CALCULATION OF THE DEPOLARIZATION OF AN INITIALLY POLARIZED BEAM IN THE LOS ALAMOS LINAC *

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Introduction

A polarized proton beam is planned for the Los Alamos Meson Facility. We have performed a numerical calculation of the spin motion for a sufficient number of orbits to obtain a reliable evaluation of the amount of depolarization to be expected in the linac.

A major advantage of linacs in the field of polarized beams is the ability to propagate the beam with any orientation of the polarization vector. Thus it is possible to provide a longitudinally polarized beam at a target area off the main beam line by tilting the polarization vector at injection to compensate for the change in orientation occurring at the bending magnet. For this reason we have used skewed orientations as well as transverse and longitudinal orientations in the calculations.

Equation of Motion for the Spin

The expectation value of the proton spin obeys a classical equation of motion. The covariant equation derived by Bargmann, Michel, and Telegdi[1] is

$$\frac{ds}{d\tau} = \frac{e}{m_o c^2}\left[\frac{g}{2} F^{ij} s_j - \left(\frac{g-2}{2}\right)(F^{jk} u_j s_k) u^i\right]$$

in which τ is the proper time, g is the g-factor (equal to 5.59 for the proton), s^i is the spin four vector, u^i is the four velocity, and F^{ij} is the electromagnetic field tensor. Explicitly, s^i and u^i are

$$s^i = (\vec{s}\ ,\ \vec{s}\cdot\vec{\beta})$$
$$u^i = (\ \gamma\vec{\beta},\ \gamma\ )\ .$$

In the orbit programs used, the independent variable is z, the distance along the machine axis. We transform the spin equation by the relations

$$d/d\tau = \gamma d/dt = \gamma\beta_z\ d/dz,$$

where t is the laboratory time.

Orbit Codes and Initial Conditions

For the drift tube section of the linac (.75 - 100 Mev) the Yale code EXACT MOTION was modified to include the components of the spin four vector. The cavity r.f. fields are described by the first four harmonics of a fourier analysis of the fields obtained by the Los Alamos relaxation code MESSYMESH. The integration of the six orbit variables (transverse positions and momenta, and energy and phase) and the four spin components is done by a Runge-Kutta method.

For the waveguide section (100-800 Mev) the Los Alamos code LINAC was used. This code uses a single harmonic description of the r.f. fields.[2] The integration method is also Runge-Kutta.

Bunches of twenty particles were generated by random selection. For the drift tube section the phase space parameters were

$$x_{max} = .206 \text{ cm}$$
$$x'_{max} = 19.4 \text{ mrad}$$
$$y_{max} = .318 \text{ cm}$$
$$y'_{max} = 12.6 \text{ mrad}$$
$$\Delta E = \pm\ 41 \text{ kev}$$
$$\Delta\varphi = \pm\ 26 \text{ degrees}$$

The above limits give a transverse phase space area of 4π mrad-cm with non-rotated phase space ellipses. The 4π mrad-cm bunch was expanded to a 10π mrad-cm bunch by multiplication.

*Research supported by the U.S. Atomic Energy Commission under contract AT (30 - 1) 2726.

For the waveguide section a separate bunch was generated with the following limits:

$$x_{max} = 1.206 \text{ cm}$$

$$x'_{max} = 3.841 \text{ mrad}$$

$$y_{max} = 1.587 \text{ cm}$$

$$y'_{max} = 4.423 \text{ mrad}$$

$$\Delta E = \pm 1 \text{ Mev}$$

$$\Delta\varphi = \pm 45 \text{ degrees.}$$

The ellipses were oriented to describe a converging beam with phase space areas of 4.4π mrad-cm. This is the acceptance limit of the waveguide section.

The initial conditions for the rest frame spin are shown in Table I. We assume initially 100 % polarization in all cases. The rest spin is first Lorentz transformed to the laboratory frame for integration. At convenient intervals, the spin vectors for the individual orbits are transformed back to the rest frame and combined to give the net polarization magnitude and orientation.

Results

Table I. shows the cumulative loss of beam polarization at the exits of the two sections. The depolarization is seen to be greatest for a longitudinally polarized beam. This is because of the dominance of the quadrupole fields which are always nearly normal to the spin vector in the case of longitudinal polarization. We note that less than 0.2 % depolarization occurs for a 4π mrad-cm beam.

Figures 1. and 2. show detailed results for the waveguide section. The oscillations correspond to the betatron oscillation frequency, demonstrating the cancellation of spin precession by succeeding quadrupoles. The depolarization is greater for $s_y = 1$ than for $s_x = 1$ because the phase space ellipse in y,y' was narrower than the x,x' ellipse. Thus in the first oscillation, stronger magnetic fields in the x-direction are felt and the s_y vector is most strongly precessed.

These results confirm that the depolarization due to focusing and accelerating fields in the linac is neglible.

Acknowledgements

I thank Dr. Shoroku Ohnuma for indispensable help at all stages of this calculation. I am also grateful to Dr. Donald Swenson and Mr. Kenneth Crandall of Los Alamos for providing orbit codes and machine data.

References

1. V. Bargmann, L. Michel, and V.L. Telegdi, Phys. Rev. Letters 2, 435 (1959). I have followed the notation of the text by Panofsky and Phillips, Classical Electricity and Magnetism, Second Edition, Addison-Wesley, 1962. pp. 437-444.
2. L. Smith, "Linear Accelerators," Handbuch der Physik, Vol. XLIV, Springer-Verlag, Berlin (1959), p.345.

TABLE I. DEPOLARIZATION IN THE LOS ALAMOS LINAC

A. Drift Tube Section

Case	Initial Spin s_x	s_y	s_z	Transverse Phase Space Area (mr-cm)	Depolarization Parts in 10^6
1	1	0	0	4π	1245*
2	0	0	1	4π	1870
3	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$	4π	1554
4	1	0	0	10π	3251
5	0	0	1	10π	4680
6	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$	10π	3933
				B. Waveguide Section	
7	1	0	0	4.4π	134
8	0	1	0	4.4π	308
9	0	0	1	4.4π	425
10	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$	4.4π	325

* The accuracy of these results is better than 10 ppm.

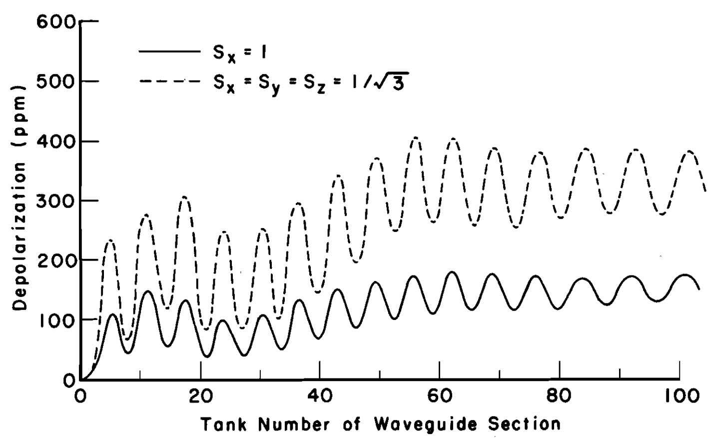

Fig. 1. Depolarization in the waveguide section of the linac. The initial conditions for the rest frame spin are shown. The curves are hand drawn to the points calculated at the exit of each waveguide tank.

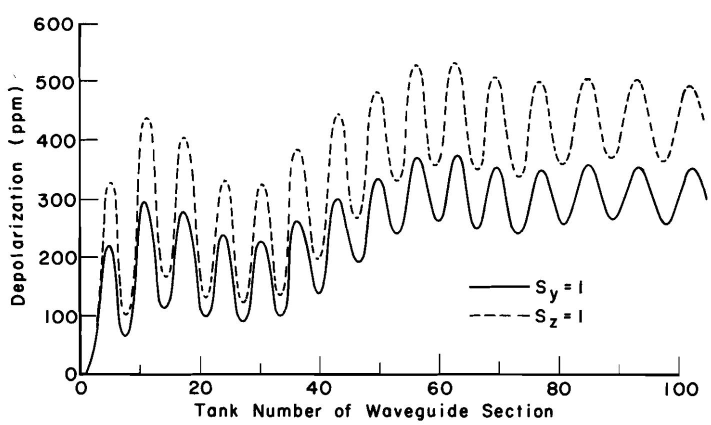

Fig. 2. Same as Fig. 1. with different initial conditions on the spin vector.