ACCEPTANCE CALCULATIONS FOR ACCELERATOR BEAM LINES

F. P. Blackstein
Atomic Energy of Canada Limited
Chalk River, Ontario, Canada

Abstract

A computer program to determine the first order phase plane acceptance of accelerator beam lines is described. It is based on an iterative technique for scanning the phase space and computing time is reduced by an order of magnitude when compared to similar methods employing an ordered scan in rectangular coordinates.

An example system is analyzed; acceptance polygons are presented and the energy dependence of the beam line acceptance is computed.

Introduction

In the design and analysis of charged particle transport systems, the compatability of a given beam for transport through the system is determined by comparing the beam's emittance and the beam line acceptance. Provided the phase plane contours representing the beam are contained by the transport system's acceptance polygons there will be complete transmission of the input beam.

The program to be described computes the phase plane acceptance in two mutually orthogonal independent planes. The acceptance polygons are plotted and the energy dependence of the system's acceptance is determined. Beam line elements are generally represented by linear transformations, however the basic technique described here may be applied when the effects of higher order aberrations are required. The linear transformations for several beam transport devices have been developed in a number of texts[1,2] and will not be repeated here.

Theory

To determine the acceptance of a given beam line a grid work is generally chosen in the two orthogonal phase planes, XX' and YY'. Particles on the grid are then traced through the system using matrix transformations and the displacements are examined at each aperture restriction along the line. Particles whose displacements exceed these restrictions are not recorded while those passing through the entire line are plotted. This method is tedious, even on a high speed digital computer, since 10^4 particles must be traced to obtain a relative accuracy of 1%. The technique reported here relies on two modifications to the above procedure. First the phase plane areas to be scanned are converted to polar coordinates and then the selection of particles to be traced is modified by an iterative method, in this case a Bolzano Bisection[3].

Suppose for the moment that the XX' acceptance polygon is known for a given system and is as seen in Fig. 1. In polar coordinates each point on the polygon's boundary is denoted by $P_i\ (r_i, \theta_i)$ and the displacement-divergence relations are simply:

$$X_i = r_i \cos \theta_i$$
$$X'_i = r_i \sin \theta_i \qquad (1)$$

To determine r_i for a given value of θ_i two extreme values for r_i are selected, namely the origin $(r_{i,1})$ and some value which is certain to be outside the acceptance polygon $(r_{i,2})$. At $\theta_i = 0$, $r_{i,2}$ can be set at the beam tube radius to satisfy this condition.

These values are then subject to the Bolzano technique with beam tracking being performed after each iteration. Figure 2 shows the bisection as it appears in the phase plane for one point on the desired polygon. The process is continued until the desired accuracy is achieved, a new angle is selected and the method repeated to cover the entire 360° of the phase plane. The flow diagram of Fig. 3 illustrates the method and an accuracy of 1% is obtained in only 8 iterations at each of 100 angles; thus the computation time is reduced by an order of magnitude over the previous method.

A further reduction in computing time is possible by initiating the bisection at successive angles between smaller limits as determined from the final value ($r_{i,f}$) at the preceding point. For example if we define a reduction factor "RF" whose value is continuously updated, then at any general point ($i+1$) we have:

$$r_{i+1,1} = RF \times r_{i,f}$$
$$r_{i+1,2} = (2-RF) \times r_{i,f} \qquad (2)$$

"RF" is initially set at 0.9 thereby defining a narrow band in which to begin the bisection at successive points. To update "RF", a test is performed at each angle to make certain the band does in fact contain the polygon's boundary. If not, the "RF" value is reduced before the iteration is initiated. The bisection is then continued to determine $r_{i+1,f}$ at θ_{i+1} and the dotted section of Fig. 3 shows the point at which this computation is inserted.

Energy Dependence

To obtain a spectrum of beam line acceptance as a function of beam energy, the phase plane areas common to the central energy (E_o) and the various energies in question (E_j) must be computed. Since the data is stored in polar coordinates this task is a trivial one and requires very little additional programming. The flow diagram of Fig. 4 shows the method used and when the results are normalized with respect to the central energy the spectrum is obtained.

The discussion thus far has dealt with a single transverse plane and in examining a given beam line the procedure is repeated in the other plane.

Example System

The beam line of Fig. 5 was analyzed with the preceding program giving rise to the results of Figs. 6-9. Since computing time is small the effect of target size, magnet strengths and apertures can be rapidly determined and the system optimized for maximum acceptance.

Conclusions

The program described can presently accommodate quadrupole and bending magnets, drift spaces, slits, collimators and targets. As well as plotting acceptance polygons and determining the energy dependence of the acceptance, the program prints out the element responsible for defining each of the polygon's boundaries. In this way elements which critically limit the system's acceptance can be modified to improve the optical properties of the entire beam line.

References

1. "High Energy Beam Optics" by K. G. Steffen. Interscience, 1965.

2. "Focusing of Charged Particles" Edited by A. Septier. Academic Press, 1967.

3. "Numerical Methods for Science and Engineering" by R. G. Stanton, Prentice-Hall, 1961.

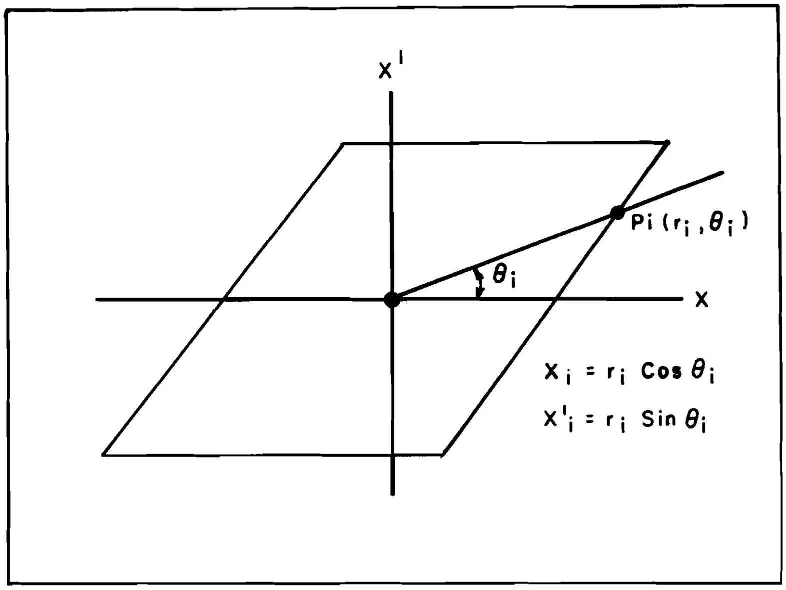


FIG. 1 Acceptance Polygon.

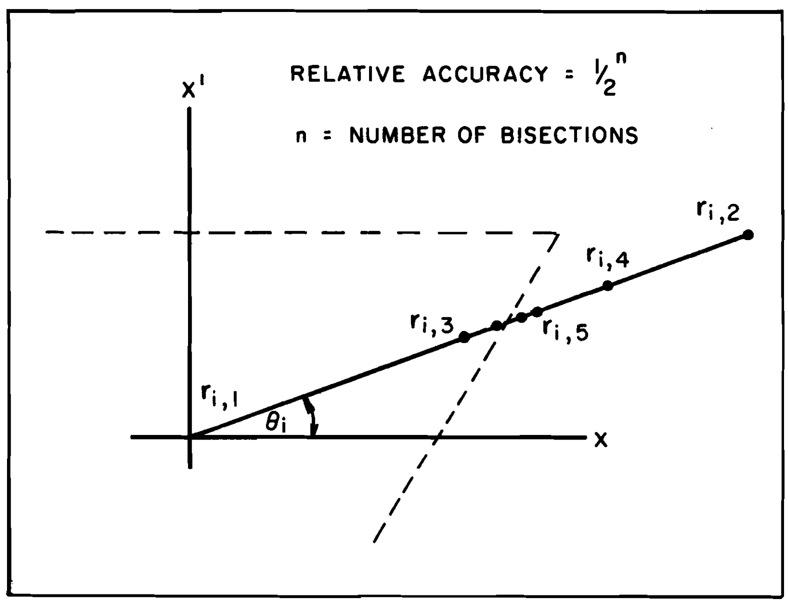


FIG. 2 Bisection Technique.

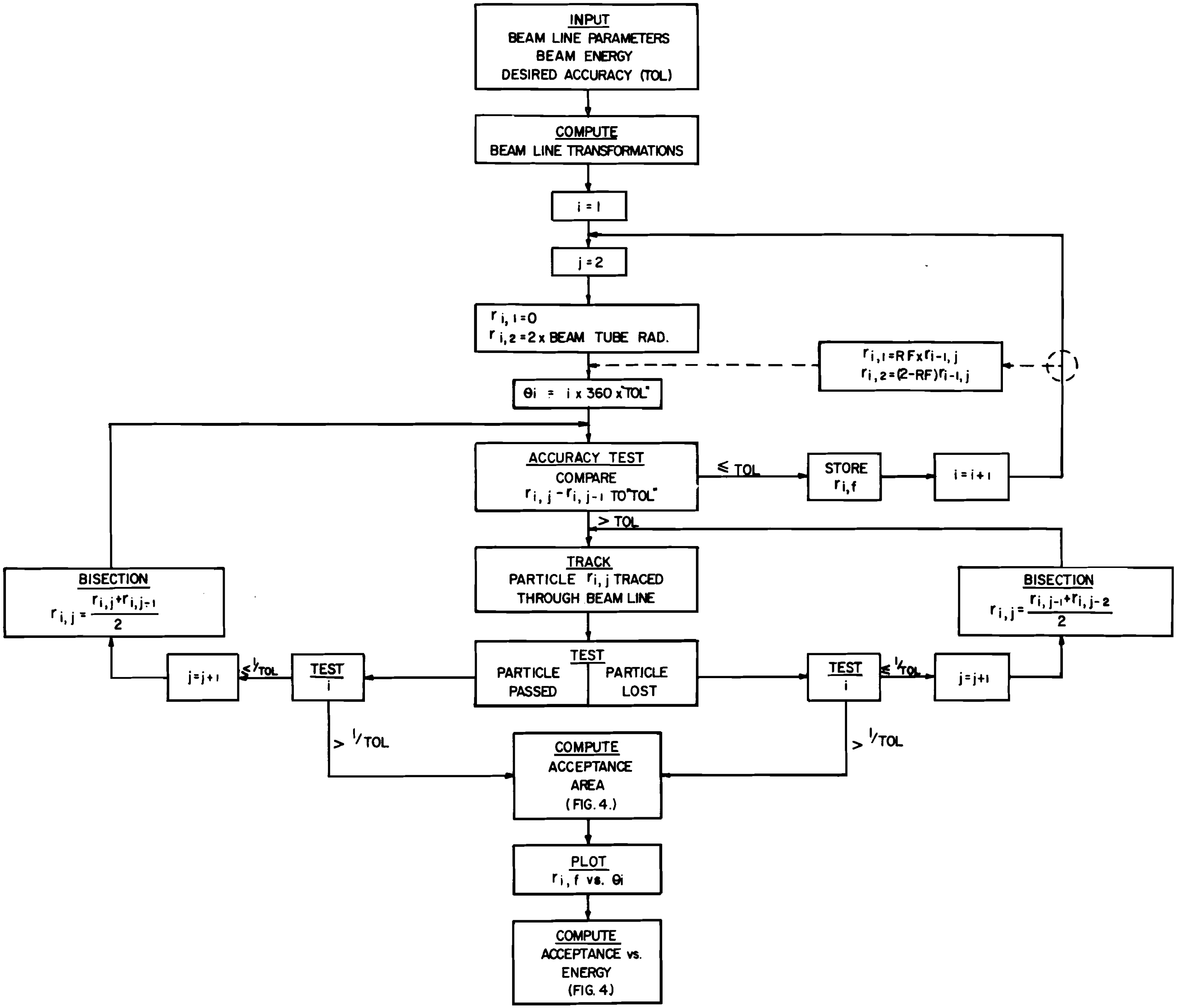


FIG. 3 Acceptance Calculations in One Transverse Phase Plane.

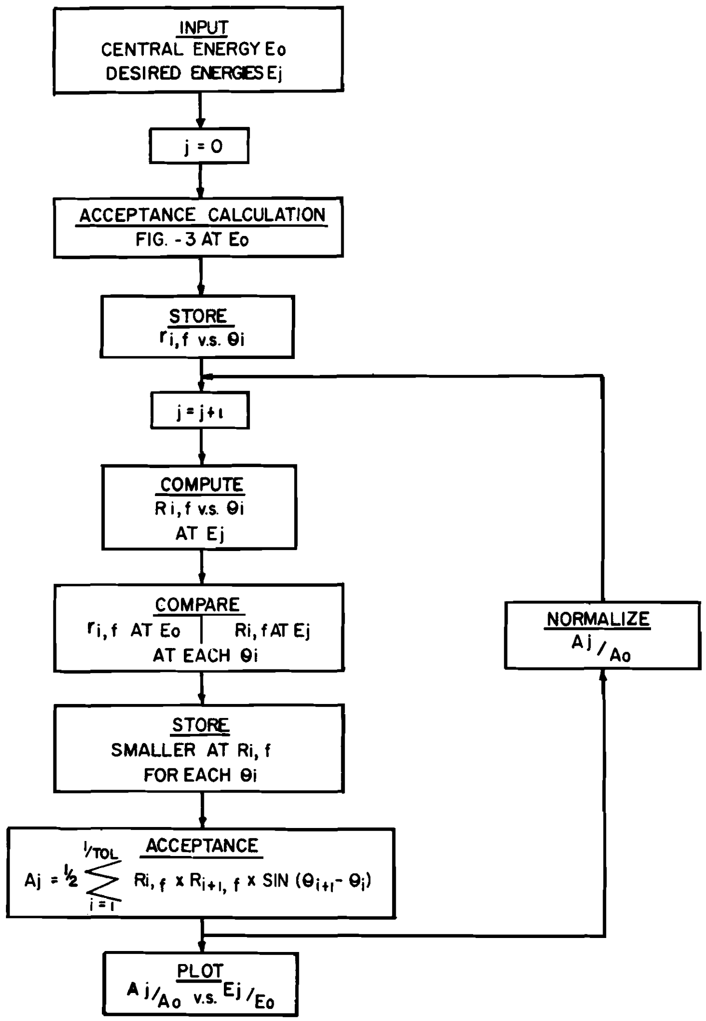
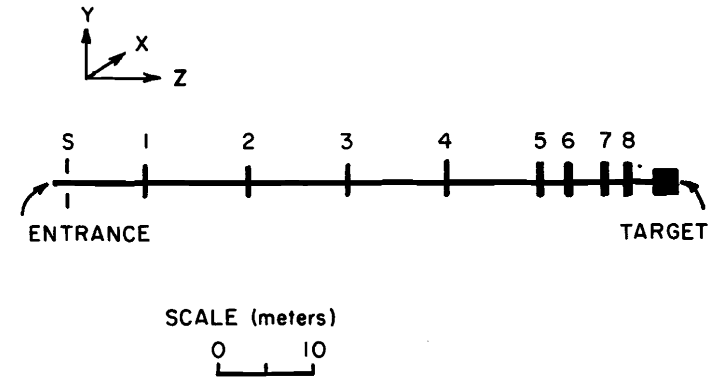


FIG. 4 Calculations of Acceptance v.s. Energy.



QUADRUPOLE	FOCUS	GRADIENT
1	X PLANE	0.08 kG/cm
2	Y "	0.16 "
3	X "	0.16 "
4	Y "	0.16 "
5	X "	0.08 "
6	Y "	0.19 "
7	X "	0.48 "
8	Y "	0.48 "

— QUADRUPOLE APERTURE = 12 cm
— EFFECTIVE LENGTH = 50 cm
"S", POINT AT WHICH APERTURE INSERTED (FIG. 8)

FIG. 5 Example Beam Line.

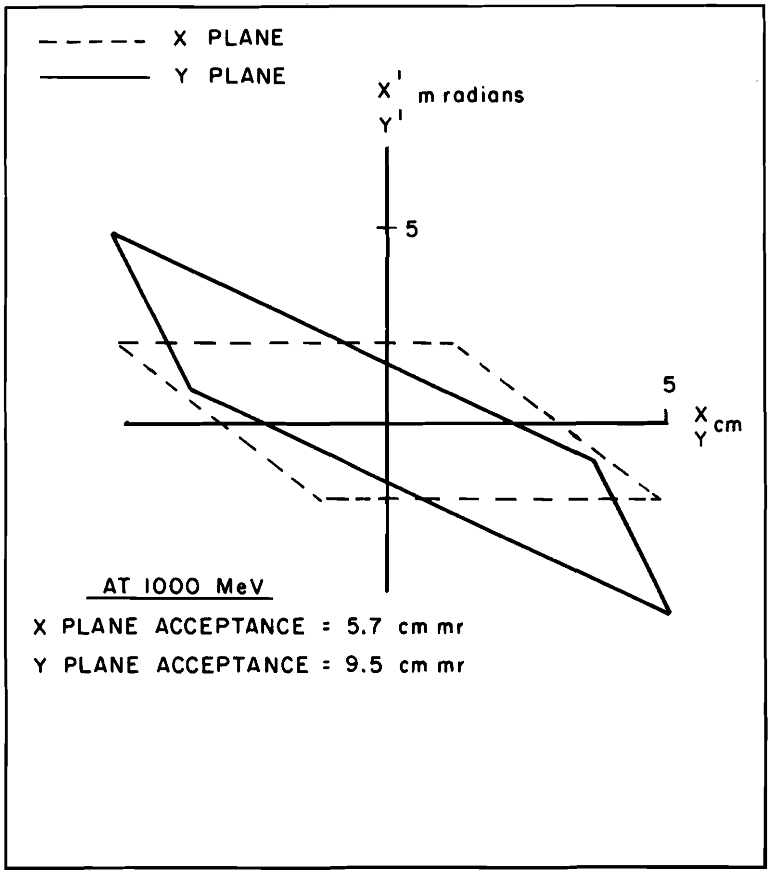


FIG. 6 Acceptance of Beam Line, Target Reward.

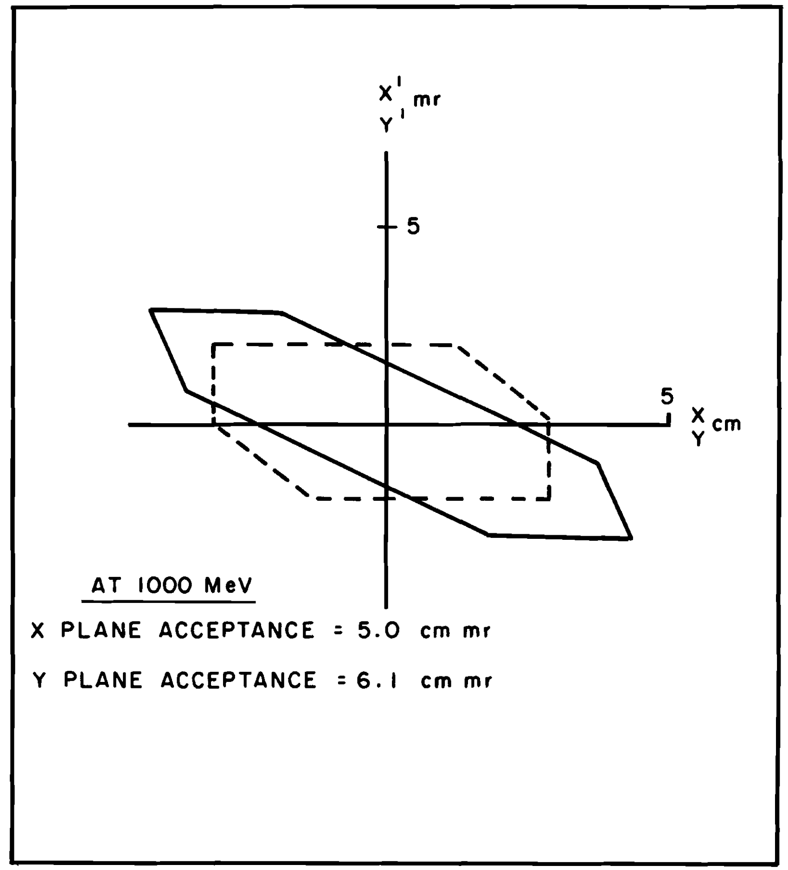


FIG. 7 Acceptance of Beam Line, 2.5 cm. dia. Target.

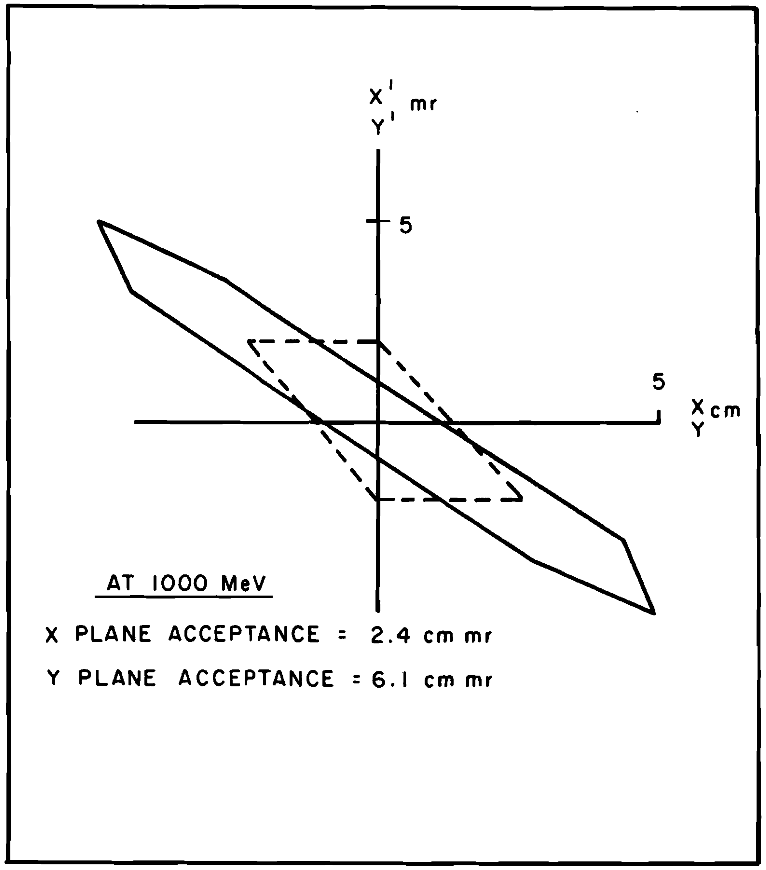


FIG. 8 Acceptance of Beam Line, 2.5 cm Aperture at S.

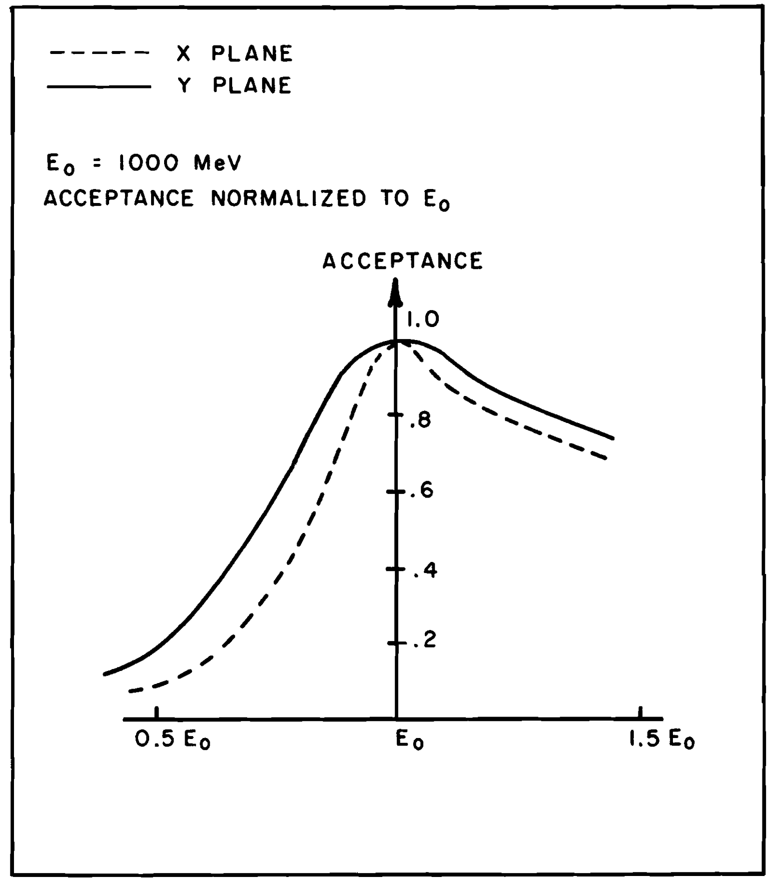


FIG. 9 Acceptance v.s. Beam Energy.