaps distributed around the ring.

Although the variable δ is used in the computation, we print out, instead, the variable w conjugate to ψ and related to δ by

$$w = -\frac{mcR_s}{h\beta_s}\,\delta. \qquad (3)$$

To compute $Ndf/d\psi$ at a given particle the numbers of particles in two ψ-bins of equal width on either side of the given particle are counted and the difference is taken. This gives the value of $Ndf/d\psi$ averaged over two bin widths. The local value of $Ndf/d\psi$ is strictly correct only when the typical wavelength of the variation of f is infinitely large compared to the vacuum chamber radius. Using the average value improves the approximation for short wavelength variations in f which occur at the ends of a beam bunch and when the beam bunch is broken up by negative-mass instability. Comparison of numerically computed exact longitudinal field with that given by Eq.(2) using the average density gradient $\langle Ndf/d\psi\rangle$ shows that the bin size should be approximately equal to 3/4 of the vacuum chamber radius b.

We assume a linear dependence of γ_s on n, namely that the synchronous energy gain per turn is constant. This corresponds to a guide magnetic field rising linearly in time. The origin of n is defined by $\gamma_{s,n=0}$ = unperturbed γ_t. We choose a starting point n_o far enough before transition where all forces are approximately linear. For initial conditions at n_o we use those given in Ref. 1 with a distribution in the (ψ,w) phase space covering an elliptical area and which is parabolic when projected on either the ψ or the w axis.

Standard checks for accumulated round-off errors and for the choice of particle number are made. For the cases studied 2000 particles are used.

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As a check, we linearized the rf force [replacing $\sin(\phi_s+\psi)-\sin\phi_s$ by $(\cos\phi_s)\psi$] and compared the rms phase ellipse of the computed distribution with that obtained in Ref. 1 using the rms envelope equation. Close quantitative agreement was obtained for all cases in which the particle bunch is stable against the negative-mass instability.

B. RESULTS AND DISCUSSION

All the results presented here are for the NAL booster synchrotron with the Sørenssen parameter[2] $\eta_0(0) = 3.8$. Other pertinent parameters are given in Table 1.

The initial equilibrium distribution at the starting point $n_0 = -750$ is shown in Fig. 1, where N* is the number of particles included in the graph. The area covered by the distribution is about 66% of the no space-charge rf bucket area. This high bucket filling factor clearly exhibits the effect of the nonlinear rf force on the distribution at transition as shown in Fig. 1. These distributions are stable against the negative-mass instability. The results for the various matching schemes studied in Ref. 1 are given below, except, now, with nonlinear forces and negative mass instability the parameters are readjusted for each case to optimize matching.

Case 1

This is the case of normal transition crossing--no γ_t-jump and simple ϕ_s-switch from ϕ_s to $\pi-\phi_s$ at transition. In addition to the familiar bunch length oscillation after transition we also observe the bunch breakup and particle loss due to the negative-mass instability shortly after transition. These results are shown in Fig. 2.

Case 2

The case of the single γ_t-jump is shown in Fig. 3. The reoptimized parameters are such that γ_t continues the unperturbed value of 5.446 to $n_1 = 240$, jumps linearly up to 5.576 at $n_2 = 270$, and stays constant from thereon. In this case the negative-mass instability results in a blowup of the phase-space area by about 90%. The small tail in the distribution caused by the nonlinear rf force leads to some particle loss and is ignored. Nevertheless, this particle loss and dilution of phase-space density is tolerable and this scheme is considered useable.

Case 3

In the case of double γ_t-jump shown in Fig. 4 the negative-mass instability is reduced by the downward γ_t-jump and leads to a phase-space blowup of only about 20%. The readjusted parameters are: γ_t jumps linearly downward from 5.446 at $n = 0$ to 5.236 at $n_1 = 105$, then linearly upward back to 5.446 at $n_2 = 210$.

Case 4

The triple ϕ_s-switch shown in Fig. 5 is extremely unstable. The rf defocusing introduced after transition keeps the bunch length large and the energy spread small. This greatly reduces the Landau damping action. The beam bunch is completely broken up and a large number of particles are lost. This is presumably the reason why the triple-switch scheme was found to be ineffective when tried on the CERN PS. On the other hand, a scheme in which a compromise is made between negative-mass instability and matching may prove to be advantageous.

Case 5

The double ϕ_s-switch is no better than the triple ϕ_s-switch.

Case 6

In the original Q-jump scheme[3] the bunch length is kept large by a big γ_t jump. Although the energy spread is kept small the big downward γ_t jump greatly increases the particle revolution-frequency spread for given energy spread, hence greatly increases the Landau damping action. This is the case most stable against negative-mass instability and is shown in Fig. 6 where γ_t jumps downward linearly from its unperturbed value at transition to 4.506 at $n = 20$. However, for our choice of ϕ_s the very long beam bunch after transition causes excessive particle loss out of the rf bucket. This particle loss can presumably be reduced by increasing the rf bucket size.

The criterion for negative-mass instability given by Neil and Sessler[4] states that a beam bunch is stable above transition if

$$\hat{\psi}\hat{w}^2 > 4hNg\,\frac{r_p}{R_s}\left(\frac{mcR_s}{h}\right)^2\frac{1}{\gamma_s\eta} \equiv \Lambda \qquad (4)$$

where $\eta \equiv \gamma_t^{-2} - \gamma_s^{-2}$, and $\hat{\psi}$ and $\hat{w}$ are the full width and height of the bunch. This criterion is plotted in Fig. 7 for all the schemes using values of $\hat{\psi}$ and $\hat{w}$ obtained in Ref. 1. Examination of these plots gives a clear understanding of the negative-mass instabilities exhibited by the multiparticle computation.

In general, we can make the following

observations:

1) At these high values of $\eta_0(0)$ the chief benefit derived from applying these so-called "matching" schemes is the reduction of negative-mass instability rather than the matching of bunch length. Any matching scheme in which the bunch length is kept large, hence the energy spread small, and in which nothing is done to increase η is ineffective because of the negative-mass instability.

2) Even with the γ_t-jump the negative-mass instability still results in some dilution of the phase-space density. Clearly, the larger the downward γ_t jump the better it is for stability. The Q-jump scheme is most effective if such a large γ_t jump can be accommodated.

3) The effect of the negative-mass instability depends on the particle distribution in the bunch. Hence only the qualitative features of the results are meaningful. In any case, the bunch breakup caused by the negative-mass instability makes the matching parameters rather insensitive.

4) Although the nonlinearity of the rf force must be considered in providing a sufficiently large rf bucket it does not affect the qualitative features of the matching schemes.

REFERENCES

1. W.W. Lee and L.C. Teng, IEEE Trans. on Nucl. Sci. NS-18, No. 3, 1057 (1971).

2. A. Sørenssen, Proc. 6th Int. Conf. on High Energy Accelerators, 474 (1967).

3. D. Möhl, CERN-ISR-300/GS/69-62 (1969).

4. V.K. Neil and A.M. Sessler, Rev. Sci. Instr. 36, No. 4, 429 (1965).

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Table 1
PARAMETERS FOR THE
NAL BOOSTER SYNCHROTRON

$h = 84$	$R_s = 75.47\text{m}$
$g = 4.5$	
$N = 4.17 \times 10^{10}$	$b = 0.03\text{m}$
ϕ_s(unperturbed) $= \begin{cases} 70° & n<0 \\ 110° & n>0 \end{cases}$	
γ_t(unperturbed) $= 5.446$	
$\gamma_s = 5.446 + 0.655 \times 10^{-3} n$	

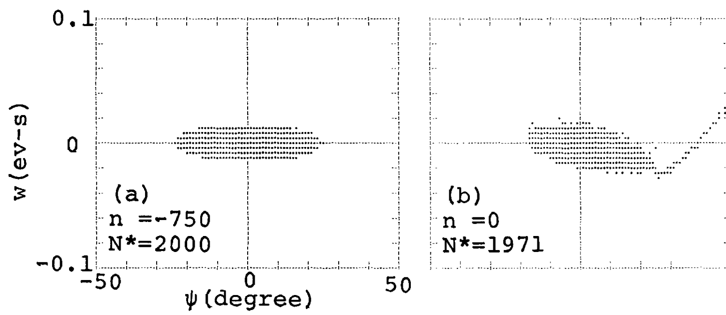

Fig. 1† Distributions before transition.

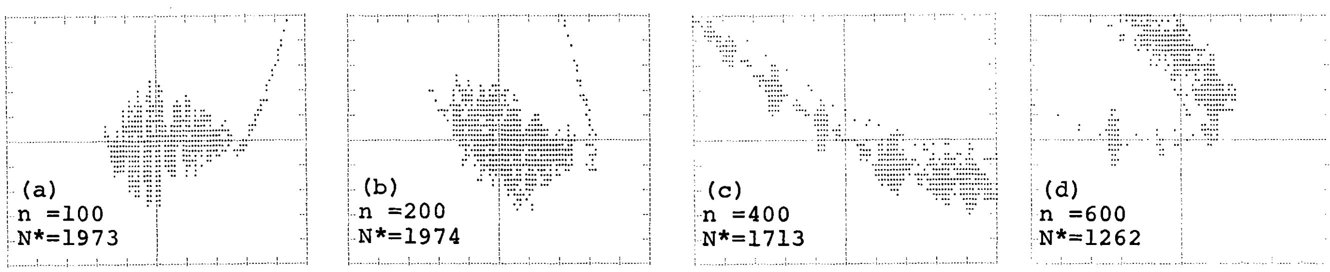

Fig. 5† Distributions for the triple ϕ_s-switch.

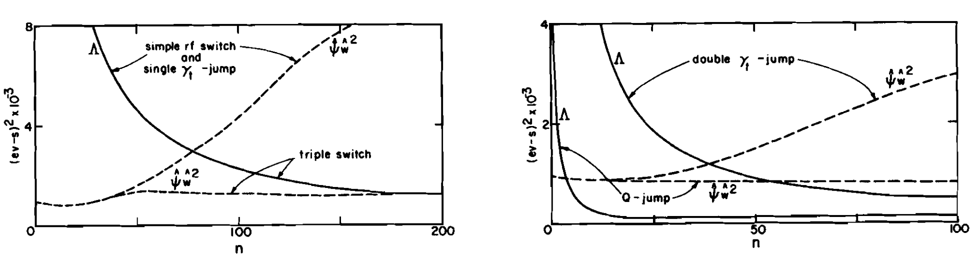

Fig. 7 Stability criterion for the matching schemes.

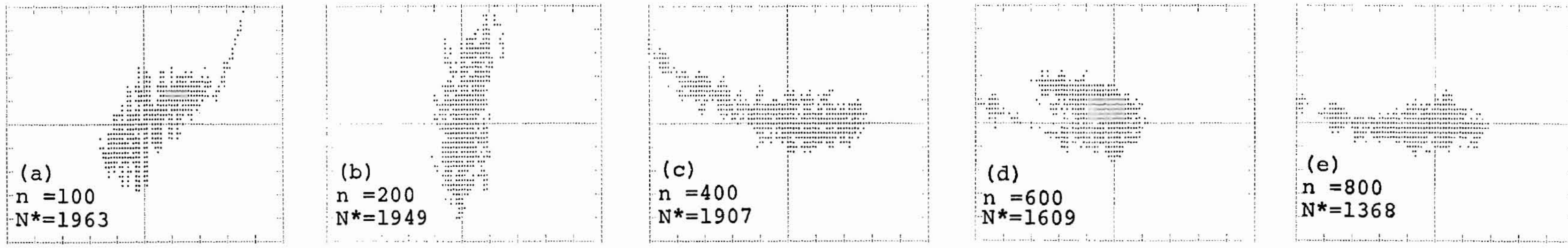

Fig. 2† Distributions for the simple ϕ_s-switch.

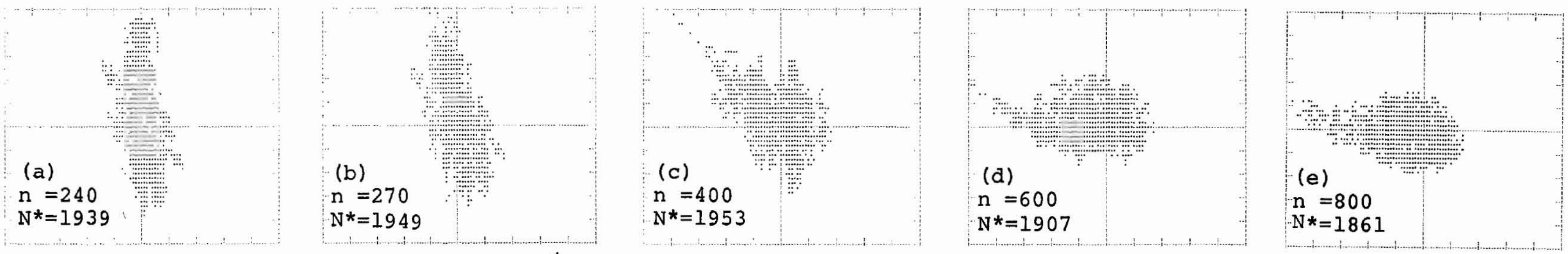

Fig. 3† Distributions for the single γ_t-jump.

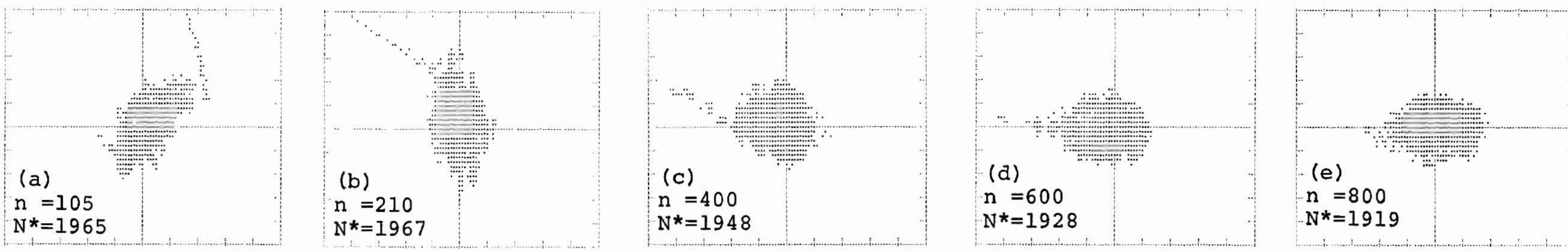

Fig. 4† Distributions for the double γ_t-jump.

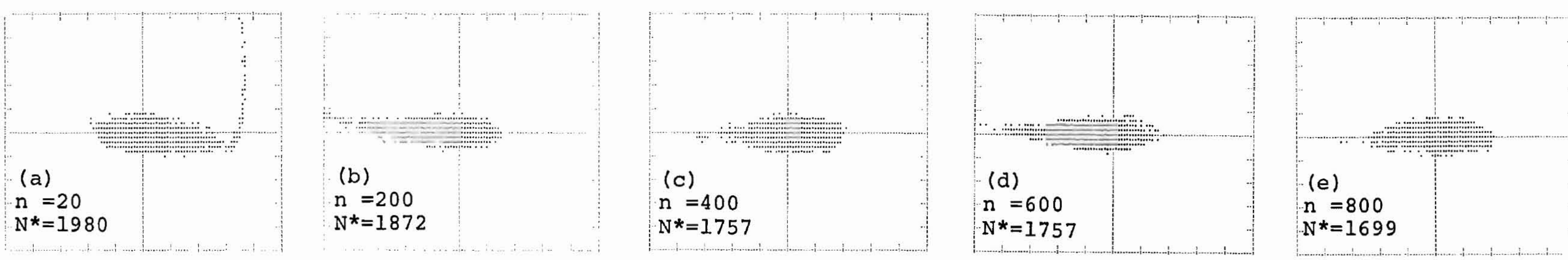

Fig. 6† Distributions for the Q-jump.

†The scales are the same as in Fig. 1-(a).