MAGNETIC-FIELD SHAPING BY SUPERCONDUCTING SURFACES

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This report is a preliminary description of some of the results we have obtained in our work on the possible use of superconductors for the shaping of magnetic fields. Since the work is still in progress, there are, of necessity, some details missing.

The investigations were begun at MURA about a year ago. Dr. S.C. Snowdon contributed part of the theory and assisted in the general discussions of the problem. Professor J. Dillinger (University of Wisconsin), who is in charge of the low-temperature laboratory, provided a number of suggestions and the necessary facilities for low-temperature measurements; Mr. I. Sviatoslavsky (MURA) assisted in the design of parts of the cryogenic equipment.

The purpose of this study was to examine and evaluate the possibilities of using superconducting materials for producing an FFAG-type magnetic field or some other field configurations of use in high-energy laboratories such as bending and focusing magnets,[1] etc. We suggest that this may be a possible new approach to the problem of producing the required magnetic fields. It appears, in the author's opinion, that difficult problems

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[1] A preliminary study of the use of superconducting magnets in FFAG accelerators was made by K.R. Symon, p. 162, Experimental Program Requirements for a 300 to 1000-Bev Accelerator and Design Study for a 300 to 1000-Bev Accelerator, BNL 772, August 1961 (revised December 1962).

will be faced in connection with the fringing fields associated with, say, 100-kilogauss superconducting magnets, in an accelerator laboratory. The present proposal effects some control over these fields.

It may be appropriate to review some of the basic properties of superconductors. We shall restrict ourselves to those which are directly relevant to the discussion which follows, rather than duplicating the report by Kruger.[2]

It is known that a soft superconductor has the following properties:

a) Zero electrical resistance. $R = 0$.

b) The magnetic flux is excluded from the material.

$B = H = 0$ (Meissner effect).

To clarify this last statement, let us consider the simple case of a long superconducting cylinder with its axis parallel to an external magnetic field H_e (Fig. 1). There are in fact two ways of describing the magnetic properties of the superconductor. One might say that inside the material $B = H = 0$; the Meissner effect is then explained by the presence of surface currents induced in the superconductor. Alternately, we might say that $B = 0$ and $H = H_e$ as the demagnetizing

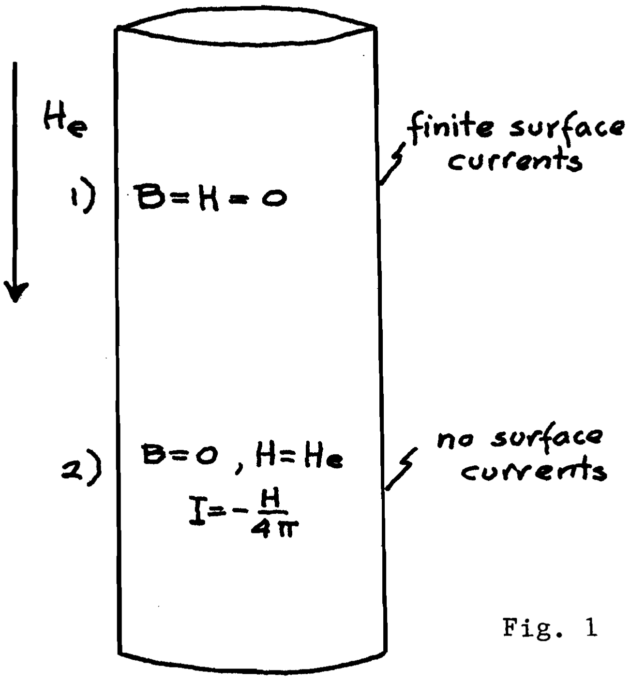

Fig. 1

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[2] P.G. Kruger, p. 417 of this volume.

coefficient is zero. This requires a volume magnetization given by

$$I = -\frac{H}{4\pi} \qquad (1)$$

Both descriptions are equivalent.

There is an additional difficulty when one tries to combine Ohm's law $E = \rho J$ for a superconductor where $\rho = 0$ with

$$\nabla \times E = -\frac{1}{c}\frac{dB}{dt} \qquad (2)$$

and the Meissner effect. This situation was corrected by London[3] with his famous equations. They are, however, valid only for the soft superconductors. A high critical field or hard superconductor presents zero resistance but an incomplete Meissner effect.

A Proposed Superconducting FFAG Magnet

The arrangement that will be described is a possible form of a model magnet which appears simple enough to be constructed at a reasonable cost. It may provide an indication of the future of superconductors in the construction of FFAG magnets. The present model may be suitable for a radial-sector accelerator. It has the potential of modification to a spiral-sector magnet.

The basic scheme consists of a source coil for producing the magnetic field and superconducting surfaces to trim or shape it to the desired manner.

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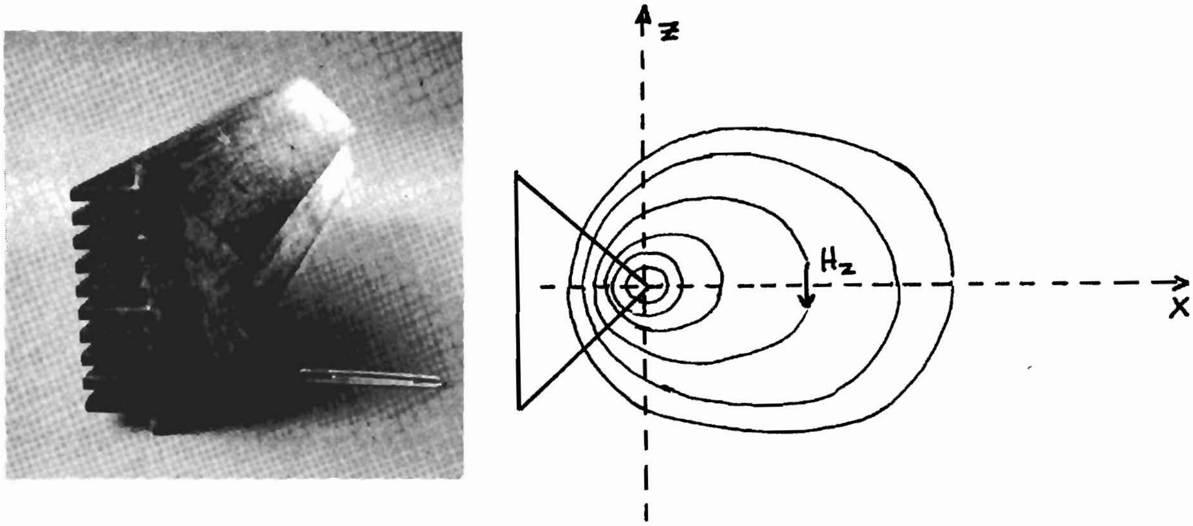
[3] F. London, Superfluids Vol. I, p.29, Dover Publications, New York, 1961.

A coil form which was used for this purpose is shown in Fig. 2a. The winding goes along the curved portion and the returns are distributed in the slots. This configuration will yield a main bundle of current which produces the magnetic field that will be shaped later on. We will represent this configuration and the resulting field by the scheme shown in Fig. 2b.



(a) (b)

Fig. 2

If one visualizes such a coil as part of one magnet for an FFAG radial-sector accelerator, the azimuthal extent will be given by the length of the coil's form, and the field region will be along the line x which will also lie on the median plane of the magnet.

The field expected from the configuration of Fig. 2 may be approximated by

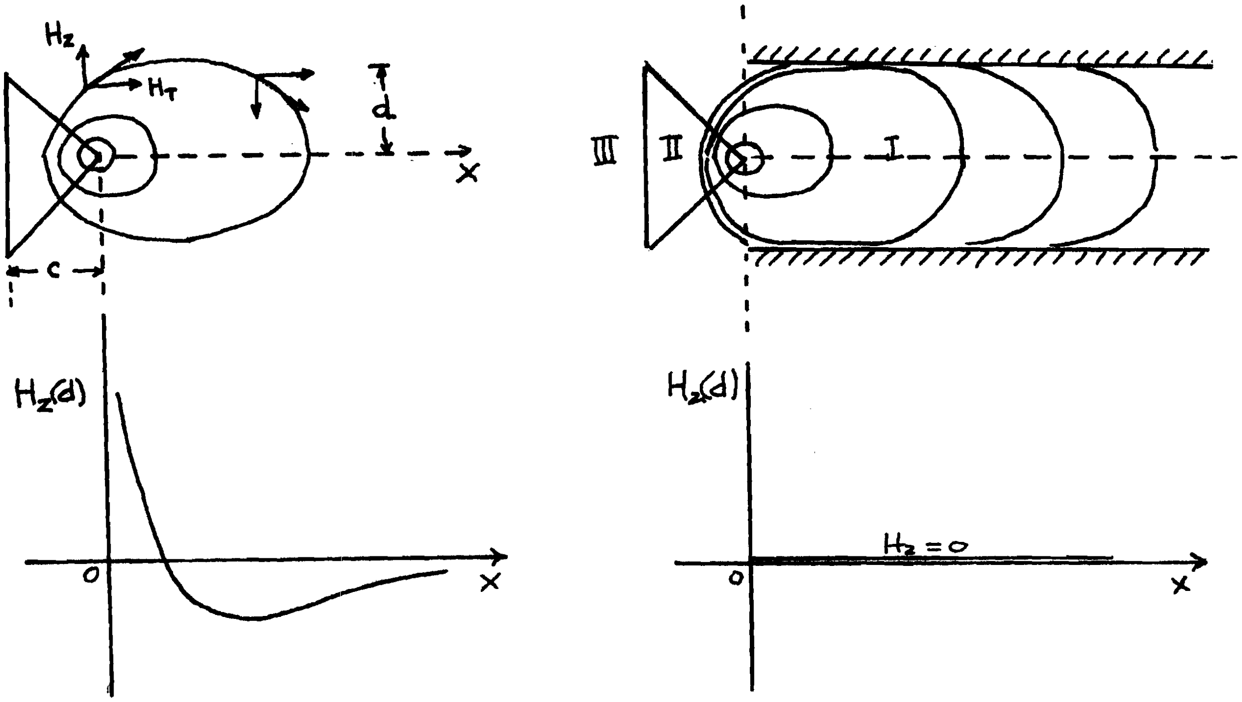
$$H_z \approx (1/r) - (1/r + c) \approx 1/r \qquad (3)$$

where r is along the x direction. This natural variation of the field with r is not appropriate for a scaling field; we wish to have field decreasing more rapidly, i.e.,

$$H = H_o \left(\frac{r}{r_o}\right)^k \approx H_o \, e^{\frac{k}{r_o} x} \,. \qquad (4)$$

The field index k = 9.3 corresponds to MURA's 50-Mev electron model.

Let us now examine the effects of introducing two superconducting plates. A naive picture of the situation provides some useful information. Consider Fig. 3. In Fig. 3a we show some of the field lines



(a) Distribution of the normal components of H at distance d from median plane.

(b) Normal components of H_z at distance d vanish, when plates are present.

Fig. 3

and the distribution of the normal components of the field at the distance d from the median plane before the superconducting plates are inserted. In Fig. 3b one can see the field that results after the plates are inserted. The Meissner effect will prevent the field from penetrating into the plates, by inducing a surface current distribution which will cancel the H_z components at the boundary.

The problem can be solved analytically by evaluating the potentials corresponding to the three regions indicated in Fig. 3b. These potentials are solutions of a two-dimensional Laplace's equation with the appropriate boundary conditions in each of the regions. In particular, the normal component of H vanishes at the surface of the superconductor. On the median plane the vertical field is then given by:

$$H_z = \frac{2\pi I}{d} \sum_n \left\{ 1 - (-1)^n \frac{e^{-\mu_n c}}{\mu_n d} \right\} e^{-\mu_n x} . \tag{5}$$

This is in fact a form of a scaling field; μ_n is a measure of the field index. As μ_n is a function of the plate separation,

$$\mu_n = \frac{(2\ n + 1)}{2d} \tag{6}$$

We then have the capability of changing the field index by merely moving the plates.

The plot of equation (5) is shown in Fig. 4. One can see that for large values of x we have an exponential field; however, near the current bundle (x = 0) the 1/r dependence predominates. This is undesirable as one would like to have a "good" field up to the high-field region. A partial correction of this effect is described later on.

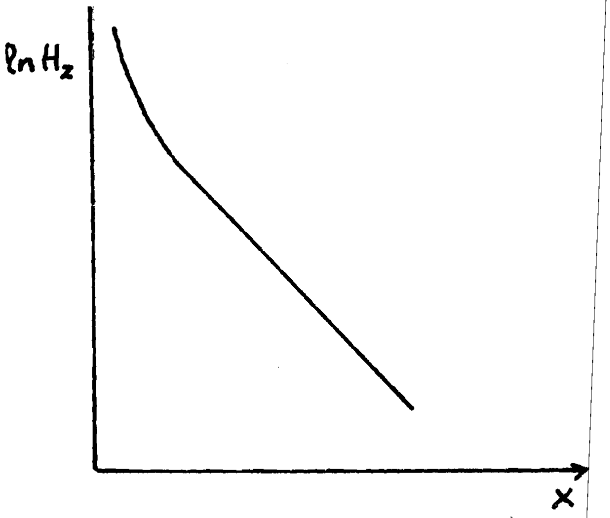


Fig. 4

In order to examine this problem experimentally without the use of superconducting materials and expensive cryogenic apparatus, an audio-frequency analog model was constructed. The magnet consisted of a copper winding excited by a signal of about 15 kc. The superconducting plates were simulated by copper plates. The electrical skin effect here represents the Meissner effect of the superconductor. The results of the measurements taken in this manner are shown in Fig. 5. The upper curve represents field measurements when the coil was excited with dc current and 15 kc without copper plates. Both agree within the experimental error.

The middle curve represents data taken at 15 kc with the plates in position. The lower curve is the result obtained from equation (5). The curve is displaced for comparing the slope only. The discrepancy in μ may be interpreted as due to a poor measurement of the plate separation. Further experiments will clarify this point; however the agreement is good.

The effect of change in μ due to plate separation is shown in Fig. 6.

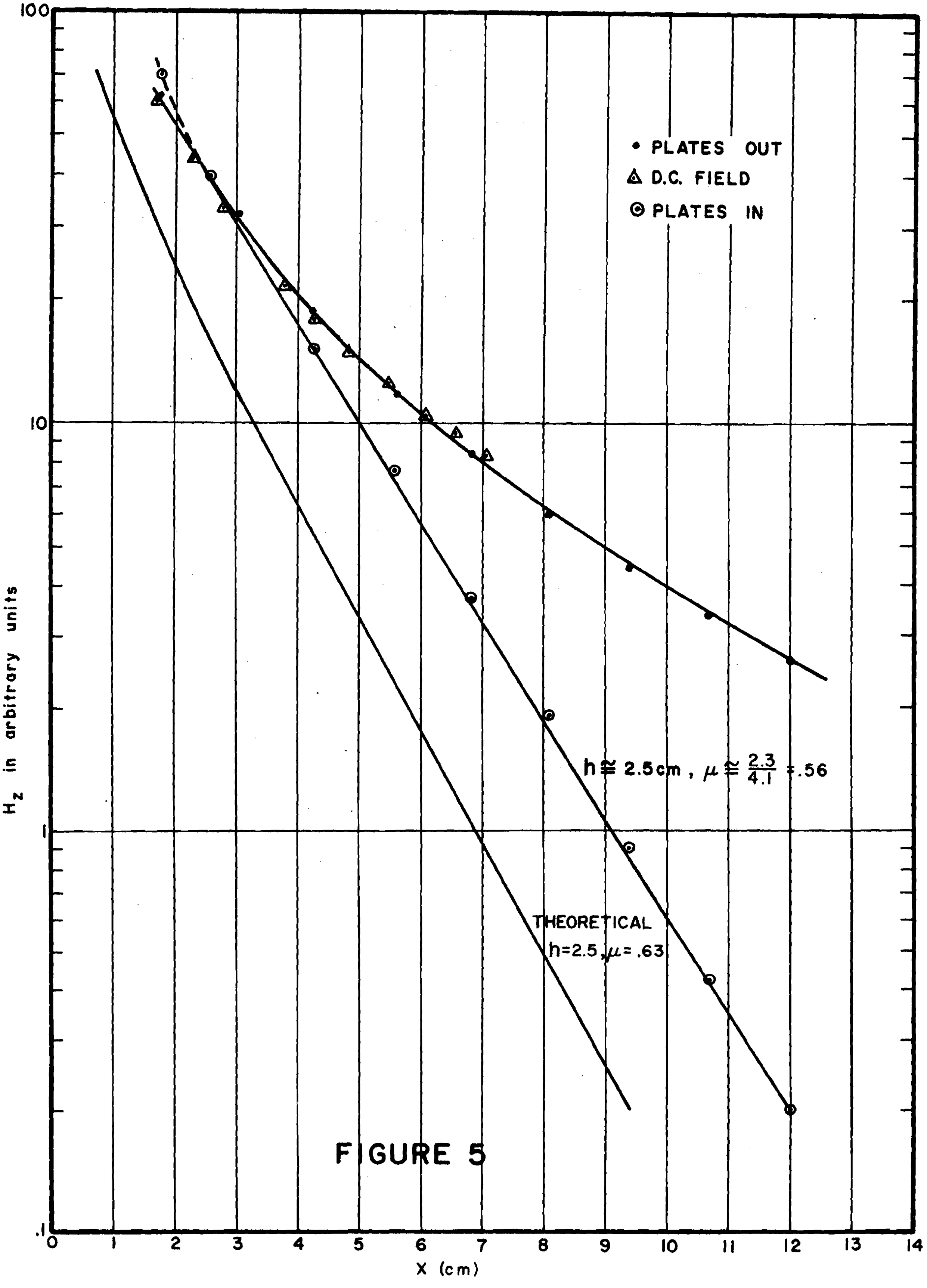

100
10
1
.1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
X (cm)
H_z in arbitrary units
• PLATES OUT
△ D.C. FIELD
⊙ PLATES IN
h ≅ 2.5cm , μ ≅ 2.3/4.1 ≅ .56
THEORETICAL
h=2.5, μ= .63

FIGURE 5

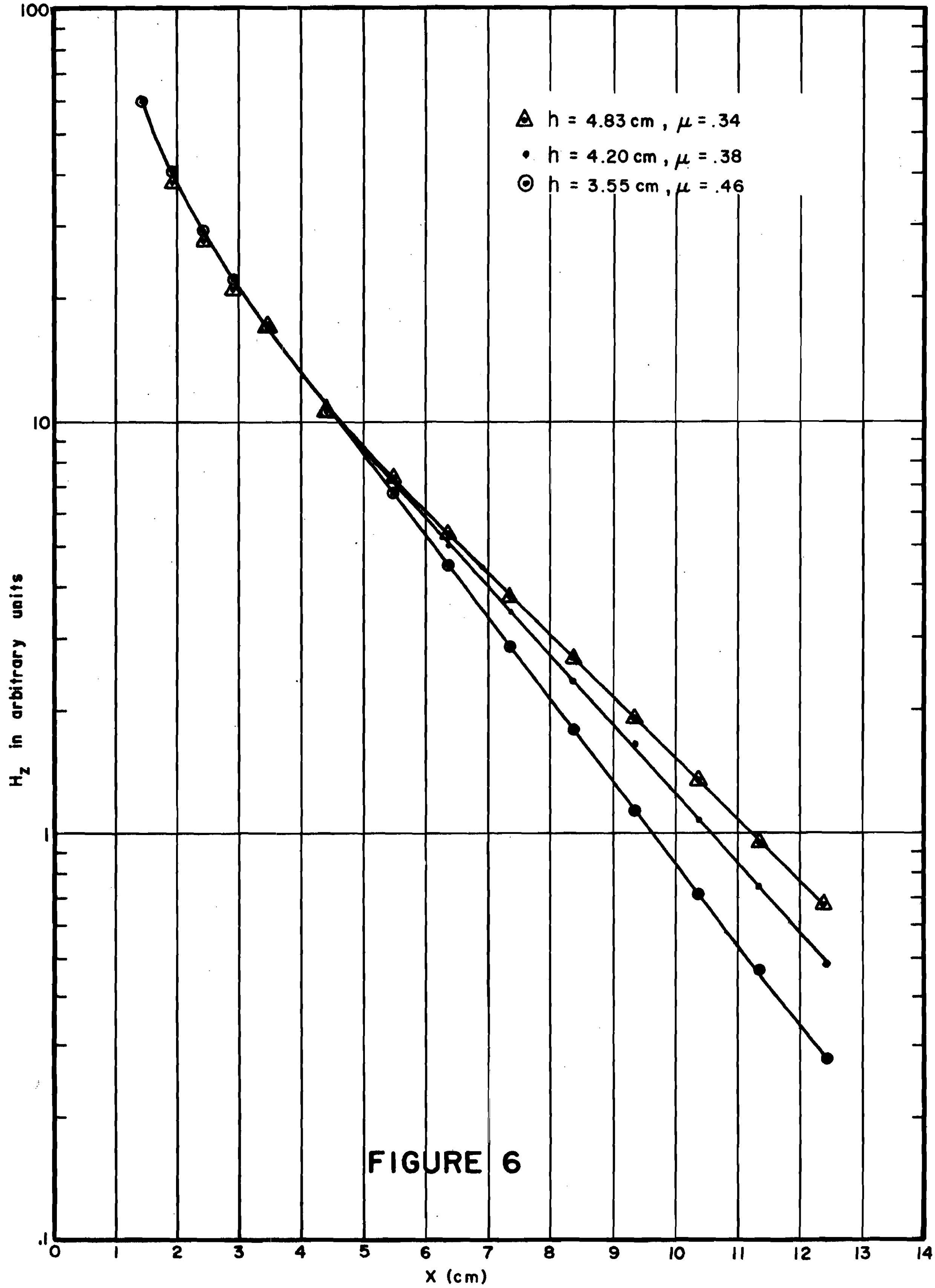
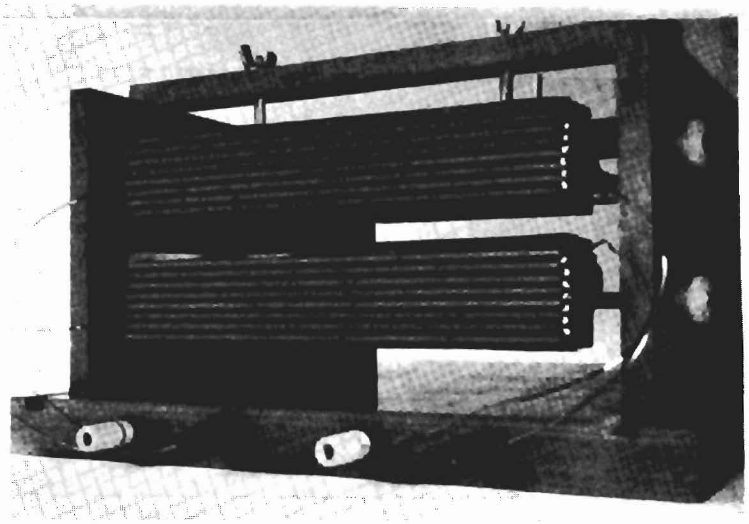
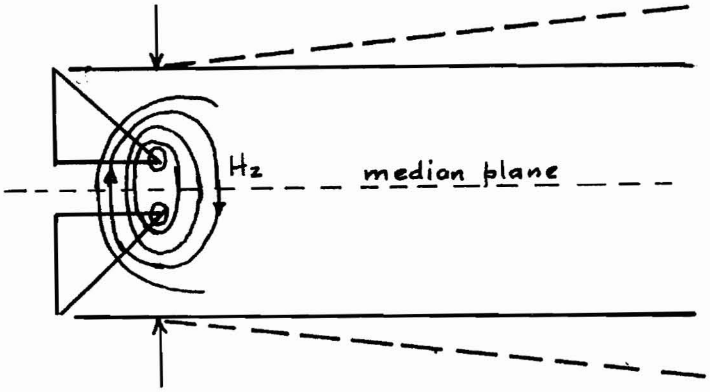

△ h = 4.83 cm , μ = .34
• h = 4.20 cm , μ = .38
⊙ h = 3.55 cm , μ = .46
100
10
1
.1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
X (cm)
H_z in arbitrary units

FIGURE 6

The results of this experiment were very encouraging. The geometry, however, is not suited for use in an accelerator. We can avoid several difficulties by using the geometry shown in Fig. 7. This consists of two similar units, one above the other. Fig. 7a is a view of the audio-frequency analog. This has several advantages: the median plane is open;



(a)



(b)

Fig. 7

there is a strong reversed field beyond the maximum of the exponential field which in this case extends to smaller values of x as can be seen in Fig. 8. In this figure the curve marked with triangles shows the correction of the field obtained by tilting the plates. The other curves show the results obtained under slightly different plate separation (parallel plates).

It is possible that more elaborate geometries may provide additional advantages but there may be additional problems also.

The next step is to establish a correspondence between the audio-frequency analog and an actual superconducting magnet. This will prove, first, that it is possible to shape the magnetic field by the use of superconducting surfaces and, second, that an audio-frequency analog can be used for modeling superconducting magnets.

B.F. Touschek (Frascati): Why use open plates?

G. del Castillo: One may use closed surfaces, but if one tries to force the shape of the field into a closed surface, one may end up with large tangential components of the field at the surface which may exceed the critical field or the critical current induced in the plate. There are additional problems associated with such geometry.

Voice: In which direction is the field most critical?

G. del Castillo: You can look at it in both ways; one may say that the induced current comes from cancelling the component normal to the plate, or one can say that magnitude of the tangential component at the surface increases above the critical field.

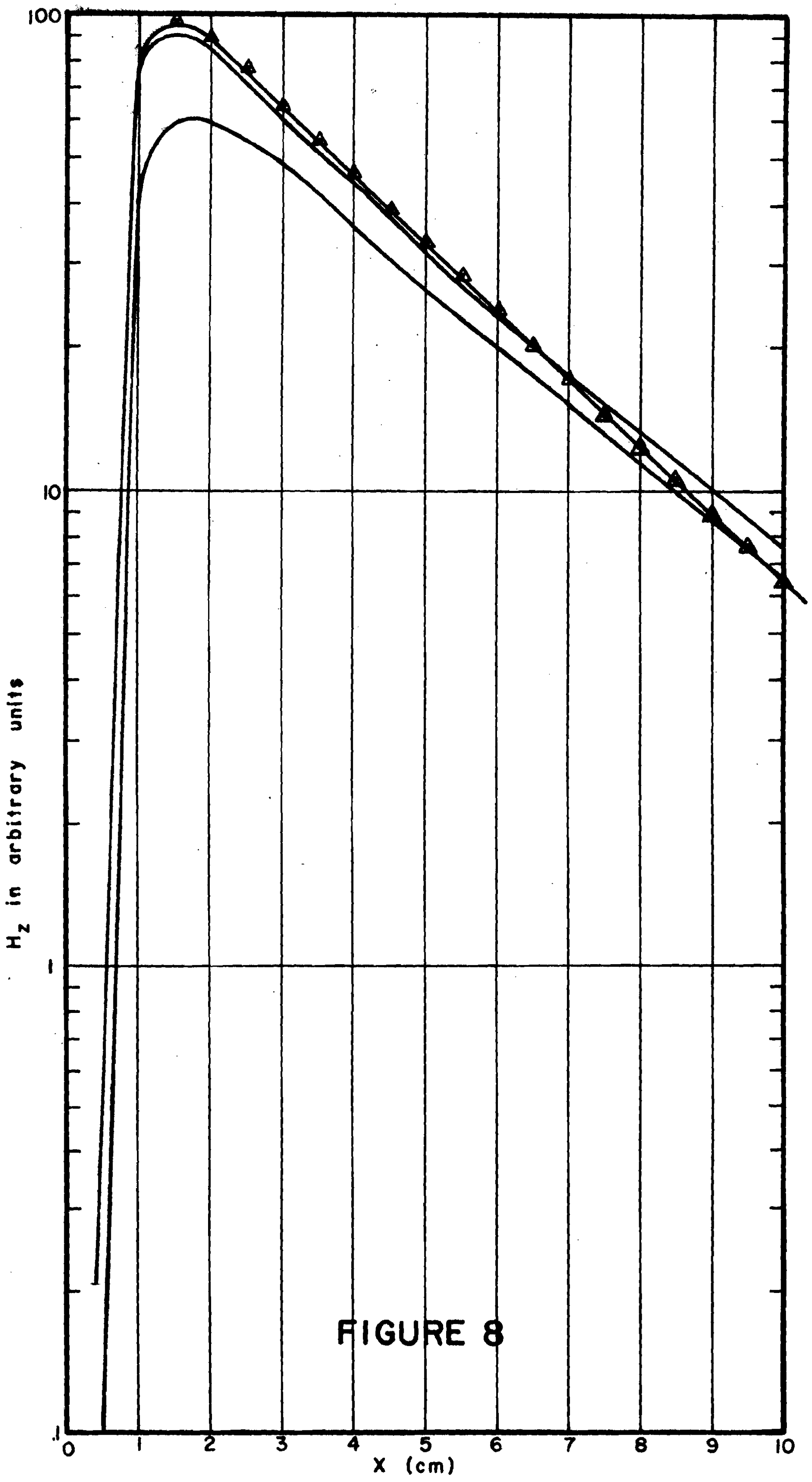


FIGURE 8

P.G. Kruger (Illinois): It appears that the critical field in a direction parallel to the surface is somewhat lower than the perpendicular component. They are about the same order.

G. del Castillo: In relation to Touschek's question, I must point out that there are advantages in using open plates. For instance, one may think that the gap of the magnet is fixed for a desired value of the field index; however this is not quite true. If one ends the plates near the arrows in Fig. 7, the field index is practically determined by the separation of the plates at this point; then one can tilt the plates as shown by the dotted line to get the required aperture at the low-field region to satisfy injection conditions. If you make a fixed-geometry surface, such as a closed surface, you lose the advantage of being able to change k by changing the plate separation and the angle between plates.

There are, however, numerous other problems which we must consider before attempting to build such an accelerator. I will outline a few of them.

The most important one that needs to be clarified soon is the effect of radiation on superconducting materials. Thus far there are only two evidences. Babcock[4] and co-workers exposed a superconducting solenoid in a 400 Mev proton flux. They use Nb-Zr wire to produce a 37-kilogauss field at a critical current of $I_c = 15.8$ amp. The integrated flux was 8×10^{10} protons/cm^2. No effect was observed due to irradiation.

Voice: Over what period of time did this irradiation take place?

P.G. Kruger: It makes no difference since this is an integrated dose.

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[4] R. Babcock and H. Riemersma, Appl. Phys. Letters 1, 43 (1962).

E.D. Courant (BNL): Was the superconductor warm or cold?

G. del Castillo: It was at liquid-helium temperature.

The second evidence comes from de Feo and Sacerdoti.[5] They prepared a source of Po^{210} by making a deposit of polonium on a platinum hollow cylinder 5 cm long and 1 cm diameter. This cylinder was inserted in a second concentric cylinder that had a Pb film of about 6 microns thick. The lead is a superconductor at liquid He temperature; it was left in a persistent current state with a field of 60 gauss. The activity of the α particle source was about 6 millicuries. They observed that the trapped flux decay, as shown in Fig. 9, had a half-life of about 14 hours.

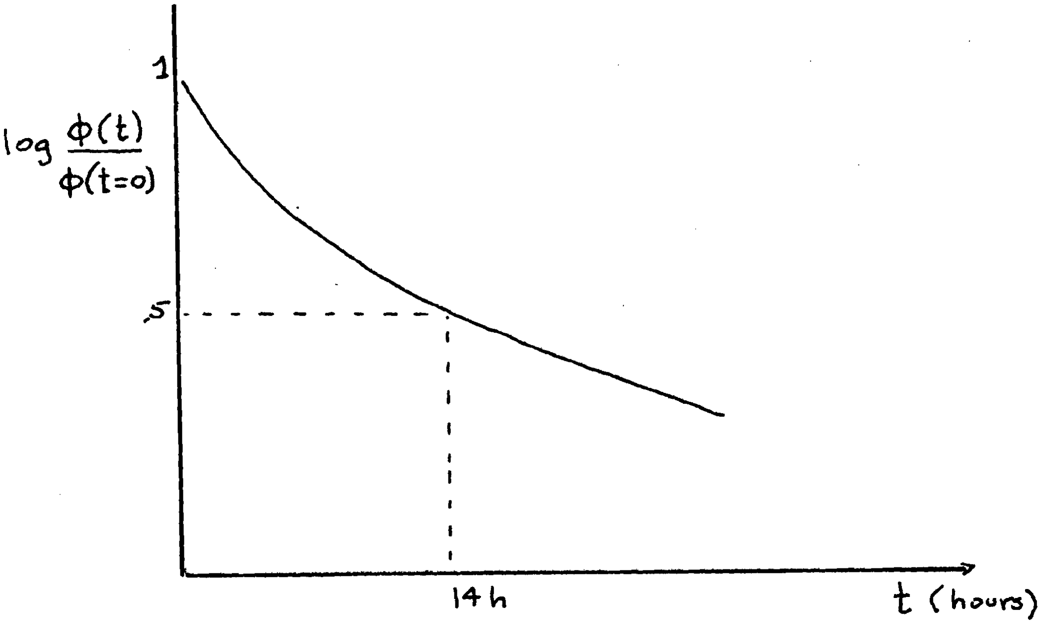

Fig. 9

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5. P. de Feo and G. Sacerdoti, Phys. Letters 2, 264 (1962).

G.K. Green (BNL): How long does the decay take without the source?

G. del Castillo: In principle, it will never decay. There is evidence, I think from MIT, that the flux does not change within years.

Voice: What is the mechanism of current reduction?

G. del Castillo: There is a paper by Cabibbo[6] from CERN where they offer a qualitative explanation of this effect, by assuming that the α particles bombarding the superconducting surface produce heat spikes that destroy locally the superconducting state; thus part of the flux leaks out through this hole. They estimate that flux leaks out at the rate of 10^4 quanta/sec.

Voice: How long do these holes last?

G. del Castillo: I do not know.

E.D. Courant: It is probably related to the thermal conductivity in some manner.

Magnetic Measurements at Liquid Helium Temperature

Another problem that we faced, which has been practically solved except for a few points of academic nature, is the measurement of magnetic field at liquid He temperature. We wanted to measure the field and position with an accuracy of at least 0.1%.

The field-measuring element is a single crystal of bismuth made in the form of a thin fiber 1/4 in. long by a few ten-thousandths of an inch

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[6] N. Cabibbo and S. Doniach, Phys. Letters 4, 29 (1963).

in diameter. These fibers were previously reported by Donovan and Conn.[7] The field is measured by using the magnetoresistance effect. We have managed to produce fibers with sensitivities up to 60 μvolt/gauss at fields of the order of 1 kilogauss. This is about one order of magnitude better than Donovan's report.

These fibers, being single crystals, show a magnetoresistance effect which is strongly dependent on the orientation of the fiber with respect to the magnetic field.

Two types of anisotropy are observed: one with respect to rotations around the principal axis of the crystal, which in our case is directed along the length of the fiber, and the second with respect to rotations around a secondary axis normal to the principal axis. We will call these asymmetries ϕ and θ respectively. Typical curves of the percent change in resistance versus ϕ and θ appear in Fig. 10.

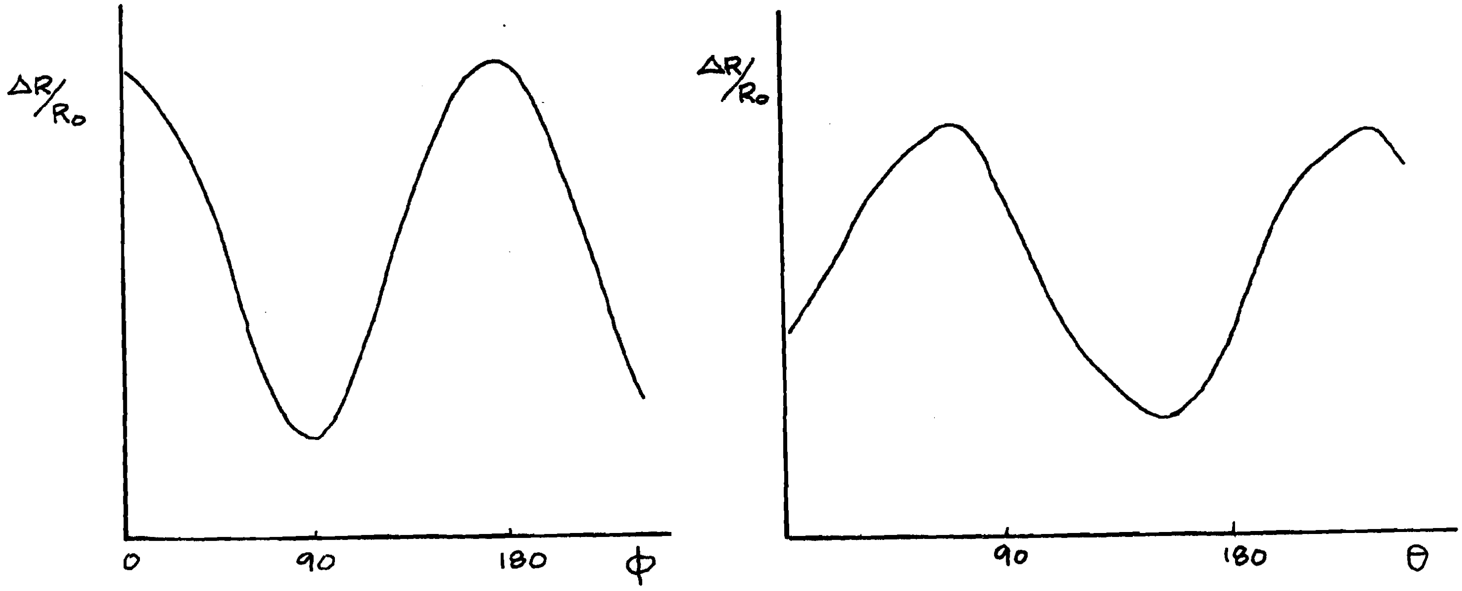

Fig. 10

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[7] B. Donovan and G.K.T. Conn, Phil. Mag. 40, 283 (1949).

The observed anisotropies suggested that one should try to mount the fiber in a support such that one could search for the maximum in both axes. By doing this one can work at the point of maximum sensitivity and at the same time make the instrument less sensitive to changes in resistance due to small misalignments. A few words should be said on the procedure for mounting one of these fibers.

Fig. 11 shows the arrangement of the microscope, illumination lamp, soldering iron and a simple support for mounting the fiber. This support is also shown in Fig. 12. Here we can see the two copper wires A and B where the fiber will be mounted. The fiber (invisible in the picture) is glued to the tips of the wire fork. By using a simple screw mechanism the fork is lowered gently until the fiber is in contact with both copper terminals. A fine soldering iron is used to deposit a small amount of Wood's metal to make the junction.

An idea of the size of the fiber can be obtained from Fig. 13, which is a view through the microscope; the diameter of the copper wire is 0.034 in.

It was previously mentioned that it is desirable to orient both axes of the fiber to obtain maximum sensitivity. The following Figs. 14 and 15 show the way this is accomplished. The fiber is mounted along the axis of the hollow cylinder (dotted line in Fig. 14) used to protect the fiber from mechanical damage. The crankshaft arrangement allows for a ϕ rotation when mounted in the support shown in Fig. 15. The maximum in ϕ is obtained at room temperature; once this is fixed, the support is placed in the Dewar with liquid He and the maximum in θ is obtained by rotating the assembly around the θ axis of Fig. 15. One can also see in both figures the current and potential leads.

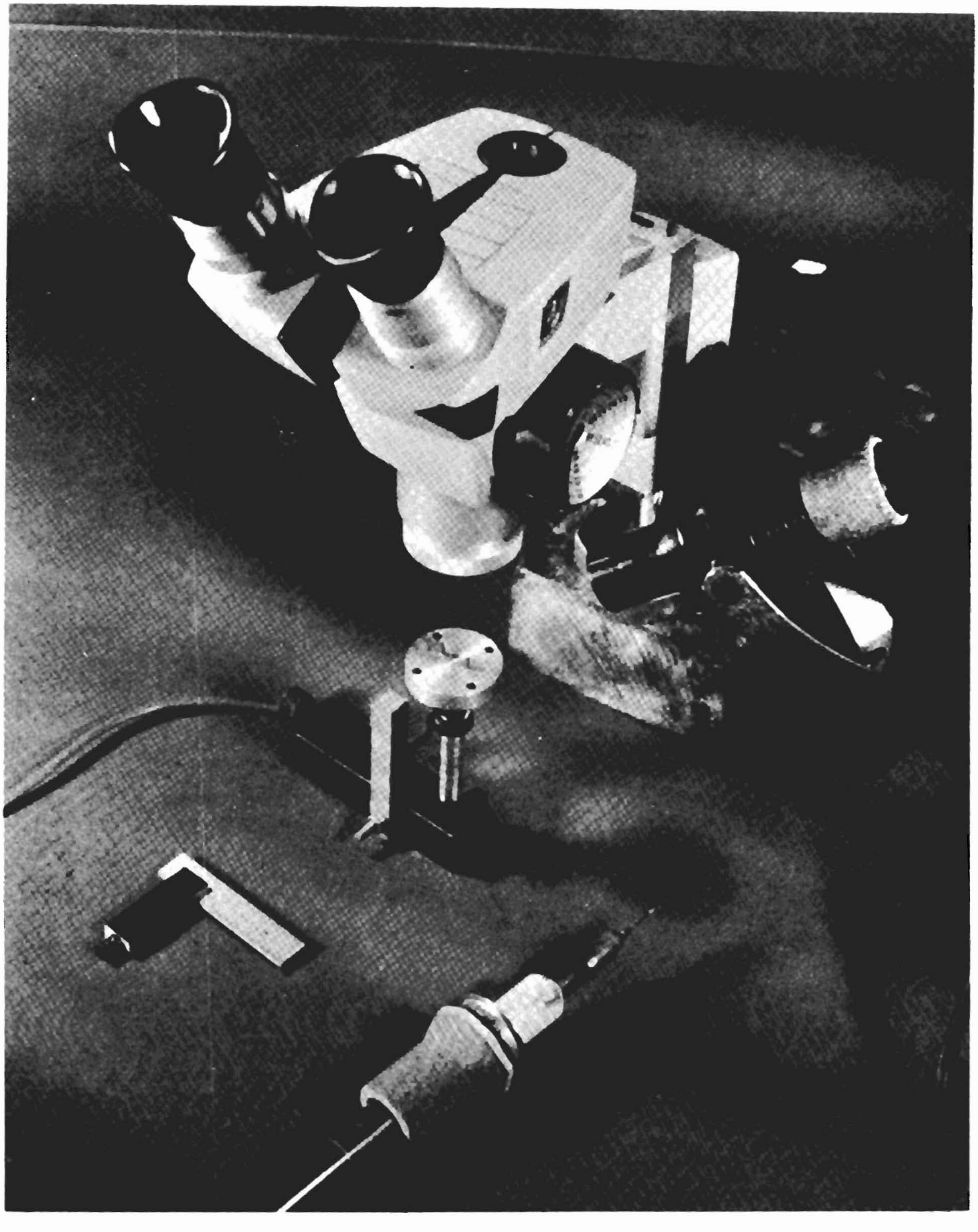


Fig. 11 - Support for single crystal fiber, also microscope, illuminating lamp and soldering iron used in mounting fiber

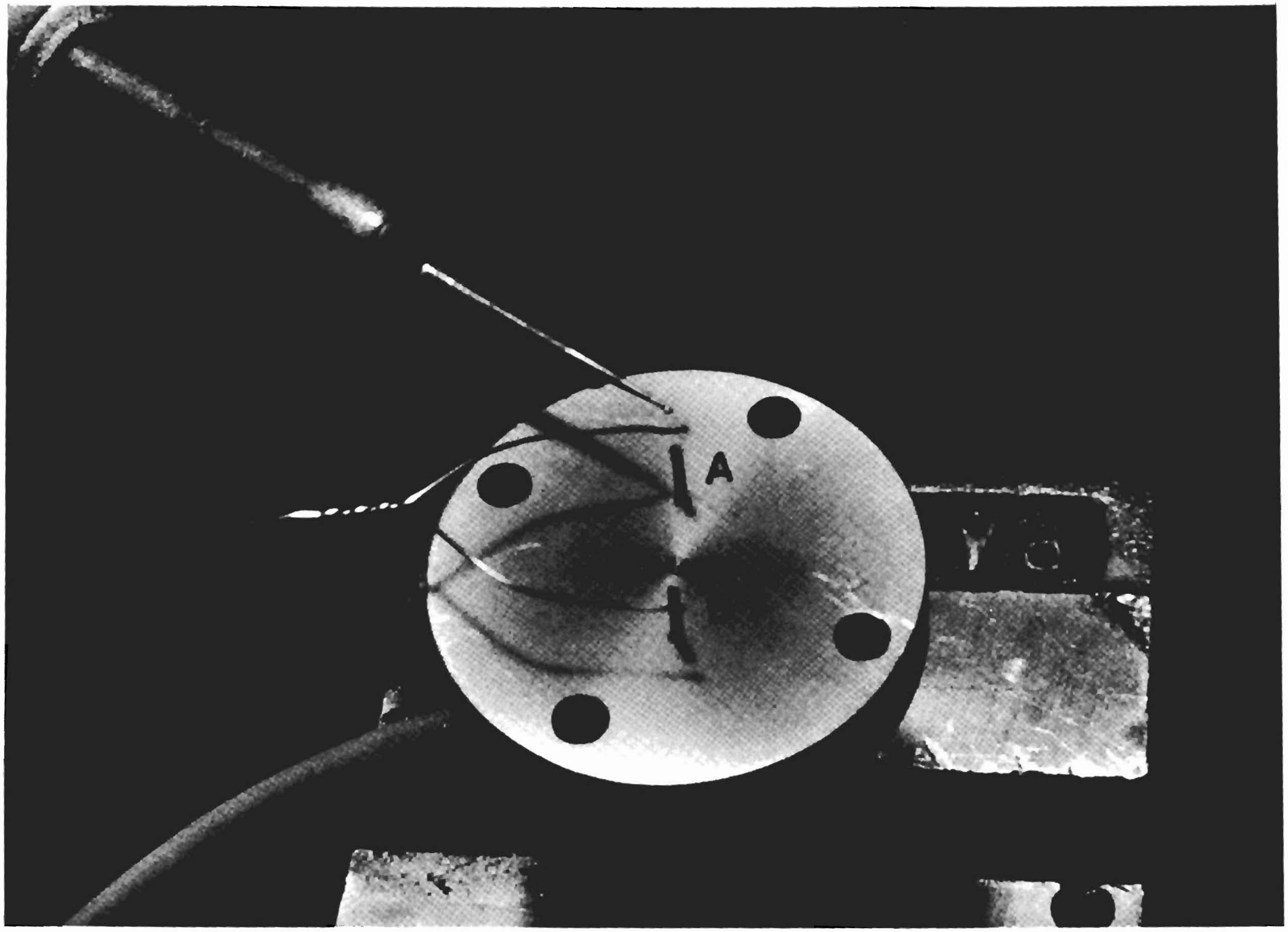


Fig. 12 - Fiber support. The fiber will be mounted between copper terminals A and B.

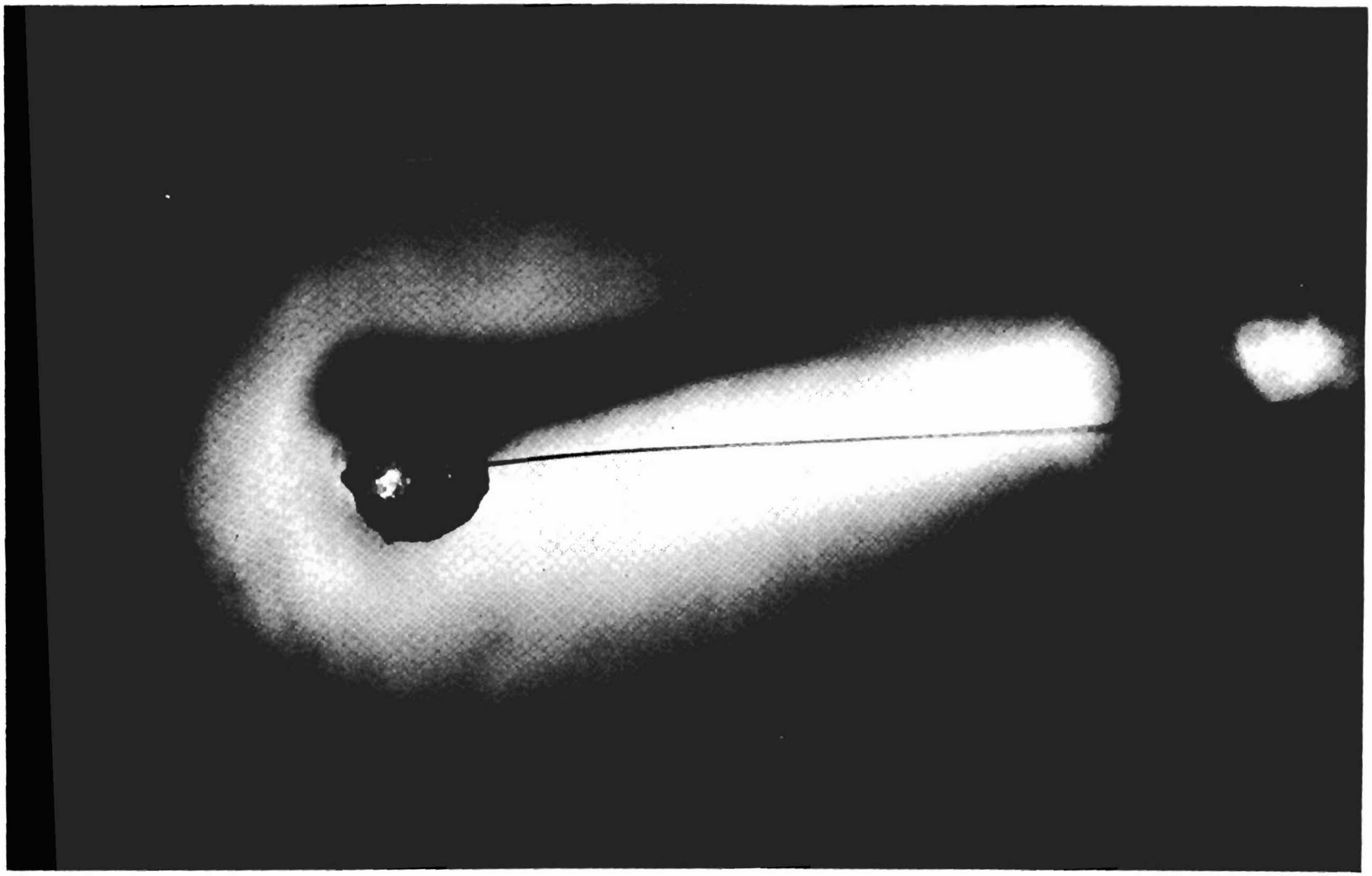


Fig. 13 - Microscopic view of single crystal fiber (copper wire diameter is 0.034")

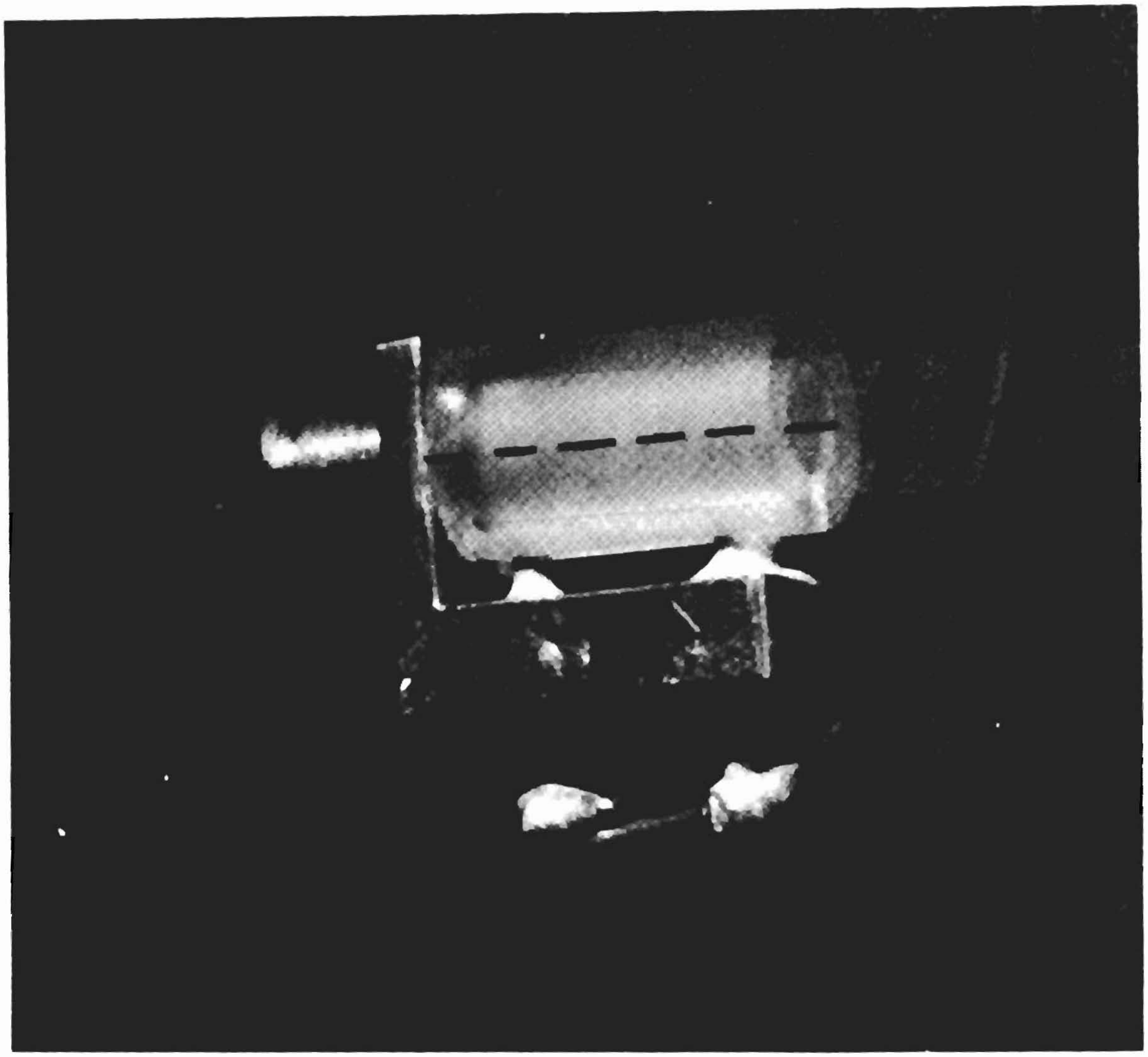


Fig. 14 - Hollow cylinder to protect fiber. Dotted line shows axis along which the fiber is mounted.

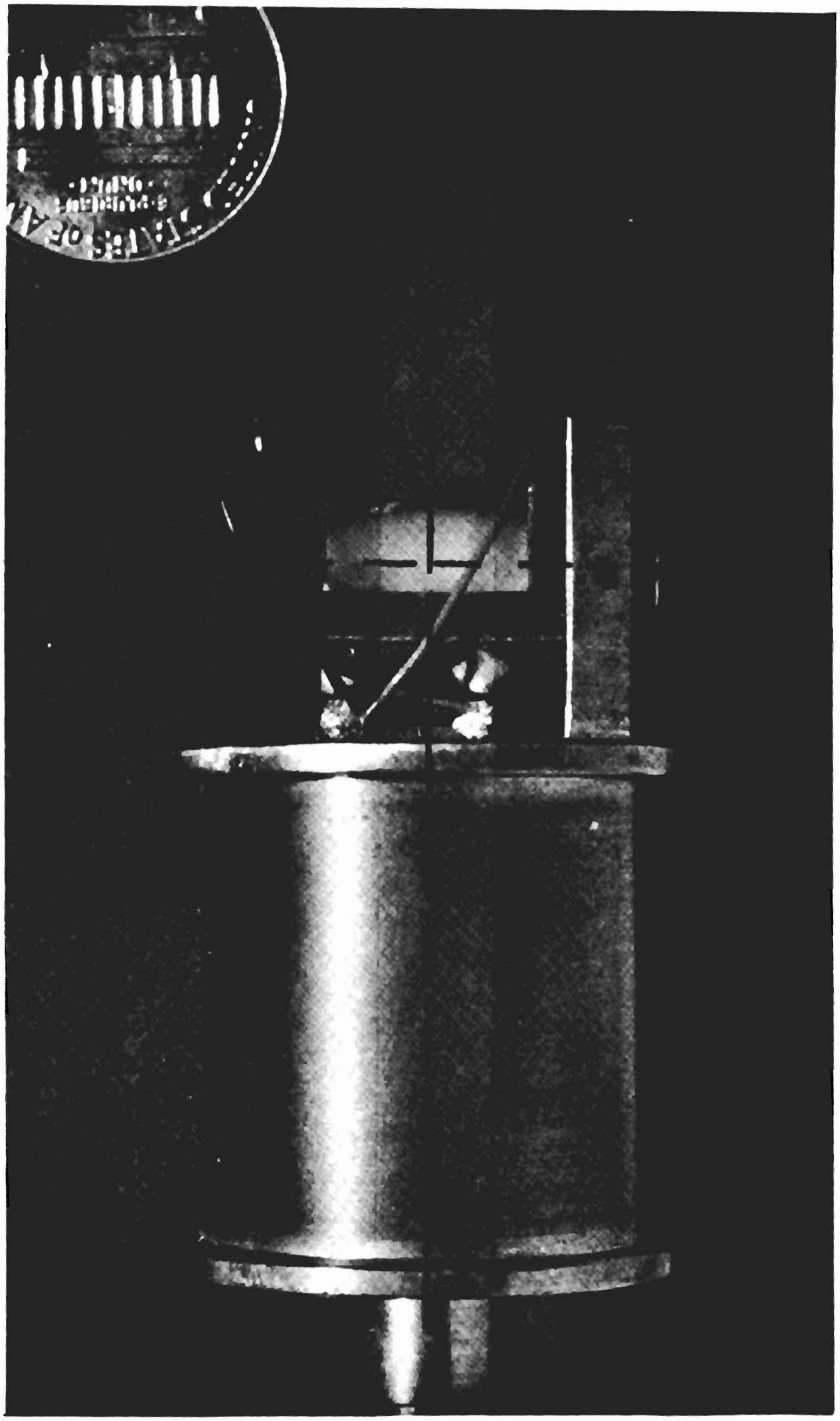


Fig. 15 - Support which permits rotation about ϕ axis and θ axis.

In Fig. 16, one can see part of the device that will be used for positioning the measuring probe. The column may be rotated as indicated by the arrow to reach for the maximum in θ. One can also see part of the He Dewar. The rest of the cryogenic equipment is essentially completed. Thus in a few months we may be able to report final results. This summarizes briefly the work that we are doing at MURA with respect to superconducting magnets.

D.C. Rahm (BNL): Is the field strength limited by the current which the superconductors can carry?

G. del Castillo: Yes, besides it depends on the coil geometry. Also, it will be limited by the maximum field that the plates can shield. It may be necessary to use thick plates; in this case we have to consider a volume distribution of currents in the plates instead of the simple case we have studied which is a surface distribution.

P.G. Kruger (Illinois): It might be useful to inquire if NRC could make Nb_3-Sn plates.

G. del Castillo: Yes, it will be very interesting; thus far we have tried to get a rough idea of the flux penetration in Wood's metal - a lower limit seems to be of about 1.4 kilogauss. We intend to use this material for the plates because of simplicity. Rather than obtaining high fields, we feel that, for the moment, we would like to establish the principle of field shaping, and the correspondence between the audio-frequency analog and the superconductor.

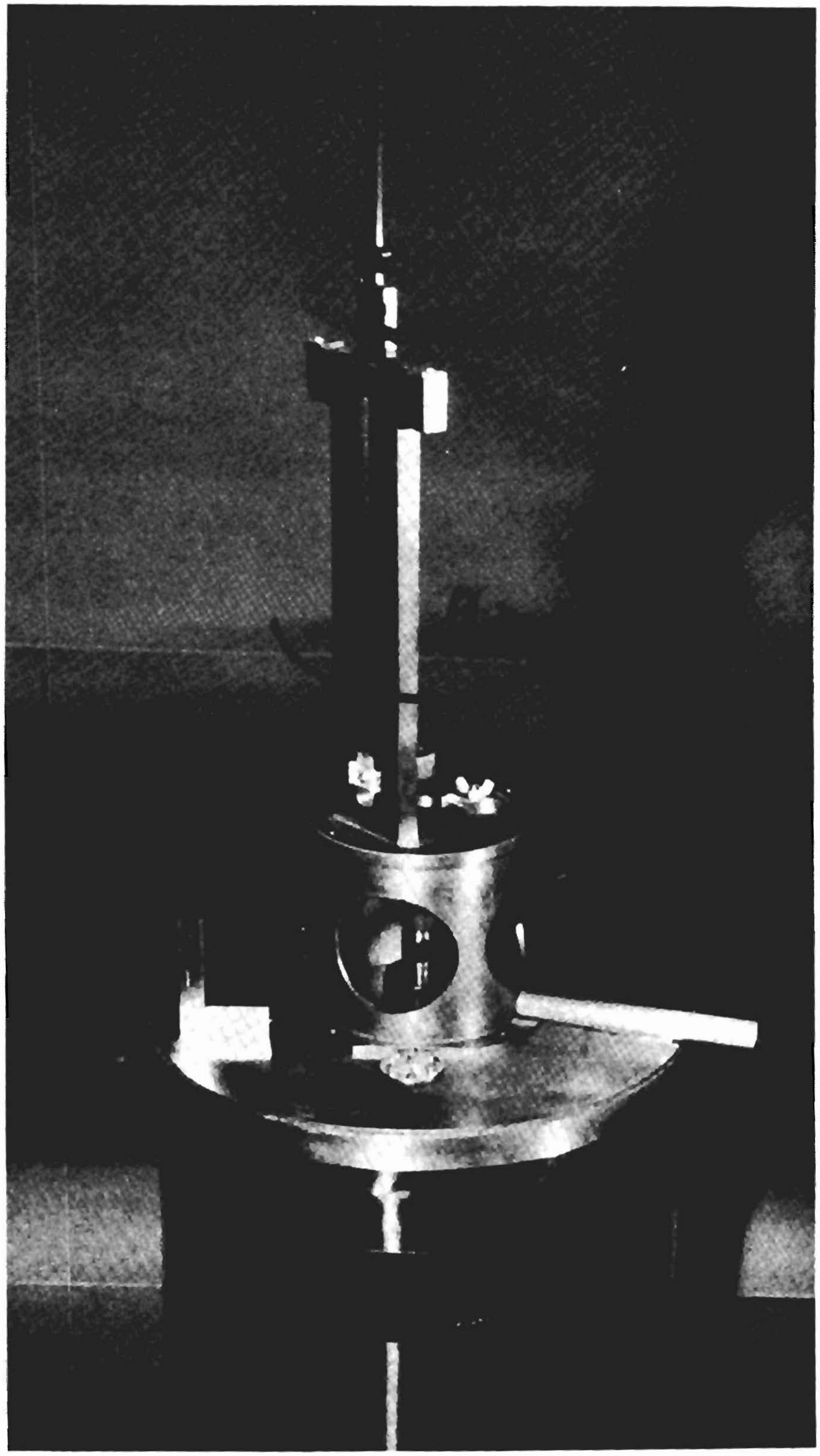
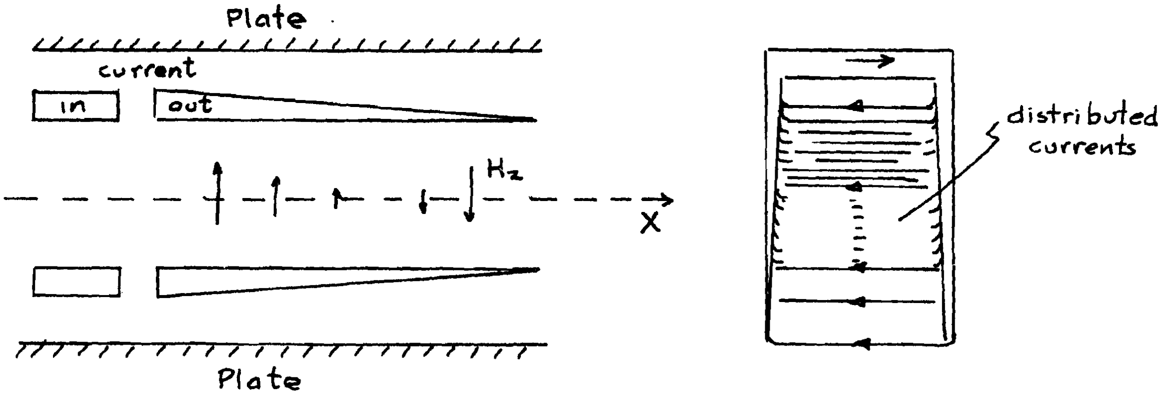


Fig. 16 - Device for positioning the measuring probe

L.J. Laslett: Is there a way to compromise in shaping fields by using both localized sources and distributed current elements instead of sources and plates?

G. del Castillo: In answering this question I must mention that Snowdon[8] made a very thorough theoretical study of such a case. Again an audio-frequency model was built and tested. I am not reporting the results, however, because of two reasons. First, the preliminary measurements of the field yielded somewhat discouraging results, and second, it has not been tested enough to reach final conclusions. I will outline this model briefly.

The magnet consisted of two units, one above the other, each having a required current distribution that is represented schematically in Fig. 17. Two copper plates were located above and below these units.



SIDE VIEW

TOP VIEW

Fig. 17

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[8] S.C. Snowdon, MURA Internal Reports TN-317, TN-328 and TN-332.

The winding is somewhat similar to the one used in our radial-sector iron magnets.

Several field measurements were taken at the median plane under different conditions of plate separation, separation between units, etc. A typical result appears in Fig. 18. One can see a deviation from the theoretical curve and a surprising field reversal near the front (weak field) end of the magnet. This resembles very much the field reversal observed in the iron magnets due to reluctance effects.

These results cannot be considered as final because the magnet is not properly terminated with the required copper (superconducting) surfaces. Their shape has been calculated also, but is rather complicated.

G.T. Danby (BNL): There is really no difference in principle.

A. Schoch (CERN): What fraction of the volume between the plates is useful?

G. del Castillo: I believe you are referring to the vertical or z-direction. This is a very important question but we still do not have the answer. We don't know what will be the actual behavior of the superconducting plates.

M.Q. Barton (BNL): Wouldn't plates involve easier calculations?

G. del Castillo: Yes, in fact, that is the idea behind the first model using flat plates. As I mentioned before, the calculations and the resulting surfaces related to the second model are rather complicated.

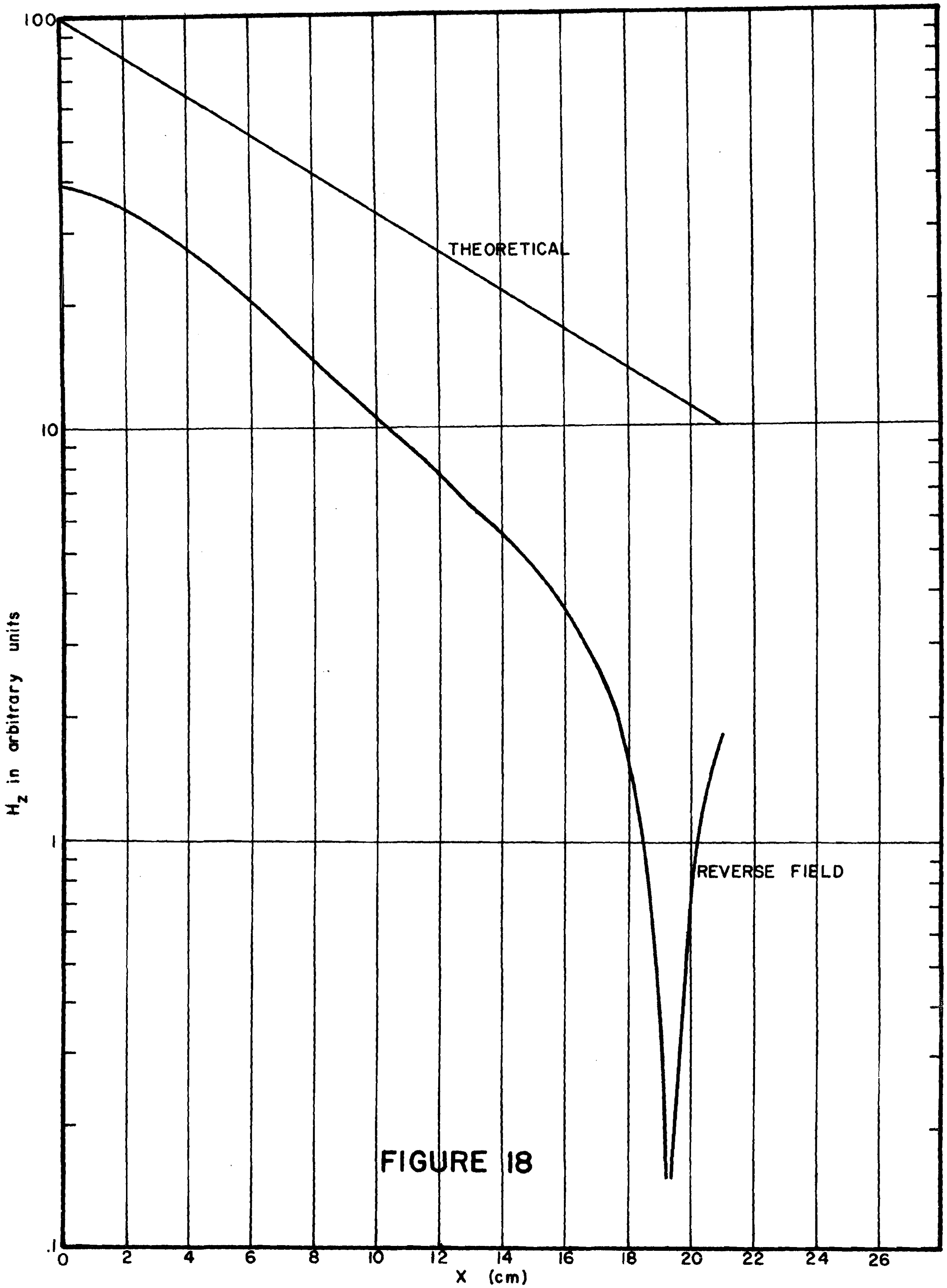
100
10
1
.1
THEORETICAL
REVERSE FIELD
H_z in arbitrary units
0
2
4
6
8
10
12
14
16
18
20
22
24
26
X (cm)

FIGURE 18

P.G. Kruger: A radial sector machine has a large circumference factor. Wouldn't a spiral sector machine minimize the cost of the superconducting material?

G. del Castillo: Large circumference factor may not be a bad disadvantage. One would like to have a certain size machine so that other components can get in. But thinking in cost, it is certain that the spiral sector magnet is better.

J.P. Blewett (BNL): When making the magnetic-field measurements which you described, will you immerse the entire structure in a Dewar?

G. del Castillo: Yes, the magnet and the measuring probe will be in liquid He. A rough sketch of the system is shown in Fig. 19.

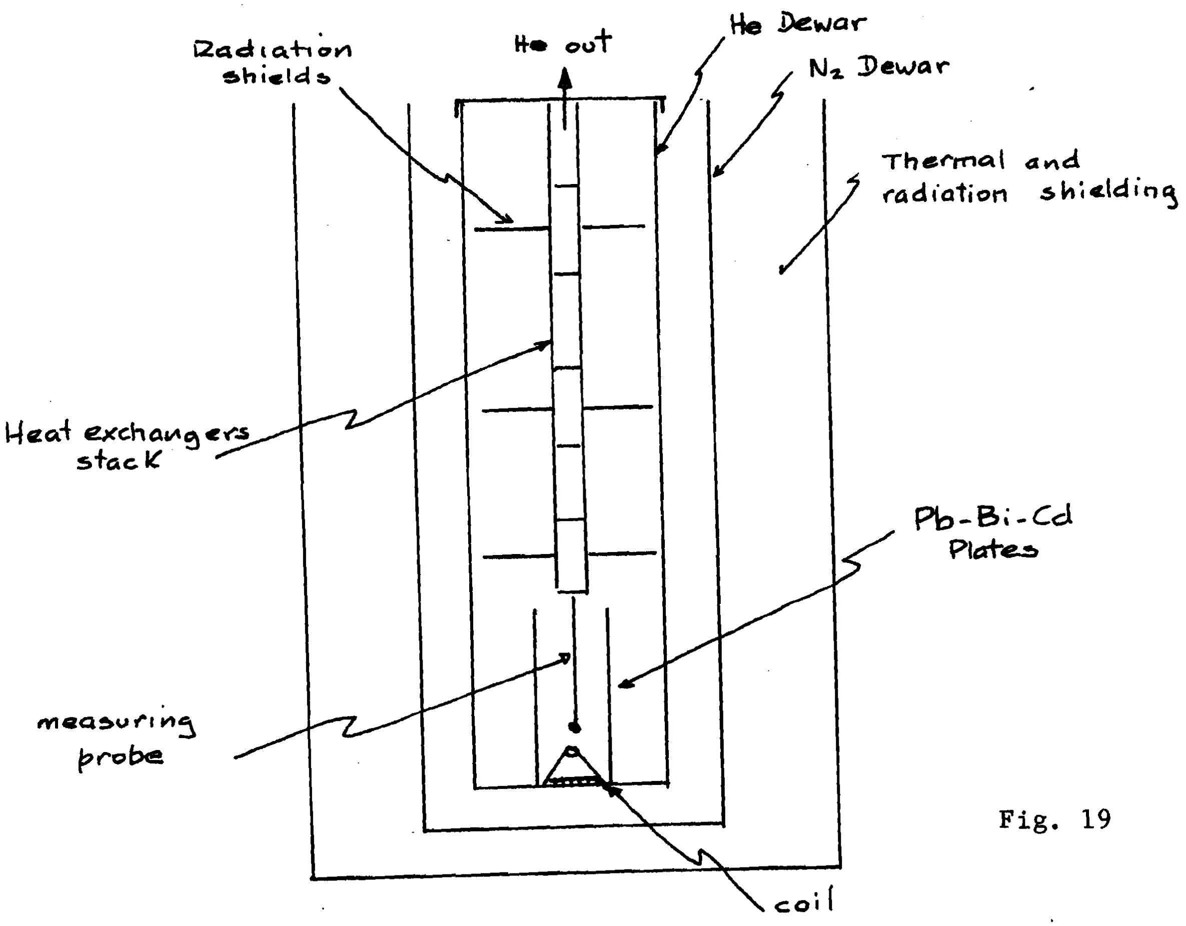

Fig. 19

G.K. Green (BNL): What materials are used as conductors?

G. del Castillo: Nb-Zr wire for the superconductors, heavy copper wire for feeding current to the coil. The cooling of these leads will be made by a heat-exchanger column as shown in Fig. 19.