OPTIMIZATION OF ALTERNATING-PERIODIC IRIS-LOADED DEFLECTORS FOR SUPERCONDUCTING RF BEAM SEPARATORS*

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Abstract

Analytical expressions for transverse shunt impedance, peak fields, group velocity, etc. of alternating-periodic iris-loaded deflector cavities operating in the confluent $\pi/2$ mode are derived from the TM_{11}-like small-pitch approximation. It is shown that the smallest peak fields and the largest shunt impedance are obtained in structures with vanishingly small width of the coupling cell. An optimization of the structure determines the ratio of the beam hole radius divided by the wavelength to be 0.3, implying a forward-wave operation. Possible complications due to the degenerate dispersion diagram are discussed.

I. Introduction

We pointed out in a previous report,[1] that superconducting deflectors are preferably designed as alternating-periodic iris-loaded cavities. Operation in the confluent $\pi/2$ mode with the largest possible width of the excited cells leads to performance characteristics comparable to that of a resonant ring but without the latter's well-known problems. The purpose of this paper is to work out the design criteria for superconducting cavities permitting a systematic optimization of the deflector geometry. Since the operating frequency is determined by the momentum range of the particles to be separated and the deflector length by the angular deflections considered desirable,[2,3] the optimization procedure is essentially limited to the choice of the beam hole radius.

In the design of a deflector a compromise solution is indicated in order to respect several conflicting requirements:

1) The stability of the fields in the presence of machining errors, mechanical vibrations, or similar perturbations, is enhanced by operating in the confluent $\pi/2$ mode with the largest possible group velocity.

2) Superconducting deflectors are intended for use in separated counter beams and the particle fluxes are of greatest importance.

3) The equipment and operating costs of a superconducting rf beam separator depend strongly on the refrigerator required, and economic considerations must enter the optimization procedure.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Application of the above requirements to a systematic deflector design will be facilitated by introducing three figures of merit. Their relative weight, however, will remain subject to discussion and no universal solution can be found.

We will attempt to find analytical expressions for the figures of merit introduced. This will require a simple representation of the fields in iris-loaded deflectors. Many properties of alternating-periodic structures can be explained by the help of the small-hole approximation used in Ref.1. However, the dependence of the deflector parameters on the beam hole radius is more naturally obtained from the small pitch approximation. The particular version of this approximation elaborated on in this paper will make use of the fact that the bandpass of the HEM_{11} deflecting mode is in the immediate vicinity of the TM_{11} cut-off frequency (TM_{11}-like approximation). The expressions obtained from this simple field approximation are probably only qualitatively correct and their verification by experiments or a rigorous theory is desirable and necessary.

II. TM_{11}-Like Small Pitch Approximation

In this section, analytical expressions for transverse interaction parameter, shunt impedance, peak fields, etc. of alternating-periodic (AP) cavities will be derived. They will be obtained directly from the corresponding expressions for uniform-periodic (UP) waveguides, which lend themselves more easily to analysis. The investigations will be based on the TM_{11}-like small pitch approximation, in which use is being made of the fact that the HEM_{11} passband is close to the TM_{11} cut-off frequency of a cylindrical waveguide with identical radius. The notations used for the analysis of UP waveguides and AP cavities are shown in Figs. 1 and 2. The width of the excited cell is designated as d, both in UP and AP cavities, whereas the period is designated as h in UP and 2h in AP cavities. By making the distinction between cell width and period, we allow for a finite iris thickness in the expressions obtained. All numerical examples, however, will be based on infinitely thin irises. In addition, all considerations will be limited to deflectors of extreme relativistic particles with $\pi/2$ phase shift per cell, leading to the condition which remains valid for UP and AP structures

$$kh = \pi/2 \qquad (1)$$

where k is the wave number in the guide.

For the purpose of analysis it is convenient to cut the iris-loaded waveguide into two regions, i.e., the axial and the cell region. Approximate field representations are found independently for each region. The subsequent matching of the tangential electric component along the common boundary yields the relative amplitude of the fields in each region. The deflector characteristics are then obtained simply by integration.

The field representation in the axial region requires, in general, two linearly independent solutions. The boundary condition $E_\theta = 0$ on the iris along r = a entails a coupling between solutions and leaves only one expansion coefficient, B_o, which is proportional to the deflecting field strength acting on synchronous particles. The fields in the axial region of a wave traveling with a phase velocity equal to the velocity of light can in natural units ($c = \mu_o = 1$) be represented by

$$\vec{E}^I = B_o \begin{bmatrix} \left\{\left(\frac{kr}{2}\right)^2 + \left(\frac{ka}{2}\right)^2\right\} e^{-i\theta} \\ \left\{\left(\frac{kr}{2}\right)^2 - \left(\frac{ka}{2}\right)^2\right\} ie^{-i\theta} \\ -j\,kr\,e^{-i\theta} \end{bmatrix} e^{-jkz} \qquad (2)$$

and

$$\vec{B}^I = B_o \begin{bmatrix} -\left\{\left(\frac{kr}{2}\right)^2 - \left(\frac{ka}{2}\right)^2 + 1\right\} ie^{-i\theta} \\ \left\{\left(\frac{kr}{2}\right)^2 + \left(\frac{ka}{2}\right)^2 - 1\right\} e^{-i\theta} \\ j\,kr\,ie^{-i\theta} \end{bmatrix} e^{-jkz} \qquad (3)$$

The time factor e^{jkt} will be omitted from all equations. Note that $i^2 = j^2 = -1$, but $ij \neq -1$. The transverse force on an extreme-relativistic particle is equivalent to a magnetic field which is given by

$$\vec{B} = j\,k^{-1}\,\nabla_T\,E_z \quad . \qquad (4)$$

Together with (1) it follows that $|\vec{B}| = B_o$; the expansion coefficient equals the equivalent deflecting field strength.

Taking into account the fact that the entire bandpass of the HEM_{11} mode is in the vicinity of the TM_{11} cut-off frequency, we are able to represent the fields in the slot by

$$\vec{E}^{II} = c_o \begin{bmatrix} 0 \\ 0 \\ j\,J_1(kr)\,e^{-i\theta} \end{bmatrix} \qquad (5)$$

and

$$\vec{B}^{II} = c_o \begin{bmatrix} \frac{J_1(kr)}{kr}\,ie^{-i\theta} \\ J_1'(kr)\,e^{-i\theta} \\ 0 \end{bmatrix} \qquad (6)$$

with the wave number given by $k = j_{11}/b$. The essence of the "TM_{11}-like" small pitch approximation lies in the use of $E_z \propto J_1(kr)$ rather than the usual[4] $E_z \propto J_1(kr) - N_1(kr)\,J_1(kb)/N_1(kb)$. The errors caused by this simplification are smallest if $a/b \to 0$, but a preliminary estimate showed that the use of (5) and (6) leads to qualitatively correct results as long as $ka < j_{11}'$, with j_{11}' being the first zero of J_1'.

The ratio of the coefficients c_o/B_o is obtained by matching of E_z at r = a according to

$$\int_{-\frac{1}{2}h}^{\frac{1}{2}h} E_z^I\,e^{jkz}\,dz = \int_{-\frac{1}{2}d}^{\frac{1}{2}d} E_z^{II}\,e^{jkz}\,dz \qquad (7)$$

which leads to

$$\frac{c_o}{B_o} = -\left\{\frac{J_1(ka)}{ka}\,\frac{d}{h}\,C_{oo}\right\}^{-1} , \qquad (8)$$

where the transit time factor

$$C_{oo} = \frac{\sin\frac{\pi}{4}\frac{d}{h}}{\frac{\pi}{4}\frac{d}{h}} \quad . \qquad (9)$$

The transverse interaction parameter of a traveling-wave UP deflector is in natural units defined as

$$\frac{R_{TW}}{Q} = \frac{B_o^2}{kW} \quad , \qquad (10)$$

where W represents the total stored energy per unit length, k the frequency or wave number in free space, and B_o the equivalent deflecting field strength. It is more convenient to investigate the dimensionless quantity $kQ/R_{TW} = k^2W/B_o^2$, which is a function of ka and d/h only, since we limit our considerations to $\pi/2$-mode operation. By simple integration, the following explicit expression was obtained:

$$\frac{k^2W}{B_o^2} = \frac{\pi}{4}(ka)^2\left\{1 + \frac{1}{6}(ka)^4\right\} + \frac{\pi}{4}\frac{d}{h}\left(\frac{c_o}{B_o}\right)^2\left\{j_{11}^2\,J_o^2(j_{11}) - (ka)^2\,J_o^2(ka) + \left[1-(ka)^2\right]J_1^2(ka) + ka\,J_o(ka)\,J_1(ka)\right\} . \qquad (11)$$

This result reduces for $ka \to 0$ to the expression previously derived from the small-hole approximation, that is

$$\frac{R_{TW}}{kQ} = \frac{1}{\pi}\frac{d}{h}\left\{ \frac{C_{oo}}{j_{11}\, J_o(j_{11})} \right\}^2 \quad . \tag{12}$$

In lossless structures, the group velocity equals the energy velocity[5] which is defined as

$$v_E = \frac{P}{W} \quad , \tag{13}$$

where P represents the axial power flow. It is more convenient to express the energy velocity as a quotient of two dimensionless quantities,

$$v_E = \left\{ k^2 P/B_o^2 \right\} / \left\{ k^2 W/B_o^2 \right\} \quad , \tag{14}$$

where the denominator is given by (11) and the numerator by

$$\frac{k^2 P}{B_o^2} = \frac{\pi}{8}(ka)^4 \left\{ \frac{1}{3}(ka)^2 - 1 \right\} \quad . \tag{15}$$

Note that this quantity is related to the series impedance of a waveguide which is usually defined as $Z = B_o^2/P$.

The transverse shunt impedance of a UP waveguide is defined as

$$R_{TW} = \frac{B_o^2}{\partial_z P} \quad , \tag{16}$$

where $\partial_z P$ represents the power loss per unit length. It appears more convenient to determine the dimensionless quantity $R_{TW}\, r_s/k = r_s\, B_o^2/k\, \partial_z P$, with r_s being the surface impedance. After some manipulations, we find that

$$k\, \partial_z P/B_o^2\, r_s = \frac{\pi}{2}\frac{t}{h} ka \left\{ 1 + \frac{1}{4}(ka)^4 \right\}$$

$$+ \left(\frac{c_o}{B_o}\right)^2 \left\{ \frac{\pi}{2}\frac{d}{h} j_{11}\, J_o^2(j_{11}) + j_{11}^2\, J_o^2(j_{11}) \right.$$

$$\left. + \left[2 - (ka)^2 \right] J_1^2(ka) - (ka)^2\, J_o^2(ka) \right\} , \tag{17}$$

where t represents the iris thickness.

The quality factor of a structure is defined by

$$Q = \frac{kW}{\partial_z P} \quad . \tag{18}$$

The losses are proportional to the surface resistance r_s and it is more convenient to determine the dimensionless quantity Qr_s, which is obtained as the ratio

$$Qr_s = \left\{ \frac{kQ}{R_{TW}} \right\} / \left\{ \frac{k}{R_{TW}\, r_s} \right\} \quad . \tag{19}$$

The expressions entering into (19) are given by (11) and (17). Equation (19) reduces for vanishing beam holes, i.e. $ka \to 0$, to the equation

$$Qr_s = \frac{1}{2}\, \frac{j_{11}\, \frac{\pi}{2}\frac{d}{h}}{j_{11} + \frac{\pi}{2}\frac{d}{h}} \quad , \tag{20}$$

which is identical to the expression previously derived from the small-hole approximation.

In waveguides with a beam hole radius $ka < j_{11}'$, the peak electric field $\hat{E}$ occurs in the slot at $ka = j_{11}'$ and its value is

$$\frac{\hat{E}_{TW}}{B_o} = J_1(j_{11}')\, \frac{c_o}{B_o} \quad . \tag{21}$$

It is more difficult to find the value of the peak magnetic field on the metallic walls, but it is possible to indicate an upper limit which is

$$\frac{\hat{B}_{TW}}{B_o} \le J_1'(0)\, \frac{c_o}{B_o} \quad . \tag{22}$$

The pertinent aspect of Eq. (21) and (22) is that the peak fields are proportional to c_o/B_o.

The characteristic parameters of a UP standing-wave cavity follow directly from the above expressions. We adopt the convention that the transverse momentum p_T imparted to a synchronous extreme-relativistic particle traversing the cavity is given by

$$p_T = q\, (R_{SW}\, P_o\, \ell)^{\frac{1}{2}} \quad , \tag{23}$$

where q is the particle charge, ℓ the deflector length, $P_o = \partial_z P\, \ell$ the total power loss in the cavity walls, and R_{SW} the shunt impedance of the cavity, which is related to the previously defined R_{TW} according to

$$R_{SW} = \tfrac{1}{2} R_{TW} \quad . \tag{24}$$

Furthermore, it can be shown that the quality factors of waveguides and cavities are identical, and that the peak fields in cavity and waveguide are related by

$$\frac{\hat{E}_{SW}}{B_o} \approx 2\, \frac{\hat{E}_{TW}}{B_o} \quad . \tag{25}$$

However, it was pointed out by Lengeler,[6] that the following inequality holds,

$$\frac{\hat{E}_{SW}}{B_o} \le 2\, \frac{\hat{E}_{TW}}{B_o} \quad . \tag{26}$$

Even though in the stationary state no energy propagates in a cavity, the group velocity of the corresponding waveguide remains indicative of the sensitivity of the structure to perturbations.

The AP cavity is obtained from a UP cavity operating in the $\pi/2$ mode by increasing the width of the excited cells. The slot region of the coupling cells are in first approximation field free. A short reflection will show that the same expressions remain valid for AP as well as for UP cavities, if d is taken to be the width of the excited cell and t the iris thickness. The group velocity, however, is only meaningful if confluence was achieved by proper adjustment of the coupling cell radius. Otherwise, the group velocity equals zero.

Numerical results for deflector cavities with infinitely thin irises (t = 0) were obtained from the above expressions and are depicted in Figs. 3 to 5, where transverse shunt impedance, peak fields, and group velocity are plotted as a function of beam hole radius with the cell size as parameter. We see that the group velocity has a relative extremum for $ka \approx 1.3$, but the absolute maximum is achieved by choosing $ka > \sqrt{3}$, which implies a forward-wave operation. It is worthwhile noting, that the group velocity is insensitive to the value of d/h. Furthermore, one finds that the smallest peak fields occur in small hole deflectors, i.e. $ka \to 0$, with $d/h \to 2$. Figure 3 shows a maximum of the shunt impedance at $d/h \approx 1.8$, but this value depends somewhat on the iris thickness. One finds that in real structures, which have a finite iris thickness, the maximum shunt impedance is obtained by adopting the smallest possible width of the coupling cell. One approaches, in the limit of vanishing cell width, a geometry in which the coupling cells are removed from the beam line, resulting in a side-coupled structure.[7] However, problems connected with the fabrication of superconducting surfaces exclude this solution. The width of the coupling cell in AP iris-loaded cavities has in practice a lower limit due to the confluence condition and excessive dimensional tolerance requirements on the coupling cell. Nevertheless, it is justified to base the optimization of the beam hole size on a deflector geometry with a very small coupling cell width; in other words, we will take $d/h = 1.8$ in the subsequent optimization procedure.

III. Optimization

In this section, an analytical optimization of the deflector for superconducting rf beam separators will be attempted. Operating frequency (1.3 GHz) and deflector length (3 m) were determined in a previous design study.[3] The results obtained in Section II of this paper support the earlier conclusion that alternating-periodic iris-loaded cavities operating in the confluent $\pi/2$ mode appear to be ideally suited for superconducting rf beam separators, and, in addition, they yielded the desirable ratio d/h. The only freedom left for the optimization procedure is the normalized beam hole radius ka.

Insensitivity of the resonant frequency of the cavity to machining errors, mechanical vibrations and similar perturbations is achieved by large absolute values of the group velocity. It was pointed out that this condition would lead to forward-wave operation. In the design of traveling wave structures, forward waves were shunned by most laboratories in view of their degenerate dispersion diagram,[8] but it was proven at CERN that successful operation is nevertheless possible.[9] Degenerate dispersion diagrams are less objectionable in standing wave operation, since, in general, the degeneracy can be removed by imposing the propagation constant through a correct choice of the deflector length. For the purpose of this paper, we will assume that no restrictions on the choice of the sign of v_g exist, but we should keep in mind that forward-wave structures will entail some complications in the design due to the increased number of resonances in the vicinity of the operating frequency.

The number of wanted particles n_W transmitted through the separator stage is proportional to the equivalent deflecting field and the beam hole radius, that is[3]

$$n_W \propto B_o\, ka \quad . \qquad (27)$$

The equivalent deflecting field is limited by the critical magnetic field and by field emission, if superconducting lead is used.[10] But we showed, in the previous section, that $\hat{E}/B_o$ and $\hat{B}/B_o$ exhibit in first approximation the same dependence on ka, whence finally

$$n_W \propto J_1(ka) \quad . \qquad (28)$$

The transmission of wanted particles is obviously maximum if $ka = j'_{11} \approx 1.84$, which would imply forward-wave operation at a reasonable value of the group velocity, $v_g \approx 0.025$.

The costs for equipment and operation of a superconducting rf beam separator depend strongly on the refrigerator required, and, therefore, on the rf power losses in the cavity, P_o. The cost figure of deflectors must be compared at equal performance levels. The rf power losses at equal $B_o$ka and equal length depend on transverse shunt impedance and beam hole radius according to

$$P_o \propto \frac{1}{R_{SW}(ka)^2} \quad ; \qquad (29)$$

they are, in view of (17), a function of ka, which is depicted in Fig. 6. The most economical structure is obtained at a beam hole radius $ka \approx 1.9$, resulting in a group velocity $v_g \approx 0.042$. The ratio $J_1(1.9)/J_1(j'_{11}) \approx 0.999$ indicates that the choice of $ka \approx 1.9$ leads to the maximum particle fluxes and, at the same time, to the most economical solution.

IV. Conclusion

Analytical expressions for transverse shunt impedance, peak fields, group velocity, etc. of alternating-periodic iris-loaded deflectors were derived from the TM_{11}-like small pitch approximation. It was concluded that alternating-periodic cavities operating in the confluent $\pi/2$ mode are well suited for use as superconducting deflectors in rf-separated counter beams.

It was shown that the smallest peak fields and the largest shunt impedance are obtained in structures with vanishingly small width of the coupling cells, but it was pointed out that, in practice, the requirement for confluence and considerations of dimensional tolerances impose a lower limit on the width of the coupling cells.

The optimization of transmitted particles and of the costs depending on the rf power losses determined the value of the normalized beam hole radius $ka \approx 1.9$. This value implies forward-wave operation with possible complications due to the degenerate dispersion diagram. On the other hand, backward-wave operation with $ka \approx 1.3$, which was suggested in Ref. 3, would result in a price increase by about 30% at otherwise equal performance levels.

Although the approximations made will limit the accuracy of the results obtained, we believe the conclusions to be qualitatively correct, and expect that future work, which is clearly necessary, will follow the direction pointed out in this paper.

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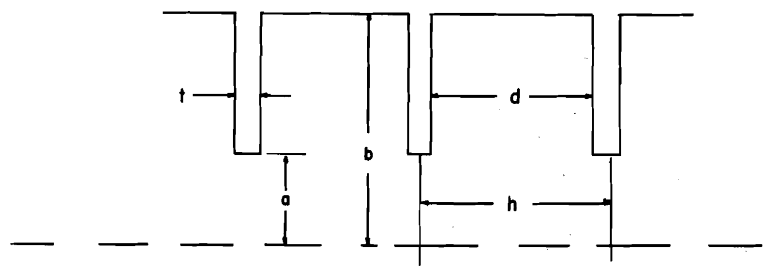

Fig. 1. Geometry of uniform-periodic waveguide.

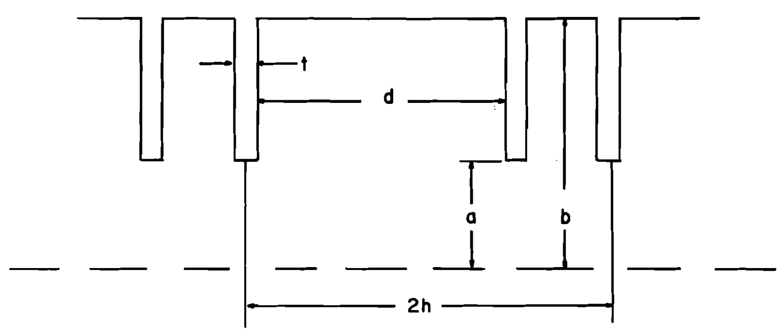

Fig. 2. Geometry of alternating-periodic waveguide.

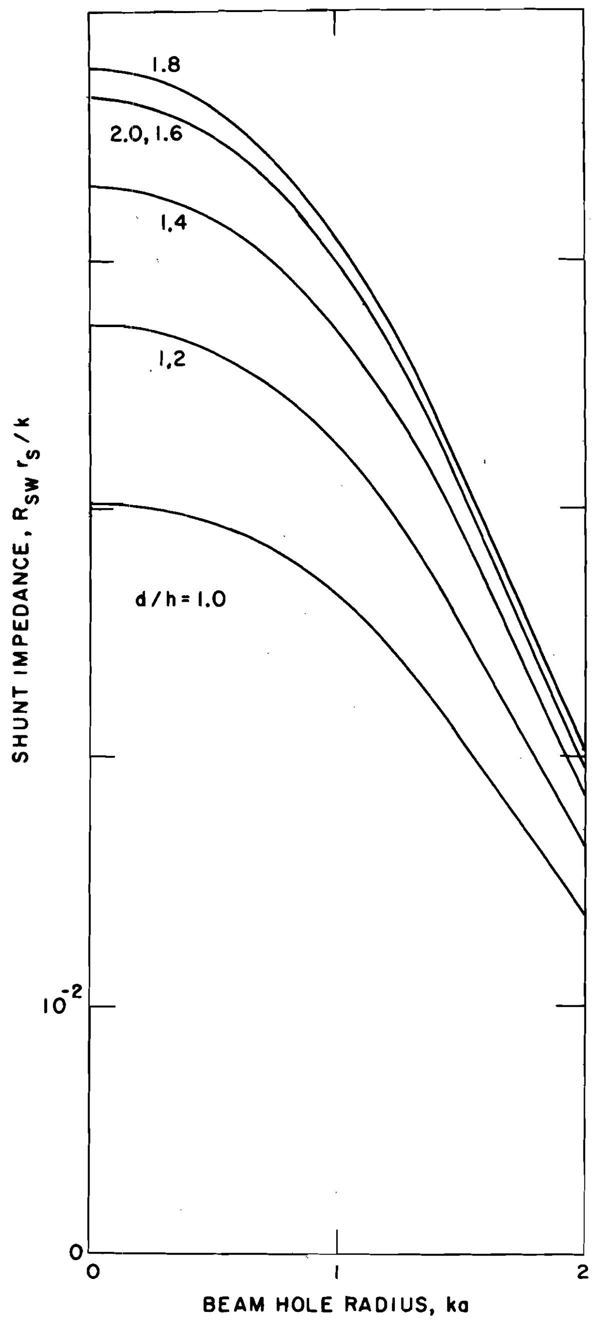

Fig. 3. Transverse shunt impedance vs beam hole radius of deflector cavities. d/h = 1.0 represents UP cavity, d/h > 1.0 AP cavity.

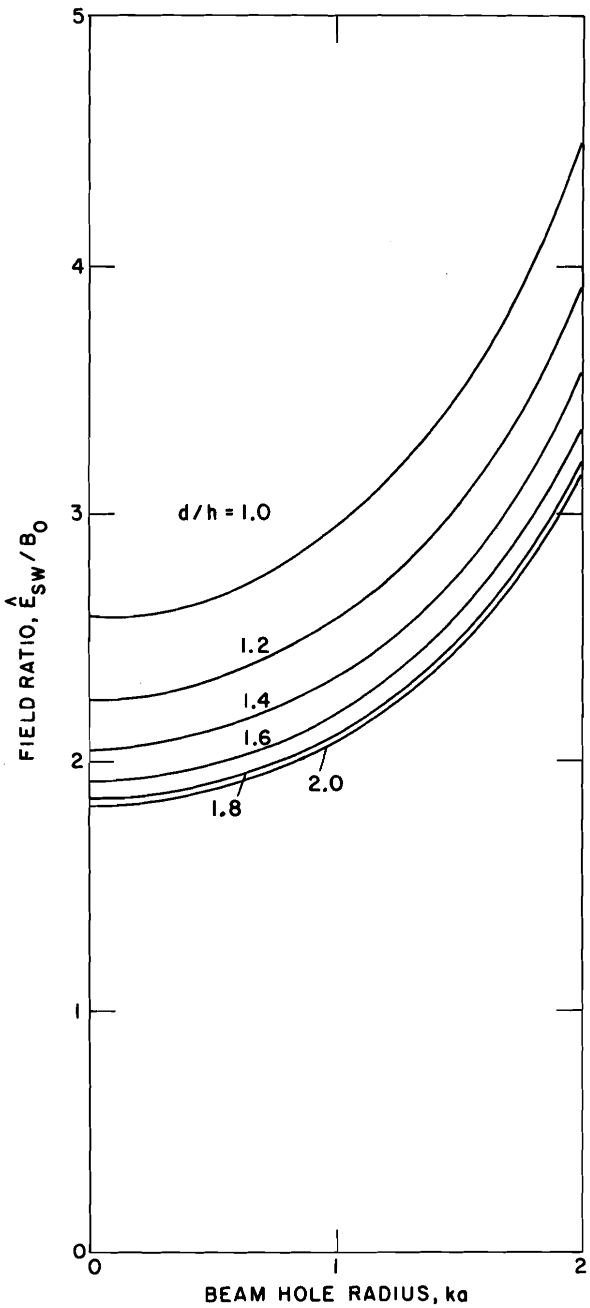

Fig. 4. Electric peak field ratio vs beam hole radius. The magnetic peak field ratio exhibits in first approximation the same functional dependence on ka. The curves are most accurate for ka → 0 and are invalid for ka > j'_{11}.

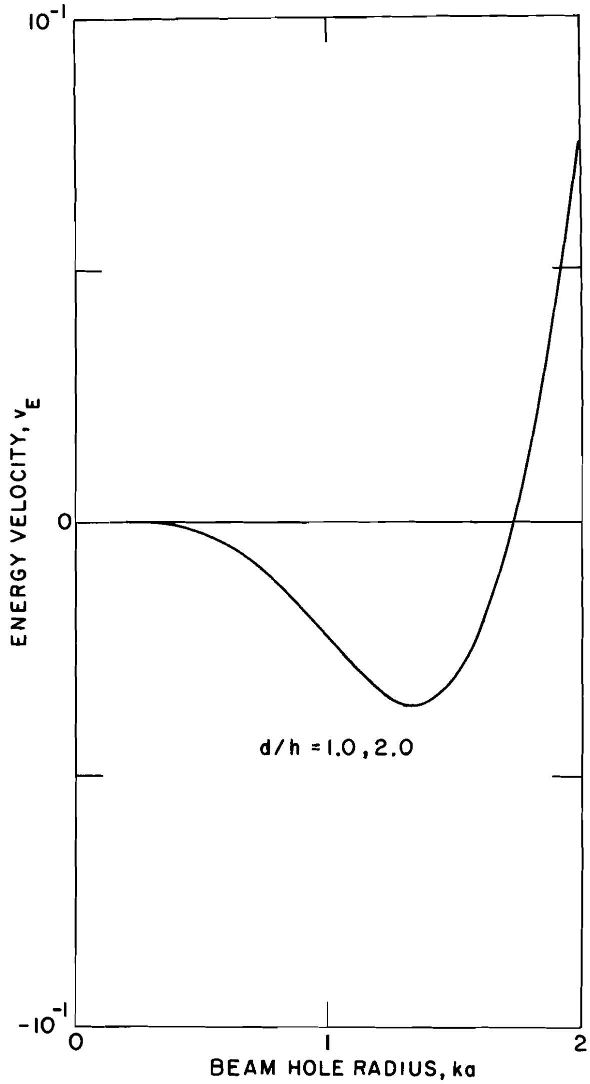

Fig. 5. Group velocity vs beam hole radius of deflector cavities. ka > $\sqrt{3}$ implies forward wave operation.

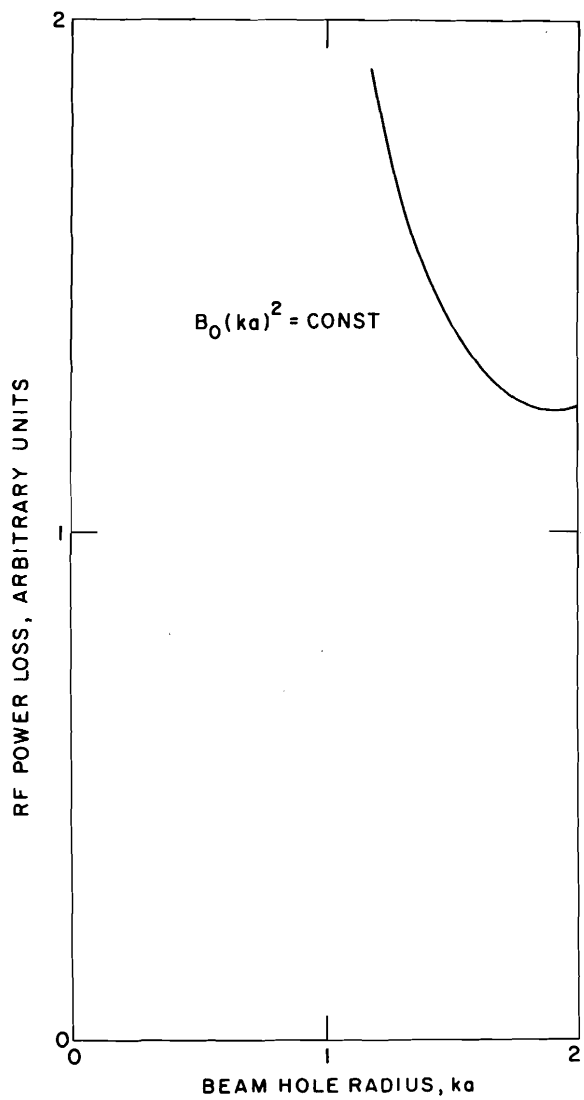

Fig. 6. Rf power loss per unit length in AP cavity with d/h = 1.8 at equal performance level. The optimum structure has ka $\approx$ 1.9.