EXPERIMENTAL INVESTIGATION OF NON-LINEAR RESONANCES AT THE IHEP 70 GeV ACCELERATOR

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The present paper is devoted to the investigation of non-linear resonances at the maximum energy of the IHEP accelerator. Knowledge of the magnetic field properties is extremely important to make a correct choice of the conditions, that would provide maximum efficiency during fast, and especially during slow ejection of the accelerated beam from the accelerator. The first investigations have already exhibited that at high magnetic fields the influence of non-linearities becomes considerable. The dependence of the horizontal and vertical dimensions of the beam during acceleration at the central orbit at different moments of the accelerating cycle is given in Fig. 1a. The dimensions have been measured in the focusing units by means of an internal target. At the necessary moment during the accelerating cycle the target was introduced into the vacuum chamber and fixed at the required coordinates either in the external or internal parts of the chamber with an accuracy not worse than ± 0.5 mm. The position of the target could be varied vertically and horizontally. The beam edge corresponded to such a position of the target when, by signals from the pick-up electrodes with an accuracy up to 1-2%, no limitations on the intensity of the accelerated beam were observed.

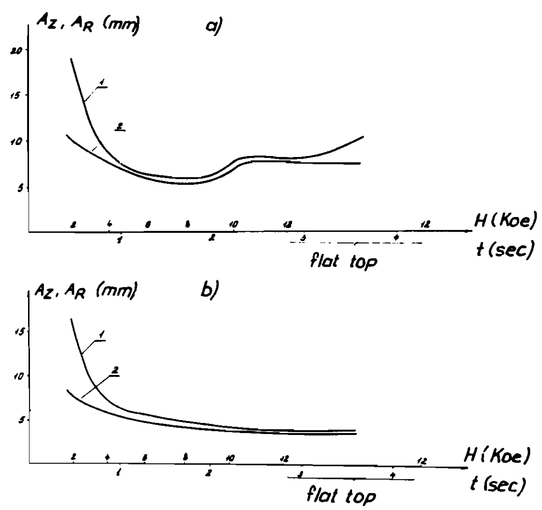

Fig. 1 The dependence of the radial (curve 1) and vertical (curve 2) half dimensions of the accelerated beam at different moments of the accelerating cycle. a) with no magnetic field corrections, b) with dynamic corrections of the gradients.

From Fig. 1a it can be seen that at the field of 9 koersted, when there appear effects of saturation the magnet iron, one observes a considerable increase of the beam dimensions in both planes. This increase is due to the influence of non-linear resonances. Indeed, when analyzing the motion of the working point in the Q_r, Q_z diagram at the end of acceleration (trajectory a-a, Fig. 2) it is found that at the field from 1 koersted up to 11 koersted , the resonance lines $3Q_z = 29$ and $2Q_z + Q_r = 29$ are intersected one after the other and at the maximum field the resonance $3Q_r = 29$. In order to avoid an undesirable influence of non-linear resonances, a method for magnetic field correction at fast and slow ejection[2,3] was worked out. Just starting from low fields, the working point was displaced into the region free from resonances. For this purpose, the radial position of the required orbit was fixed and for this orbit a necessary law of gradient changing during the acceleration time was chosen. The required shift of the betatron oscillations was achieved by independent change of the gradient value in the focusing and defocusing units of the accelerator by means of the pole windings now available. The method worked out for the magnetic field corrections has been checked experimentally. The trajectory b-b in Fig. 2 illustrates the motion of the working point when the beam is accelerated at the central orbit and when the system for dynamic gradient corrections is on.

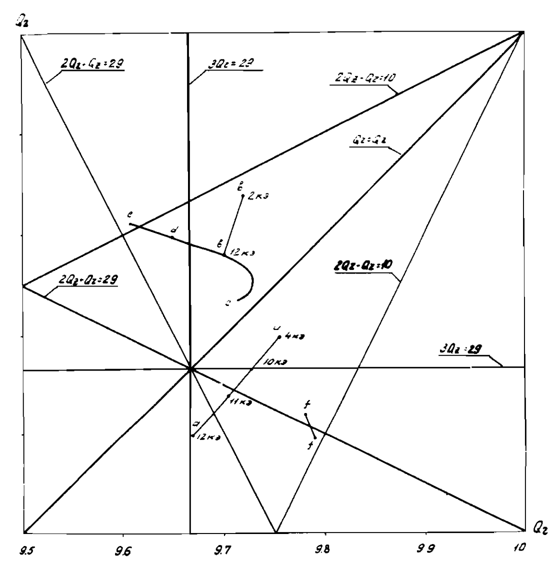

Fig. 2 Q_r, Q_z diagram with the trajectories of the working point with no corrections (a-a) and with dynamic corrections for gradients (b-b). Other curves illustrate the trajectories of the working point at crossing the resonances investigated.

The dependence of the accelerated beam dimensions in both planes on the magnetic field value, that corresponds to this mode are given in Fig. 1b. One can easily see that any increase of the beam dimensions because of non-linear effects are fully avoided. As it became clear from the calculations the optimum conditions at ejection are at the frequency values of Q_r and Q_z 9.7 and 9.78 [2)3)] respectively. In this the working point is far enough from the lines of the adjacent resonances. On the other hand from this starting point one can easily get to resonance $3Q_r = 29$, that is an operational one at slow ejection. Basing on the measurement results, one may expect that the most distinct non-linear resonances will be the ones of the third order, that are excited by quadratic non-linearity of the vertical component of the magnetic field. The magnetic measurement data were analysed proceeding from the measured values of the quadratic non-linearities in the open and closed units, assuming that the spread of the values in the units corresponds to the ordinary spread with RMS deviation $\sim$ 50%. It allows us to estimate the harmonic values of the real quadratic perturbation

$$C_k = \frac{1}{|w|} \int_0^L \frac{1}{H_o R_o} \frac{d^2 H_z}{dr^2} |\phi|^3 \, e^{-i\,k\,2\pi\, x/L} \, dx$$

where k is the number of the harmonics, $H_o R_o$ is the beam stiffness, $|\phi|$ is the modulus of the Floquet function, L is the orbit length, $|w| = 2m$ normalization of the Floquet function. The value of the 29-th harmonic $|C_{29}|$ appeared to be equal to $\sim$ 10 and the value of the 10-th harmonic $|C_{10}|$ equaled $\sim$ 16. Resonances $2Q_z + Q_r = 29$, $3Q_r = 29$ and $2Q_z - Q_r = 10$ were investigated experimentally. Before studying them it was defined how much essential the coupling between horizontal and vertical oscillations was at the maximum induction of the magnetic field at $Q_r = Q_z$. For this purpose the maximum value of the current in the windings for gradient corrections was changed step by step and the working point was displaced to the resonance line $Q_r = Q_z$. As is known the values of the betatron oscillations change due to the coupling in agreement with relation

$$Q_{1,2} = \frac{Q_r + Q_z}{2} \pm \frac{1}{2} \sqrt{(Q_r - Q_z)^2 + P^2} \;,$$

where

$$P = \frac{2}{|w|} \int_0^L \frac{1}{H_o R_o} \frac{dH_z}{dr} \Delta\phi \, |\phi|_r \, |\phi|_z \, dx$$

describes an average slope of the median plane of the accelerator and $\Delta\phi$ is the slope of the median plane in some units. In Fig. 2, the curve b-c illustrates the trajectory of the working point when approaching resonance $Q_r = Q_z$. Based on these measurements the estimated value of P is 5.10^{-3}. Thus the influence of oscillation coupling should be taken into account only in the close neighbourhood of the line $Q_r = Q_z$.

Changes of the horizontal dimensions of the beam as a function of the detuning at crossing the resonance line $2Q_z + Q_r = 29$ is given in Fig. 3 (trajectory f-f in Fig. 2). The beam dimensions were measured at the end of the magnetic field flat-top, that comprises 1.5 sec. During this period of time the accelerating RF voltage was on, therefore instabilities of the beam radial position was better than 0.5 mm. Momentum spread of the particles at the end of acceleration is $\pm\, 2.10^{-4}$ [4)]. Displacement of the working point because of the influence of these destabilizing factors does not bring us to any considerable mistake in defining the resonance band, as it goes on in parallel with the resonance line.

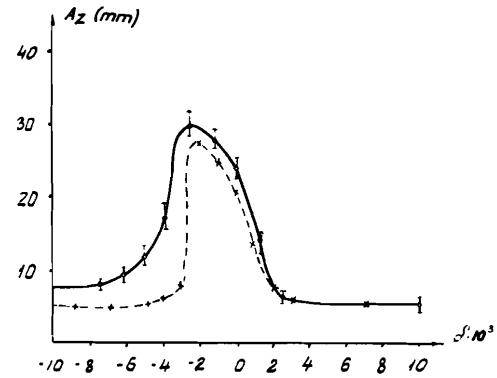

Fig. 3 The dependence of the beam half dimensions in the focusing units on the detuning when crossing the resonance line $2Q_z + Q_r = 29$. The calculated values are marked with crosses.

In Fig. 3 is also presented a calculated curve, obtained by numerical integration of the equation of motion in the vicinity of this resonance, the results on magnetic measurements at the field of 12 koersted taken into account. As is seen the experimental curve is in good agreement with the curve calculated for the value $|C_{29}| = 10$. Asymmetry of the curves may probably be explained by the influence of the cubic non-linearity. Some discrepancies between the curves in the region of negative detunings is connected with the previous fast crossing of the resonance line by the trajectory f-f. We come to the conclusion that the resonance harmonic $|C_{29}|$ is close to 1C.

From this value we can get an explanation for the results obtained in crossing the resonance line $3Q_r = 29$ (trajectory b-d, Fig. 2). The dependence of horizontal and vertical dimensions of the beam upon the detuning is given in Fig. 4. At the values of destabilizing factors mentioned above, the band of the frequencies of betatron oscillations, where there appears the resonance under investigation, widens approximately by 0.01. The width of the resonance band equals 6.10^{-4} at $|C_{29}| = 10$ and the amplitude of betatron oscillations equals 6 mm, therefore the width of the frequency band will mainly be determined by the particle spread over the momentum and by radial instabilities of the beam. This is in good agreement with the results presented in Fig. 4.

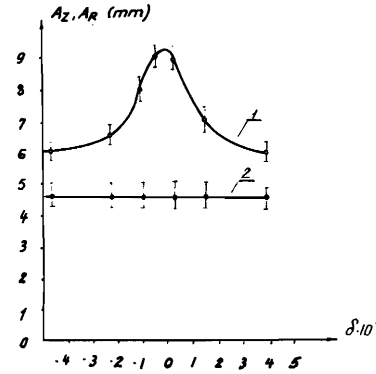

Fig. 4 The dependence of the horizontal (curve 1) and vertical (curve 2) half dimensions of the beam in the focusing units on the detuning when crossing the resonance line $3Q_r = 29$.

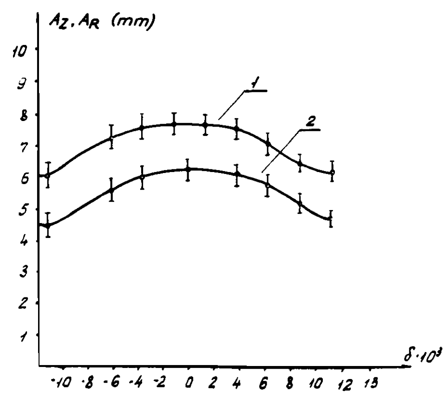

Fig. 5 The dependence of the horizontal (curve 1) and vertical (curve 2) half dimensions of the beam in the focusing units on the detuning at passing the resonance line $2Q_z - Q_r = 10$.

Besides an adjacent resonance of the third order $2Q_z - Q_r = 10$ was studies. In Fig. 5 are given the measurement results on the beam dimensions in the focusing unit in the crossing of this resonance line along the trajectory (d-e, Fig. 2). Taking into account the influence of destabilizing factors, the dimensions of the region over the frequencies where one observes the influence of the resonance are approximately in agreement with the one of the 10-th harmonic of quadratic non-linearity, obtained in magnetic measurements. The maximum value of the beat amplitudes A_r and A_z satisfies with good accuracy the integral of motion at this resonance.

The technique of measuring beam dimensions with the help of a target does not allow one at the present moment to measure the vertical dimensions larger than 10 mm. For this reason the vertical dimensions of the beam are not presented. Increase of the vertical dimensions of the beam was observed together with the horizontal one. From these investigations it became clear that the influence of the natural non-linearities at the maximum energy is very considerable and may result in undesirable beam perturbations during ejection. The values of the resonance harmonics determined through the results on magnetic measurements in the magnet units agree approximately with the experimental results.

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