USING OF LONGITUDINAL ACCELERATING FIELD FOR FREQUENCY MEASUREMENTS AND DAMPING OF RADIAL BETATRON OSCILLATIONS

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Abstract

A theory is briefly outlined of a feedback system in which longitudinal accelerating field is used to affect the radial betatron motion. Experimental results obtained at the Serpukhov accelerator are also given. The system can be used both for exitation of the betatron oscillations to facilitate their frequency measurements and for damping of the self-exited oscillations (in case of resistive instability or non-zero initial conditions at injection).

Introduction

Both damping and exitation of coherent oscillations can be caused by means of an amplitude or phase modulation of the accelerating field[1,2].

In fact, accelerating field modulation leads to a change in the momentum of the particles and therefore a radial displacement is observed according to the well-known equation $X''+Q^2\Omega^2X=r_0\Omega^2\frac{\Delta p}{p}$ where x is the radial displacement, Q - the number of betatron oscillations in the closed orbit, Ω - the revolution frequency, and r_0 - average radius of the accelerator.

The effect produced by the modulation of the accelerating field depends upon the source of the modulation signal. If an external source is used with a frequency equal to one of the beam oscillation modes, a resonant build-up is observed. If a radial beam position pick-up signal is used for the modulation, either exitation or damping of the oscillations is possible depending upon the phase shift in the feedback circuit.

Theoretical aspects

Let us consider a beam continuous in the azimuthal direction. Oscillations of its center of mass are in hydrodynamic approximation described by the expression[3].

$$D^2\bar{X}+Q^2\Omega\bar{X}=\bar{L}(\theta,t) \qquad (1)$$

where $\bar{X}(\theta,t)$ is the radial displacement of the beam center of mass (radial and vertical motions are considered to be independent); θ - azimuth, $D=\frac{\partial}{\partial t}+\Omega\frac{\partial}{\partial\theta}$, Ω is the revolution frequency supposed the same for all the particles (a monoenergetic beam).

The disturbance $L(\theta,t)$ can be written in a general form as[3]

$$L(\theta,t)=\iint N(\theta;\theta';t;t')\,X(\theta';t')\,d\theta' dt' \qquad (2)$$

where $N(\theta;\theta';t;t')$ is the transfer function of the feedback circuit.

We suppose that the transfer function is a homogeneous function of time $N(\theta;\theta';t;t')=N(\theta;\theta';t-t')$ which means that the feedback parameters do not change with time.

If Laplace and Fourier transformations with respect to time and θ correspondingly are applied to equations (1),(2) one can obtain

$$\left[(p+in\Omega)^2+Q^2\Omega^2\right]\hat{X}_n=\sum_{m=-\infty}^{\infty}\hat{N}_{nm}\hat{X}_m \qquad (3)$$

where $\wedge$ means Laplace transformation, N_{nm} are double Fourier harmonic expansion coefficients for the transfer function.

For homogeneous disturbance[3] $\hat{N}_{nm}=\hat{N}_n\delta_{nm}$, where δ_{nm} is Kronecker symbol, the equations for the harmonics are separated. In real systems $\hat{N}_{nm}\neq\hat{N}_n\delta_{nm}$ due to pick-ups and correctors operation in discrete mode. Hence a coupling arises between the azimuthal harmonics of the coherent oscillations.

General relationships having been considered, specific application problems will be discussed below. Assume that the feedback system consists of S equally spaced pick-ups and RF-stations used as correctors. Each pick-up signal is used to modulate output voltage of one RF-station only. Then harmonics of the transfer function may be given in the form[4].

$$\hat{N}_{nm}=\frac{\Omega}{4\pi^2}\frac{K(p)S}{p+in\Omega}\cdot e^{-in\Delta\theta+i(m-n)\theta_0}\cdot\xi_{nm} \qquad (4)$$

where

$$\xi_{nm}=\begin{cases}1 & m=n+rS\\ 0 & m\neq n+rS\end{cases} \qquad (5)$$

and r - any integer, K(p) - transfer function of the circuit between a pick-up and the corresponding corrector, $\Delta\theta$ - the distance between the pick-up and the corresponding corrector, and θ_0 - location of the 1-st pick-up. The determinant of the infinit set of equations (3) with the harmonics of the transfer function can be calculated, and the dispersion equation may be represented as

$$1=\frac{\Omega K(p)S}{2\pi}\cdot\sum_{r=-\infty}^{\infty}\frac{e^{-i(m+rS)\Delta\theta}}{[p+i(m+rS)\Omega]\{[p+i(m+rS)\Omega]^2+Q^2\Omega^2\}} \qquad (6)$$

(m = 1,2 ... S)

If the feedback is weak (i.e., the transfer coefficient K(p) is small) the solutions of this equation are near to the zeroes of denominators in the sum (6) and can be written as $p = p_n + \varepsilon_n$, where $p_n = -i(n\pm Q)\Omega$, and ε_n are small corrections. Using the disturbance method to determine the values of these corrections we can find the decrements (or increments) for the coherent oscillations harmonics

$$\omega_n = i p_n = (n\pm Q)\Omega$$

$$\mathscr{X}_n = \mathrm{Re}\,\varepsilon_n \frac{2\pi}{\Omega} = SR\left[(n\pm\theta)\Omega\right]\cos\left\{n\Delta\theta - \varphi\left[(n\pm\theta)\Omega\right]\right\} \quad (7)$$

In expression (7) $\mathscr{X}_n$ is decrement (or increment) of oscillations per one turn

$$R(\omega) = \frac{|K(\omega)|}{2Q^2\Omega^2} \quad (8)$$

and $K(\omega)$, $\varphi(\omega)$ are amplitude and phase responces of the feedback curcuit.

There is a principal limitation on the value of increments that can be achieved. It is imposed by the fact that the modulation level of the RF voltage cannot exeed 100%. The evaluation of this limitation can be made in the following way. The expressions for the transfer function harmonics (4) are given under the assumption that the deflecting force applied to the particles passing the RF station is $F_{cor} = Kx_d$, where x_d is the deflection measured by the pick-up and K is the transfer coefficient from the pick-up to the beam. On the other hand, this force can be expressed in the variation of the longitudal momentum. Hence $|Kx_d| \leqslant \tau_0\Omega^2\left|\frac{\Delta p}{p}\right|_{max}$, where $\left|\frac{\Delta p}{p}\right|_{max}$ is the relative variation of the momentum corresponding to the 100% modulation.

Taking (7) and (8) in consideration, the limitation for the maximum descrements will be the followings:

$$\left|\mathscr{X}_{n\,max}\right| = SR\left[(n\pm Q)\Omega\right] \leqslant \frac{S}{2Q^2}\cdot\frac{\tau_0}{x_0}\left|\frac{\Delta p}{p}\right|_{max} \quad (9)$$

where x_0 - the amplitude of the coherent oscillations existed before the automatic correction system was introduced - is substituted for x_d. It follows from the right-hand sides of (9) that decrements attainable are the more the less is initial amplitude i.e. the system in consideration is effective in damping small oscillations.

Experimental results

An experiment has been carried out at the Serpukhov proton synchrotron to test the method for damping and exiting of coherent betatron oscillations. One pick-up and one RF station were used in the experiment, its block-diagram being given at Fig.1.

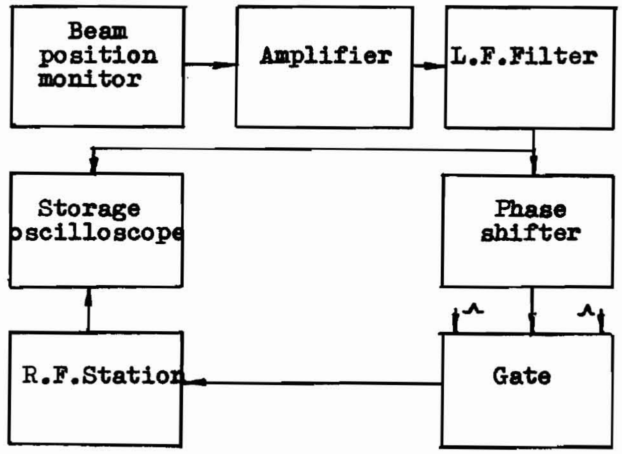

Fig.1.Experimental apparatus block-diagramme.

Setting necessary phase shift by means of a phase shifting circuit either damping or build-up of coherent betatron oscillations could be obtained during a time interval determined by means of a gating circuit. A two-trace storage oscilloscope was used to observe the processes, the upper trace being used for the filtered signal of the damped or exited mode of the frequency $\Delta Q\Omega$ (ΔQ is the complement to the nearest integer), and the lower trace for the RF voltage. An oscillogramme is given at Fig.2 of betatron oscillations with the feedback switched off. At intencities more than $7\cdot10^{11}$ particles per pulse the oscillations exist for a prolonged time with the amplitude of 2-3 mm at 2 GeV.

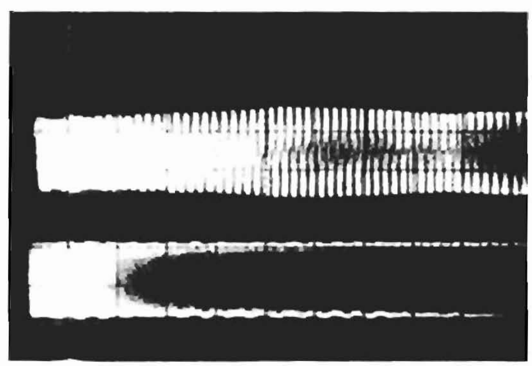

Fig.2.Feedback is cut off, the upper trace-betatron oscillations; the lower trace - RF voltage; 0.5 msec/div.

Damping of the oscillation can be seen at Fig.3. From the moment of switching the feedback on (t_1 corresponds to the triggering of the oscilloscope) RF modulation is changing in accordance with changes in the oscillations amplitude. This process can be better seen at Fig.4, which corresponds to the case of positive feedback switched on for the time of t_2 = 1.5 msec, the initial amplitude being zero.

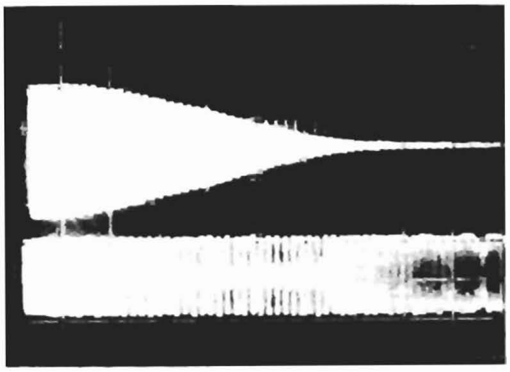

Fig.3.Feedback is closed. Oscillations damping mode; 0.5 msec/div.

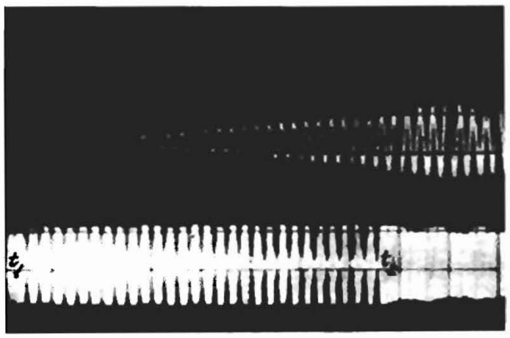

Fig.4.Feedback is closed. Oscillations excitation mode. 250 μsec/div.

Decrement measured at Fig.3 equals $(1.5\ \text{msec})^{-1}$, which is 2 times more than the calculated given by (9). The discrepancy can be explained by the fact that clipping exists in the feedback circuit, so that the transfer coefficient increases as the oscillations are damping untill the clipping ends. This process is illustrated by Fig.3 where a 100% modulation can be seen during most of the time base.

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