

STEADY-STATE BEAM LOADING OF A STANDING WAVE LINAC RF SYSTEM\*

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Introduction

The effects of steady-state beam loading on a linac rf system have been studied by many authors.<sup>1-5</sup> The rf system is composed of an rf generator feeding an accelerator cavity via a section of transmission line and a coupling loop. In particular, Murin and Kvasha<sup>1</sup> have investigated methods of stabilizing the amplitude and phase of the accelerating cavity voltage by proper choice of circuit parameters and rf supply operating conditions. It was shown theoretically that for different beam current values constancy in both amplitude and phase of the cavity voltage can be maintained by altering the value of the internal resistance of the generator.

Alternately, steady-state beam loading compensations can be obtained by varying the amplitude and phase of the generator output and not altering the generator resistance. More recently, however, Murin et al<sup>3</sup> demonstrated experimentally the possibility of finding values for the system parameters so that only amplitude variation is required for beam loading compensation. In this paper, a method of selecting optimum values for the system parameters under these conditions is considered and some requirements on the steady-state beam loading compensations are described.

Model of the Linac Rf System

Figure 1 shows a circuit diagram of the rf system<sup>6</sup> to be studied. The linac cavity is represented as a parallel RLC network characterized by an admittance  $G_2 + jB_2$ . (For a loop-coupled cavity, the resistive loss and self-inductance of the loop are included in  $G_2$  and  $B_2$ , respectively.) The generator is represented by an ideal current source  $\vec{I}_g$  in parallel with an admittance  $G_g + jB_g$ . Power from the generator is delivered to the cavity via a section of a lossless transmission line of length  $l$  and an ideal transformer of turns ratio  $(1:n)$ . The beam is taken to be an ideal current source  $\vec{I}_b$  connected across the cavity. The value of  $\vec{I}_b$  is given by the product of the transit time factor and the fundamental Fourier component of the beam current.

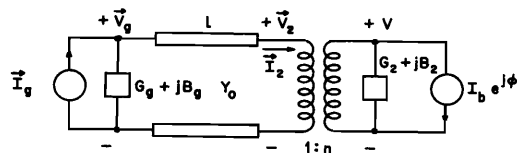


Fig. 1. Equivalent Circuit of a Linac Rf System.

Assuming the phase and amplitude control systems are operating correctly, in steady state, the phase angle\* between the beam current and the cavity voltage is kept equal to  $\phi_s$ . The amplitude of the cavity voltage is also kept constant. Thus, taking the voltage across the cavity,  $\vec{V}$ , as the phase reference, we have

$$\vec{V} = V \quad (1)$$

and

$$\vec{I}_b = I_b e^{j\phi_s} \quad (2)$$

where  $V$  is the peak value of the required cavity voltage. This condition is illustrated in Fig. 2.

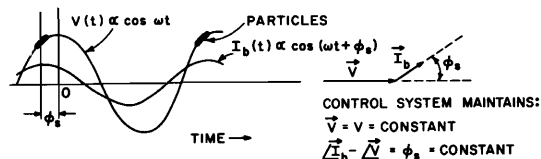


Fig. 2. Effective Cavity Voltage and Beam Current Waveforms.

For the accelerator cavities, the power required by the beam,  $P_b$ , and the power loss in the cavity walls,  $P_w$ , are specified from

\* Work performed under the auspices of the U. S. Atomic Energy Commission.

\* At low beam current,  $\phi_s$  is equal to the synchronous phase angle.

MESSYMESH calculations.<sup>7</sup> In terms of the cavity voltage and current

$$P_b = \frac{1}{2} V I_b \cos \varphi_s \quad (3)$$

and

$$P_w = \frac{1}{2} G_2 V^2 \quad (4)$$

where

$$R_2 = \frac{1}{G_2} = \text{effective cavity resistance.}$$

Thus, the effective cavity voltage and resistance are given by

$$V = \frac{2 P_b}{I_b \cos \varphi_s} \quad (5)$$

and

$$R_2 = \frac{V^2}{2 P_w} \quad (6)$$

where  $V$  and  $I_b$  are peak values.

#### Conditions for Maximum Power Transfer

The rf power required for particle acceleration is proportional to the beam current. Thus, maximum beam current is limited by the maximum available power which can be delivered to the cavity from the generator. This occurs when the cavity admittance as seen by the generator is equal to the complex conjugate of the generator admittance. In the design of the rf system, the values of the system parameters such as the cavity coupling, cavity detuning and the line length may be chosen so that the maximum power transfer condition is satisfied at maximum beam current:

$$Y_{im} = Y_g^* \quad (7)$$

where  $Y_{im}$  is the cavity admittance as seen by the generator at maximum beam current. This condition is satisfied if the cavity admittance as seen at the end of the line,  $Y_{lm}$ , is given by

$$\begin{aligned} Y_{lm}(\ell) &= Y_o \frac{Y_{im} \cos \beta \ell - j Y_o \sin \beta \ell}{Y_o \cos \beta \ell - j Y_{im} \sin \beta \ell} \\ &= Y_o \frac{G_g \cos \beta \ell - j (B_g \cos \beta \ell + Y_o \sin \beta \ell)}{(Y_o \cos \beta \ell - B_g \sin \beta \ell) - j G_g \sin \beta \ell} \end{aligned} \quad (8)$$

where  $Y_o$  = characteristic admittance of the line,

$\beta$  = propagation constant of the line,

$\ell$  = length of the line.

But

$$\begin{aligned} Y_{lm} &= \frac{I_2}{V_2} \\ &= n^2 \left( G_2 + jB_2 + \frac{I_{bm}}{V} e^{j\varphi_s} \right) \end{aligned} \quad (9)$$

Equating these two expressions for  $Y_{lm}$  and solving for  $n^2$  and  $B_2$ , we obtain:

$$n^2(\ell) = \frac{G_g}{\left( G_2 + \frac{I_{bm}}{V} \cos \varphi_s \right) \left( \frac{G_g}{Y_o^2} \sin^2 \beta \ell + F^2 \right)} \quad (10)$$

and

$$\begin{aligned} B_2(\ell) &= \frac{Y_o}{G_g} \left[ \frac{G_g}{Y_o^2} \sin \beta \ell \cos \beta \ell \right. \\ &\quad \left. - F \left( \sin \beta \ell + \frac{B_g}{Y_o} \cos \beta \ell \right) \right] \\ &\quad \left( G_2 + \frac{I_{bm}}{V} \cos \varphi_s \right) - \frac{I_{bm}}{V} \sin \varphi_s \end{aligned} \quad (11)$$

where

$$F(\ell) = \cos \beta \ell - \frac{B_g}{Y_o} \sin \beta \ell \quad (12)$$

The value of  $B_2$  as given by Eq. (11) represents the susceptance of the LC resonant network

$$B_2 = \omega C \left( 1 - \frac{\omega_o^2}{\omega^2} \right) \quad (13)$$

where

$\omega$  = frequency of the generator,

$\omega_o = \frac{1}{\sqrt{LC}}$  = undamped natural frequency of the cavity.

If we let

$$\omega_o = \omega + \Delta\omega \quad ,$$

for  $\omega \approx \omega_o$ , we find

$$\Delta\omega \approx - \frac{B_2}{2 G_2} \text{ (B.W.)} \quad ,$$

which corresponds to detuning of the cavity

$$\delta = \frac{\Delta\omega}{B.W.} = - \frac{B_2}{2 G_2} \quad (14)$$

where

$$B.W. = \text{bandwidth} = \omega/Q, \\ Q = \text{cavity } Q.$$

In order to minimize the interaction of the phase and amplitude control systems, zero detuning of the cavity may be desirable. In that case, an external susceptance  $B_{ex}$  must be added to the end of the line to provide the same load susceptance to the line. Reflecting  $B_2$  across the transformer gives the value for  $B_{ex}$ :

$$B_{ex}(\ell) = n^2(\ell) B_2(\ell). \quad (15)$$

This situation is illustrated in Fig. 3.

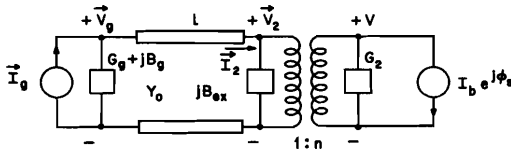


Fig. 3. Equivalent Circuit of a Linac Rf System for Cases in which the Cavity Resonant Frequency is Equal to the Rf Excitation Frequency ( $\omega_o = \omega$ ).

Equations (10), (11) and (15) show that the maximum power transfer requirement alone does not determine uniquely the values of the system parameters  $n$ ,  $\delta$ , and  $B_{ex}$ . The values of these parameters depend upon the value of the length of the transmission line used in the design. A criterion for the selection of line length is considered in the next section.

#### Choice of Line Length

In steady states, the generator voltage and current are related to the cavity voltage and beam current by:

$$\begin{bmatrix} \vec{V}_g \\ \vec{I}_g \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ G_g + jB_g & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ I_b e^{j\phi_s} \end{bmatrix} \quad (16)$$

where

$$A = \frac{1}{n} \cos \beta\ell + j \frac{n}{Y_o} (G_2 + jB_2) \sin \beta\ell \\ B = j \frac{n}{Y_o} \sin \beta\ell \\ C = j \frac{Y_o}{n} \sin \beta\ell + n (G_2 + jB_2) \cos \beta\ell \\ D = n \cos \beta\ell$$

There are two quantities which are of particular interest. One is the VSWR on the line,  $S$ , and the other is the power dissipated in the generator,  $P_d$ . At maximum beam current, due to the matching procedure used,  $S$  and  $P_d$  are independent of the values of  $\ell$ . However, at other values of beam current

$$S(\ell) = \frac{1 + |\rho|}{1 - |\rho|} \quad (17)$$

and

$$P_d(\ell) = \frac{1}{2} G_g |\vec{V}_g(\ell)|^2 \quad (18)$$

where

$$\rho(\ell) = \frac{Y_o - Y_\ell}{Y_o + Y_\ell} \quad (19)$$

$$Y_\ell = n^2 \left( G_2 + jB_2 + \frac{I_b}{V} e^{j\phi_s} \right) \quad (20)$$

Solving for  $\vec{V}_g$  and substituting the result in Eq. (18) gives

$$P_d = \frac{1}{2} G_g \left\{ \frac{V^2 \cos^2 \beta\ell}{n^2} + \frac{n^2 \sin^2 \beta\ell}{Y_o^2} \left[ (G_2 V + I_b \cos \phi_s)^2 + (B_2 V + I_b \sin \phi_s)^2 \right] - \frac{V \sin^2 \beta\ell}{Y_o} (B_2 V + I_b \sin \phi_s) \right\} \quad (21)$$

In particular,  $P_d$  and  $S$  can be evaluated for zero beam current:  $P_{d0}$  and  $S_0$ .

Finally, we come to the question of choosing the line length. One approach is to calculate the quantities  $n$ ,  $\delta$ ,  $B_{ex}$ ,  $S_0$ ,  $P_{d0}$  for systems having

\*For the special case of  $Y_g = Y_o$ , the only quantity which varies with  $\ell$  is  $P_{d0}$ . An experimental procedure for the adjustment of  $n$  and  $B_{ex}$  may be found in Ref. 6.

different values of  $\ell$  and select those for which the values of these quantities are acceptable. A criterion in this selection may be to make the value of  $n$  large in order to reduce the inductance and losses in the coupling loop, while making the values of  $\delta$ ,  $B_{ex}$ ,  $S_o$ , and  $P_{do}$  small.

In particular, for the case of  $B_g = B_2 = B_{ex} = 0$ , the value of  $\ell$  satisfies the condition:

$$\sin 2\beta\ell = \frac{2 \frac{I_{bm}}{V} \sin \varphi_s}{\left(G_2 + \frac{I_{bm}}{V} \cos \varphi_s\right) \left(\frac{G_g}{Y_o} - \frac{Y_o}{G_g}\right)}, \quad (22)$$

provided that the right-hand side of this equation has an absolute value less than unity, i.e.,

$$\left| \frac{G_g}{Y_o} - \frac{Y_o}{G_g} \right| > \left| \frac{2 \frac{I_{bm}}{V} \sin \varphi_s}{G_2 + \frac{I_{bm}}{V} \cos \varphi_s} \right|. \quad (23)$$

The corresponding value of  $n$  is given by Eq. (10) with  $B_g = 0$ .

#### Beam Loading Compensations

In the operation of the linac, the generator current is adjusted so that the cavity voltage reaches its steady-state value. Just before the beam pulse is turned on, the generator current is increased to provide additional rf power to the cavity to compensate for beam loading. This change in the generator current is made in such a way that the steady-state value of the cavity voltage is the same before and during the beam pulse (after the transients due to turning-on of the beam and the compensating rf pulse have faded). The variations in the values of the generator current required for producing this condition are considered in this section.

Substituting the expressions for  $A$ ,  $B$ ,  $C$ , and  $D$  in Eq. (16) and rearranging terms, we find

$$\vec{I}_g = (aV + bI_b) + j(cV + dI_b), \quad (24)$$

where,

$$a = nF(\ell)G_2 + \frac{G_g}{n} \cos \beta\ell - \frac{n G_g B_2}{Y_o} \sin \beta\ell$$

$$b = nF(\ell) \cos \varphi_s - \frac{n G_g}{Y_o} \sin \beta\ell \sin \varphi_s$$

$$c = nF(\ell)B_2 + \frac{B_g}{n} \cos \beta\ell + \left( \frac{n G_g G_2}{Y_o} + \frac{Y_o}{n} \right) \sin \beta\ell$$

$$d = nF(\ell) \sin \varphi_s + \frac{n G_g}{Y_o} \sin \beta\ell \cos \varphi_s.$$

Thus, the amplitude and phase of  $\vec{I}_g$  are given by:

$$|\vec{I}_g(I_b)| = \left[ (a^2 + c^2) V^2 + 2(ab + cd) V I_b + (b^2 + d^2) I_b^2 \right]^{\frac{1}{2}} \quad (25)$$

and

$$\varphi_g(I_b) = \tan^{-1} \left( \frac{cV + dI_b}{aV + bI_b} \right). \quad (26)$$

The changes required in the amplitude and phase of  $\vec{I}_g$  for beam loading compensation are given by:

$$\Delta |\vec{I}_g(I_b)| = |\vec{I}_g(I_b)| - |\vec{I}_g(0)| \quad (27)$$

and

$$\begin{aligned} \Delta \varphi_g(I_b) &= \varphi_g(I_b) - \varphi_g(0) \\ &= \tan^{-1} \left[ \frac{(ad - bc) I_b}{(a^2 + c^2) V + (ab + cd) I_b} \right]. \end{aligned} \quad (28)$$

In particular, for the maximum power transfer design, substitution of Eqs. (10) and (11) into the above equations yields:

$$(a^2 + c^2) = \alpha G_g \left[ \left(2G_2 + \frac{I_{bm}}{V} \cos \varphi_s\right)^2 + \frac{I_{bm}^2}{V^2} \sin^2 \varphi_s \right]$$

$$(ab + cd) = G_g \left( 2 \cos \varphi_s - \frac{\alpha I_{bm}}{V} \right) \quad (29)$$

$$(ad - bc) = 2G_g \sin \varphi_s$$

$$(b^2 + d^2) = \alpha G_g,$$

where

$$\alpha = \frac{1}{G_2 + \frac{I_{bm}}{V} \cos \varphi_s}.$$

It may be noted that the values of these quantities do not depend on  $\ell$  and  $B_g$ . Thus, the same compensations are required for systems having the same value of generator conductance,  $G_g$ . Furthermore, the phase compensation,  $\Delta \varphi_g$ , is the same regardless of the value of  $G_g$ , and the amplitude compensation,  $\Delta |\vec{I}_g|$ , is proportional to  $\sqrt{G_g}$ .

#### Conditions for Zero Phase Compensation

In practice, it may be desirable to compensate for beam loading by varying only the value of the amplitude of  $\vec{I}_g$  but not its phase. This possibility is investigated in this section.

From Eq. (28), the condition for zero phase compensation is given by

$$ad - bc = 0. \quad (30)$$

Equation (29), however, shows that the condition for zero phase compensation and the condition for maximum power transfer are mutually exclusive. Therefore, for a system designed for zero phase compensation, the maximum beam current will always be less than that for a system designed for maximum power transfer.

Four special cases are considered here. In each case, the value of one of the system parameters is adjusted to satisfy Eq. (30). We take for Case i:  $B_{ex}$  adjustable, Case ii:  $B_g$  adjustable, Case iii:  $l$  adjustable\* and Case iv:  $n$  adjustable.

Case i:  $B_{ex}$  adjustable

The value of  $B_{ex}$  required to obtain zero phase compensation is given by:

$$B'_{ex} = \frac{Y_o \left[ \frac{G_g^2}{Y_o^2} \sin \beta l \cos \beta l - F \left( \sin \beta l + \frac{B_g}{Y_o} \cos \beta l \right) \right]}{\left( \frac{G_g^2}{Y_o^2} \sin^2 \beta l + F^2 \right)} + \frac{\tan \varphi_s \left[ G_g + n^2 F^2 G_2 + \frac{n^2 G_g^2 G_2}{Y_o^2} \sin^2 \beta l \right]}{\left( \frac{G_g^2}{Y_o^2} \sin^2 \beta l + F^2 \right)} \quad (31)$$

Case ii:  $B_g$  adjustable

The values of  $B_g$  are solutions of the equation:

$$a_1 B_g'^2 + a_2 B_g' + a_3 = 0 \quad (32)$$

where

$$a_1 = \frac{1}{Y_o^2} \left[ \left( n^2 B_2 \sin^2 \beta l - \frac{Y_o}{2} \sin 2\beta l \right) \cos \varphi_s - n^2 G_2 \sin^2 \beta l \sin \varphi_s \right]$$

$$a_2 = \frac{1}{Y_o} \left[ \left( Y_o \cos 2\beta l - n^2 B_2 \sin 2\beta l \right) \cos \varphi_s + n^2 G_2 \sin 2\beta l \sin \varphi_s \right]$$

\*This condition has been experimentally demonstrated by Murin et al. (Ref. 3).

$$a_3 = \left[ n^2 B_2 \left( \frac{G_g^2}{Y_o^2} \sin^2 \beta l + \cos^2 \beta l \right) - \left( \frac{G_g^2}{Y_o^2} - 1 \right) \frac{Y_o}{2} \sin 2\beta l \right] \cos \varphi_s - \left[ G_g + n^2 G_2 \cos^2 \beta l + \frac{n^2 G_g^2 G_2}{Y_o^2} \sin^2 \beta l \right] \sin \varphi_s .$$

Real solutions for this equation exist provided that the condition

$$a_2^2 - 4a_1 a_3 > 0 \quad (33)$$

is satisfied.

Case iii:  $l$  adjustable

The values of line length required ( $l'$ ) must satisfy Eq. (31) with  $B'_{ex}$  replaced by  $B_{ex}$  and  $l$  replaced by  $l'$ . Real-valued solution for  $l'$  may not always exist.

Case iv:  $n$  adjustable

The value of coupling coefficient ( $n'$ ) required for obtaining zero phase compensation is given by:

$$n' = \left\{ \frac{1}{G_2 \tan \varphi_s} \left[ B_{ex} - \frac{Y_o \left[ \frac{G_g^2}{Y_o^2} \sin \beta l \cos \beta l \right]}{\left( \frac{G_g^2}{Y_o^2} \sin^2 \beta l + F^2 \right)} - \frac{F \left( \sin \beta l + \frac{B_g}{Y_o} \cos \beta l \right) + G_g \tan \varphi_s}{\left( \frac{G_g^2}{Y_o^2} \sin^2 \beta l + F^2 \right)} \right] \right\}^{\frac{1}{2}} \quad (34)$$

provided that the quantity under the square root sign is positive.

Example

In the design of an rf system, the values of  $V$  and  $R_2$  are first calculated from the power requirements given by Eqs. (5) and (6). Then for a selected value of  $\beta l$ , the values of  $n$  and  $B_{ex}$  are calculated from Eqs. (10), (11), and (15)

and the values of  $S_0$ ,  $P_{d0}$ ,  $|\vec{I}_g|$  and  $\Delta\phi_g$  from Eqs. (17), (21), and (25) - (28). The values of  $B_{ex}$ ,  $B_g$ ,  $l'$  and  $n'$  required for zero phase compensation are determined from Eqs. (31) - (34). This design procedure is illustrated by an example below:

For this example, we take:

$$I_p(\text{max}) = 100 \text{ mA (current average over a pulse)}$$

$$g/L = \text{Gap width cell length} = 0.4 \text{ (Ref. 8)}$$

$$\tau/L = \text{Bunch width/rf period} = 0.04$$

$$G_g = 0.015 \Omega^{-1} \text{ (Ref. 9)}$$

$$Y_o = 0.02 \Omega^{-1}$$

$$Q = 60,000$$

$$\phi_g = 30^\circ$$

$$P_w = 2.75 \text{ MW (Tank 5) (Ref. 8)}$$

$$P_b = 2.4 \text{ MW (Tank 5) (Ref. 8)}$$

For a uniformly bunched beam, the effective current is given by

$$I_b = \frac{2 \sin\left(\frac{\pi g}{L}\right) \sin\left(\frac{\pi \tau}{T}\right)}{\left(\frac{\pi g}{L}\right) \left(\frac{\pi \tau}{T}\right)} I_p \quad (35)$$

Using Eqs. (5), (6), and (35), we find:

$$I_{bm} = 1.51 \text{ A}$$

$$R_2 = 245 \text{ M}\Omega$$

$$V = 36.7 \text{ MV}$$

The quantities  $n$ ,  $\delta$ ,  $B_{ex}$ ,  $S_0$ , and  $P_{d0}$  have been calculated for systems having different values of  $\beta l$ . The results are given in Figs. 4-8 for three cases: Case 1, 2, and 3 for  $B_g = +0.015$ , 0.0 and  $-0.015 \Omega^{-1}$ , respectively.

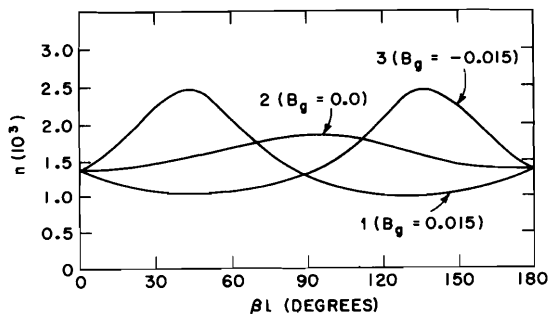


Fig. 4. Cavity Coupling Coefficient ( $n$ ) for Systems Having Different Line Length.

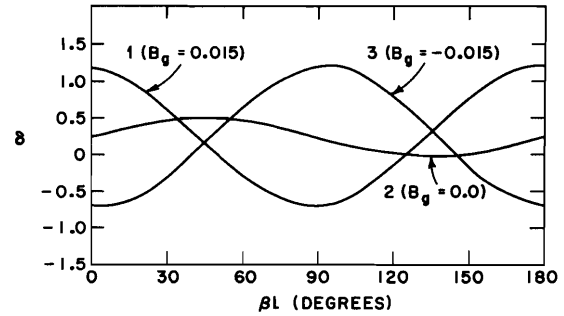


Fig. 5. Cavity Detuning ( $\delta$ ) for Systems Having Different Line Length.

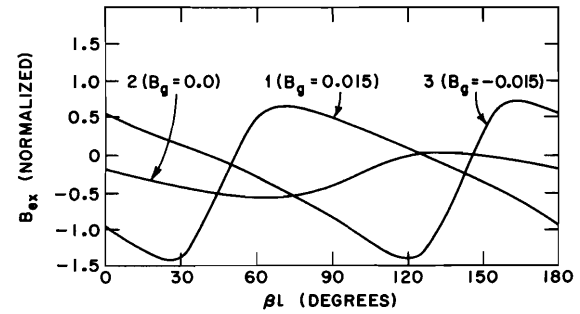


Fig. 6. Matching Susceptance ( $B_{ex}$ ) for Systems Having Different Line Length.

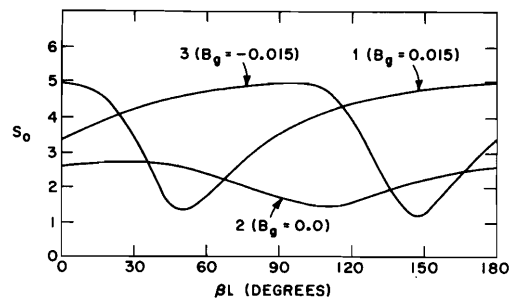


Fig. 7. VSWR ( $S_0$ ) for Systems Having Different Line Length ( $I_b = 0$ ).

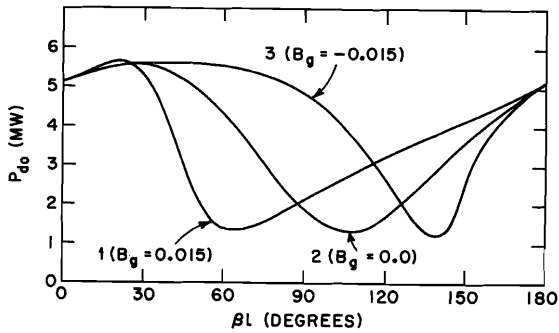


Fig. 8. Power Dissipated in Generator ( $P_{d0}$ ) for Systems Having Different Line Length for Systems Having Different Line Length ( $I_b = 0$ ).

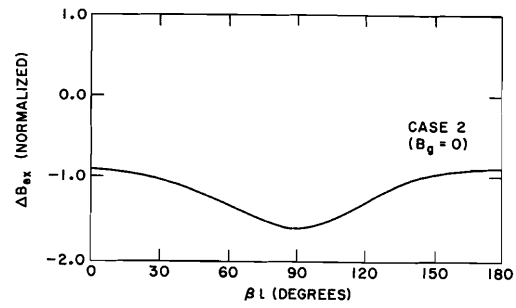


Fig. 11. The Change in Load Susceptance ( $\Delta B_{ex}$ ) Required for Zero Phase Compensation  $\Delta B_{ex} = B_{ex} - B'_{ex}$  where the Values for  $B_{ex}$  are Given in Fig. 6.

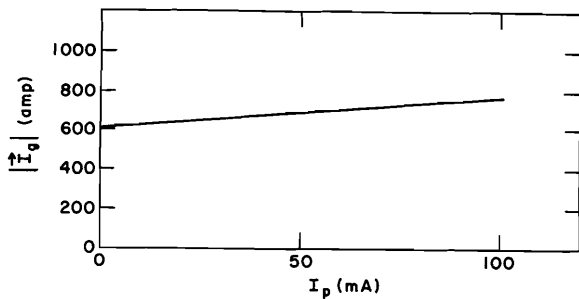


Fig. 9. Amplitude of Generator Current Required to Compensate Steady-State Beam Loading for Different Values of Beam Current ( $I_p = \text{Avg. Beam Current Over a Pulse}$ ).

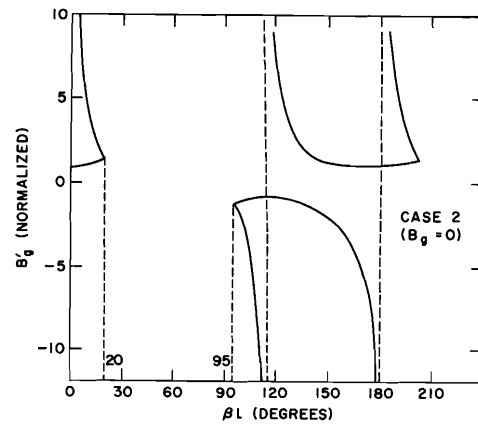


Fig. 12. Generator Susceptance Required ( $B'_g$ ) for Zero Phase Compensation. The Corresponding Change in  $B_g$  is Given by  $\Delta B_g = -B'_g$ .

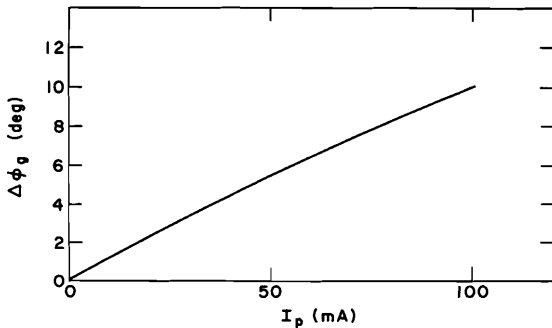


Fig. 10. Phase Change of Generator Current Required to Compensate Steady-State Beam Loading for Different Values of Beam Current.

Figures 4 - 6 show that as the value of  $\beta l$  is varied over a  $180^\circ$  range, minimum variations in the corresponding values of the system parameters  $n$ ,  $\delta$  and  $B_{ex}$  occur for the case of  $B_g = 0$ . In particular, the condition  $B_g = B_{ex} = B'_g = 0$  [Eq. (22)] is satisfied for  $\beta l = 135^\circ$ .

Plots of  $|\vec{I}_g|$  and  $\Delta \phi_g$  required for beam loading compensations as functions of beam current are shown in Figs. 9 and 10. Since the values of these quantities are independent of both  $l$  and  $B_g$ , the plots are the same for all systems. The generator current is 612 A and 786 A for  $I_p = 0$  and 100 mA ( $I_{bm} = 1.51$  A), respectively. The change in  $\phi_g$  required is 10 degrees.\*

\*  $\Delta \phi_g = 1.5^\circ$  for a change of  $I_p$  from 0 to 30 mA has been observed in the existing AGS Linac by K. Batchelor et al. (Ref. 10).

The adjustments required for zero phase compensation for Case 2 have been calculated and the results are shown in Figs. 11 and 12. It is found that, for this case, the zero phase condition cannot be obtained by varying the line length or cavity coupling coefficient alone. In addition, for  $20^\circ < \beta l < 95^\circ$ , no real solution for  $B'_g$  exists as shown in Fig. 12. For the other values of  $\beta l$ , two solutions for  $B'_g$  are possible. In the neighborhood of  $\beta l = 0$  and  $115^\circ$ , the absolute value of one of these solutions becomes large, while the other reaches a minimum.

A summary of results for the special case of  $\beta l = 135^\circ$  is given below:

System	$n (10^3)$	$B_{ex}$	$B_g$	Remarks
A	1.6	0	0	System designed for maximum power transfer ( $I_{bm} = 1.51$ A).
i	1.6	-1.1	0	System A with $B_{ex}$ adjusted for zero phase compensation.
ii	1.6	0	2.0 -1.2	System A with $B_g$ adjusted for zero phase compensation.
iii		No solution		System A with $l$ adjusted for zero phase compensation.
iv		No solution		System A with $n$ adjusted* for zero phase compensation.

\*It may be shown that it is impossible to obtain zero phase compensation by adjusting the cavity coupling alone in a system designed for a maximum power transfer.

### Summary

Two criteria which take into account the effects of steady-state beam loading have been considered theoretically for the design of a linac rf system. In one, the power transferred to the linac cavity is maximized for maximum beam current. In the other, the system is designed so that phase compensation for beam loading is not required. In general, it is not possible to have a system which meets both criteria.

It has been shown that for any system, zero phase compensation is always obtainable provided that the load susceptance is adjustable. However, zero phase compensation is not always attainable for a system in which the generator susceptance, line length or cavity coupling coefficient is the only adjustable parameter.

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