FIELD STABILIZATION IN PERIODIC SYSTEMS*

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Introduction

Recent theoretical and experimental studies[1-14] on the effects of distributed tuning errors, losses and beam loading have emphasized the need for stabilization of field amplitude and phase in linear accelerators. (Such effects are, of course, not peculiar to linacs and some occur in all periodic systems.) One of the most significant results of these analyses was the prediction that all such deleterious effects on the accelerating field could be minimized by operating in a $\pi/2$ space mode. This followed because of the wider separation between accelerating and higher order modes. However, wider mode separation can be achieved even at π and 2π modes in any band. The role of fields in the stabilizing reaction has not been described in a physically revealing way. The use of equivalent circuits introduces valuable simplifications which are highly productive and suggestive. But circuit elements, which represent integrated field effects, obscure the detailed behavior of fields and may mask other deleterious effects. One of the purposes of this report is to suggest a physical model and an appropriate measure for field stabilization in any periodic system. Additionally an approximate representation of the dispersion relationship, consistent with measured curves for stabilized cavities, will be applied to the expression of fields in such cavities.

Field Stabilization

First it is recognized that stabilization is not peculiar to $\pi/2$ mode operation. Such characterization is neither essential nor desirable. The fundamental requirement is the periodic modification of a structure to produce a joining (or coupling) of two passbands at a band edge where the dispersion curves have opposite curvature, as shown in Fig. 1.

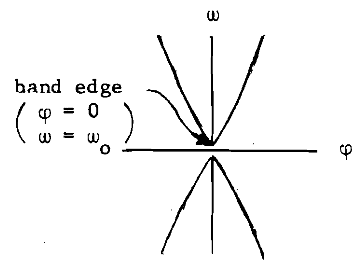

Fig. 1. Dispersion curves for stabilized cavities.

Such periodic perturbations may also be viewed as converting the original periodic structure into an alternating or bi-periodic structure. (An equivalent statement is that cell fields in one band are bounded by or coupled to alternately aiding and opposing fields from the other band.) Then propagation or resonant cavity oscillation in one band will induce periodic oscillation, with the same phase shift, in the adjacent band. These latter oscillations, representing non-propagating evanescent modes, driven at a frequency far beyond its resonance will be in time quadrature with, and have amplitudes proportional to, the resonant driving mode oscillation at the same point of the periodic structure. Despite its small amplitude, such non-propagating oscillations can significantly influence the phase shift of propagating modes across coupling regions. If the propagating mode lies in the upper band, the induced oscillation in the lower band is driven from above its resonance and most of its stored energy lies in its magnetic field. This changes when the relative frequency position of propagating and evanescent modes reverses. As described below, the effect of this coupling between bands is to produce a wider mode separation at the band edge, varying linearly[3,4,9] with periodic phase instead of quadratically. As a result we obtain a non-vanishing group velocity at the previous cutoff frequency and the stabilization effects referred to above.

Dispersion Relationship

The shape of the dispersion curve near cutoff may be obtained as a degenerate case of the bi-periodic structure.[13] In the usual manner[15] the

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field in a cell is expanded in the normal modes satisfying electric wall boundary conditions and the Maxwell equations then lead to the following forced oscillation equation for the n^{th} field coefficient, $V_n = \int \bar{E} \cdot E_n \, dv$,

$$\frac{d^2 V_n}{dt^2} + \omega_n^2 V_n = - \frac{\omega_n}{\sqrt{\epsilon\mu}} \int_s \bar{E} \times \bar{H}_n \cdot \bar{n} \, ds \quad .$$

(An additional driving integral may be added on the right to account for beam interactions.) Tangential E on the boundary may be resolved into three components, $E = E_1 + E_2 + E_3$:

- E_1, due to cell to cell field variation of a normal mode with negligible coupling to other bands in a lossless structure
- E_2, due to losses in real boundary walls
- E_3, due to non-propagating mode excitations in adjacent bands.

If we first consider a band with negligible coupling to other bands in a lossless periodic structure, $(E \approx E_1)$, then application of Floquet's theorem to the drive integral leads to the common dispersion relationship[13]

$$\frac{\omega_n^2 - \omega^2}{\omega^2} = B\,(1 - \cos k L_o)$$

giving a quadratic dependence of ω on cell to cell phase, $k L_o$, at cutoff.

Wall losses are accounted for by adding the integral

$$\frac{\omega_n}{\sqrt{\epsilon\mu}} \int_s \bar{E}_2 \times \bar{H}_n \cdot \bar{n} \, ds = (1 + j)\, \frac{\omega_n}{Q_{on}} \, \frac{d V_n}{dt} \quad ,$$

which produces a complex frequency dependent term proportional to $1/Q_o$ and makes the propagation constant complex, in the dispersion relationship,[2]

$$\omega_n^2 - \omega^2 - (1 - j)\, \frac{\omega\, \omega_n}{Q_{on}}$$

$$= B\omega^2 \left[1 - \cos (k_1 + jk_2)\, L_o\right] \quad .$$

The imaginary component gives an additional phase shift per cell, which removes the cutoff points, opening up the ends of the band. The original frequency band is now contained within a slightly reduced phase band with apparent 0 and π edges bordered by open regions limited to phase shifts of order $[f_o/Q_o \, (\Delta f)_{band}]^{\frac{1}{2}}$. However, such border regions are very small for large bandwidth, high-Q structures such as drift tube linacs.

Finally the effect of coupling to an adjacent band is obtained from the integral, $\int \bar{E}_3 \times \bar{H}_n \cdot \bar{n} \, ds$, including the effect of non-propagating modes having the same cell to cell phase shift as the propagating mode. Consider a single cell approximated with discontinuous boundary regions and fields represented in Fig. 2.

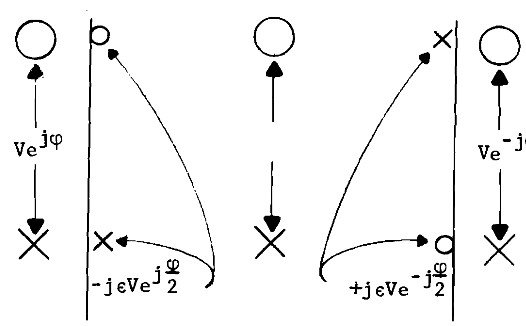

Fig. 2. Cell, showing boundary fields.

V represents the propagating mode and ϵ is a small number measuring the amplitude of the evanescent mode relative to the propagating mode, which falls off rapidly with band separation. The cutoff oscillations are in time quadrature with and symmetrically distributed in phase with the propagating mode oscillations and the sign reversal corresponds to the requirement of stabilizing fields which alternately aid and oppose the oscillating field in the propagating band. Evaluation of the driving integral then leads to the proportionality

$$\int (\bar{E}_1 + \bar{E}_3) \times H_o \cdot \bar{n} \, ds$$

$$\propto \left[2V - Ve^{j\varphi} - j\epsilon Ve^{j\frac{\varphi}{2}} + Ve^{-j\varphi} + j\epsilon Ve^{-j\frac{\varphi}{2}} \right]$$

$$\propto \left[2V\,(1 - \cos \varphi) + \epsilon\, 2V \sin \frac{\varphi}{2} \right]$$

and the dispersion relationship becomes

$$\frac{\omega_o^2 - \omega^2}{\omega^2} V = BV \left(1 - \cos \varphi + \epsilon \sin \frac{\varphi}{2} \right) ,$$

whence, defining $4S \equiv B\epsilon$,

$$\frac{\omega_o^2 - \omega^2}{\omega^2} = 4S \sin \frac{\varphi}{2} + B\,(1 - \cos \varphi) \quad .$$

The coupling to the adjacent stabilizing band produces a linear dependence of $(\omega - \omega_o)$ on φ in the cutoff neighborhood around $\varphi = 0$. Thus the slope of the dispersion curve at $\omega = \omega_o$ becomes $d\omega/d\varphi = -\omega_o S$ and the measure of mode separation or stabilization is

$$- S = \frac{1}{\omega_o}\frac{d\omega}{d\varphi} \quad .$$

Similar results may be derived from the analyses of Nishikawa[3,4] and Nagle, Knapp and Knapp.[8] We can readily show that $|S| \simeq K_1/2$ in Ref. 8 notation and $|S| \simeq \sqrt{B/8}$ where $B \simeq 2/\sqrt{(C/L)\cdot(L_s/C_s)}$ in Nishikawa's notation.

The stabilization factor S decreases rapidly as the bands separate and the dispersion curves revert to the original form given above for the isolated band.

The stabilization factor also depends on the spatial distribution of the propagating and evanescent fields, which in turn depends on the geometry of the periodic perturbations in the structure. Comparison of different stabilizing modes arising from different periodic perturbation geometry is obtained by measuring the linear slope of the dispersion curve at the cutoff point of the accelerating band. If the fundamental periodicity also changes, as in a variable β accelerating structure, there will be corresponding changes in the curvature of the dispersion curve and stabilization factor, $S(\beta)$.

Steady State Field

Stabilization effects on cavity field amplitude and phase distribution are shown by substitution of the dispersion relationship into the field expressions. Applying Nishikawa's results[1] to a lossy cavity with two in-phase drives spaced L/4 from each end, we may write the field in the following manner:

$$\frac{E(z,t)}{E_o e^{j\omega_o' t}} = 1 + 2j\frac{\omega_o'^2}{Q_{oo}} \sum_{n=1,2,3,---} (-1)^n \left[\frac{\cos(4n\pi z/L)}{\omega_{4n}'^2 - \omega_o'^2} + \eta_{(4n-2)} \frac{\cos(4n-2)\pi z/L}{\omega_{(4n-2)}'^2 - \omega_o'^2} + \eta_{(2n-1)} \frac{\cos(2n-1)\pi z/L}{\omega_{(2n-1)}'^2 - \omega_o'^2} \right] \quad .$$

In this expression, which is valid for small n, the coefficients, η, measure the amplitude of higher order modes relative to the dominant mode. If the symmetry of the cavity and the L/4 + 3L/4 drive completely suppress the odd-numbered higher order modes, the steady state field may be approximated by the first two even terms above. Thus, using the previously derived approximation for the dispersion relationship of an N-cell stabilized cavity near cutoff, we find

$$\omega_4^2 - \omega_o^2 \simeq 2\left(\omega_2^2 - \omega_o^2\right) \simeq 8\omega_o^2 S \frac{\pi}{N}$$

and

$$\frac{E(z,t)}{E_o e^{j\omega_o t}} \simeq 1 - j\frac{1}{4\pi Q_o S}\left(\cos\frac{4\pi z}{L} + 2\,\eta_2 \cos\frac{2\pi z}{L}\right) \quad .$$

It can be seen that the quadrature field and corresponding phase shifts within a cavity operating at cutoff varies inversely with the stabilizing factor. The same conclusion may be drawn for the perturbations due to distributed tuning errors.

Conclusions

Stabilization of periodic fields has been described together with a measure of such stabilization. Dispersion relationships and fields have been expressed for stabilized cavities.

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