DAMPING OF THE LONGITUDINAL INSTABILITY IN THE CERN PS

D. Boussard and J. Gareyte

CERN, Geneva, Switzerland

Abstract

After a summary of the theory of the coherent longitudinal instability observed in the CPS, three different compensation techniques are presented : spread in the synchrotron frequencies of individual bunches, Landau damping by RF non linearities, and active feedback. Experimental results are described.

1. Introduction

Coherent bunch oscillations become unstable in the CPS when intense short bunches are accelerated. The instability is attributed to a coupling from bunch to bunch via some external structure of impedance Z: a bunch induces in this structure a voltage which perturbs the motion of the following bunches. The system being closed after h=20 (harmonic number) bunches may be unstable.

2. Mathematical description of the instability

The perturbed synchrotron equations for a particle having a momentum deviation $u=p-p_o$ and a phase deviation φ are:

$$\dot{u} = \frac{e}{2\pi R}\left[V\,(\sin\varphi - \sin\phi_s) + v\,(\varphi)\right] \qquad (1)$$

$$\dot{\varphi} = \frac{u}{MR}\ ;\ M = \frac{p_o}{R\,\omega_o\,h\eta} = \text{equivalent mass of the phase oscillator}$$

ω_o = revolution frequency

φ = $h\theta$, where θ is the azimuth and h the harmonic number. V is the accelerating potential and v the perturbing potential.

2.1 Simple approach [1,2,3]

If we assume that the bunches are rigid, equations (1) apply to the phase φ_m and the momentum deviation u_m of the centre of mass of bunch m, subject to the perturbing potential v_m (φ). Linearizing (1) and expanding v_m to first order in φ we get:

$$\ddot{\varphi}_m + \Omega^2\ \varphi_m + \sum_{n=1}^{h} \beta_{mn}\ \varphi_n = 0 \qquad (2)$$

Looking for harmonic solutions of the form ϕ_m exp (- $j\omega t$), eq. (2) becomes algebraic:

$$(-\ \omega^2 + \Omega^2)\ \phi_m + \sum_{n=1}^{h} \beta_{mn}\ \phi_n = 0 \qquad (3)$$

The h solutions for ω^2 can be found from the eigenvalues of the coefficient matrix of (3). The growth rates are $1/\tau = \mathrm{Im}\ (\omega)$. If the h bunches are equal and if there is no synchrotron frequency spread, the matrix is cyclic and adjacent bunches oscillate with a phase difference $k(2\pi/_h)$; k = 1,2,...h, will be called the mode number.

The β_{mn} are easily found by a Fourier analysis method. If I_{om} is the mean current of bunch m, we have:

$$\beta_{mn} = \frac{I_{om}e}{\pi R^2 M}\ \mathrm{Re} \sum_{p=0}^{\infty} A_p^{\ 2}\ Z(p)\ j\frac{p}{h}\ \exp[j\frac{p}{h}\cdot(n-m)2\pi] \quad (4)$$

where A_p is the Fourier coefficient of the bunch, defined, if i(t) is the current, by

$$i(t) = I_o \sum_{p=-\infty}^{+\infty} A_p \exp(j\ p\,\omega_o\ t) \qquad (5)$$

2.2 The influence of the beam control system

It was demonstrated[2,3] that in absence of synchrotron frequency spread from bunch to bunch the beam control loops (radial and phase) do not affect the stability conditions of the system. They introduce a coupling between bunches, but which is apparent only for the k = 0 mode.

On the contrary, if a small spread is present, the beam control is able to "see" the motion of the bunches also for the other modes, and has an important influence on the dipole instability[3,4]. This was demonstrated analytically for a h=4 machine, and confirmed by means of a computer programme in the case of the CPS (h=20). This computer programme solves the general case by computing the eigenvalues and eigenvectors of a hxh complex matrix, in which, in addition to the wake fields, the action of phase lock, radial control, bunch to bunch spread in frequency and population is taken into account.

A noteworthy consequence of the action of beam control is that suppressing some bunches in the machine does not ensure stability, even in the case of low Q wake fields: the beam control "bridges the gap" provided that some spread in synchrotron frequencies is present[5].

3. Experimental observations in the CPS.

Different observation techniques are used:
- Direct observation of bunches on a fast oscilloscope with "mountain range" display (photo 1),
- observation of the phase difference between bunches and the RF voltage, by means of phase discriminators,
- spectral analysis of the signal of a radial PU station to separate the different modes (photo 3),
- observation of the peak detected PU signal showing filamentation (photo 2).

Clean oscillations of the first moment of the 13 ns long bunches are observed soon after transition, reaching a maximum peak to peak amplitude of 7 ns before filamentation and higher order instabilities come into play.

As can be seen on photo 3, in general mode numbers from 1 to 5 are present with e folding times τ around 50 ms. The parasitic resonances of our 14 accelerating cavities ($Q \simeq 20$, $F \simeq 48$ MHz, Ro = shunt resistance $\simeq 800\ \Omega$ each) may give rise to mode numbers 3,4,5 with $\tau \simeq 80$ ms. Electrostatic septum extractor tanks ($Q \simeq 700$; $F \simeq 69$-90 MHz, R_o=18 kΩ) introduced recently in the machine, where found to be responsible for very powerful instabilities with e-folding times of 10 ms. The mode numbers are related to the resonant frequency of the tank, which changes with the position of the electrodes. Their influence was very much reduced by damping the main resonance by means of a magnetic coupling loop and an external resistor.

It is felt that other structures are involved in the excitation of the observed instability, and investigation is going on.

4. Stabilization by frequency spread

A very general method of damping the instability is to make the synchrotron frequency of the bunches slightly different in order to reduce the influence of parasitic couplings.

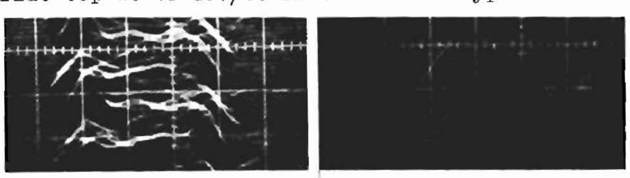
4.1 Selection of a modulation pattern

In the case of a machine without beam control a direct solution of the problem can be found for a sinusoidal modulation of bunch frequencies, (1 period per turn) using Chesbyshev functions[6].

In our case the effect of some modulation patterns is strongly attenuated by the beam control system [3,4]. The computer program already mentioned has allowed us to select the more efficient schemes. For a mode 1, a sinusoidal modulation gives the stronger reduction of growth rate, and the effect is not reduced by the beam control. For a mode 5 (short range wake fields), a meander pattern (h/2 periods per turn) should be preferred because it is not affected by the beam control. On the contrary, the effect of the sinusoidal spread is in this case strongly reduced. For typical e-folding times of τ=70 ms the effect of a frequency modulation of ± 3.5% is to multiply τ by about 4, which is sufficient to ensure stability within a 500 ms acceleration cycle. For a stronger instability, (τ < 70 ms) the multiplying factor falls off rapidly, so that unpractically large modulation amplitudes would be needed.

4.2 Experimental results

A simple way to produce the meander pattern is to drive one cavity at half the RF frequency. Experiments were first made on a magnetic flat top at 10 GeV/c. Photos 1 show typical bunch



Photos no. 1 - Bunch shapes
a) without RF/2 b) with RF/2

shapes at the end of the 500 ms flat top without (a) and with (b) one cavity fed at RF/2. (10 kV, corresponding to 7% of the main RF voltage, i.e. $\Delta\Omega/\Omega \sim \pm 3.5\%$).

During these experiments, the instability, with measured e-folding times of 50 to 150 ms, was completely damped. Further experiments during a normal acceleration cycle gave not so favorable results, either because the growth rates were then a little larger, or for another unknown reason.

Experiments both on a flat top and during acceleration with a sinusoidal amplitude modulation of the RF at the revolution frequency (producing the sinusoidal pattern) showed no visible damping, in agreement with the theory for the case of short range wake fields and beam controlled acceleration.

5. Stabilization by Landau-damping

A spread in the unperturbed synchrotron frequencies of the individual particles may introduce a damping of the instabilities.

5.1 Analytical calculations [7]

Following a paper by A.N. Lebedev [8], we use a Vlasov equation approach. The fact that we consider h=20 bunches oscillating not in phase complicates strongly the problem in our case. Simplifying assumptions have to be made to make the calculations tractable: in particular a single resonator with a Q not too low is assumed to be responsible for the instability.

Let $f(u,\varphi,t)$ be the distribution function of the particles in the machine. If $f_o(u)$ is the stationary (unperturbed) distribution function, we can write:

$$f = f_o + f_t \text{ and suppose } f_t \ll f_o \qquad (6)$$

Applying the Vlasov equation to f, expressing u and φ in the amplitude a and phase Ψ of the unperturbed synchrotron motion and assuming f_t of the form:

$$\sum_{n=\infty}^{+\infty} f_n \, e^{-jn\Psi},$$

we get

$$f_t = \sum_{n=-\infty}^{+\infty} \frac{-je}{2\pi R} \frac{1}{a} \frac{\partial f_o}{\partial a} \frac{e^{-jn\Psi}}{(\omega+n\Omega)} \frac{(u\cdot v)_n}{(R\Omega M)^2} \qquad (7)$$

where $(u\cdot v)_n = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{jn\Psi} u(a,\Psi)\cdot v\,(a,\Psi)\, d\Psi$

Now let $v(\varphi)$ be the instantaneous voltage seen by a particle of bunch m where it has the phase φ. It is the sum of the ones created in impedance Z by the passage of all bunches, and can be decomposed in Fourier series in $\Theta = \frac{\varphi}{h}$ (azimuth in a frame rotating with the synchronous particles). If we consider only dipole oscillations (without changes in shape), we can write:

$$v(\varphi) = \sum_{\ell=1}^{h} I_o \sum_{p=-\infty}^{+\infty} Z(p)\, A_p\, e^{-jp\frac{\varphi}{h}}\, e^{-j\frac{p}{h}[2\pi(m-\ell)+\phi_\ell]} \qquad (8)$$

As $u\,d\Psi = \Omega M R\, d\varphi$, we get:

$$(u\cdot v)_n = \sum_\ell I_o \sum_p Z(p)\, A_p \frac{nh}{p}\, \Omega M R\, I^*_{np} \times e^{-\frac{jp}{h}[2\pi(m-\ell)+\phi_\ell]} \qquad (9)$$

where

$$I_{np} = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{jp\frac{\varphi}{h} - jn\Psi}\, d\Psi$$

Now the current $i_m\ (\varphi)$ of bunch m is given by:

$$i_m\ (\varphi) = \frac{2\pi I_o}{R\Omega M} \int f_m\ (u,\varphi)\ du$$

We can again develop both sides in Fourier series and get, with f_m given by (7), after some calculations:

$$A_k\ e^{-j\frac{k}{h}\phi_m} = \sum_n \sum_\ell \frac{-I_o je}{R^2\Omega M} \int \frac{\partial f_o}{\partial a} \frac{da}{\omega + n\Omega} \sum_p Z(p)\ A_p$$

$$\times\ e^{-j\frac{p}{h}[2\pi\ (\ell - m) + \phi_\ell]}\ I_{\frac{nh}{p}}\ I^*_{np}\ I_{nk} \qquad (10)$$

If Z is the impedance of a resonator with a Q not too low, we can make further simplifications: neglect the variation of A_p in the meaningful range of frequencies, take p=k in the I_{np}. If we restrict ourselves to n=-1, (dipole motion) and to small oscillations, we get:

$$\phi_m = \frac{e}{R^2\Omega M} \sum_\ell I_o \int \frac{\partial f_o}{\partial a} \frac{da}{\Omega - \omega} \sum_{p=o}^{\infty} 2\mathrm{Re}\ Z(p)$$

$$\times\ e^{-j\frac{p}{h} 2\pi(\ell - m)}\ \frac{p}{h}\left[\left(\frac{h}{p}\right)^2 J_1^{\,2}\left(\frac{pa}{h}\right)\right] j.\phi_\ell \qquad (11)$$

where J_1 is the Bessel function of order 1.

If $\frac{pa}{h} \gtrsim 1$, the bracket depends on p. We can again neglect this variation if the Q of the resonator is not too low, and write:

$$\phi_m = \frac{1}{\lambda} \sum_\ell \phi_\ell\ \beta_{m\ell} \qquad (12)$$

where

$$\frac{1}{\lambda} = 4\pi \frac{h^2}{p^2} \int \frac{\partial f_o}{\partial a} \frac{J_1^{\,2}\ (\frac{p}{h}\ a)\ da}{(\Omega - \omega)2\Omega} \qquad (13)$$

The $\beta_{m\ell}$ is the one of section 2, except for the Fourier coefficient of the bunch, which is now implicitely included in the integral.

Knowing λ from the eigenvalue problem (12) we have now to solve the dispersion relation (13) to get the perturbed collective frequency ω. For distributions other than the rectangular one, we have to turn to numerical calculations.

5.2 Numerical calculations

A computer programme has been written to solve (13) for any given distribution function f_o, in particular for a function of the type $(1-a^4)^2$, which gives a good fit to the observed CPS bunches.

The determination of the threshold must be carried out in any particular case, as the relative magnitude of the real and imaginary part U and V of the eigenvalues depend very much on the Q and the tune of the resonator responsible for the instability. As pointed out by Hereward [9] the space charge frequency shift should be in some way introduced in the diagonal terms of the matrix. This extra term is, for the CPS comparable with the term coming from the wake fields. It might change the threshold intensity by approximately 40%.

The practical result of our calculations is that a very strong non linearity has to be introduced to damp the instability, and this can only be achieved by bringing the extreme particles of the bunches very near to the separatrix.

5.3 Experimental results

A programmed voltage reduction has been applied from transition to high energy, such as to give a constant longitudinal acceptance, very near to the emittance measured just after transition (6 GeV/c). The result is spectacular (photo 2) provided the bucket is tightly fitted to the bunch all along the acceleration cycle. The emittance blow-up which is normally around 6 is reduced to a negligible amount. This method is presently used in operation.

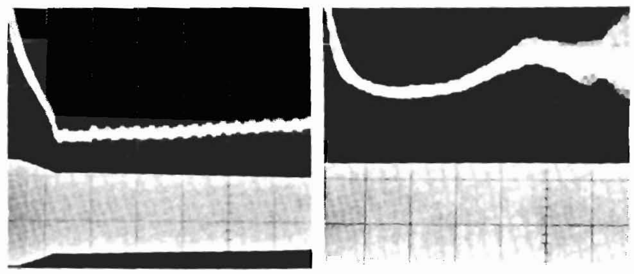


Photo no. 2 - Peak detected wide band PU signal
- RF voltage
trigger : transition timing-50 ms/cm

6. Active feed-back damping

The principle of this method is to detect any longitudinal instability by looking at the signal of a radial PU-station and to feed back this signal to the beam through a cavity. This system can be considered as an auxiliary beam control working on the RF voltage (unlike conventional systems working on the phase), which allows us to act more or less separately on each bunch depending on the tune and the bandwidth of the cavity. As the latter is not memoryless, information from bunch number m will affect not only bunch number m itself, but also all the other bunches. The total time delay of the system (PU to cavity) must be equal to the transit time of the beam.

This feed-back system will modify equations (3) in the following manner:

$$(-\Omega^2 + \omega^2)\phi_m + \sum_{n=1}^{h} \beta_{mn}\ \phi_n + \sum_{n=1}^{h} j\ \gamma_{mn}\ \omega\ \phi_n = 0 \qquad (14)$$

γ_{mn} corresponds to the influence of bunch number n on bunch number m through the feed-back loop. The γ's are proportional to the loop gain and the $j\omega$ says that the radial position of bunch n is proportional to the derivative of φ_n. For equal bunches and no synchrotron frequency spread the eigenvalues are given by:

$$\omega^2 = \Omega^2 + \sum_{\ell=1}^{h} (\beta_\ell + j\gamma_\ell\ \omega)\xi_k^{\,h-\ell} \qquad (15)$$

where ℓ = m-n and ξ_k = exp 2π jk/h. The influence of the main beam control system does not affect this result.

Υ_ℓ can be calculated using a Fourier expansion like in equation (4). The PU signal which corresponds to bunch ℓ is of the form:

$$s\, R_{o\ell} \sum_{p=-\infty}^{+\infty} A_p \exp jp\ \omega_o\, t \qquad (16)$$

$R_{o\ell}$ being the radial excursion of bunch ℓ and sR_o the DC component of the PU signal from bunch ℓ. The voltage seen by one bunch when it passes through the cavity is therefore:

$$gs \sum_{\ell=1}^{h} \sum_{p=-\infty}^{+\infty} R_{o\ell}\ A_p^2 [X(p) + jY(p)] \exp(2jp \frac{\ell\pi}{h}) \exp(jp\ \ \omega_o\, \Delta t) \qquad (17)$$

$g[X(p) + jY(p)]$ being the electronic gain of the feed-back loop and Δt the time delay error. Now, converting radial displacements into phase oscillations, one finds Υ_ℓ:

$$\Upsilon_\ell = \frac{g\ s\ h\ e}{2\pi\ \ \gamma_{tr}^2\ p_o} \sum_{p=-\infty}^{+\infty} A_p^2 [X(p) + jY(p)] \times \exp(2jp \frac{\ell\pi}{h}) \exp(jp\ \ \omega_o\, \Delta t) \qquad (18)$$

The root displacement, due to the feed-back system is given by:

$$\Delta\, \omega^2 = \sum_{\ell=1}^{h} j\Upsilon_\ell\ \omega\ \ \xi_k^{h-\ell} \qquad (19)$$

Using (18) and (19) one finds that for $p \neq qh \pm k$ (q any integer), $\Delta\,\omega^2$ vanishes (sum of the h roots of unity) whereas for $p = k$ for example one gets:

$$\Delta\omega^2 = \frac{g\ s\ h^2\ e\ \omega}{2\pi\ \ \gamma_{tr}^2\ p_o} A_k^2\ [X(k)+jY(k)] \exp(jk\ \omega_o\ \ \Delta t) \qquad (20)$$

This result shows that mode number k can be stabilized by a feed-back system working on the frequency $(qh \pm k) \frac{\omega_o}{2\pi}$.

6.2 Experimental results

We used one of the present 14 cavities of the PS as a feed-back cavity and tuned it around the 17th harmonic of the revolution frequency (h=20 in the CPS) in order to stabilize mode numbers around 3.

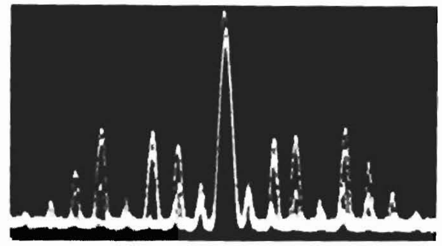
The electronic gain required to suppress instabilities having 10 ms growth times is around 10^6 at 28 GeV using our normal PU stations. Therefore the noise is an important problem and we reduced it by filtering the signal by a "comb" filter (band-pass at 17, 18 and 19th f_{rev}, band reject at f_{RF}).

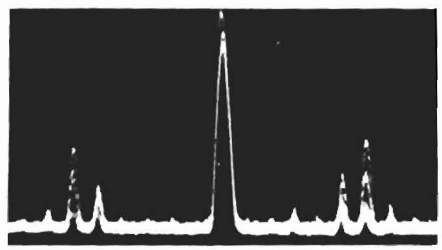
Note that the beam was brought in the centre of the radial PU electrodes in order to remove as far as possible unwanted components due to a dispersion in bunch populations.

Photos 3, taken on a spectrum analyser with multiple triggering show the stabilizing influence of the feed-back loop on modes 1,2,3 and 4. The adjacent modes (5 and 6) are outside the bandwidth of the system and are not damped. This result was confirmed by the use of the other diagnostic techniques.

7. Conclusion

The three studied schemes are efficient in damping longitudinal instabilities, and may be combined to suit particular cases. However, their use becomes very tricky if fast growth rates are involved. Therefore, care has to be taken not to introduce in the machine too harmful equipment.





Photos no. 3
Spectrum display of a radial PU signal (vert.log. scale)
a) without damping
b) with damping

Acknowledgements

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