ELASTIC SCATTERING OF $\pi^{\pm}$-p, $K^{\pm}$-p, $\bar{p}$-p, AND p-p WITH A 600 TO 1000-BEV ACCELERATOR

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I. Introduction

It is generally recognized that detailed information on the elastic scattering of the various elementary particles with protons is of particular interest in regard to the asymptotic behavior of high-energy interactions.[1] The experimental requirements for the elastic scattering studies of pions, kaons, protons and antiprotons are essentially the same, and these experiments can be performed in the secondary beams of a 600 to 1000-Bev accelerator with equal facility.

A number of studies have demonstrated the feasibility of proton-proton scattering experiments (at small momentum transfers) with colliding beams in storage rings. On the other hand, it does not seem possible to perform scattering experiments with the unstable particles in storage rings. Certain advantages in performing the experiment in a system with the center of mass at rest and disadvantages in some other aspects have been pointed out.[2] It is the purpose of this study to show that the scattering experiments using secondary beams from a 600 to 1000-Bev accelerator can be performed without resorting to very large experimental setups.

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1. T.D. Lee, R. Serber, G.C. Wick and C.N. Yang, Experimental Program Requirements for a 300 to 1000-Bev Accelerator and Design Study for a 300 to 1000-Bev Accelerator, p. 15, BNL 772, August 1961 (revised December 1962).
2. K.M. Terwilliger, p. 238 of this volume.

It is sufficient to investigate the problems in measuring the elastic and quasi-elastic scattering of protons, since experiments using other secondary particles from such an accelerator are essentially the same.

II. Kinematics

Since it will develop that essentially the only problem in this experiment is the separation of the elastic from the quasi-elastic scattering, we must compare the kinematics of the two processes. We consider an incident proton with momentum P_1 striking a proton at rest, producing a proton with momentum P_r and a system of mass M_s (which may be a proton, an excited isobar, or the residue of a complicated interaction after one proton has been picked out), with momentum P_s. Since we are interested in fairly small momentum transfers, we can use the approximations appropriate for very slow or very fast recoil protons, and obtain some simple kinematic relations. There are two cases to consider:

a) Fast recoil proton

$$P_r \simeq P_1 - \frac{M_s^2 - M_p^2}{2\,M_p}$$

(with c = 1)

independent of angle, for small momentum transfers. For example, if $M_s = 1.5$ Bev, for the lowest $T = 1/2$ isobar, and $P_1 = 600$ Bev, the separation of elastic and quasi-elastic processes on the basis of P_r alone requires momentum resolution $\leq 0.1\%$. To reach the coulomb interference region in momentum transfer requires an angular definition better than 0.1 mrad. It is possible to realize these requirements but a large setup is required.

b) Slow recoil proton

For the minimum momentum transfer (recoil proton at zero degrees)

$$P_{ro} = \frac{M_s^{\,2} - M_p^{\,2}}{2\,P_1}$$

and for $M_s = 1.5$ Bev, $P_1 = 600$ Bev,

$$P_{ro} = 1.1 \text{ Mev/c} .$$

It is convenient to obtain the angular dependence of P_r by using the longitudinal and transverse components of P_r, P_ℓ and P_t, $P_r^{\,2} = P_\ell^{\,2} + P_t^{\,2}$. $P_t \simeq$ momentum transfer. Then

$$P_\ell = \frac{P_1}{4}\left(\frac{P_t}{P^*}\right)^2 + P_{ro} ,$$

where P^* is the center-of-mass momentum of the incident proton. Then

$$\tan\theta_r = \frac{4\,P^{*2}}{P_1 P_t} .$$

In Table I, P_t and P_ℓ are given for momentum transfers up to 700 Mev/c for elastic scattering. For isobar production, P_ℓ is greater by an amount P_{ro}. Although small, this difference can be detected with reasonably simple equipment, as discussed in the next section.

III. Measurement Accuracy Needed, Measuring the Slow Recoil Proton

In order to separate scattering and isobar production, it is necessary to measure the angle and momentum of the recoil proton. In Table I we give the separation of the elastic and $M_s = 1.5$ Bev kinematics in angle and momentum. When the angle variation is calculated, the momentum is held constant and vice versa. In order to resolve two peaks, the measurement accuracy should be somewhat better than these values.

A fundamental limitation on the accuracy of measurement of these slow protons is the multiple scattering in the target and detector. Also given in Table I are the number of radiation lengths of material allowed for the stated angular accuracy, and the corresponding quantity of hydrogen. It is clear that gas targets are required.

TABLE I

P_t Bev/c	P_ℓ Bev/c	$\delta\theta$ Milliradians	$\frac{\delta p}{p}$	L_r Radiation lengths	L_{H_2} gm/cm^2
.05	.0013	20	-	10^{-5}	6×10^{-4}
.1	.0053	10	25%	3×10^{-5}	2×10^{-3}
.2	.021	5	6%		
.3	.048	3	2.5%	3×10^{-4}	2×10^{-2}
.4	.085	2.5	1.4%		
.5	.132	2	.8%		
.7	.260	1.5	.5%	6×10^{-3}	4×10^{-2}

The optimum detector changes as P_r varies over this interval. For $P_r < 200$ Mev/c, a hydrogen diffusion chamber or a gas target with angles defined by slits and proton energy measurement by semiconductor or scintillator spectrometers is indicated. For high momenta, a gas target and spark-chamber spectrometers (with very thin plates) is probably the best instrument. Counting rate does not seem to be a problem for the small momentum transfers, since the cross section is quite large.

Without a very detailed design it is difficult to say what is the smallest value of M_s which may be detected by this method. $M_s = 1.5$ Bev seems rather marginal, and $M_s = 1.2$ Bev (the 3/2 isobar) seems impossible. In order to detect small M_s or to go to momentum transfers larger than $\sim$ 700 Mev/c, a coincidence technique is necessary. Of course, if the inelastic collisions giving $M_s > 1.5$ Bev are eliminated, the background for the elastic scattering at high energies is probably quite small, and measurement of the slow recoil alone is sufficient, for small momentum transfer. It is the study of $N^*_{3/2}$ (1238) which requires the use of the coincidence technique.

IV. Coincidence Measurement

Consider an event where the slow recoil proton was found to have a $P_r \simeq 100$ Mev/c and the measurement accuracy was such that, say, $M_s < 1.7$ Bev. This is then a candidate for an elastic scattering, but to detect the forward proton is difficult because of the very small scattering angle. However, if an isobar with mass of 1.2 Bev was produced, for instance, the proton from the decay of that isobar will have a momentum spectrum reaching from ~ 600 Bev/c to ~ 570 Bev/c, while the angle it forms with the beam direction will still be small. Consequently, a measurement of the momentum distribution of particles leaving the target near zero degrees, in coincidence with slow recoils defining small M_s, would allow a statistical separation of isobar production and elastic scattering. Note that the spectrometer for measuring the fast proton with this technique needs only about ½% resolution.

V. Conclusions

The elastic scattering cross section for small momentum transfer can be measured with a 600 to 1000-Bev accelerator. The cross section for large momentum transfers can presumably be measured only with this accelerator, since the counting rate becomes too small in a storage-ring experiment. The cross section for isobar production can be determined without measuring neutral particles.