MACROPARTICLE COMPUTER SIMULATION OF MULTITURN INJECTION INTO THE CERN PS BOOSTER

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(presented by C. Bovet)

Abstract and Introduction

Multiturn injection will be necessary for filling the CERN Booster (PSB) when 10^{13} protons are to be accelerated since the injected current must then reach three times the Linac intensity. Though many aspects of this injection process have already been described by different authors[1,2] it was felt that a Monte Carlo simulation with macroparticles, could help to clarify the dependence of the capture efficiency upon various parameters (see also Ref. 3), and would lend itself to a treatment including space-charge effects.

Linac beam

A satisfactory assumption for the 50 MeV beam is that protons have a six-dimensional Gaussian distribution in the phase space volume:

$$\rho(x,p_x,z,p_z,s,p_s) = C \exp\left[-\frac{x^2}{2\bar{x}^2}-\frac{p_x^2}{2\bar{p}_x^2}-\frac{z^2}{2\bar{z}^2}-\frac{p_z^2}{2\bar{p}_z^2}-\frac{s^2}{2\bar{s}^2}-\frac{p_s^2}{2\bar{p}_s^2}\right] \quad (1)$$

Therefore projected distributions also have a Gaussian shape. In either of the transversal phase planes we have

$$\rho(x,x') = C \exp\left[-\frac{x^2-(\beta x')^2}{\beta\bar{\varepsilon}}\right] \quad (2)$$

which holds for the vertical phase plane with the same $\bar{\varepsilon}$. Note that $\bar{\varepsilon}$ is the mean value, for all particles, of the Courant and Snyder invariant,

$$w = \beta y'^2 + 2\,\alpha y y' + \gamma y^2.$$

The beam is assumed to debunch during transfer from the Linac, which is close to reality. The momentum spread has a Gaussian distribution with $\sigma = \Delta p/p$.

At the injection point (downstream end of the inflector septum) the Linac beam is steered onto the vertical closed orbit and placed in the horizontal plane at a distance d_i from the septum, with an angle d_i' (see Fig. 1).

The Linac beam is matched to the PSB lattice in the vertical phase plane but beam ellipse axis ratio β_i are studied in the horizontal phase plane.

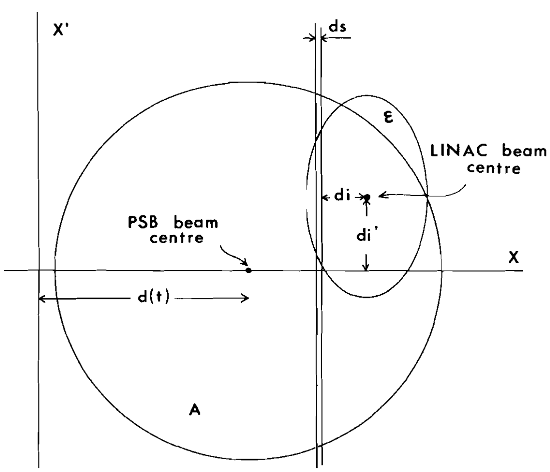

Fig. 1 - Horizontal phase plane diagram showing the Linac beam emittance, the PSB acceptance and the inflector septum.

Multiturn Process

During injection the PSB closed orbit is displaced from an external position inside the inflector towards the centre of the vacuum chamber (see Fig.1).

This closed orbit parallel displacement is a function of time

$$d(t) = d_0 - (d_0-d_f)(1 - e^{-\gamma t})/(1 - e^{-\gamma t_f}) , \quad (3)$$

where d_0 is the initial closed orbit displacement, d_f and t_f are position and time at which no further Linac particles are accepted, and γ is a parameter (Fig. 2).

d_0 must be chosen large enough so that the centre of the acceptance is filled. It should be noted that the time t_f during which particles are accepted by the PSB is longer than the actual injection duration t_m (see Fig. 2 and 3). t_f which regulates the slope of the closed orbit displacement is also to be optimized.

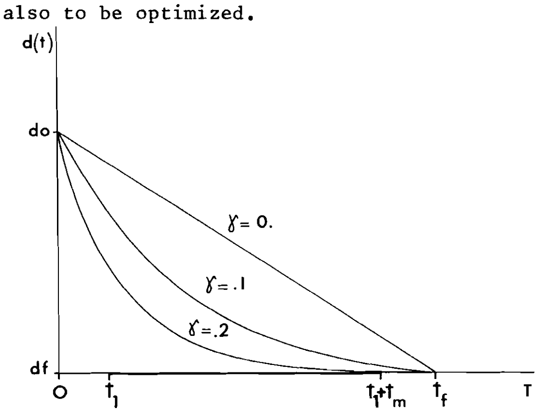

Fig. 2 - Closed orbit parallel displacement in time during multiturn injection.

For the simulation particles are injected at hundred discrete instants as slices of the Linac beam. For each slice 100 particles are taken with random coordinates in five dimensions of the phase space, generated according to Eq. 1.

Particles hitting the septum or exceeding the PSB acceptance are lost.

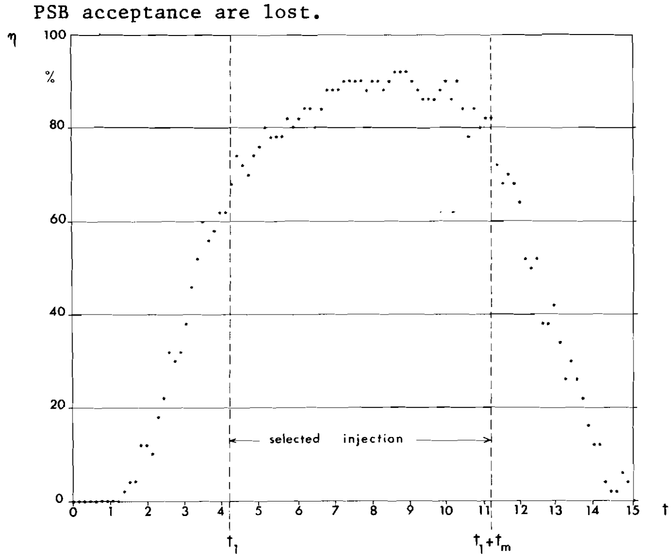

Fig. 3 - Capture efficiency during the process.

PSB dynamics without space charge

The PSB is characterized by an acceptance A, an horizontal betatron frequency Q, an amplitude function β and a momentum compaction factor α_p, at injection point.

Since no interactions are considered here every particle can be treated independently. One revolution in the PSB is performed by a matrix multiplication. Then the closed orbit is displaced according to Eq. 3 and the particle may hit the septum from the inside. If it survives it will be traced for one more revolution.

Thanks to the simple solution of the dynamics one full injection process involving 10'000 particles only uses 10 s of CP time on a CDC 6600.

PSB dynamics with space charge

When the dynamics include electric self field of the beam, computer time goes up and the number of slices is reduced to one per revolution.

Each of the 16 periods of the PSB is divided into 15 sections with a definite vacuum chamber cross section three different vacuum chambers exist i) in long straight sections, ii) in bending magnets and iii) in triplets . The slice under computation contains about 1'000 macroparticles which have more or less charge according to the current already injected. In order to lessen the statistical fluctuations of the injection efficiency, macroparticles are given a certain extension in the x direction so that they can be destroyed partially whenever they pass closely by the septum.

The slice is transferred from one section to the next by matrix multiplication. Then using fast Fourier transform technique[4,5], a two dimensional Poisson's equation is solved for the local beam distribution and with the equipotential boundary given by the vacuum chamber. For each macroparticle the electric field $\vec{E}(x,z)$ is obtained by interpolation and derivation of the potential known at the mesh points, and the space charge defocussing is introduced as a thin lens action.

Summary of parameters

The values of the relevant PSB parameters used in this analysis are shown in Table I.

Table I

Item	Parameter	Range of values	Units
Linac beam	t_m	7, 20 , 34	μs
	$\bar{\varepsilon}$	5, 8, 10	mrad mm
	$\Delta p/p$	0 -- 0.4	%
	β_i	0.6 - 4.0	m
	d_i	2.0 - 8.5	mm
	d_i'	2 - 2	mrad
Multiturn process	t_f	10 - 60	μs
	d_s	0 - 2	mm
	γ	0 - 0.2	
PSB	A	130	mrad mm
	Q	4.1 - 4.9	
	β	5.8 - 4.5	m
	$R\alpha_p$	1.52- 1.04	m

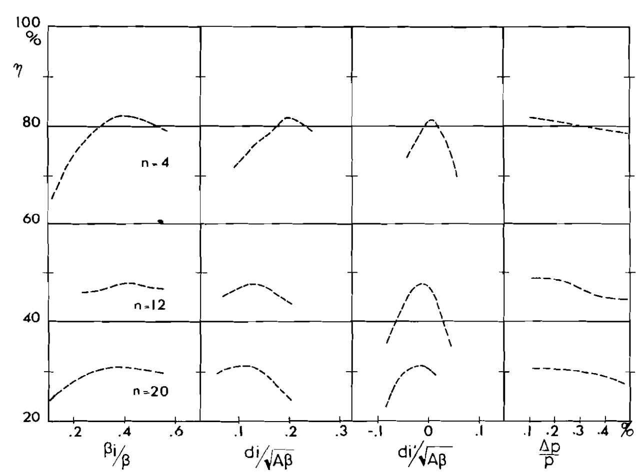

Fig. 4 - Efficiency optimization versus beam focusing β_i, beam steering d_i,d_i' and momentum spread $\Delta p/p$.

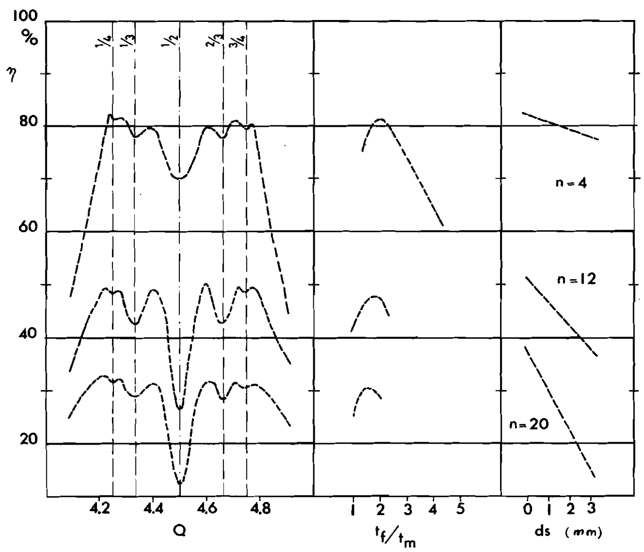

Fig. 5 - Efficiency optimization versus horizontal tune Q, septum thickness ds and closed orbit displacement t_f.

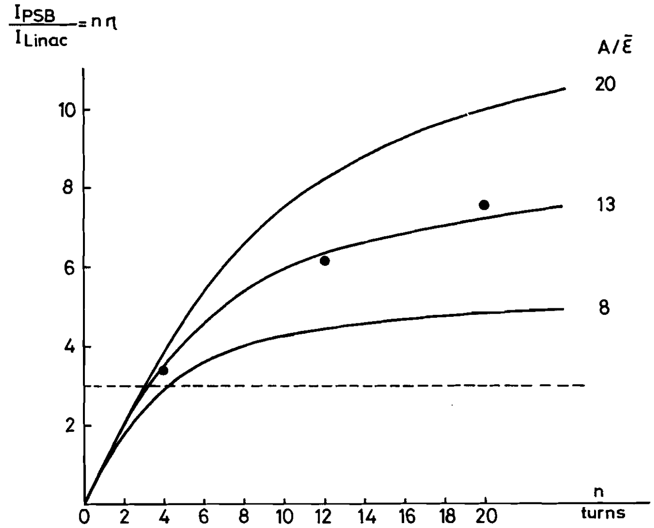

Fig. 6 - Current built up versus number of turns for zero septum thickness.

Optimization of efficiency

For a given Linac beam with a pulse length t_m, and a quality given by $\bar{\varepsilon}$ and $\Delta p/p$ we can optimize the current filling the acceptance A. The optimization shown in Figs. 4 and 5 is made for a ratio $A/\bar{\varepsilon} = 13$. On Fig. 4 injection parameters are normalized to A and β in order to ease a scaling towards other machines. On Fig. 5 one sees i) the dramatic effect of Q close to some fractional values, ii) the optimum choice of t_f for the closed orbit displacement and iii) the importance of the septum thickness. The efficiency decreases when the number of injected turns increases. This is presented on Fig. 6 where curves obtained from an analytical model are used for interpolation and scaling towards different values of $A/\bar{\varepsilon}$.

Space-charge effects

Introduction of space charge effects into the simulation did not change the picture dramatically. Of course, the spread of individual particles Qs results in smoothing the curves of η(Q) given on Fig. 5. It was observed that particles lying in phase plane between the closed orbit and the center of mass of the beam have a betatron frequency modulated around the coherent frequency of the beam, which may be definitely higher than the expected single particle frequency.

No significant blow-up was detected in the vertical phase plane.

Conclusions

In the optimization of efficiency some parameters like the horizontal ellipse aspect ratio β_i, or the momentum spread, $\Delta p/p$, have a weak influence, some other (the closed orbit displacement, t_f, the horizontal tune, Q, or the septum thickness, d_s) are critical. When injection is optimized four turns are sufficient for the PSB to reach the wanted intensity, twelve turns double this value but any longer pulse would be wasted by lack of efficiency. And when the highest intensity is not asked for the multiturn process can be optimized to produce a smaller beam (i.e. to improve the brilliance).

References

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2. L. Nielsen, CERN Int. report, MPSInt. Lin/69-13.
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4. R.W. Hockney, Methods in Computational Physics, Vol. 9, Academic Press Inc., New-York and London, 1969.
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DISCUSSION

E. REGENSTREIF : What was the shape of the initial distribution used in the solution of the Poisson equation ?

C. BOVET : The distribution was Gaussian.