PASSING TRANSITION IN THE FUTURE CPS

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Abstract

When the future CPS operates with $\sim 10^{13}$ p/p, the longitudinal space-charge parameter n is expected to be about 3, which corresponds to an $\eta_0(0)$ of about 4. In order to obtain favourable conditions for bunch matching, it is planned to combine the γ_t-jump method with the triple switch. The triple switch is used for matching, whereas the decreased γ_t generates a large bunch length at the matching point. In this way the solution is least sensitive to a spread of the space-charge parameter. Various arrangements for quadrupoles have been examined, all capable of achieving a change of γ_t of about 0.6.

1. Introduction

Passing transition at high longitudinal space charge is rather different from that at low intensity. This is because the repelling space-charge force lengthens the bunches before transition and shortens bunches above transition as a consequence of the negative mass action. Since the two different regimes follow immediately at the γ-γ_t crossing point, a matching problem arises for which various solutions have been proposed in the past[1-4]. We shall describe the system planned for the CPS. First the bunch dynamics will be considered and then possibilities for obtaining the desired $\Delta\gamma_t$. Finally we shall compare this solution with other conceivable matching procedures.

2. Bunch Dynamics

The behaviour of the bunch length Θ is discussed qualitatively, but for convenience the theory is recapitulated in the Appendix.

If the bunch motion is sufficiently adiabatic, an equilibrium bunch length Θ_{eq} can be defined, around which the bunch length Θ tends to oscillate in a non-linear manner. With space charge the equilibrium bunch length behaves entirely differently below and above transition. Below transition Θ_{eq} tends to $n^{1/3}$, whereas above transition Θ_{eq} starts from zero with an initial slope $\sim (\gamma - \gamma_t)/n$. It is not possible to decrease Θ_{eq} below transition, whereas above transition a rapid decrease of γ_t starting at the crossing point raises Θ_{eq} and thus reduces the mismatch. This is the basic principle of the γ_t- jump [which has been called Q-jump for the special case when $\Delta\gamma_t \sim \Delta Q$ [2,3]]. Important parameters of this γ_t-jump are its speed, its magnitude, and its shape. Under special conditions they can be chosen such that it is sufficient to switch the RF phase only once from the stable phase angle before transition to the new stable phase after transition. This switching of the RF phase has to be done at least once anyway. Since that switching is also a simple and reproducible operation, it can be repeated to provide more flexibility. Already with three switches, matching is possible in general. The third switch determines the matching point where Θ touches tangentially Θ_{eq}. This can be achieved by adjusting the timing of the first two phase switches. Without switching back to the unstable point, the bunch length would cross the equilibrium bunch length. In order to match, the two additional switches are necessary. The duration between these switches is the shorter, the larger the bunch length and the larger $(\gamma - \gamma_t)$. Since both these features are provided by the γ_t-decrease, matching by the triple switch is much facilitated and a short duration at the unstable point is sufficient, whereas without γ_t-jump, matching may become impossible with the triple switch. But the combination of the γ_t-jump and the triple switch is flexible enough to achieve matching under very general conditions.

3. Computational Results

A series of solutions with different values of $|\Delta\gamma_t|$ ranging from 0.215 to 0.645 has been computed[5] for n = 3. Typical cases are shown in Figs. 1 to 4 together with the corresponding curves for γ and γ_t. Since matching could be achieved in all cases, one might judge the solutions as being equally good. But this would be premature, since imperfect matching due to spread and jitter of parameters should also be examined. The solutions are mostly sensitive to a change of the longitudinal space charge which is expected to vary from cycle to cycle, from bunch to bunch, and even within a bunch due to non-linearities. As a criterion we computed as a function of the space-charge parameter n the blow-up factor D^2 of the bunch area as it would result owing to the mismatch after filamentation. These curves are plotted in Fig. 5 for the cases of Figs. 1 to 4 and show the following behaviour. The solutions become less sensitive with increasing $|\Delta\gamma_t|$, but only up to some value of $|\Delta\gamma_t|$ beyond which it is also necessary to increase $\dot{\gamma}_t$ if one wants to improve further. With the parameters considered for the CPS, a decrease of γ_t by 0.5 within $\sim$ 1 ms (or preferably faster) leads to satisfactory results from the sensitivity point of view.

Another problem is the fight against the negative mass effect which is partly responsible for bunch shape distortion already observed above transition and which could seriously limit the luminosity in the ISR. A larger value of $|\Delta\gamma_t|$ should help, but again a large $|\dot{\gamma}|$ is more important if some minimum $|\Delta\gamma_t|$ is exceeded. These problems are still being studied theoretically and experimentally.

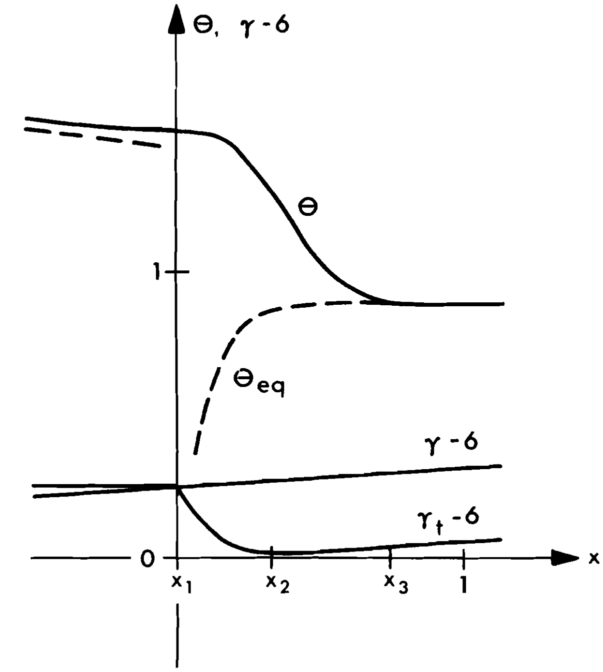

Fig. 1 γ_t-jump with $|\Delta\gamma_t|$ = 0.215; $\gamma_t'(0)$ = -1.22.
Phase switches at:
x_1 = 0.0; x_2 = 0.313; x_3 = 0.723.

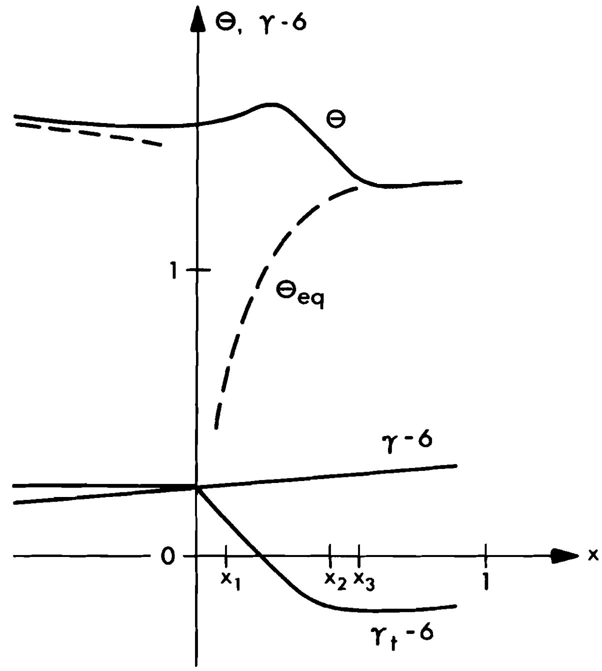

Fig. 2 γ_t-jump with $|\Delta\gamma_t|$ = 0.43; $\gamma_t'(0)$ = -1.22.
Phase switches at:
x_1 = 0.12; x_2 = 0.472; x_3 = 0.589.

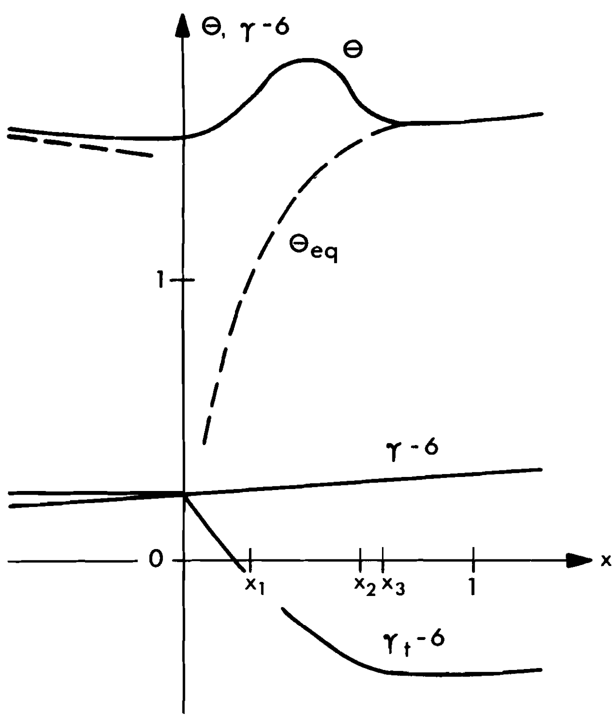

Fig. 3 γ_t-jump with $|\Delta\gamma_t|$ = 0.645; $\gamma_t'(0)$ = -1.22.
Phase switches at:
x_1 = 0.230; x_2 = 0.610; x_3 = 0.689.

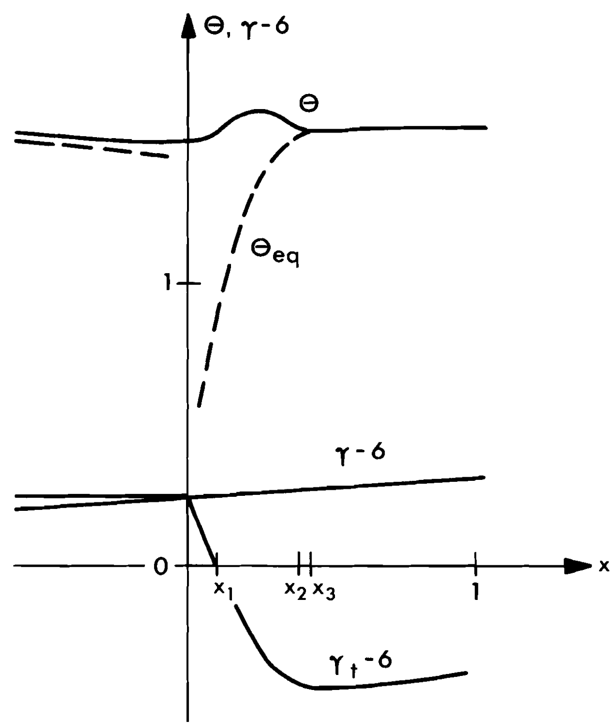

Fig. 4 γ_t-jump with $|\Delta\gamma_t|$ = 0.645; $\gamma_t'(0)$ = -0.243 (fast).
Phase switches at:
x_1 = 0.091; x_2 = 0.374; x_3 = 0.399.

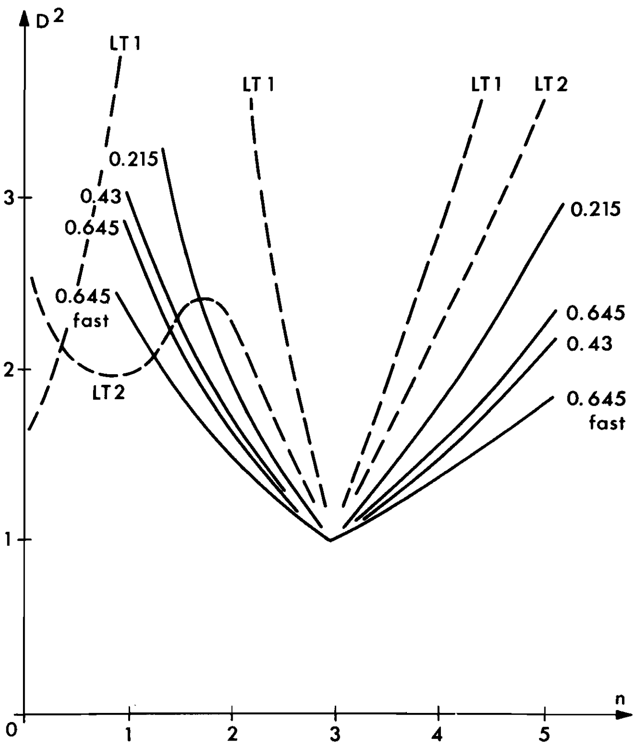

Fig. 5 Sensitivity of bunch blow-up vs.n. Label = $|\Delta\gamma_t|$; dashed curves for single jump (LT1) and double jump (LT2) of Lee and Teng[8,9].

4. Possible Lens Configurations for $|\Delta\gamma_t|$ 0.5

In order to get a large γ_t-change, Teng[4] proposed the insertion of two oppositely excited sets of quadrupoles, all spaced equidistantly, in such a way as to introduce a harmonic close to Q_H into the focusing structure. When that scheme was examined for the CPS[6] it was found that the harmonic number six created too large peaks of the momentum compaction function, but five resulted as a good harmonic which in addition fits the super-periodicity of the CPS. An economic way of operation is to increase γ_t very slowly before transition, using only one set of rather weak quadrupoles. At transition the fast γ_t decrease is generated by the other set of quadrupoles. The tolerable net $|\Delta Q_H|$ (< 0.29) fixes the magnitude of $|\Delta\gamma_t|$ ($\sim$ 0.65). The peak of the momentum compaction is then about doubled.

It is possible to avoid the Q-change[7]. Consider two equally strong quadrupoles of opposite polarity, separated by half a betatron wavelength. Such a doublet leaves the β-function unchanged outside and does not affect the Q-value. Several of those doublets can be arranged in various ways such that the new momentum compaction function leads to a large change of γ_t.

Two configurations of that kind were considered, one consisting of five such doublets and another consisting of two triplets. The triplets may be imagined as being composed of two oppositely driven adjacent doublets.

The constant-Q method is clearly limited by the excursion of the momentum compaction function. The resulting beam width at transition for the five doublets is only marginally larger than for the equidistantly spaced quadrupoles. The two triplets are most economic in lens requirements but lead to a larger beam width ($\sim$ 20% more).

Finally it was decided to select the configuration of ten equidistant quadrupoles (five slow plus five fast) mainly because lack of space in the relevent straight sections prevented the adoption of the five-doublet scheme.

5. Comparison with Other Matching Methods

It is possible to match

i) by γ_t-jump alone,
ii) by phase manipulations without γ_t-jump.

i) For the matching process itself one needs, in general, an overshoot of Θ_{eq} which implies a fast increase of γ_t before the matching point[8]. This is wasteful in $\Delta\gamma_t$-capacity and gives a harder specification for the quadrupole power supply. The triple switch is much simpler. Of course, matching without a complicated shape of the γ_t-jump and without triple switch is a special case of the proposed combined scheme when the required duration between the second and the third switch just shrinks to zero. That happens for a relatively large $|\Delta\gamma_t|$ which makes Θ_{eq} comparable to Θ at transition crossing. An example is the present Q-jump of the CPS[2] for which the corresponding curves are shown in Fig. 6.

ii) The obvious advantage of this possibility is that no hardware for lenses and their power supplies is needed. But the solutions are much more sensitive to n-variation and are likely to show more negative mass instability. Reference

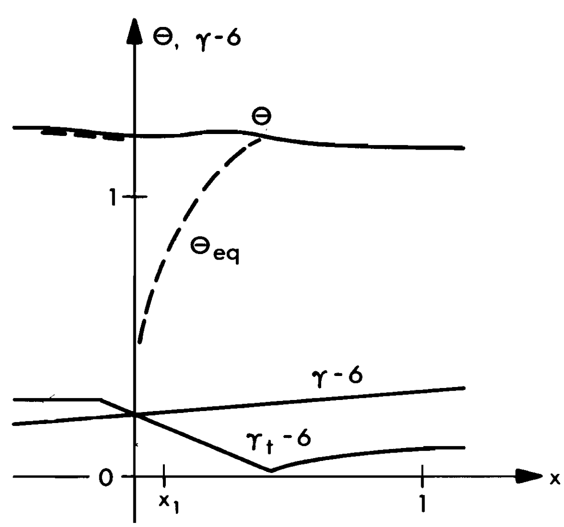

Fig. 6 CPS Q-jump with n = 1; $|\Delta\gamma_t|$ = 0.24; γ_t' = -0.525. Phase switch at: x_1 = 0.091.

is made to a conference paper by Lee and Teng[9] in which they show with a superparticle computation how the negative mass instability develops.

The insertion of a ferrite-loaded helix[10] is still considered as an additional method to facilitate transition crossing at very high space charge, which may occur when using the special PS filling schemes proposed for the ISR. Not enough work has been devoted to judge the merit of this device for the CPS.

6. References

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Appendix

The computer solves by numerical integration the system

$$p'_\Theta = \frac{f}{\Theta^3} - \frac{n}{\Theta^2} - \delta\Theta$$

$$\Theta' = f p_\Theta \; ,$$

where $' = d/dx$; $x = t/T$ is the normalized time counted from crossing $\gamma - \gamma_{t_0}$:

$$T^3 = \frac{\gamma_t^4(0)}{4\pi\nu_{RF}\dot{\gamma}^2 |\cot \Phi_s|}$$

$$f = \frac{1}{2}\frac{\gamma^3}{\gamma'}\left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right)$$

$$\delta = \begin{pmatrix} + \\ - \end{pmatrix} \text{sign (f) for} \begin{pmatrix} \text{stable} \\ \text{unstable} \end{pmatrix} \text{phase} \; .$$

The r.m.s. half bunch length $\Psi = \Phi - \Phi_s$ is related to Θ by

$$\Psi = \frac{A^{1/2}}{\left(\frac{\pi}{2}\gamma^2 |\cot \Phi_s|\right)^{1/3}} \left(\frac{\nu_{RF}}{\dot{\gamma}}\right)^{1/6} \cdot \Theta \; ,$$

and the r.m.s half bunch height is

$$\Delta(\beta\gamma) = \left(\frac{|\cot \Phi_s|\gamma^2}{2\pi^2}\right)^{1/3} \left(\frac{\dot{\gamma}}{\nu_{RF}}\right)^{1/6} A^{1/2} \left(p_\Theta^2 + \frac{1}{\Theta^2}\right)^{1/2} ,$$

where A = r.m.s. bunch area in Ψ, $\Delta(\beta\gamma)$ units. The other parameters have the following meaning:

$\nu_{RF} = \frac{h\beta c}{2\pi R}$ = RF frequency

h = harmonic number

$n = 0.77233\eta_0(0) = \frac{3\pi^2 r_p N g_0}{R A^{3/2}} \left(\frac{\nu_{RF}}{\dot{\gamma}}\right)^{1/2}$

$r_p = \frac{e}{4\pi\varepsilon_0 \mathcal{U}_p} = 1.53 \times 10^{-18}$ m = classical proton radius

N = total number of particles per pulse

$g = \frac{1}{2} + 2 \ln \left(\frac{\text{radius of vac. pipe}}{\text{beam radius}}\right)$.

β, γ are the usual relativistic parameters

$2\pi R$ = machine circumference

Θ_{eq} = positive root of:

$$\text{sign (f)} \; \Theta^4 + n\Theta - f = 0 \, .$$

$|\cot \Phi_s|$ is assumed to be constant so that only phase jumps between Φ_s and $\pi - \Phi_s$ can be treated. The formalism for the more general case of changing Φ_s together with the voltage V, such that $V \sin \Phi_s$ = constant, is found in Ref. 8. The theory is further restricted to linear forces. Methods of treating non-linear forces and first results can be found in the papers of Lee and Teng[9] and Sørenssen[11].