MISCELLANEOUS COMMENTS ON COLLIDING BEAMS

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I. Space-Charge Limits

The recent discussions at Brookhaven and CERN concerning colliding beams have assumed present intensities for the AG synchrotrons (about 5×10^{11} protons per pulse) and a reasonable (about 5 to 10 cm) radial aperture of storage rings. From rf phase-space considerations this leads to stacked, circulating currents of 10 to 20 amperes in the storage rings filling the radial aperture.

Recently there has been increasing discussion of the beam that could be accelerated in the CERN or Brookhaven accelerators by increasing the injection energy. The AGS space-charge limits are very roughly:

50 Mev injection	2×10^{12} protons per pulse
200 Mev injection	10^{13} protons per pulse
1 Bev injection	5×10^{13} protons per pulse

While the betatron phase-space density would not increase linearly with increasing current (the beam is assumed to fill the aperture for each injection energy to achieve these theoretical maximum currents), the rf phase-space density could increase more or less proportionally with current. Therefore it is relevant to inquire as to the theoretical maximum current a storage ring could accept.

L.J. Laslett[1] has recently rederived the space-charge formula including image forces and rf bunching in the form:

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[1] L.J. Laslett, p. 324 of this volume.

$$N = B\frac{\pi}{2}\frac{b(a+b)}{Rr_p}\;\frac{(\nu_{y_o}{}^2 - \nu_y{}^2)\,\beta^2\gamma^3}{\left\{\epsilon(B\beta^2\gamma^2 + 1) + \frac{\pi^2}{24}B\beta^2\gamma^2\frac{h^2}{g^2}F\right\}\frac{b(a+b)}{h^2} + 1}$$

where N = total number of beam particles, B the rf bunching factor (≤1), a and b the radial and vertical beam dimensions (assumed elliptical in cross section), ϵ a factor depending on the donut shape, h the vertical semiaperture of the vacuum tank, g the vertical semiaperture of the magnet, r_p the classical proton radius, R the accelerator radius, and F is the iron "circumference factor" (≤1) of the ring.

For the case of $\gamma >> 1$, B = 1, (as in storage rings) this reduces to:

$$N = \frac{\pi\nu_y\Delta\nu_y\gamma}{r_pR\left\{\left(\frac{\epsilon}{h^2} + \frac{\pi^2}{24}\frac{F}{g^2}\right) + \frac{1}{b(a+b)\,\gamma^2}\right\}}$$

In the present storage ring case, we may take:

$a = 2$ cm

$b = 0.5$ cm

$R = 1.5 \times 10^4$ cm

$r_p = 1.5 \times 10^{-16}$ cm

$h = 3$ cm

$g = 5$ cm

$\gamma = 36$

$\nu_o = 8$

$\Delta\nu = \frac{1}{2}$

$C = 0.172$ (donut elliptical with $w = 2h$)

$\pi^2/24 = 0.4$

$F = 0.66$

In the above formula this gives

$$N = 6.6 \times 10^{15} \text{ protons},$$

$$I = 330 \text{ amperes}.$$

Thus injector improvements in the AGS could result in maximum stored currents of up to 300 amperes (corresponding to 30 megajoules) and correspondingly higher interaction rates. Since these rates go as I^2, those experiments not limited by background or singles rates and designed to explore very small cross sections would profit considerably.

II. Beam-beam Defocusing Effects

When two relativistic, unneutralized beams cross, there is a vertical defocusing effect on each. A form of the expression giving the number of protons and the corresponding change in vertical betatron tune $\Delta\nu_y$

$$N = \frac{2\pi\gamma b\alpha\nu_y \Delta\nu_y}{r_p p}$$

where α is the beam-crossing angle, p is the number of crossing points around the circumference and the other quantities are as defined above. For $\alpha = \frac{1}{4}$, $p = 8$,

$$N = 8 \times 10^{16} \text{ protons in each ring.}$$

III. Bunching of the Stacked Beam with Rf

It is relevant for many experiments to consider the beam to be bunched by rf; however, several comments are in order.

If the fraction of azimuth occupied by beam under rf bunching is B, the energy spread is ΔE, the interaction rate is R and the background rate is P, then for a total circulating current I,

$$R \propto I/B$$

$$\Delta E \propto \Delta E_o/B$$

where P is independent of B and the subscript o refers to values without bunching. Bunching a beam with rf increases the instantaneous interaction rates as $1/B^2$ and reduces the average as B. A beam bunched by rf occupies more radial space in proportion to the total energy spread.

The consequence of all of this on experiments is that bunching leads to an improvement in data-collection rate where the limit is set by background in a device of "poor" time resolution. Thus with spark chambers or bubble chambers the time resolution is longer than 1/f of any bunching rf, and the events rates in an experiment would be improved directly as 1/B for some constant, maximum tolerable rate set by interactions with residual gas.

However, rates in scintillation counters would not in general be improved by bunching since the time resolution in counters (2-5 nsec) is shorter than 1/f for convenient rf bunching systems (10-200 Mc).

At full energy in the AGS, the beam is bunched to B = 1/20. A stacked beam would require a voltage proportional to the square root of the stack number if the particles were to occupy an rf bucket of the same "shape". From figures on rf voltages for 0^o phase angle, a beam with 40 stacks, or ΔE_o = 50 Mev could be bunched by rf to B = ½-¼ by a system supplying 50-100 kv per turn.

As a numerical example of bunching, consider an experiment using spark chambers and limited by background to I = 0.1 ampere, or 4 stacked pulses of accelerated beam. The energy spread of the unbunched, stacked beam would be 5 Mev. Now if the beam were bunched with rf to B = 1/10, the interaction rate would increase by 10 with no increase in background through the spark chambers from residual gas. The energy spread of the bunched beam would now be 50 Mev. If 1 ampere were stacked, unbunched, it would also have a 50-Mev energy spread, and would provide 10 times the interaction rate as well as 10 times the background rate as the bunched, 0.1 ampere beam, or 100 times the interaction rate as the unbunched 0.1 ampere beam.

An incidental advantage of rf bunching would be as a time base in time-of-flight determinations. Thus with 50 Mc rf and B = 1/10, the beam interacts for only 2 nsec every 20 nsec. Bunching also introduces yet another dimension to the beam which must be known in order to determine absolute cross sections.

IV. Energy and Circumference

If a storage ring is made with a circumference larger than the AGS circumference, the circulating current will be less in direct proportion and reaction rates less as the square of the ratio of circumferences. However, if the storage-ring circumference is twice that of the AGS, two successive pulses of the AGS could be "stacked" circumferentially in the ring by holding the first with rf to occupy only half the circumference until the other pulse is inflected. This double pulse would then be added to an existing stacked beam in the normal way. Thus storage rings an integral number of times larger than the AGS could stack beam as efficiently as one the same size as the AGS. Obviously, the same trick would allow stacking of beam to the same longitudinal current density as in the AGS for any ratio of circumferences, if the remaining beam is discarded. Thus, with a storage ring 1.4 times the AGS circumference, 7/10 of each of two successive AGS pulses could be put into the ring. Such a trick only improves the energy spread or radial width of the stacked beam for a given interaction rate, or the maximum which can be stacked in a given radial aperture.

If a ring of twice (or 3 times) the diameter of the AGS were built, it would be possible to conceive of stacking a beam in it at 33 Bev and then accelerating (perhaps by phase displacement) to, say, 66 Bev (or 100 Bev). If the magnet were laminated, acceleration of single pulses would be possible and the system would be indistinguishable from a 66- (or 100)-Bev accelerator pair.

In discussions of energy, the physics of the storage ring has been compared with that of a synchrotron which might be built at Brookhaven. This latter is discussed in the context of 600 to 1000-Bev, where it has been mentioned that 750 Bev is possible on the present site. Since there is also discussion at Berkeley of a 200-Bev accelerator there, it may be relevant to note that the equivalent energy available in the storage-ring case (2450 Bev) represents almost as great a jump over 750 Bev as 750 does over 200.