OBSERVATION OF THE HEAD-TAIL INSTABILITY AT THE CAMBRIDGE ELECTRON ACCELERATOR

A. Hofmann
CEA, Harvard University and Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

G. Muelhaupt
DESY, Hamburg, Germany

Introduction

The head-tail effect which has been described by several authors[1,2,3,4] is an instability caused by short-range interaction between particles of one bunch. A particle 1 at the head of the bunch induces a field in the wall which will apply a force on a later-arriving particle 2 at the tail of the bunch. This second particle will start to execute a betatron oscillation. After half a synchrotron oscillation, particle 2 will be at the front, and excite particle 1 which is now at the tail. This interplay of betatron and synchrotron oscillations can lead to a single bunch instability. In order to make such oscillations grow it is necessary that the phase of the betatron oscillation changes when the particle moves from head to tail and vice versa. Sands[3] investigated the head-tail effect using a "fast wake" which is constant during the time the bunch passes through but decays before the next one arrives. In this case the driving force has a real part which gives rise to a frequency shift and an imaginary part which represents the growth. The growth rate is given by $\Delta\omega_\mu = \alpha_\mu - i\beta_\mu$, $\alpha_\mu = 0$ for $\mu > 0$.

$$\alpha_0 = -\frac{NF}{8\pi\gamma mf}\,, \qquad \beta_\mu = \frac{N\xi F\ell}{\pi^2\alpha c(4\mu^2-1)\gamma m} \tag{1}$$

with μ = mode number, N = number of particles per bunch, F = wake force, γ = energy divided by rest energy, m = rest mass of the electron, f = betatron frequency, $\xi = (p/\nu)(d\nu/dp)$ = chromaticity (p = momentum, ν = betatron number), ℓ = bunch length and α = momentum compaction.

In the lowest mode $\mu = 0$ there is strong coherent motion, while in the higher modes different particles oscillate with quite different phases. The lowest mode is damped for positive ξ and instable for negative ξ. All other modes are instable for positive chromaticities. The real part α_μ represents a frequency shift and is very important in the presence of Landau damping. The threshold of the instability can be determined either by the real or by the imaginary part of $\Delta\omega_\mu$. In the first case it will be independent of chromaticity.

Effect of Octupole Field and Chromaticity

The threshold of the horizontal instability has been measured as a function of octupole current for different chromaticities. The beam energy was 1.6 GeV and ν_s, the number of synchrotron oscillations per turn was .04. The experiment was done with positrons to avoid ion effects. The threshold was "defined" as an increase of beam width of $\sim$ 20% and the threshold current quoted here is the instantaneous bunch current averaged over one rf cycle (1 mA $\sim$ $1.4 \cdot 10^7$ particles/bunch). The octupole magnet powered with 1000 A produced a ν-value change $\Delta\nu \sim .015$ for a particle executing a horizontal betatron oscillation of 1-cm amplitude. The chromaticity was controlled with sextupoles in the center of several synchrotron magnets. The results from several measurements are shown in Fig.1.

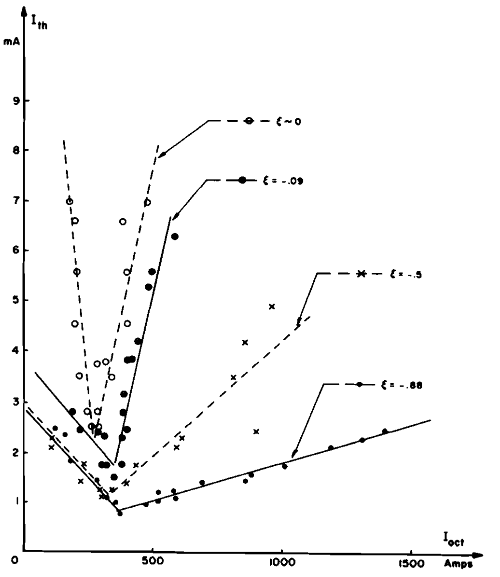

Fig.1 THRESHOLD vs. OCTUPOLE CURRENT

At $I_{oct} \sim 300$ to 400 amperes the built-in octupole field of the synchrotron is compensated. On both sides of this minimum the threshold increases linearly with total octupole field but with different slopes. The head-tail effect in the presence of Landau damping has been investigated by Pelegrini[2] and Muelhaupt[5]. Here a very simplified picture is used. At the threshold there is an equilibrium between growth rate of the head-tail effect and Landau damping plus radiation damping. If the octupole strength is increased the Landau damping increases and a new equilibrium at a higher threshold current can be established. The inverse slope of this increase (Fig. 1) is approximately proportional to the growth rate of the instability per unit current $|\Delta\omega_0'/I|$. Knowing roughly how much Landau damping is produced by the octupole, this growth rate per unit current can be calculated approximately and is plotted in Fig.2.

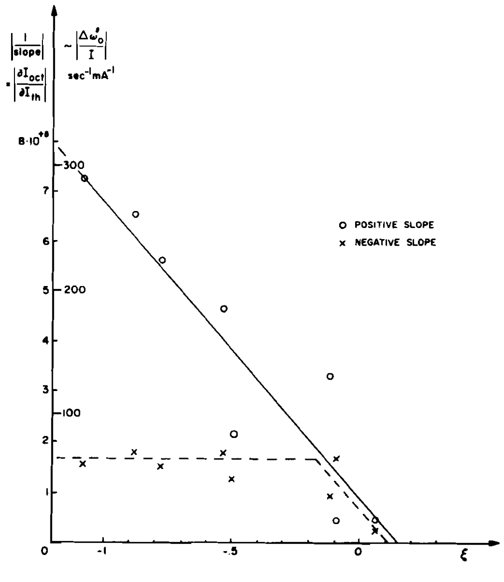

Fig. 2 GROWTH RATE PER UNIT CURRENT vs. CHROMATICITY

If the chromaticity is changed only the slopes on the right side of the minimum change. This indicates that on this side, the rate of rise is determined by the imaginary part. The other slope stays constant at first; it is determined by the real frequency shift. At very small ξ the rate of rise goes to 0. A search for a higher mode, which is unstable for positive ξ, has been carried out. The threshold is much larger. With $\xi \sim 4.5$, an instability has been observed with electron currents of about 40 mA. However the measurement was not very reproducible.

Effects of the Bunch Length

The effect of the bunch length has been measured with the technique described in the preceding chapter, however an electron beam with some ion clearing was used. The threshold current has been measured as a function of octupole current for a chromaticity of -.3 and different rf voltages. The bunch length was calculated from the measured ν_s. Figure 3 shows the unexpected result, that rate of rise decreases with increasing bunch length. According to Eq.(1) the opposite is expected. However a wake which decays fast compared to the time it takes the bunch to pass through could explain the result. In this case the strength of the force would not be proportional to the total number of particles in the bunch, but rather to the longitudinal particle density.

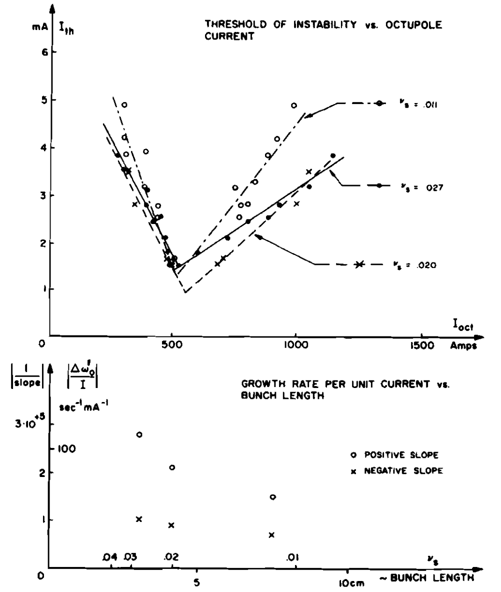

Fig. 3 DEPENDENCE OF THE INSTABILITY ON THE BUNCH LENGTH

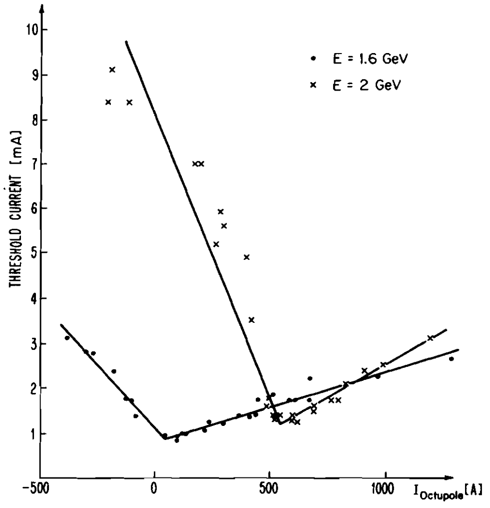

Fig. 4 INSTABILITY THRESHOLD AS A FUNCTION OF OCTUPOLE CURRENT

Effect of the Beam Energy

The threshold as a function of octupole current has been measured for two different positron energies (Fig. 4). The chromaticity was -1 and the rf voltage was kept constant and ν_s was $\sim$ .04 at 1.6 GeV. If the results from the last chapter are used to correct for the different bunch lengths at the two energies, and if the octupole current is scaled with energy, the growth rate goes like E^{-3} as expected.

Acknowledgments

We thank K. Robinson for many valuable comments and suggestions, and R. Eddy for his enthusiastic help during the experiments.

References

[1] C. Pellegrini, Frascati LMF-69/49.

[2] C. Pellegrini, Int. School of Physics, Enrico Fermi Course XLVI, p. 221 (1971).

[3] M. Sands, SLAC TN-69-8 and TN-69-10.

[4] A. Piwinski, DESY 11-70/21.

[5] G. Muelhaupt, DESY H5-71/7.