ACCELERATORS WITH NONLINEAR FOCUSING - THE ORLOV PROPOSAL

E. D. Courant
Brookhaven National Laboratory

I want to report on some papers by Y.F. Orlov[1-4] and the results of a conference at Berkeley,* which was held a few weeks ago, on the Orlov machine.

Orlov points out that one can achieve the equivalent of alternating-gradient focusing in an azimuthally-uniform field. This is accomplished by having a magnetic field which varies with r as is shown in Fig. 1.

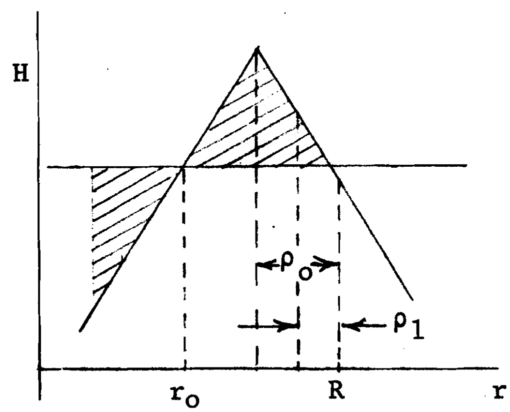

Fig. 1 - Radial distribution of field (linear approximation).

1. Y.F. Orlov, J. Exptl. Theoret. Phys. (USSR) 43, 1308-1314 (1962).
2. Y.F. Orlov, Soviet Phys. - JEPT (USA) 16, 928 (1963), translation of Ref. 1.
3. Proc. of Conf. on Problems of Elementary Particle Physics, Nor-Amberd, March-April, 1962.
4. CERN Internal Report AR/Int.SR/63-6, translation of Ref. 3.

*The participants in the conference included N.C. Christofilos, F.T. Cole, T.L. Collins, D.L. Judd, G.R. Lambertson, L.J. Laslett, E.M. McMillan, F.E. Mills, A.M. Sessler, L. Smith, L.C. Teng and W.A. Wallenmeyer.

The radius r_o may be the center of the aperture of the machine. Corresponding to a given momentum p, one can have orbits at two radii; for example, r_o and R. It is clear that small oscillations about r_o are unstable vertically while stable horizontally because of the large positive gradient. At the radius R, just the opposite is true. One can have large stable horizontal oscillations in the shaded region shown in Fig. 1. The limits of this region are determined by the condition that the shaded areas to the right and left of r_o are equal.

The large horizontal oscillations may have vertical stability because the particle spends some of its time in a region of positive gradient and some of its time in a region of negative gradient, and the net result may be focusing in both horizontal and vertical directions as in an alternating-gradient machine.

For simplicity in the calculations, we will assume that the shape of the magnetic field shown in Fig. 1 is that of a triangle with equal and opposite gradients on the sides of the triangle. One can then solve the problem analytically using linear theory, although the effect is essentially a nonlinear one.

Now, let us consider a large horizontal oscillation. Such a horizontal oscillation is shown in Fig. 2. φ_2 is the azimuthal extent of the region in which the particle experiences a field with positive gradient;

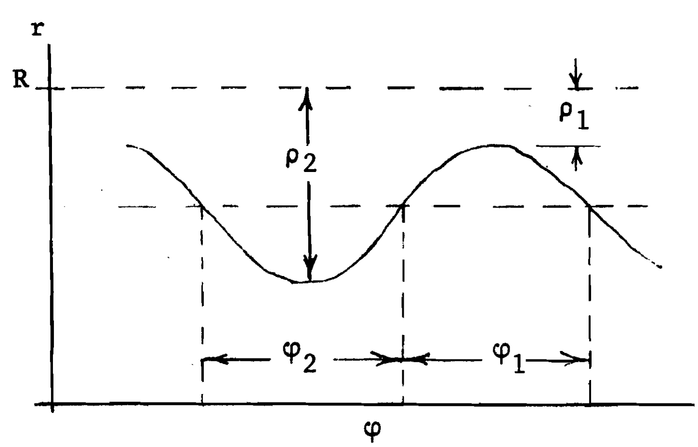

Fig. 2 - Horizontal or radial motion of particle.

φ_1 is the extent of the region where the particle experiences a negative gradient. The condition for stability of the horizontal oscillations may be written as follows:

$$\tan \frac{1}{2} \theta_2 = - \tanh \frac{1}{2} \theta_1 ,$$

where

$$\theta_2 = \sqrt{n}\, \varphi_2 , \quad \theta_1 = \sqrt{n}\, \varphi_1 .$$

An approximate solution of this equation is

$$\frac{1}{2} \theta_2 \simeq 135^\circ + k\pi .$$

Now, let us consider the condition for vertical stability. We set up the usual transformations for the vertical oscillations and we find that for one horizontal oscillation

$$\cos \mu_z = \cosh \theta_2 \cos \theta_1 .$$

In order for $\cos \mu_z$ to lie between plus and minus 1, we must have $\cos \theta_1$ near 0. Thus, the regions of vertical stability occur when θ_1 is near 90° or 270° and so on.

We can represent this result in a slightly different way by drawing a potential function, which is given by

$$U = \int (H - H_o)\, dr .$$

This potential function is shown in Fig. 3. In Fig. 3 the bands of stable vertical oscillations are shown as bands in the potential.

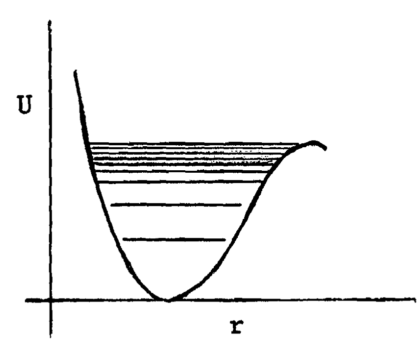

Fig. 3 - Potential function of the particle and bands of verticle stability.

Now, let us write down some of the parameters of these bands. θ_1 in band 1 is $90^o \pm 0.68^o$. In band 2, θ_1 is $270^o \pm 1^o$.

Another important parameter is the quantity x which is given by

$$x = \rho_1/\rho_0$$

x is a measure of the amplitude of the oscillation. In the first band, we have x = .735 and in the second band, we have x = .188.

The vertical motion is determined by the following differential equation:

$$\frac{d^2z}{ds^2} + \frac{1}{B\rho} \frac{dB}{dx} z = 0 .$$

In the above equation for the z motion, the magnetic field and gradient are to be evaluated along the horizontal oscillation. Thus the solutions of this equation will have bands of stability corresponding to certain amplitudes of the horizontal oscillations.

We will now make some comparisons with the alternating-gradient synchrotron. The claim that Orlov makes is that the phase space available in the stable transverse oscillations for this machine is much larger than the corresponding phase space for the AGS.

The betatron oscillation frequencies ν_x and ν_z are both roughly proportional to the square root of n in the first and second bands. The vertical phase space Ω_v is given by

$$\Omega_v = \frac{\pi z^2_{max} n^{1/2}}{168 R} .$$

The factor 168 in the denominator arises from the fact that in each cycle the particle spends a long time - corresponding to approximately

$3/2\ \pi$ radians of oscillation - in the vertically defocusing region; this leads to a very unfavorable shape factor β_{max}/β_{av} in the vertical oscillations.

The horizontal phase space is given by

$$\Omega_H = \frac{.006\ \rho_o^{\ 2}\ n^{1/2}}{R} .$$

The small factor 0.006 comes from the necessity of maintaining the amplitude of horizontal oscillations in the narrow band where vertical stability exists.

We find, then, that the total phase space is given by

$$\Omega_T = \frac{.006\ \pi\ z_{max}^2}{y\ n_{geo.}} ,$$

$$y = H_{orbit}/H_{max} ,$$

$$n_{geo.} = R/\rho_o .$$

Now, let us assume reasonable parameters and estimate the phase space available. We will assume that the vertical oscillation can reach 1 cm and we will assume that $y = .01$ at injection. We then find for the total phase space

$$\Omega_T = 10^{-4}\ cm^2 - \text{steradians} .$$

The comparable phase space for the Brookhaven AGS is also 10^{-4} sterad-cm^2. The phase space of the Orlov machine is then comparable to that of the AGS, which is surprising in view of the bad form factor for the Orlov machine.

A major difficulty of the Orlov machine would be the range of frequency of betatron oscillation that is covered over the course of acceleration. It ranges all the way from 200 per revolution to some very low value in the course of acceleration. Thus the particle will have to cross many resonances. In order to be able to cross these resonances, the tolerances will have to be 100 times more severe than for the AGS, according to Orlov's estimates.

Another interesting aspect of this machine is how the magnetic field must vary with time in order to keep the particle in the center of these bands of stability. Fig. 4 shows the variation of the peak magnetic field as the energy is increased. One observes that the peak magnetic field must first decrease. There is, then, a central flat region, and then the field must increase. The central flat region leads Orlov to suggest that this machine could be used as an FFAG accelerator.

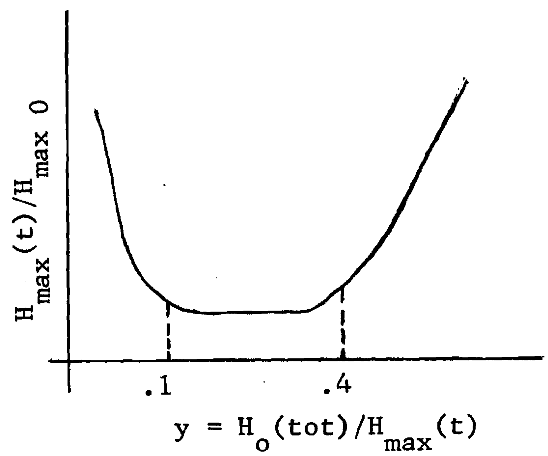

Fig. 4 - Variation of peak magnetic field as the particle enegy is increased.

One other thing that Orlov talks about is colliding beams. He proposes an accelerator with magnet poles as sketched in Fig. 5. The advantage of this kind of storage ring over other storage rings is not apparent.

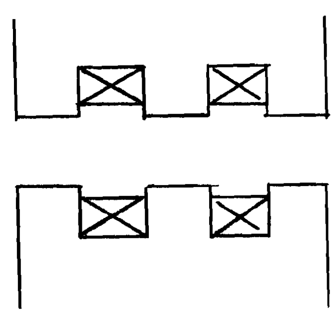

Fig. 5 - Pole configuration for a storage ring proposed by Orlov.

The following improvement over the Orlov machine was suggested by the recent

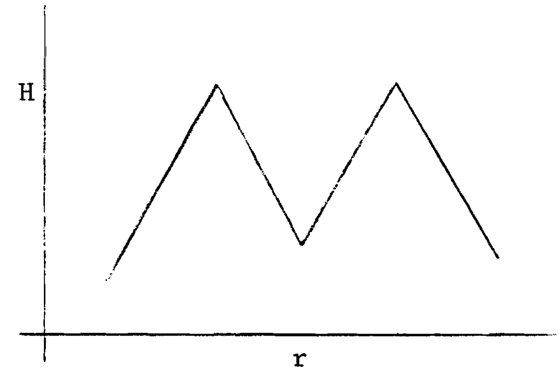

Fig. 6 - Field distribution of an Orlov-type storage ring suggested by M.S. Livingston.

trip report of M.S. Livingston. The magnetic field is changed according to Fig. 6. The particle now sees two vertical-focusing regions separated by a horizontal-focusing region. This results in a considerable improvement of the available phase space because the stability band is wider and the form factor is more favorable. The resulting increase in the available phase space is given by a factor of 20. One now has a choice of a good circumference factor with a small momentum range, or a poor circumference factor with a large momentum range. However, it might be more difficult to maintain the correct amplitude of oscillations during acceleration. Therefore, this version might be most suitable for use as a storage ring, accommodating a range of the order of 10 to 20% in momentum.

The large radial oscillations associated with this kind of an accelerator or storage ring would allow injection over a large number of turns. There would also be a corresponding advantage for ejection.

One problem whose solution is not clear is the addition of straight sections. An estimate made by the group at Berkeley seems to indicate that the largest permitted straight section would be 1½ mm long.