A UNIFIED THEORY ON BEAM DYNAMICS IN PROTON LINACS

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I. Introduction

A number of theoretical studies on beam dynamics in proton linacs have been developed for analyzing longitudinal-transverse coupling effects,[1,2] space-charge effects,[3-9] etc. It is, however, worthwhile to treat the various effects in terms of a unified theory which gives physical insight into the coupling mechanisms and the relations between those effects independently discussed.

For this purpose, as has been discussed in detail by Nielsen, Sessler and others for circular machines,[10,11] we shall start from the reduced Boltzmann equation, or the Vlasov equation, on the transport problems in a collisionless assembly of charged particles. Introducing the density distribution in the phase space as ψ, we write the Vlasov equation

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + (\vec{v}\cdot\vec{\nabla})\psi + \left(\frac{\vec{K}}{M}\cdot\frac{\partial\psi}{\partial\vec{v}}\right) = 0 , \qquad (1)$$

where the force $\vec{K}$ is assumed to be independent of the particle velocity and the motion of particles is treated nonrelativistically.

The coordinates moving with the synchronous particle on the accelerator axis (z-axis) are chosen as the reference system. The relativistic correction can be made by using the longitudinal and transverse masses for the motion of z-direction and x (or y)-direction, respectively. First, we shall discuss the stationary state, for which $\partial\psi/\partial t = 0$, and then proceed to the time-varying case by perturbation analysis.

II. Stationary State Boundary Equations

For a stationary state, we may assume the charge distribution in a rather simple form. Nielsen and Sessler[10] assumed two-dimensional constant distribution within the region bounded by a phase trajectory and zero outside. Kapchinsky-Kronrod[3] and Morton,[4] independently, applied Nielsen-Sessler's method to the analysis on the longitudinal space-charge effect in proton linacs. To discuss the longitudinal and transverse motions in a unified theory, we may assume constant distribution in the six-dimensional phase space as

$$\psi = \sigma\, U(|v_x| - |v_x^B|) \times U(|v_y| - |v_y^B|) \times U(|v_z| - |v_z^B|) , \qquad (2)$$

where U is a step function which is unity for negative arguments and zero otherwise. σ is a constant and v_x^B, v_y^B and v_z^B are boundary velocity parameters, each of which is, in general, the function of x, y, and z. At a spatial point (x,y,z), this gives constant distribution inside a rectangular box in the velocity space. The charge density in the usual space can be given by

$$\rho(x,y,z) = \int \psi(x, y, z, v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z = 8\sigma\, |v_x^B|\, |v_y^B|\, |v_z^B| , \qquad (3)$$

where $v^B_{x,y,z}$'s are assumed to be symmetrical about $|v| = 0$. Equation (3) means that the spatial distribution will have a peak at the origin of the coordinates and decrease towards each direction. Inserting Eq. (2) into Eq. (1), we get

$$\left[\frac{K_x}{M} - \left(v_x\frac{\partial v_x^B}{\partial x} + v_y\frac{\partial v_x^B}{\partial y} + v_z\frac{\partial v_x^B}{\partial z}\right)\right]\delta(v_x - v_x^B)\, U(|v_y| - |v_y^B|)\, U(|v_z| - |v_z^B|) + \left[\frac{K_y}{M} - \left(v_x\frac{\partial v_y^B}{\partial x} + v_y\frac{\partial v_y^B}{\partial y} + v_z\frac{\partial v_y^B}{\partial z}\right)\right]\delta(v_y - v_y^B)\, U(|v_x| - |v_x^B|)\, U(|v_z| - |v_z^B|)$$

$$+ \left[\frac{K_z}{M} - \left(v_x\frac{\partial v_z^B}{\partial x} + v_y\frac{\partial v_z^B}{\partial y} + v_z\frac{\partial v_z^B}{\partial z}\right)\right]\delta(v_z - v_z^B)\, U(|v_x| - |v_x^B|)\, U(|v_y| - |v_y^B|) = 0 .$$

This equation is identically satisfied inside the boundary. In order to satisfy the Vlasov equation at any point on the boundary, it is required that each bracket term should be zero on the boundary, or

$$\begin{aligned}
&\frac{K_x}{M} - \left(v_x\frac{\partial v_x^B}{\partial x} + v_y\frac{\partial v_x^B}{\partial y} + v_z\frac{\partial v_x^B}{\partial z}\right)\Bigg|_{v_x = v_x^B} = 0 \\
&\frac{K_y}{M} - \left(v_x\frac{\partial v_y^B}{\partial x} + v_y\frac{\partial v_y^B}{\partial y} + v_z\frac{\partial v_y^B}{\partial z}\right)\Bigg|_{v_y = v_y^B} = 0 \qquad (4)\\
&\frac{K_z}{M} - \left(v_x\frac{\partial v_z^B}{\partial x} + v_y\frac{\partial v_z^B}{\partial y} + v_z\frac{\partial v_z^B}{\partial z}\right)\Bigg|_{v_z = v_z^B} = 0 .
\end{aligned}$$

Using the relations as

$$v_x\frac{\partial v_x^B}{\partial x} + v_y\frac{\partial v_x^B}{\partial y} + v_z\frac{\partial v_x^B}{\partial z} = \frac{dv_x^B}{dt} = v_x\frac{dv_x^B}{dx} , \text{ etc.,}$$

we can rewrite the above equations as

$$\frac{K_x}{M} - v_x^B \frac{dv_x^B}{dx} = 0 \qquad (5\text{-}1)$$

$$\frac{K_y}{M} - v_y^B \frac{dv_y^B}{dy} = 0 \qquad (5\text{-}2)$$

$$\frac{K_z}{M} - v_z^B \frac{dv_z^B}{dz} = 0 \quad , \qquad (5\text{-}3)$$

or in the integrated form as

$$\frac{1}{2} M(v_x^B)^2 - \int K_x dx = \text{const} \qquad (6\text{-}1)$$

$$\frac{1}{2} M(v_y^B)^2 - \int K_y dy = \text{const} \qquad (6\text{-}2)$$

$$\frac{1}{2} M(v_z^B)^2 - \int K_z dz = \text{const} \ . \qquad (6\text{-}3)$$

Equations (5) and (6) are the boundary equations in the stationary state. A set of (v_x^B, v_y^B, v_z^B) need not be the components of a single particle velocity, so that these are not the trivial equations of motion. It should be noted, however, that the boundary velocity in each direction satisfies the equation of motion in that direction.

For example, we shall consider the transverse motion. Usually, the gradients of Q-magnets are chosen to be approximately $B' \propto 1/v_s$, so that the force in the smoothed approximation does not explicitly depend on t. The rf defocusing force at the synchronous phase angle, with neglection of the slow damping of the amplitude, can also be treated as a stationary force. Thus, including the space-charge effect, we get the stationary solution by writing K_x as

$$K_x = - k_x^2 \, x - e \frac{\partial V}{\partial x} \ , \qquad (7)$$

where k_x^2 is the smoothed force constant of betatron oscillations and V the space-charge potential. Inserting Eq. (7) into (6-1), we get

$$M|v_x^B|^2 + k_x^2 \, x^2 + 2eV(x,y,z) = \text{const.} \qquad (8)$$

For the case without space charge this gives the usual elliptic boundary. The space-charge potential can be written as

$$V(x,y,z) = \frac{e}{\epsilon_0} \int G(x,y,z \mid x',y',z') \times \rho(x',y',z') \, dx'dy'dz' \ , \qquad (9)$$

by using the density function $\rho(x,y,z)$ in Eq. (3) and the Green function, or the kernel, $G(x,y,z \mid x',y',z')$. In a recent work, Hirawaka[12] has performed a numerical computation of the space-charge potential in drift tubes for several typical ellipsoidal bunches. Analytically, the Green function in a cylindrical metallic tube of radius a is given by

$$G(r,\theta,z|r',\theta',z') = \frac{1}{2\pi a} \sum_{m=0}^{\infty} \sum_{\ell=1}^{\infty} \frac{J_0\left(\lambda_{m\ell} \frac{r}{a}\right) J_0\left(\lambda_{m\ell} \frac{r'}{a}\right)}{\lambda_{m\ell} \, J_{m+1}^2(\lambda_{m\ell})}$$

$$\times \, S_m \cos m(\theta - \theta') \exp\left[- \frac{\lambda_{m\ell} \, |z - z'|}{a}\right] , \qquad (9')$$

in the cylindrical coordinates. In this expression, $\lambda_{m\ell}$ is the ℓ-th root of the m-th Bessel function $J_m(u)$, and $S_m = 1$ for m = 0 and $S_m = 2$ for $m \neq 0$. The self-consistent solutions will be obtained by combining Eqs. (3), (9), and (9′) with the boundary equations. It should be noted here* that, if we disregard the space-charge potential, the boundary equation (8) yields a square cross-sectional beam. In general, the beam shape is determined from the boundary equations (6) including space charge, so that to obtain the self-consistent solutions we need to find the consistent limits of the integral in Eq. (9). From an approximate calculation, however, $\langle x^B \rangle_{max}$ of 0.6 ~ 0.7 cm is given for a 100 mA beam with $v_s = 0.04c$ in the typical (SNSN) focusing system. Such a beam will also give a limit for the (SSNN) system.

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III. Perturbation Treatment on Nonstationary State

In the perturbation treatment, the density function in nonstationary state will be written as

$$\psi(\vec{r},\vec{v},t) = \psi_0(\vec{r},\vec{v}) + \psi_1(\vec{r},\vec{v},t) \ , \qquad (10)$$

where the suffix 0 denotes the stationary-state value. It is assumed that $\psi_1 << \psi_0$, and the force $\vec{K}$ is also written as

$$\vec{K}(\vec{r},t) = \vec{K}_0(r) + \vec{K}_1(\vec{r},t) \ , \qquad (11)$$

where $|\vec{K}_1| << |\vec{K}_0|$.

Using the equation for the stationary state, and neglecting the higher-order terms, we get

$$\frac{\partial \psi_1}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \, \psi_1 + \left(\frac{\vec{K}_0}{M} \cdot \frac{\partial \psi_1}{\partial \vec{v}} \right) + \left(\frac{\vec{K}_1}{M} \cdot \frac{\partial \psi_0}{\partial \vec{v}} \right) = 0 \ . \qquad (12)$$

For convenience, we expand ψ_1 in terms of the Fourier series contained in a finite box of the phase space, and consider a component

$$\psi_1 \propto \exp j(\vec{\kappa} \cdot \vec{r} + \vec{\mu} \cdot \vec{v}) \ . \qquad (13)$$

The perturbing force K_1 may be divided into two parts; the time-varying external force, $K_1{}^e$, and the perturbed space-charge force, $K_1{}^s$. $K_1{}^s$ is given by

$$\vec{K}_1{}^s = -e\,\vec{\nabla}\,V_1 \tag{14}$$

and

$$\nabla^2 V_1 = -\frac{1}{\epsilon_0}\rho_1 = -\frac{1}{\epsilon_0}\int \psi_1\, d\tau , \tag{15}$$

where $d\tau = dv_x dv_y dv_z$. Equations (13) and (15) yield $V_1 \propto \exp j\vec{\varkappa}\cdot\vec{r}$.

As a result, Eq. (12) becomes

$$\frac{\partial\psi_1}{\partial t} + j\left(\vec{\varkappa}\cdot\vec{v}+\frac{1}{M}\vec{\mu}\cdot\vec{K}_0\right)\psi_1 - j\frac{e}{M}V_1\left(\vec{\varkappa}\cdot\frac{\partial\psi_0}{\partial\vec{v}}\right) + \frac{1}{M}\left(\vec{K}_1{}^e\cdot\frac{\partial\psi_0}{\partial\vec{v}}\right) = 0 . \tag{16}$$

We shall take the Laplace transform of Eq. (16),

$$(p+q)\,\psi_p - \psi_1{}^0 - j\frac{e}{M}\left(\vec{\varkappa}\cdot\frac{\partial\psi_0}{\partial\vec{v}}\right)V_p + \frac{1}{M}\left(\vec{K}_p{}^e\cdot\frac{\partial\psi_0}{\partial\vec{v}}\right) = 0 , \tag{17}$$

where

$$q \equiv j\left[(\vec{\varkappa}\cdot\vec{v}) + \frac{1}{M}(\vec{\mu}\cdot\vec{K}_0)\right] \tag{17'}$$

and $\psi_1{}^0$ is the initial value of ψ_1. ψ_p, V_p, and $K_p{}^e$ are the Laplace transforms of ψ_1, V_1 and $K_1{}^e$, respectively. The Laplace transform of Eq. (15) is

$$|\vec{\varkappa}|^2\, V_p = \frac{1}{\epsilon_0}\int \psi_p\, d\tau . \tag{18}$$

Eliminating ψ_p from Eqs. (17) and (18), we get

$$V_p = \frac{1}{\epsilon_0|\vec{\varkappa}|^2}\,\frac{\int \frac{\psi_1{}^0}{p+q}\,d\tau - \frac{1}{M}\left(\vec{K}_p{}^e\cdot\int\frac{1}{p+q}\left(\frac{\partial\psi_0}{\partial\vec{v}}\right)d\tau\right)}{1 - j\frac{e^2}{\epsilon_0 M|\vec{\varkappa}|^2}\left(\vec{\varkappa}\cdot\int\frac{1}{p+q}\left(\frac{\partial\psi_0}{\partial\vec{v}}\right)d\tau\right)} \tag{19}$$

In this expression, the terms due to $\psi_1{}^0$ and each component of $K_p{}^e$ can be calculated independently and be summed up afterwards.

The integral both in the denominator and numerator in Eq. (19) is obtained as

$$I_x = \int\frac{1}{p+q}\left(\frac{\partial\psi_0}{\partial v_x}\right)d\tau = \frac{\sigma}{\varkappa_y\varkappa_z}\sum_{\ell,m,n=1,2}(-1)^{\ell+m+n} \times A_{\ell mn}\log A_{\ell mn} \tag{20}$$

with

$$A_{\ell mn} \equiv p + \frac{1}{M}j(\vec{\mu}\cdot\vec{K}_0) + j\left[(-1)^\ell\varkappa_x|v_x^B| + (-1)^m\varkappa_y|v_y^B| + (-1)^n\varkappa_z|v_z^B|\right], \tag{20'}$$

where the stationary distribution in Eq. (2) is used. Similar results can be obtained for I_y and I_z.

If we neglect smaller oscillations within the phase boundaries, we may concentrate our interests into the long-wave components for which $|\vec{\varkappa}\cdot\vec{r}^B| \leqslant 1$ and $|\vec{\mu}\cdot\vec{v}^B| \leqslant 1$. Then, the integral is approximated as

$$I_x \approx j\,\varkappa_x\,\frac{\rho_0(x,y,z)}{p^2} \tag{21}$$

at the long-wave limit ($|p| >> |q|$). Using the expression of the proton plasma frequency, $\omega_p{}^2 = e^2\rho/\epsilon_0 M$ and neglecting $\psi_1{}^0$, we get

$$eV_p \approx \frac{j(\vec{\varkappa}\cdot\vec{K}_p{}^e)}{|\vec{\varkappa}|^2}\,\frac{\omega_p{}^2}{p^2+\omega_p{}^2} . \tag{22}$$

It is noted that V_p has a pole at $p = \pm j\omega_p$. A 100 mA beam bunched in the volume of 10 cm^3 has the plasma frequency of about 4 Mc.

Now we shall ask how the phase-space boundary will change with time. The boundary motion will be investigated by considering the motion of particles on the boundary. For the x-direction,

$$\frac{dv_x^B(x,y,z,t)}{dt} = \frac{\partial v_x^B}{\partial t} + \left[\frac{\partial v_x^B}{\partial x}v_x + \frac{\partial v_x^B}{\partial y}v_y + \frac{\partial v_x^B}{\partial z}v_z\right]_{v_x=v_x^B} = \frac{K_x}{M} . \tag{23}$$

At the stationary state, this reduces to Eq. (4). In the perturbation treatment, we again let

$$v_x^B(x,y,z,t) = v_{x0}^B(x,y,z) + v_{x1}^B(x,y,z,t) \tag{24}$$

and $|v_{x1}^B| << |v_{x0}^B|$. We also consider the particles for which $|v_y|$, $|v_z| << |v_{x0}^B|$. Neglecting higher-order terms, we get

$$\frac{\partial v_{x1}^B}{\partial t} + v_{x0}^B\frac{\partial v_{x1}^B}{\partial x} + v_{x1}^B\frac{\partial v_{x0}^B}{\partial x} - \frac{K_{1x}}{M} = 0 . \tag{25}$$

Again, we take a Fourier component of v_{x1}^B and the Laplace transform of Eq. (25), obtaining

$$v_{xp}^{B} = \frac{K_{px}^{e} - e\frac{\partial V_p}{\partial x}}{p \pm j(\omega_x \pm \varkappa_x' \, |v_{x0}^{B}|)} , \tag{26}$$

where ω_x is the angular frequency of the unperturbed transverse oscillations and the relation

$$\frac{\partial v_{x0}^{B}}{\partial x} = \pm j\,\omega_x$$

is used. The initial value of v_{x1}^{B} is assumed to be zero. If v_{xp}^{B} has a pole higher than the first order, then the perturbation builds up and beam will blow up during the acceleration. This will occur if the numerator of the right-hand side has a pole around ω_x at the long wave limit. From Eq. (22), it is shown that in a high-intensity linac such a condition is particularly satisfied as ω_p approaches to ω_x.

Similar analysis can be made on the longitudinal component, but coupling between the longitudinal and the plasma oscillations will be much stabilized due to the relatively fast change in ω_z during acceleration.

IV. Coupling Between Longitudinal and Transverse Motions with Space Charge

As an example, we shall consider the coupling between the longitudinal and transverse motions including space-charge effect. The term in K_{1x}^{e} due to the coupling by rf field is written as

$$K_{1x}^{e} \approx \frac{\omega_0 x}{2v_s} \; eET\,\Phi\cos\varphi_s \tag{27}$$

in a nonrelativistic and linear approximation. $\Phi = \varphi - \varphi_s$ is the phase of the longitudinal oscillations measured from the synchronous phase angle, φ_s. In the first approximation, we may assume the time dependence of the parameters as

$$v_s = at + v_0 \quad \text{(constant acceleration)}$$
$$x = x_0^{\pm}\, e^{\pm j\omega_x t}$$
$$\Phi = \left(\frac{v_s}{v_0}\right)^{\alpha} \Phi_0^{\pm}\, e^{\pm j\omega_z t} \quad , \alpha > -1$$

and constant for others. If we neglect the space-charge effect, then $\alpha \approx -3/4$ is obtained from the adiabatic approximation. With space charge α becomes larger, although the damping may still remain.[8] In general, frequencies of betatron oscillations (ω_x) and phase oscillations (ω_z) will undergo slower changes with time. The Laplace transform of K_{1x}^{e} is given by

$$K_{px}^{e} \approx -\frac{\omega_0 eET}{2a} \sum_{\pm} \frac{x_0^{\pm}\,\Phi_0^{\pm}}{\alpha} \cos\varphi_s \left\{1 - \left(\frac{a}{v_0}\right)^{\alpha} \times \frac{\Gamma(\alpha+1) - \gamma\left(\alpha+1, \frac{v_0}{a}\right)}{\left[p - j(\pm\omega_x \pm \omega_z)\right]^{\alpha}} \exp\left[p - j(\pm\omega_x \pm \omega_z)\right]\frac{v_0}{a}\right\} , \tag{28}$$

where $\Gamma(u)$ and $\gamma(u,v)$ are the gamma, and the Legendre imperfect gamma functions, respectively. The summation is taken for all possible combinations of $\pm\,\omega_x$ and $\pm\,\omega_z$; the suffix of x_0 corresponds to the sign before ω_x and the suffix of φ_0 to the sign before ω_z.

In these equations, we considered only the term proportional to Φ. The nonlinear phase oscillations, however, will add to K_{1x}^{e} the terms oscillating with the frequencies of $|\omega_x \pm n\,\omega_z|$, where n is an integer and the case of n = 2 is particularly important.

Inserting K_{px}^{e} thus obtained into Eqs. (22) and (26), we get coupling effects. Gluckstern[1] and Ohnuma[2] investigate in detail the coupling without space charge, which is given by the first term of the numerator on the right-hand side of Eq. (26). It is noted here that the so-called resonant effect, which would occur when the condition, $\omega_z = \omega_x$ or $2\omega_x$, are satisfied, will result in damped oscillations due to the parameter change during acceleration.

On the other hand, if we consider the second term which gives the coupling through space charge, then the perturbation is strongly enhanced and built-up at $\omega_x \approx \omega_p$. The resonant coupling will also cause build-up oscillations even when the phase oscillations are damped ($\alpha < 0$). These are explained in that the longitudinal oscillations excite a sort of plasma oscillations, or a collective motion of the particle assembly, and couple to the transverse motion.

V. Conclusion and Discussion

Using the Vlasov equation, we have discussed the longitudinal and the transverse motions in proton linacs in a six-dimensional phase space. Although we made some simplified assumptions, a unified theory is derived for the physical analysis on space-charge effects, longitudinal and transverse coupling effects, and other high intensity effects. In particular, it is pointed out that the coupling between the longitudinal and the transverse motions will be considerably enhanced through the collective motion of the particle assembly. The following points should be noted concerning the above-presented analysis.

1) In the perturbation analysis on space-charge effects, we only discussed the long-wave limit of plasma oscillations. Actually, however, the shape and the charge distribution of bunches will show complicated local fluctuations leading to a number of higher modes of plasma oscillations with the characteristic values of $\vec{\varkappa}$ and $\vec{\mu}$. Thus a number of possible poles will appear besides $p = j\omega_p$ in Eq. (22). In addition, it is shown that some of them may have a positive real part giving an exponential build-up; a part of them, however, will be cancelled out by the general effect of the Landau damping in plasma oscillations.

2) There are some possible ways to stabilize the build-up oscillations. In general, frequencies of the phase and the betatron oscillations (ω_z and ω_x, respectively) will change during acceleration. The plasma frequency, ω_p, will also change corresponding to the change of the bunch shape and the charge density. These effects will give the major stabilizing mechanism. Effects of acceleration will also result in some degree of reductions of the rf defocusing force and the space-charge defocusing force given by Eq. (7).

3) We have specifically discussed the effects of longitudinal oscillations and space charge on the transverse motion. It should be pointed out, however, that any other effects on the transverse and longitudinal motions can be treated in the perturbation method reported here. In particular, the coupling of the x and y motion due to an imperfectness of the quadrupole focusing system may cause such a build-up perturbation as discussed above on the longitudinal and transverse coupling through space charge. Finally, the dynamic beam instabilities due to the field imposed on the accelerator cavity by beam can also be included in the present theory as investigated by Sessler and others for circular machines.

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