SCATTERING AT VERY HIGH ENERGIES

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Part I - Dispersion Theory of Strong Interactions

This will be a review of the dispersion theory of strong interactions insofar as it makes predictions at very high energies, particularly above 30 Bev. Of course, the dispersion theory at high energies is intertwined with the subject of Regge poles so that the talk will also be concerned with the current status of Regge poles. Specifically, I thought it of interest to consider two possibilities: (a) Regge poles dominate high-energy behavior; (b) the complex angular-momentum plane is important but there are moving cuts as well as poles. I will take up, point by point, the qualitative differences you expect in these two cases and what sort of definite predictions you can make if there are cuts.

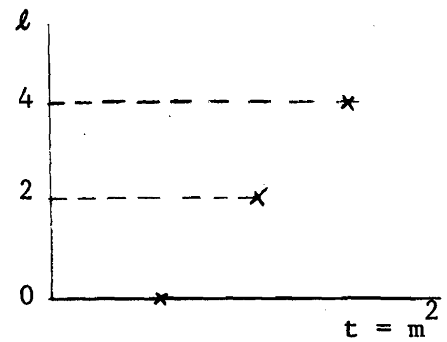
We will begin with a qualitative review of the Regge-pole ideas, how the ideas are tested and how the cuts come into the picture.

Low-Energy Aspects

One starts by thinking about a centrifugal barrier:

$$V_C = \frac{\hbar^2 \ell(\ell + 1)}{2mr^2} . \tag{1}$$

This barrier makes it more difficult to resonate in a state with a high angular momentum than one with a low angular momentum. If one has a large attraction, then we expect to find families of resonances and bound



states as shown in the sketch, where $\ell = 0, 2, 4, \ldots$. As the angular momentum increases, the centrifugal barrier increases and, therefore, the particles have to be heavier. In such a family of particles, all members will share the same baryon number, strangeness, isotopic spin, parity and other quantum numbers, except spin. It is the parity that causes "ℓ" to increase in steps of two instead of steps of one.

To determine the spacing between members of the family, i.e., $\ell = 0$ and $\ell = 2$, we require a qualitative estimate of the mass of the system which is bound, or resonating, and also the characteristic distance of separation. An example of such families, in atomic physics, is the hydrogen atom, where the characteristic distance is the Bohr radius. In nuclear physics, an example of such families is given by rotational levels where the characteristic distance is the "size of the nucleus". In particle physics, this distance will be much smaller than in the preceding examples; it is bounded by the pion-Compton wavelength. The mass of the bound or resonating particles will also be at least the pion mass,

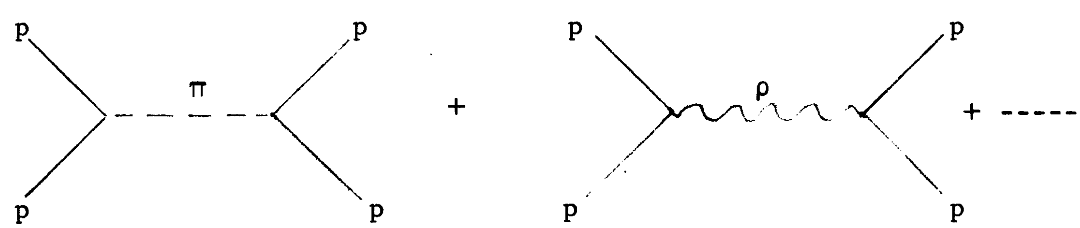
$$\text{i.e.,} \quad r \lesssim \frac{\hbar}{m_\pi c}$$

$$m \gtrsim m_\pi$$

Using such values, one arrives at separations between members of the family of several hundred Mev. This is much larger spacing than in nuclear or atomic physics and, hence, it is harder to establish that you have an entire family. However, in π-p scattering, K-n scattering, etc., you do have a sequence of resonances which tend to be spaced a few hundred Mev above the ground states. Here we may be looking at families, but we cannot be sure at present because in most cases the parity and spin of the higher members has not been established. The trouble is that these resonances are up in the inelastic regions where there are several partial waves operating and it is hard to separate the different effects. In any case, the prediction of families can eventually be tested at present energies with high-intensity beams of π's, K's, and protons.

High-Energy Aspects

In the past one described high-energy scattering in terms of exchanges of particles. For example, p-p scattering was described in terms of the exchange of a π plus the exchange of a ρ and whatever other particles one knew about. This procedure had the advantage of expressing high energies



in terms of low-energy parameters such as the pion-nucleon coupling. But there is a difficulty: the scattering amplitude tends, at large energies S, to go as

$$A_{S \to \infty} \propto S^{J} \tag{2}$$

where J is the spin which is being exchanged. Then the differential cross section is

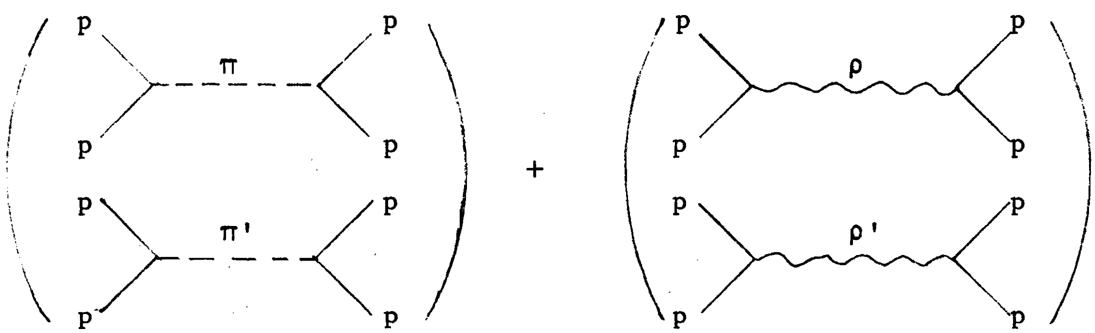
$$\frac{d\sigma}{dt} \propto S^{2J - 2} \tag{3}$$

and the effect on the total cross section is

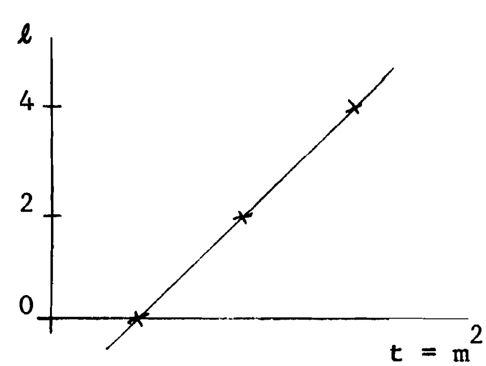
$$\sigma_{total} \propto s^{J - 1} . \tag{4}$$

The trouble is that there are systems which could be exchanged with spin greater than one. These exchanges by themselves would make the cross section rise indefinitely as a power of the energy, contradictory to experiment and contradictory to the intuitive idea that a cross section is a geometrical size of the target and should not rise with the energy. We have here simply the old divergence problem of field theory for spins greater than one, restated in terms of cross sections.

Another way to organize high-energy scattering is to lump together the exchange of all members of the same family. For example, one might exchange the pion plus any higher members of the pion family (designated by π', etc.). Now one can exchange nonresonating as well as resonating systems, so a more complete description is necessary. A basis for such a



description is supplied by Regge's ideas about poles which interpolate between members of the family. The idea is that if the angular momentum is considered to increase continuously, the centrifugal barrier and the mass of the bound state or resonance also increase continuously. Mathematically,



RADIATION RESEARCH 19: 187, 1963 BNL 6800

The Effect of Chronic Gamma Irradiation on Cell Proliferation and Growth Inhibition in Root Meristems of Pisum. J. Van't Hof* and A. H. Sparrow, Biology Department, Brookhaven National Laboratory, Upton, New York.

A series of experiments were performed to determine whether the minimum mitotic cycle time was altered in root meristem cells of *Pisum sativum* by chronic gamma irradiation exposures sufficient to produce growth inhibition. Reduction in cell number per meristem and inhibition of root elongation were used as indices of radiation effect. Colchicine was used to produce a tetraploid population of cells and the minimum cycle time was determined by noting the period of time between the production of tetraploid cells and the time they first appeared in the subsequent mitosis. Cycle time measurements were carried out after 3 successive days of irradiation. At this time the number of cells per meristem was reduced by approximately 5, 25, and 45% of control in roots exposed to 250, 500, and 1000 r per day respectively. Root elongation was also inhibited after 3 days at 500 and 1000 r per day, but there was no difference in the minimum cycle time between irradiated and control roots. Reduction in the number of cells per meristem proved to be a valuable index of radiation effect since the data showed a dose relationship much earlier than did growth measurements. Cytological analysis indicated that the number of abnormal anaphases produced after 3 days of irradiation increased with dose. Since the minimum cycle length was not changed by doses which caused growth inhibitions, it is suggested that cell proliferation was reduced in the exposed meristems by cell death due to chromosome damage or by permanent mitotic arrest and not primarily by an increase in duration of the mitotic cycle.--Research carried out at Brookhaven National Laboratory under the auspices of the U. S. Atomic Energy Commission.

continuous line of new poles (a cut in angular momentum) implying a more complicated description at high energies, with both cuts and poles exchanged. This question is not fully settled, but without resolving it we can state that high-energy behavior is affected in several definite ways. In the first place, if there are cuts, they will raise the energy where asymptotic behavior sets in. In the second place, the qualitative behavior is going to be different. In particular, for pole exchange, one had shrinking diffraction peaks which shrank more and more as the energy increased. With a simple model of cut exchange, these peaks will shrink less and less with increasing energy. In the third place, the addition of the cuts complicates the phenomenological description by introducing more parameters. However, none of the conjectured cuts raise the power of J upon which the total cross section is dependent:

$$\sigma_{total} \propto s^{J(0) - 1} \qquad (5)$$

or the differential cross section in the forward direction. This means that the cross sections remain bounded even in the presence of cuts.

Now let us see how these considerations work out in some specific processes, and then discuss the properties of the conjectured cuts in more detail.

The simplest problem is that of the total cross section, (σ_{total}), which is related to the forward elastic amplitude by the "optical theorem". Here it is appropriate to review Pomeranchuk's theorem. Pomeranchuk assumed that, at very high energies,

(a) the total cross section approaches a constant value,

(b) the forward-scattering dispersion relation holds,

(c) the forward amplitude is purely imaginary.

From these assumptions he deduced that the total cross section for the scattering A-B should approach that for A-$\overline{B}$,

$$\text{i.e.,}\quad \begin{aligned} \sigma(pp) &\rightarrow \sigma(p\overline{p}) \\ \sigma(p\pi^+) &\rightarrow \sigma(p\pi^-) \\ \sigma(pK^+) &\rightarrow \sigma(pK^-) \end{aligned} \tag{6}$$

The present experimental situation is as follows: $\sigma(pp)$ is not equal to $\sigma(p\overline{p})$, but they seem to be approaching each other. The $\sigma(p\pi^+)$ and $\sigma(p\pi^-)$ are close together, but do not seem to be getting closer. The $\sigma(pK)$'s are somewhere in between. Thus Pomeranchuk's theorem is definitely not satisfied at energies as high as 20 Bev, but it might be "tending" in that direction. Evidently higher-energy accelerators are needed to settle the question.

It is straightforward to express the above relations in terms of the exchange of a single Regge pole (the Pomeranchuk pole). The rate of approach to the limit is then a question of competition between Pomeranchuk pole exchange and other exchanges. Below is a list of the predictions one obtains for both the case of poles only and the case of poles plus cuts.

(1) How fast does the total cross section approach its constant limit?

(a) For poles only: $\sigma^{\text{total}}_{(AB,S)} - \sigma^{\text{total}}_{(AB,S\,=\,\infty)} \approx S^{\alpha_2(t=0)\,-\,1}$ (7)

where α_2 is the highest exchanged spin other than the Pomeranchuk spin.

(b) If cuts are present, one expects a slower approach to the constant limit.

$$\sigma^{\text{total}}_{(AB,S)} - \sigma^{\text{total}}_{(AB,S\,=\,\infty)} \approx \frac{1}{\ln S} \tag{8}$$

(The competition is more formidable here.)

(2) How fast are the Pomeranchuk limits approached?

(a) For poles only, these limits are approached as

$$\sigma^{\text{total}}_{(AB)} - \sigma^{\text{total}}_{(A\bar{B})} \approx s^{\alpha_3(t=0) - 1} \tag{9}$$

where $\alpha_3(t)$ is the spin of the first pole that can "tell" the difference between (AB) scattering and ($A\bar{B}$) scattering. For example, ρ or ω exchange is different for particles and antiparticles and could contribute here.

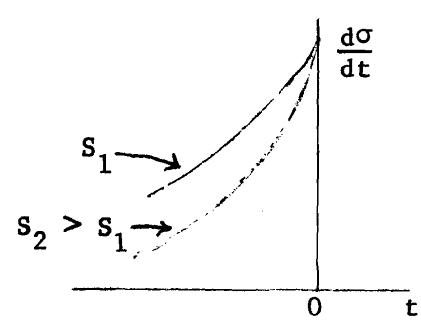
(b) Cuts do not change this prediction. The cuts which made the total cross section approach its constant limit so slowly do not distinguish between (AB) and ($A\bar{B}$) scattering and therefore cancel.

(3) Variation of the diffraction peak with energy.

(a) For the one-Regge-pole exchange

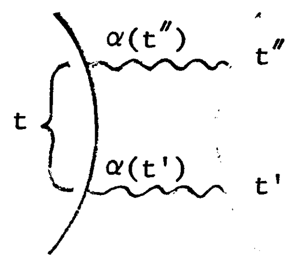
$$\frac{d\sigma}{dt} = f(t)\ s^{2\alpha(t) - 2} . \tag{10}$$

The Pomeranchuk pole has been given spin one at $t = 0$ in order to agree with the constant cross section. The property $d\alpha/dt > 0$ at $t < 0$ leads to the prediction of a shrinking diffraction peak: This prediction was qualitatively verified in p-p scattering but not verified in π-p scattering. I have not seen any convincing argument which simultaneously explains these two experiments. Some of the possible explanations are: first, that several Regge poles are involved and that their parameters



vary more exotically than we had expected; second, that there are moving cuts as well as poles. Of course, it is also possible that some entirely different explanation is needed!

Let us discuss the second possibility, (i.e., there are moving cuts as well as poles). We will consider first the origin of these cuts. Amati, Fubini and Stanghellini[1] have studied two-Regge-pole-exchange diagrams where we make up a total exchange of momentum transfer t in two steps t', t''. That is, we exchange $\alpha(t')$ and $\alpha(t'')$:



The total momentum transfer can consist of a small t' and large t'' or vice versa. There is a continuum of combinations. The total spin exchange $J(t)$ in terms of the separate exchanges is given by

$$J(t) = \alpha(t') + \alpha(t'') - 1 \; . \qquad (11)$$

Since there exists a continuum of t' and t'', there is also a continuum of spins exchanged. This is the origin of the cut, i.e., instead of exchanging one spin, we are exchanging an entire line of spins. If another diagram representing m-pole exchange were considered, one would get another cut and a similar formula would result.

In the work of Amati et al., all momentum transfers t, t' and t'' are physical ($t \leq 0$). Also recall that α is an increasing function of t in this region, and $\alpha(t = 0) \leq 1$. Therefore, $J(t)$ is maximum when t', $t'' = 0$. Then the highest value the cut can reach is one. The cut does not spoil

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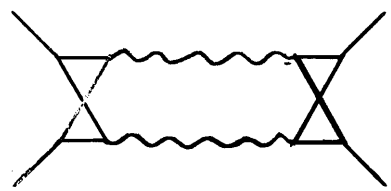
1. D. Amati, S. Fubini, A. Stanghellini, Nuovo Cimento 26, 896 (1962).

our ideas that the cross section should not be rising as a power of the energy.

Some of us[2] had studied these same m-pole exchange diagrams at unphysical momentum transfers $t > 0$, and we had noticed that although $J(t)$ is well-behaved in the physical region, it increases at positive t without bound. This led us to suspect that the cut should cancel with other diagrams. Last winter, S. Mandelstam[3] gave an argument that the cut does in fact vanish. Simultaneously, however, Mandelstam proposed that there are more complicated diagrams (see sketch) in which the cuts do not cancel, and which satisfy Eq. (11) again. Here straight lines represent ordinary particles and wavy lines represent Regge poles. The new feature is that one has a multiparticle intermediate state with a crossed structure, instead of just a two-particle intermediate state. This diagram is more complicated since it involves more particles and it is not clear that the same objection would not apply to this diagram as to the first one. Theoretically, the whole subject is still rather unsettled.



In any case, let us suppose that cuts really exist, and inquire what important qualitative properties they might possess.

The first important consideration is that the cut would lie above the pole, except at zero momentum transfer. In the sketch below the solid

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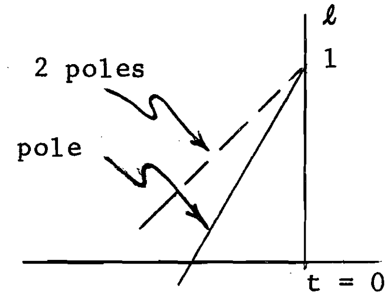
2. G. Chew, S. Frautschi, S. Mandelstam, Phys. Rev. 126, 1202 (1962).
3. S. Mandelstam, private communication.

line represents the pole and the dotted line represents the highest obtained from exchanging two poles. If more poles are exchanged, one gets a still higher upper limit, eventually approaching one. To see how this comes about, consider the formula previously given:



$$J(t) = \alpha(t') + \alpha(t'') - 1 . \tag{11}$$

The quantity t is really momentum transfer squared, i.e., the momentum transfer is $\sqrt{|t|}$, and there are "triangular" inequalities such as

$$\sqrt{|t|} \leq \sqrt{|t'|} + \sqrt{|t''|} . \tag{12}$$

If we consider a model in which the pole is moving in a straight line, with

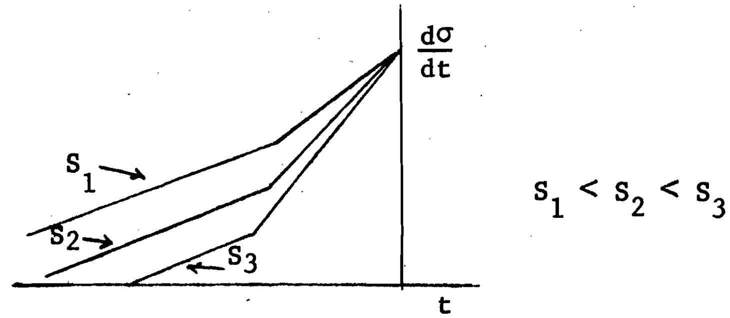
$$\alpha(t) = 1 + \alpha'(0)t , \tag{13}$$

then the constraint (12) allows the cut to extend up to the value given by $t' = t'' = t/4$:

$$J(t) = 2 \left[1 + \frac{t}{4}\alpha'(0)\right] - 1 = 1 + \alpha'(0)\frac{t}{2} . \tag{14}$$

That is, if we compare the upper limit of the cut with the position of the pole, we see at each t that the cut is twice as close to J = 1 as the pole. Therefore the cut produces less shrinking of the diffraction peak.

The details are model-dependent. Suppose, for example, that the coupling of the pole is quite strong and the coupling of two poles is weaker. Then, at some intermediate energy, the pole exchange would dominate. As the energy is increased, the cut exchange would begin to dominate because of its higher spin, first at large $|t|$ and then at progressively smaller t. Experimentally, a shrinking diffraction peak would be observed at intermediate energies, but the rate of shrinkage would diminish, especially with increasing energy, at large t as shown in the sketch.



The proton-proton data are uncertain at the present time, but some have suggested that in fact the shrinking vanishes as the energy increased. To be certain of this, one must have smaller errors in the data at high energies and at momentum transfers of about 0.5 to 1 Bev.

Another consequence is that, in the forward direction, the cut comes right up to the pole. As has been previously stated, this indicates that the forward scattering (also the total cross section by the optical theorem) has heavier competition than it did before. If one looks carefully, one finds that, at sufficiently high energy, the pole "wins out" over the cut*, but just logarithmically. This is the basis of the previous statement that the cross section would approach a constant logarithmically.

Experimentally, π-p scattering resembles the old-fashioned diffraction-scattering theory. It is possible that π-p is much more asymptotic than p-p scattering and the shrinking is much less, for this reason, as Moffat and others have suggested.[4] There is a question here of what normalization

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*Provided we assume that the discontinuity across the cut does not diverge as $J \to 1$.

4. e.g., Gatland and Moffat, to be published (Phys. Rev.).

to use in the asymptotic formulae. The traditional one for p-p is

$$\frac{d\sigma}{dt} = f(t)\left(\frac{S}{2M_p^{\,2}}\right)^{2J(t) - 2} \tag{15}$$

as suggested by the expansion of the cosine of the angle in the cross channel,

$$\cos\theta_t \sim \frac{S}{2M_p^{\,2}} \tag{16}$$

On the same basis in π-p scattering,

$$\cos\theta_t \sim \frac{S}{2M_p m_\pi} \tag{17}$$

suggests

$$\frac{d\sigma}{dt} = g(t)\left(\frac{S}{2M_p m_\pi}\right)^{2J(t) - 2} . \tag{18}$$

This new normalization makes the cross section at given S more asymptotic.

Unfortunately, there is, in the residue of the Regge-pole term, another factor which precisely cancels the normalization factor suggested by the expansion of cos θ_t. Therefore, we have no theoretical basis for the normalization as described above.

If π-p scattering is asymptotic above 7 Bev, then perhaps, in the intermediate region below 7 Bev, there is a shrinking peak. But experiment provides no evidence of such a peak. There is another difficulty as well. One would expect π-p scattering to become asymptotic before p-p in the present picture. However, p-p scattering becomes smooth at a relatively low energy while π-p exhibits resonances at energies up to about 2.5 Bev. From this point of view, π-p appears to become asymptotic more slowly. Therefore the suggestion that π-p scattering is simply more asymptotic is not very convincing.

As mentioned before, the cuts introduce more parameters; in fact, it was mentioned that there was an entire chain of cuts, so it is evident that the picture is hopelessly complicated until a theory is found which, for example, would permit the calculation of cuts if one were given a pole. We will not have a reliable theory until we settle the question of which cuts cancel and which do not. Thus, at present, it is difficult to predict what will happen at 60 Bev or higher energies.

Another interesting question in high-energy scattering is the search for exchanges of fixed J's, i.e., J = 1, which are supposed to correspond to the exchange of elementary particles according to some indications from field theory. It is obvious that the problem of differentiating between exchange of a fixed spin and exchange of a moving spin is made harder by moving cuts.

Pomeranchuk has also suggested that charge-exchange reactions, for example

$$\pi^- + p \rightarrow \pi^o + n \tag{19}$$

would have cross sections which would approach zero as the energy increased. Physically, this is an inelastic reaction which competes with an increasing number of inelastic reactions for the constant total cross section. In Regge terms, the distinction between such a charge-exchange cross section and an elastic one is that the charge-exchange cross section will still have the structure

$$\frac{d\sigma}{dt} \propto s^{2J(t) - 2} \tag{20}$$

but the spin "J" involved here has to carry across isotopic spin = 1 in order to affect the change from π^- to π^o. The isotopic spin involved in the elastic scattering, on the other hand, could be either 1 or 0. Pomeranchuk's idea indicates that the pole with isotopic spin one should have a smaller spin than the leading pole with isotopic spin zero, thereby allowing the elastic reaction to dominate.

We can now see how this suggestion holds up in the presence of cuts. If we consider the diagrams which give cuts, we can look for an I = 1 exchange. There are two ways to achieve this exchange. First, one can exchange I = 0 once and I = 1 the other time; or, second, one can exchange I > 0 both times. In either case, we apply the formula

$$J(t) = \alpha(t') + \alpha(t'') - 1 \tag{11}$$

remembering that $\alpha(I = 0) \leq 1$ and $\alpha(I = 1) < 1$. The upper limit of the cut at t = 0, in the first instance, will be the value of the I = 0 pole

which is just one, plus the value of the I = 1 spin, minus one. So once again the cut just comes up to the pole

$$\left[J(t{=}0) = \alpha_{I=0}(0) + \alpha_{I=1}(0) - 1 = 1 + \alpha_{I=1}(0) - 1 = \alpha_{I=1}(0)\right].$$

In the other case, one can convince oneself that the cut falls short and lies below the pole. This means that Pomeranchuk's idea is preserved even if there are cuts. The spins that are involved are still less for higher "I" spin exchanges. It was on this basis that it was stated earlier that the approach to the Pomeranchuk limit for particle and anti-particle cross sections would still depend upon a power law even in the presence of cuts.

There are two ways to see these other exchanges. One is through interference with diffraction. For example, interference due to exchange of ρ, ω, etc., is presumably responsible for the differences between p-p and p-$\overline{\mathrm{p}}$ cross sections.

One can also look at reactions where diffraction cannot take place. For example,

$$\pi^- + p \rightarrow K^+ + \Sigma^- \tag{21}$$

$$n + p \rightarrow p + n \tag{22}$$

In the first case, you exchange something with strangeness and in the second case, something with isotopic spin equal one so Pomeranchuk exchange is not allowed. The pole predictions are modified in the following way by possible cuts. As usual, the asymptotic region will set in more slowly and where previously one expected shrinking, the shrinking will slow down. It is still true that these cross sections should go to zero as a power of energy because, as was pointed out, the cut does not extend beyond the pole at zero momentum transfer. These cross sections are thus expected to become very small and hard to see.

In conclusion, the remaining region, which we have not discussed, is the large-momentum-transfer region. The reason this was not discussed is that there is no reason for one Regge term to dominate, even if cuts are absent at very large momentum transfers. Instead, a number of poles probably get close to each other and compete. Some new idea is needed to organize the theory. This should not obscure the fact that this region is a direct test of very-small-distance behavior and is almost certain to be very important. The existing experiments suggest that: high intensity will be necessary if one is to see anything; also that the cross section is becoming very small and that small-distance behavior is cutting off sharply.

Discussion

L.W. Jones (Michigan): In Drell's paper last summer, he started out by mentioning something that Gribov had done, indicating that a simple optical model of diffraction is contradictory to unitarity. Is this still true?

S.C. Frautschi: Yes. Gribov, Lehmann, and others pointed out that if $d\sigma/dt$ depends only on t over a range of S, the dispersion relations cannot be satisfied. However, $d\sigma/dt$ might closely approach the simple optical-model result as $S \to \infty$, without violating the dispersion relations.

L.W. Jones: Is this in conflict with Serber's model?

S.C. Frautschi: As I understand it, Serber's model is intended to be a representation of a dominant trend at high energies and, of course, does not attempt to explain small deviations from the dominant trend. He would regard the existing shrinking in p-p scattering as a secondary effect.

J. Orear (Cornell): Are you saying, as you go toward more negative t's, that the shrinkage would become less in the pole-plus-cut situation?

S.C. Frautschi: Yes. The ratio $(d\sigma(S_1,t)/dt)\ /\ (d\sigma(S_2,t)/dt)$, with $S_1 > S_2$, decreases as $|t|$ increases, but most of this decrease occurs at small $|t|$ and the ratio levels off (stops shrinking) at larger $|t|$.

Part II - Nearly-Elastic and Highly-Inelastic Scattering at High Energies

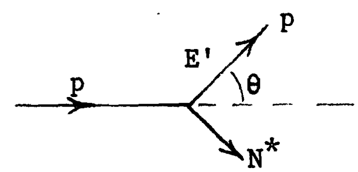
Nearly-Elastic Scattering[5]

Nearly-elastic scattering, exemplified by the reaction

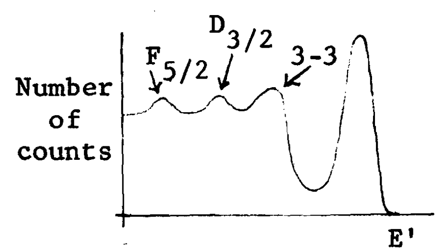
$$p + p \rightarrow p + N^* , \tag{23}$$

is a means of getting more information of essentially the same kind that one got from two-body reactions. A resonance is produced which decays so that the final state has three particles. For orientation purposes one can treat the resonance as a particle and proceed as in the case of two-body reactions.

In experiments of this nature, done to date, the energy of the proton coming out of the reaction, and its angle relative to the incident proton are measured. One holds the energy of the incident proton and the angle fixed and varies the energy E' of the outgoing proton. For each E' the mass of the recoil N^* system can be deduced. The number of counts is plotted against E'.

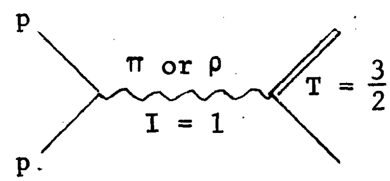


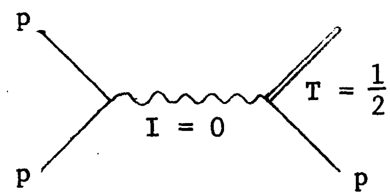
There is first an elastic peak then an inelastic continuum in which the inelastic proton came off with less energy. Superimposed, on the continuum, are bumps corresponding to the 3-3 resonance, the $D_{3/2}$ and $F_{5/2}$ resonances.



5. A more detailed account can be found in: A.P. Contogouris, S.C. Frautschi and H.S. Wong, Phys. Rev. 129, 974 (1963).

In experiments below 5 Bev one saw all three of these bumps, the 3-3 bump being especially prominent.[6] However, at experiments above 15 Bev, the 3-3 peak had gone away but the other two peaks remained.[7] This difference in energy variation is consistent with the notion that, in order to make the 3-3 resonance, one had to exchange an $I = 1$ system which could be the π or ρ trajectories. To make the $T = \frac{1}{2}$ resonances one could exchange the Pomeranchuk trajectory, which has a higher spin and therefore has a higher cross section at high energies. This is the kind of prediction (as mentioned in Part I of this report) that still holds in the presence of cuts. The cuts will not remove the dominance of $I = 0$ exchanges.





There are some interesting matters here which deserve further experimental study. The first question is: if one makes a more complete survey of the energy and angle dependence, will one see something like an "old-fashioned diffraction peak", or a shrinking peak? That is, one can ask the same question as for the elastic reactions. The nearly-elastic reactions have one disadvantage. The accuracy will be less because we have

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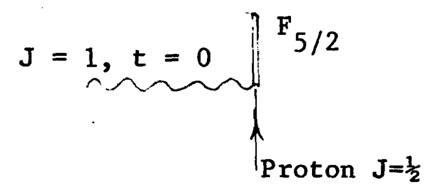
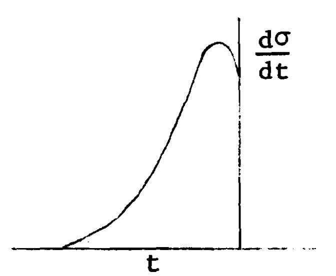
6. G.B. Chadwick, G.B. Collins, P.J. Duke, T. Fujii, N.C. Hien, M.A.R. Kemp, and F. Turkot, Phys. Rev. 128, 1823 (1962).

7. G. Cocconi, A.N. Diddens, E. Lillethun, G. Manning, A.E. Taylor, T.G. Walker, and A.M. Wetherell, Phys. Rev. Letters 7, 450 (1961).

to pick these peaks out of the background which is an inherently ambiguous procedure. All one can look for is large-scale effects.

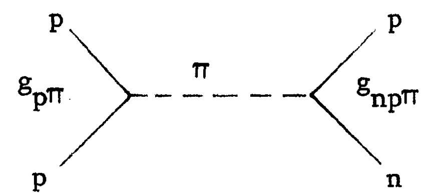
A second question (a "footnote" to the theory of diffraction) is concerned with diffraction dissociation. The Pomeranchuk trajectory was supposed to be responsible for elastic diffraction; now we exchange the same trajectory, with the possible addition of associated cuts, to make an isobar. Isobar production, however, looks inelastic, and one might interpret Pomeranchuk's ideas about the competition among inelastic processes as implying that this cross section should eventually approach zero. In the case of $F_{5/2}$ production we are quite sure that this must be so. We look at the vertex where the isobar is produced. For scattering with zero momentum transfer, a proton comes in, collides with a Regge pole carrying spin 1, and an $F_{5/2}$ isobar emerges. But at zero momentum transfer, no orbital angular momentum is available to help the incoming spins add up to J = 5/2. In other words, there is a zero in the coupling of this exchange just because of angular momentum requirements. This means that we expect this reaction to appear as something that starts out like a diffraction peak but gets "timid" as it approaches t = 0. It follows that the integrated cross section will decrease slowly, in agreement with a literal interpretation of Pomeranchuk's ideas about inelastic reactions.



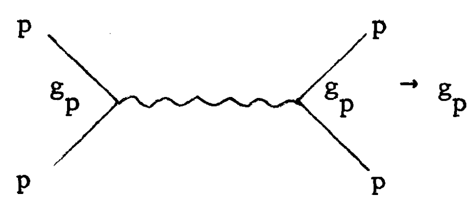


In the case of the other resonance, $D_{3/2}$, we do not have this angular momentum argument available to give us a zero, so it is an open question whether the differential cross section fails in the forward direction or not. These questions should be investigated more thoroughly at the present energy range and, until that has been done, we cannot be sure if it is important to go above 30 Bev to get more information.

Another point arises. It can be introduced by remembering that in an ordinary particle exchange, for example a π exchange, one has the property that the matrix element factors into a coupling on the left side times the coupling on the right side. It was shown by V. Gribov, I. Pomeranchuk[8] and M. Gell-Mann[9] that the same sort of factorization holds for Regge trajectories. For example, exchange of the pion trajectory still factors into a product of couplings.

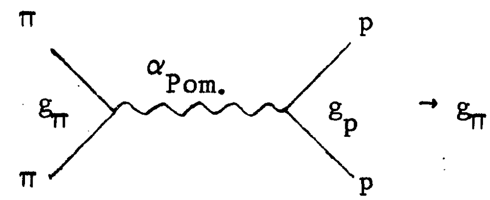


For elastic reactions, it is difficult to check this idea. Consider for example, Pomeranchuk exchange. We commence with the p-p diffraction peak; exchanging the Pomeranchuk trajectory we have a coupling between the proton and the Pomeranchuk trajectory on the left and the same coupling on the right. Thus, in principle, the measurement of the cross section would determine

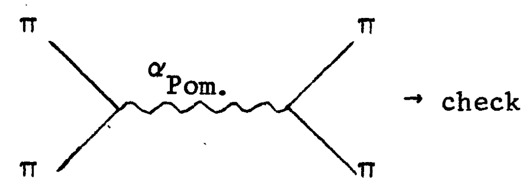


8. V.N. Gribov and I. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).
9. M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962).

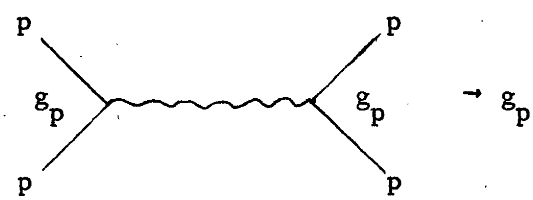
this coupling. We can then perform a π-p experiment. This time we use pion coupling on the left and proton coupling on the right. In principle, these two experiments should then determine the pion coupling.



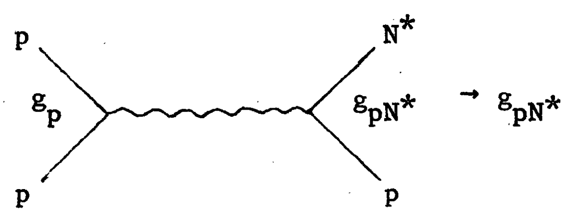
We have, in addition, π-π scattering which depends only on the pion coupling, which we have already determined. We thus have a check on this "factorization idea". The difficulty, of course, is the unavailability of stable pion targets.



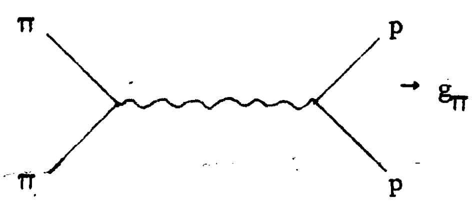
If we consider the production of isobars, we can devise sequences where we can get a real check. We commence as above with p-p scattering.



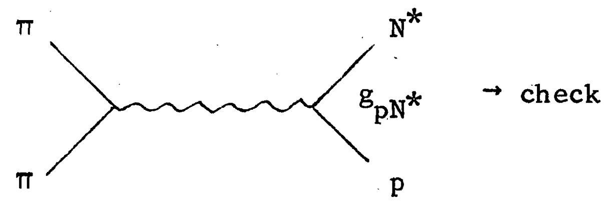
and then make an isobar, giving us the p-N*-Pomeranchuk coupling, and then



repeat the same two experiments with a pion



and



Using the pion to produce an isobar provides a check. The check will not work if there are cuts, however, because interference from the cuts will complicate the interpretation of the observed cross sections.

One of our difficulties, in obtaining checks of the factorization rules, was the lack of stable targets. Aside from the proton, the stable targets available in nature are the heavy nuclei. For example, one can compare

$$p + p \rightarrow p + p \quad (\rightarrow g_p) \tag{24}$$

with

$$p + d \rightarrow p + d \quad (\rightarrow g_d) \tag{25}$$

and then

$$d + d \rightarrow d + d \quad (\rightarrow \text{check}) \tag{26}$$

These would give us, respectively, the proton coupling, the deuteron coupling and a check.

If we consider this class of experiments, we must decide what region is asymptotic. Naively one supposes that v/c must approach one. This means that if one deals with heavy nuclei, higher energies are required. This is confirmed by investigation of the condition under which cos θ becomes large. For heavy nuclei, energies in excess of 30 Bev are required.

If there are no cuts, then at 10^3 Bev one should certainly be in an asymptotic region and one would expect the amplitude could be factored into a number of couplings. On the other hand, down at energies with which we are more familiar, the total cross section (if it is geometrical) will be

$$\sigma_{total} \approx 2\pi(R_1 + R_2)^2 \tag{27}$$

where R_1 and R_2 are the radii of the scattering objects. This does not factor into a product of "things involving one particle" times those of the other. In order to proceed to an asymptotic picture, where we had a "product of couplings", we must go through a transition region.

If there are cuts, the size of the transition region increases but a factored form still emerges in the high-energy limit. A specific example was provided by Gell-Mann and Udgaonkar,[10] who found a cut with a different origin than those discussed in Part I. This cut may be due to the spatial structure of a complex nucleus, which extends beyond the range of the forces. In dispersion language, this means that one gets anomalous thresholds in addition to the normal ones. Gell-Mann and Udgaonkar gave the new cuts in angular momentum the physical significance of eclipse terms, i.e., one of the nucleons in the deuteron shielding the other one. The possible existence of these cuts is just as uncertain as the existence of other cuts. Gell-Mann and Udgaonkar were able to make a numerical estimate, however, of the length required for the transition from the familiar region to the unfamiliar one. It turned out to be such a slow transition, that increasing the energy to 10^3 Bev would be of little help. With rather light nuclei such as carbon, an increase from 10 Bev to 10^3 Bev

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10. M. Gell-Mann and B.M. Udgaonkar, Phys. Rev. Letters 8, 346 (1962).

would change the expected cross section by only 10%. If cuts are not important, then, one might see this transition in the region between 10 and 10^3 Bev, but if cuts are important, the transition will probably be extremely slow.

Highly-Inelastic Collisions

If a high-energy accelerator is built, highly-inelastic events will be the most common. Some information on these events is now available from cosmic rays and a theory exists which corollates this information, but the theory is descriptive and too crude to pose any fundamental questions. Until a few years ago, the most popular theories for interactions in this region were those by Fermi, Landau and Heisenberg, all of which were based on the idea that a large number of particles would be produced in the interaction and one could make some simplifications. The Fermi theory involved statistical calculations. Others used hydrodynamical approaches. They all expected quantum effects to become unimportant, and large numbers to "take over" at high energies.

In actual fact, it has been found that nearly all of the particles come out either in the forward or backward direction and much of the time they seem to come out with a structure which has been called fireballs. This suggests that the process splits into several vertices and each of these produces only a few particles instead of a very large number. For this reason, it seems that approaches based on large numbers are inadequate. Present theories follow the same lines that are applied at lower energies in terms of exchange of particles, etc., using definite quantum effects.

The following is a list of fairly well-accepted experimental points, applying to π-p and p-p scattering.

1) The total cross section appears to be approximately constant $\pm$ 50% up to 10^4 Bev. This is consistent with Pomeranchuk's ideas.

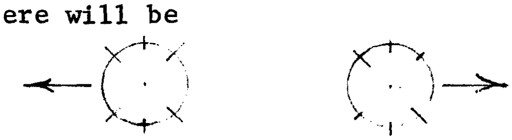
2) The average number of secondaries produced in an interaction $\overline{n}_S$ has traditionally been considered to vary as the ¼ power of the laboratory energy, but the particle exchange theories favor $\overline{n}_S \propto \ln(E_{lab})$. It increases very slowly in either event. At 10^3 Bev, $\overline{n}_S \approx 12$.

3) Most of the secondary particles are pions.

4) Experiments are emphatic in showing that the transverse momentum $p_{(t)}$ of the π's has a peak at about 0.4 Bev and is strongly damped at larger transverse momenta.

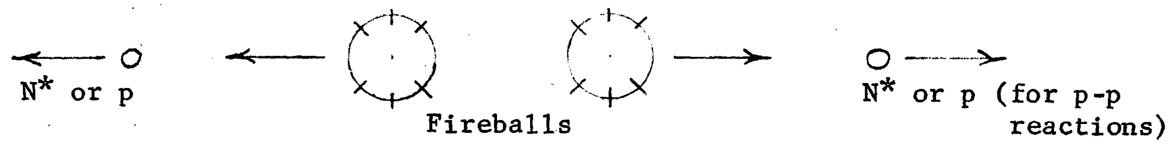
5) At laboratory energies above 10^3 Bev, one often sees structures called fireballs; one does not always see them and they are not always symmetric. These are illustrated in the center-of-mass sketches below. If you have two protons going in:

then coming out there will be

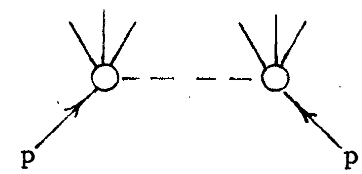


a collection of pions moving forward and another moving backwards. If one looks at the forward-moving collection, one will find, in a frame of reference moving with this collection, that the transverse momenta of these pions is very small ($\approx$ 0.3 Bev). Relative to this frame, the longitudinal momenta are also small. Thus, all of these pions in the forward direction appear to come from a common center.

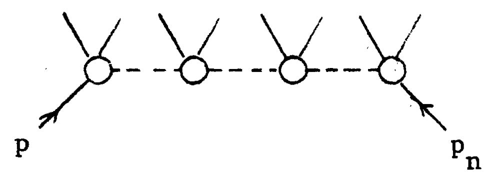
6) People think they see isobars preceding these fireballs, carrying off a considerable fraction of the energy



When considering these points theoretically, one would start as usual by exchanging some particle,[11-13] a π or K or a Regge pole. Then the mathematical form will factor into two vertices, one on each side, which can be treated separately.



You can then factor each side in terms of another exchange and so on. This can be continued if desired until only a few particles emerge from each vertex.



The advantage of this process is that one can fractionate a high-energy process into a series of lower-energy processes where the treatment is more familiar. Mathematically, each of these exchanges can be represented by some factor which, if one was exchanging a particle, would be, for example:

$$\frac{1}{t - M^2} \times (\text{form factor}) \tag{28}$$

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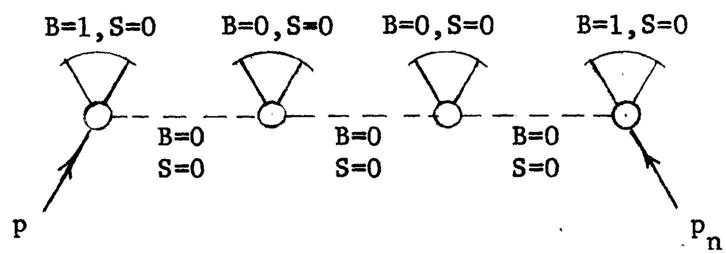
11. F. Salzman and G. Salzman, Phys. Rev. 120, 599 (1960) and 121, 1541 (1961).
12. C. Goebel, Proceedings of the 1961 International Conference on Theoretical Aspects of Very-High-Energy Phenomena at CERN, p.353.
13. D. Amati, S. Fubini, A. Stanghellini, and M. Tonin, Nuovo Cimento 22, 569 (1961); D. Amati, S. Fubini, and A. Stanghellini, Phys. Letters 1, 29 (1962).

or, if one is exchanging a Regge trajectory, the factor would be:

$$\frac{\beta s^{\alpha(t)}}{\sin \pi\alpha} \quad . \tag{29}$$

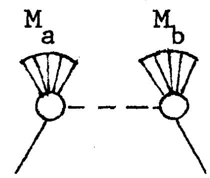
In either case, the denominator gets larger if the exchange mass increases so that the exchange of a heavy object is inhibited.

It is reasonable to expect, then, that the objects exchanged are relatively light, producing relatively long-range forces, and that means that we can specify the quantum numbers which are being exchanged most of the time as baryon number = 0 and strangeness = 0. One then knows the quantum numbers emerging from each vertex. The first vertex carries the



original baryon number along. The second one produces no baryons and no strangeness, etc. We then have a series of several processes. For example, the second one has two objects coming in and producing several particles at a fairly low energy, with no net baryon number or strangeness. We can be fairly certain that, most of the time, these are all pions. The situation is similar, all along the chain, except for the original baryon at either end. This is, then, the qualitative explanation of why one gets mostly pions.

If we consider again the exchange factors and suppose that they damp large momentum transfers fairly strongly (this is automatically insured by the Regge form, or by a rapidly-falling form factor), then the momentum transfers will be small. This can directly explain why the transverse momenta are small, since large transverse momenta would inevitably build up a large momentum transfer. To be more specific, suppose in the previous illustration the forward-going clump of particles had a mass M_a and the backward clump M_b, then one has a definite relationship for the momentum transfer between the proton and the clumps of particles.



$$t = -p_t^2 - \frac{M_a^2 M_b^2}{S} + \text{small terms}, \qquad (30)$$

where p_t is the transverse momentum of the clump of particles. This relation indicates that if the momentum transfer to the clump of particles is small, then the transverse momentum of the clump is necessarily small.

Now, for the case of our "multiply-factored interaction", the argument can be repeated for each of the clumps. We have four clumps of particles all moving along the original directions of motion, each of them representing a fairly-low-energy interaction. One repeats this argument until one arrives at vertices corresponding to only a few Bev and then one considers what sort of pions come out of such a low-energy vertex. We know experimentally that these have transverse momenta of hundreds of Mev. This sort of argument indicates why the transverse momenta should be about the same, independent of the initial momentum.

The preceding discussion can be related to the production of fireballs.[14] Suppose we have a laboratory energy of about 10^3 Bev and produce an exchange

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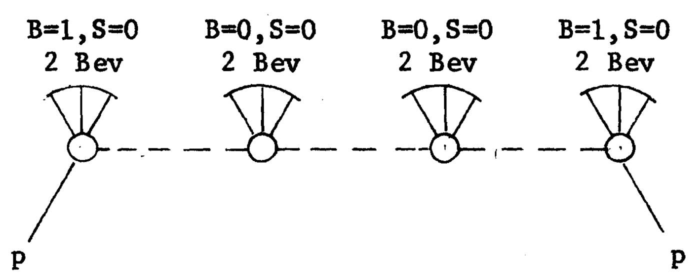
14. S.C. Frautschi, Nuovo Cimento <u>28</u>, 409 (1963).

of some object with the emission of forward and backward clumps of particles with masses M_a and M_b. Suppose the momentum transfer is limited to values $|t| < 1\ \text{Bev}^2$ by the form of the propagator. We have, from Eq. (30),

$$M_a^{\,2} M_b^{\,2} \le S\,|t|\,. \tag{31}$$

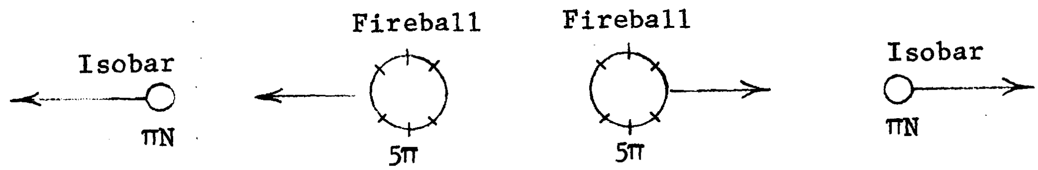
This implies that M_a and M_b will each be bounded by approximately 30 Bev. Thus the clumps or factored processes have much lower energies than the original reaction.

Now consider the forward clump, produced by the incoming proton and the exchanged system. This reaction has enough energy (~ 30 Bev) to make the outgoing particles peak strongly fore-and-aft. Therefore, we break the forward clump into two groups, each with mass of a few Bev according to Eq. (31) (more careful evaluation yields a mass of about 2 Bev). Similarly the backward clump breaks into two groups. The clumps could be



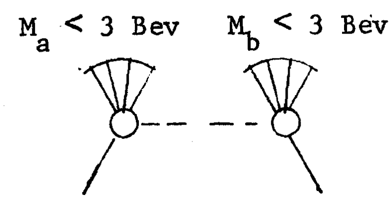
factored further, but at 2 Bev none of them are strongly peaked fore-and-aft so no further clearly separated groups would be found.

It is easy to identify the outside clumps (B = 1, S = 0, 2 Bev) as πN isobars. The inside clumps (B = 0, S = 0, 2 Bev) contain a number of π, ρ, ω, etc., which decay into several π's. Since the energy is comparable to that liberated in $p\bar{p}$ annihilation, the number of π's emerging is probably about five. This group of pions is clearly to be identified with the "fireball". So we have two fireballs and two isobars, in accord



with the experimental picture.* The total number of secondary particles is about 14, consistent with experiment.

We now have to consider how this picture changes with energy. We had started at 10^3 Bev. Suppose the incoming energy were 10 Bev instead. Then we would have forward and backward peaking and we would make one exchange. Two groups of particles result. Each of



these groups of particles (according to $M_a^2 M_b^2 \leq S|t|$) would have an energy less than 3 Bev. Further exchanges do not yield distinct groups of particles. Thus, at 10 Bev, we obtain only one group forward and one group backward. As the energy increases from 10 Bev to 1000 Bev, the mass of each side slowly increases and, for example, the group on the left will slowly separate itself more distinctly into forward and backward groups. Thus the two clumps which were originally more or less isotropic will develop

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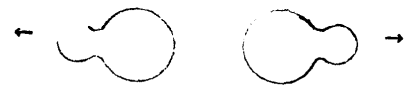
*In some cases the mass of one of the original clumps might be much less than the other, so that the clump would be fairly isotropic. We would then split up only the more massive, anisotropic clumps, resulting in a total of three instead of four groups, and one fireball would be missing.

like dumbbells as the energy rises and can eventually be considered as distinct backward and forward groups. If we continue to increase the energy above 10^3 Bev, the four groups separate into still more backward and forward groups. Thus, the prediction is that the number of fireballs slowly increases with energy.



The only part, of all this conjectured structure, that one could test with a 10^3 Bev accelerator is the first transition from a simple backward-forward division to a division containing two fireballs and two isobars.

Discussion

V.W. Hughes (Yale): Would you comment on what you would expect to learn from p-p scattering if you took into account the spin-dependent effect?

S.C. Frautschi: Polarization effects are mathematically the result of interference. In low-energy terms, it is interference between different partial waves. In high-energy terms, it can be interference between exchange of different Regge poles, cuts, etc. This is a source of information which would complement the differential cross section in a very useful way.