PHYSICS TO BE DONE WITH COLLIDING BEAMS

L. W. Jones
University of Michigan

Brief Historical Introduction

In September, 1955, Kerst considered the possibility of using colliding beams with the high circulating currents associated with FFAG accelerators. Later that year, D.B. Lichtenberg, R.G. Newton and M.H. Ross suggested the use of storage rings. At that time, it did not seem probable that one could transfer the beam from the accelerator to these rings without large increases in transverse phase space. Independently, G.K. O'Neill also proposed storage rings and, subsequently, the delay-line extractors and inflectors to get the beam out of the accelerator and into the rings. Ohkawa invented a two-way FFAG accelerator with multiple intersections and a model has since been constructed at MURA. Symon and Sessler in the meantime developed the rf theory of stacking and filling rf phase space. All of this had taken place prior to 1957.

Since that time, the main advances in the field of storage rings consist of: the practical development of AGS accelerators at CERN and Brookhaven, capable of delivering 5 to 7 $\times$ 10^{11} protons per pulse and potentially capable of still more intensity; fast beam extraction with good emittance characteristics using pulse-line extractors; and rf beam stacking, as demonstrated in the MURA electron model. Another development of great importance of course has been the spark chamber, fulfilling the role of a visual detector with fast time resolution which can be triggered "ex post facto". The point is: all of the technology for the design, construction and use of storage rings now exists! Consequently, the entire attention of those interested in proton-proton colliding beams has focused on the use of storage rings in conjunction with proton alternating-gradient synchrotrons.

Terwilliger has discussed elastic scattering, the most extensively considered experiment in connection with colliding beams.* It is one experiment that will certainly be of interest when storage rings are built.

Experiments Accessible to Colliding Beams

It is difficult to guess what topics will be of interest by the time storage rings are built, since almost all experiments are now being done at only a few Bev. Some are suggested in a paper by Lee, Serber, Wick and Yang.[1] It will be of interest to determine which of these can be done with 30-Bev storage rings.

1) Weak interactions.

If one had the equivalent of 1000 Bev, one could make intense neutrino and muon beams. Now these cannot be made with storage rings at all. However, if the intermediate boson exists and its mass is very large, i.e., $>$ 10 Bev, then storage rings will become very important, as discussed below.

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*Editor's note: Descriptions of very small and large-angle elastic-scattering experiments are also given in Appendix A of this paper.

1. "Some Theoretical Considerations on the Desirability of a 300 to 1000-Bev Proton Accelerator", Experimental Program Requirements for a 300 to 1000-Bev Accelerator and Design Study for a 300 to 1000-Bev Accelerator, p. 15, BNL 772, August, 1961 (revised December, 1962).

2) General character of interactions at very high energies.

A study of this type is well-adapted to colliding beams. All of the momenta of the secondary particles are accessible in the laboratory. One may even identify all of the particles using time-of-flight, threshold Cerenkov counters, etc. This would be more difficult with a single 1000-Bev accelerator and a bubble chamber.

3) Pomeranchuk theorems and related questions.

One is interested in investigating total cross sections and the width of the diffraction peak as a function of energy. Since $\pi^{\pm}$-p and $K^{\pm}$-p interactions cannot be measured, the usefulness of colliding beams is somewhat limited in this case. One may possibly investigate $\bar{p}$-p total cross sections and small-angle elastic scattering. However, a stacked $\bar{p}$ beam would have a factor of 2×10^{8} less current than a proton beam and would make any $\bar{p}$-p experiments quite marginal. These minimal measurements of $\bar{p}$-p interactions would be of considerable interest in connection with Pomeranchuk theorems.

4) Production of new particles or resonances.

One expects that, as one goes to higher energies, cross sections tend to become more or less constant or decrease and resonances occur less frequently. However, if heavier mass systems do exist, then colliding beams could be used to discover them with a degree of difficulty comparable to a 1000-Bev accelerator. Such an experiment will be discussed below.

5) Rare processes.

a) High momentum transfer.

Here storage rings with an equivalent of 2000 Bev can be used to produce large momentum transfers in p-p scattering as existing accelerators can with momentum transfers in π-p scattering. One could do as well as any experiment done to date below 30 Bev even in p-p scattering. It will probably be important to look at these high t events.

Voice: You don't have to restrict yourself to elastic scattering. You could get larger cross sections at high momentum transfers using inelastic events.

L.W. Jones: This is a relevant comment; however, at these energies, no one knows yet whether the elastic or inelastic large t processes are more frequent.

b) Direct production of strange particles (i.e., leptons, intermediate bosons).

An experiment producing intermediate bosons will be described later in this paper.

c) Rare decays of known objects, i.e., leptonic decays of some of the heavy mesons.

One cannot do nearly as well as with a single-beam accelerator.

d) Breakdown of symmetries.

One might look for a reaction such as

$$p + p \rightarrow \Sigma^+ + p \quad \text{or}$$
$$p + p \rightarrow \Sigma^+ + \Sigma^+$$

I do not know how one would do such an experiment with either type of machine.

e) Hidden symmetries.

This topic is even more nebulous than the previous topic. I do not know what role colliding beams may play.

One finds that storage rings are useful in investigating perhaps half of the topics mentioned above. This is more than many people had expected. If one compares the costs and time scales involved in building both storage rings and "equivalent" single accelerators, then this is a remarkably inexpensive way to see the future. We are experiencing the same difficulty, in foreseeing the areas of interest in a new energy range, that people have always had in the past. This is particularly true today, for although we have 30-Bev accelerators available, with the exception of elastic scattering and total cross section measurements, most of the experiments are being done at energies much less than 30 Bev.

Colliding-Beam Peripheral-Production Experiment

Good and Walker[2] have pointed out that at very high energies, small momentum transfer collisions may be highly inelastic. Thus, if two protons of momentum p_o collide head-on (so that p_o is the center-of-mass momentum), one proton may continue forward with $(p_o - dp)$, while the other proton undergoes an increase in the mass to km, where m is the proton rest mass, with the following relationship:

$$dp/p_o = (k^2-1)\ (m/2p_o)^2\ .$$

2. M.L. Good and W.D. Walker, Phys. Rev. 120, 1857 (1960).

Thus a 1% change in momentum of a 30-Bev/c proton could result in excitation of the other proton to 6-Bev mass, or creation of a 5-Bev particle (or system) moving with the other proton.

To study such processes with colliding beams we would propose a counter telescope for one proton similar to that described for the very small-angle elastic-scattering experiment*, however in this case with the counter matrices about $\lambda/2$ downstream from the target on the side of the donut toward the machine center. In this way protons which have lost 1/4 to 2% of their energy but deflected through comparatively small angles would be efficiently detected.

The other "leg" of the experiment would be designed about a spark chamber-magnet system as sketched in Fig. 1. This system would have an azimuthal angle acceptance of about 1/30 of 2π, and a θ acceptance of from 0.025 rad to 0.2 rad (adjacent to other beam) or 0.35 rad (on the clear side of the ring). The magnetic field would fall off roughly as 1/r both to hold the azimuthal angle acceptance more nearly constant and to give more nearly constant percentage momentum measurements of secondary particles, making use of the empirical rule-of-thumb that higher-momentum secondaries appear at smaller angles. Thus, if $p_\perp$ is roughly independent of $p_\parallel$, $\theta \propto 1/p$.

As a numerical example, consider a boson of mass 5 Bev produced peripherally and therefore accompanying the parent proton along the beam axis with a $\gamma = 5$. If the dipion decays at 90° CM, each pion will have $p_\perp$ of 2.5 Bev and $p_\parallel$ of 12 Bev, and therefore come off at $\theta = 0.2$ rad.

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*Very small and large-angle elastic experiments are described in Appendix A.

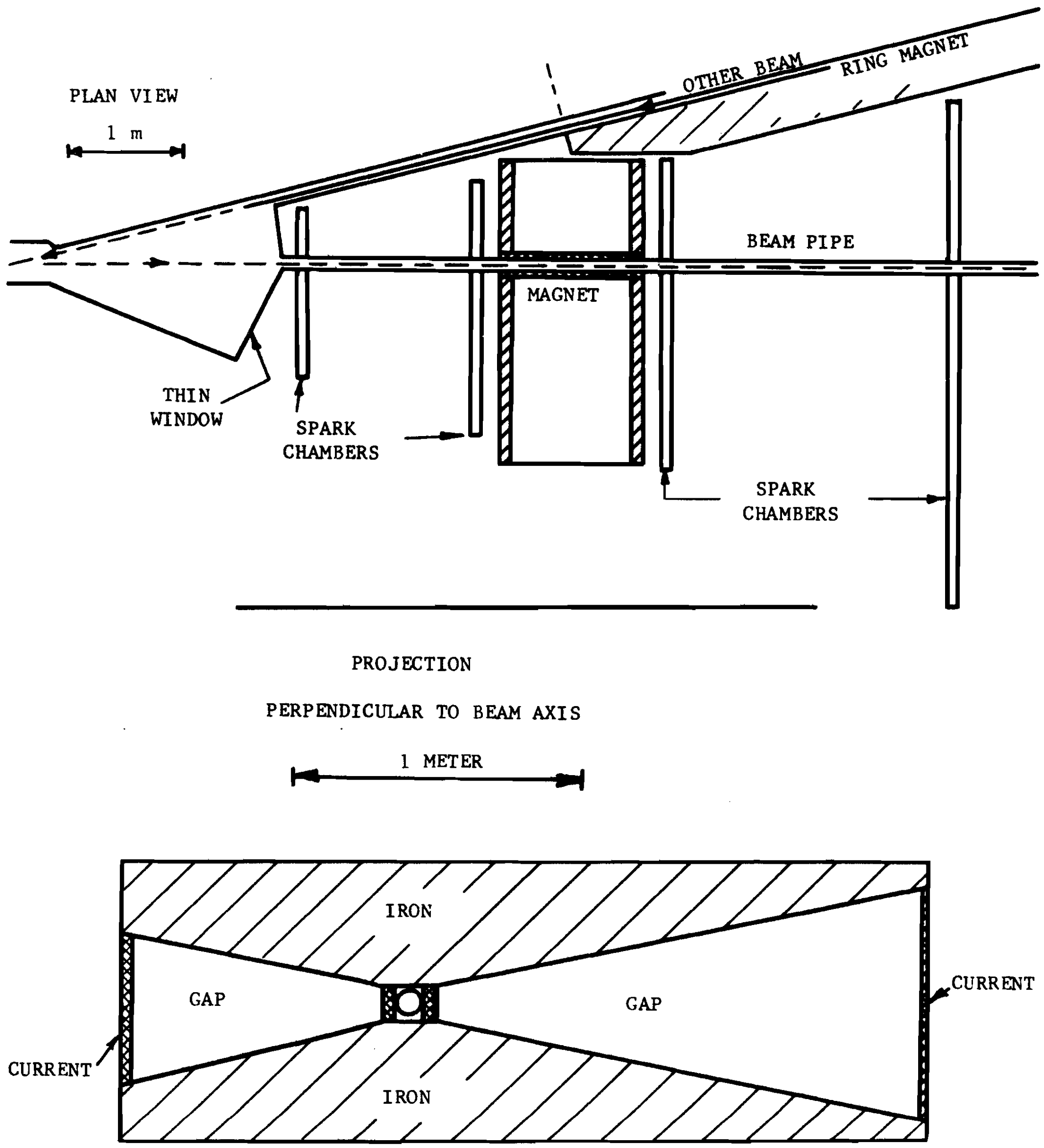


Fig. 1 - Configuration at one side of colliding-beam target region for studying products of peripheral reactions

Generally one pion would emerge at smaller angle and higher momentum, etc. If the production cross section for such an object were 10^{-28} cm^2 and the branching ratio for two-pion decay were 10^{-3}, an experiment with 5 amperes in each beam would yield 10^{-3} sec^{-1} or 4 per hour. The background flux through a 50 × 50 cm^2 spark chamber would be less than one particle per μsec on the average. The momentum resolution of the pions would be about 2% corresponding to a 2% measurement of the diboson mass.

Excited nucleon levels as well as three-or more-body decays could also be studied with this geometry. This particular magnetic configuration is illustrative of another producing no first-order perturbation on the circulating beam. Lesser momentum resolution with larger solid angles could be obtained with the magnet described in Section II, Appendix A. At the expense of putting corrections on the circulating proton orbits, magnets such as one recently started at the Brookhaven AGS could be used. In this particular case, the vertical gap is 30-60 cm, the dimension along the beam is close to 1 m, and the width of the gap is 3 m.

Good and Walker emphasize that such processes might be expected to occur with rather large cross sections, since the very small momentum transfers involved can come from very peripheral interactions. As a numerical example, even coulomb excitation of the $T = \frac{1}{2}$ nucleon isobars can be estimated to have a cross section of several microbarns in the collisions of two 30-Bev protons.

Experiment to Detect the Intermediate Boson

O'Neill has discussed an experiment for detecting an intermediate boson, where such a boson might have a mass of, say, 5-15 Bev. His suggestion was to use a heavy iron shield 3 m thick between the target and spark chambers to stop all muons of less than 3 Bev. This is sketched in Fig. 2 showing the location of inner spark chambers about 30 cm from the target and outer, thick plate chambers 3 m from the target. The geometry subtends $45^{\circ} \leq \theta \leq 135^{\circ}$ from the beams, so that forward or backward, high-energy particles would not be detected. The physics of the experiment would be that an intermediate boson, produced in an inelastic p-p collision, would decay a fraction of the time into a muon plus neutrino only. If the γ of the boson were about 2, about half of the time the muon from the decay would enter the solid angle subtended by the detectors.

Van Hove has estimated that, if the intermediate boson were this massive, its production cross section could be as high as 10^{-29} cm^2 in p-p collisions. If we assume $\sigma_{\text{prod.}} = 2 \times 10^{-31}$ cm^2, I = 20 amperes circulating in each storage ring, and a branching ratio for this decay mode $B = 3 \times 10^{-3}$, the rate for detection is:

$$R = 10^{28} I^2 \sigma(\delta\Omega/4\pi) B = 1.2 \times 10^{-3} ,$$

or about one every 15 minutes. The rate for producing pions with $p_{\perp}$ of 3 Bev/c in p-p collisions at this energy, as calculated from the Cocconi, Perkins and Koester formula by Burhop, would be 5×10^{-6} per collision, or 0.8 per second. The decay mean free path of 3-Bev pions is about 10^4 cm so that, with shielding close about the interaction region, only about 10^{-3} of the pions would decay before interacting, and only half of them with their muons forward, or with momentum comparable to the pion momentum. Thus the rate of background muons from p-p collisions might be comparable to the rate from intermediate bosons. By magnetizing the

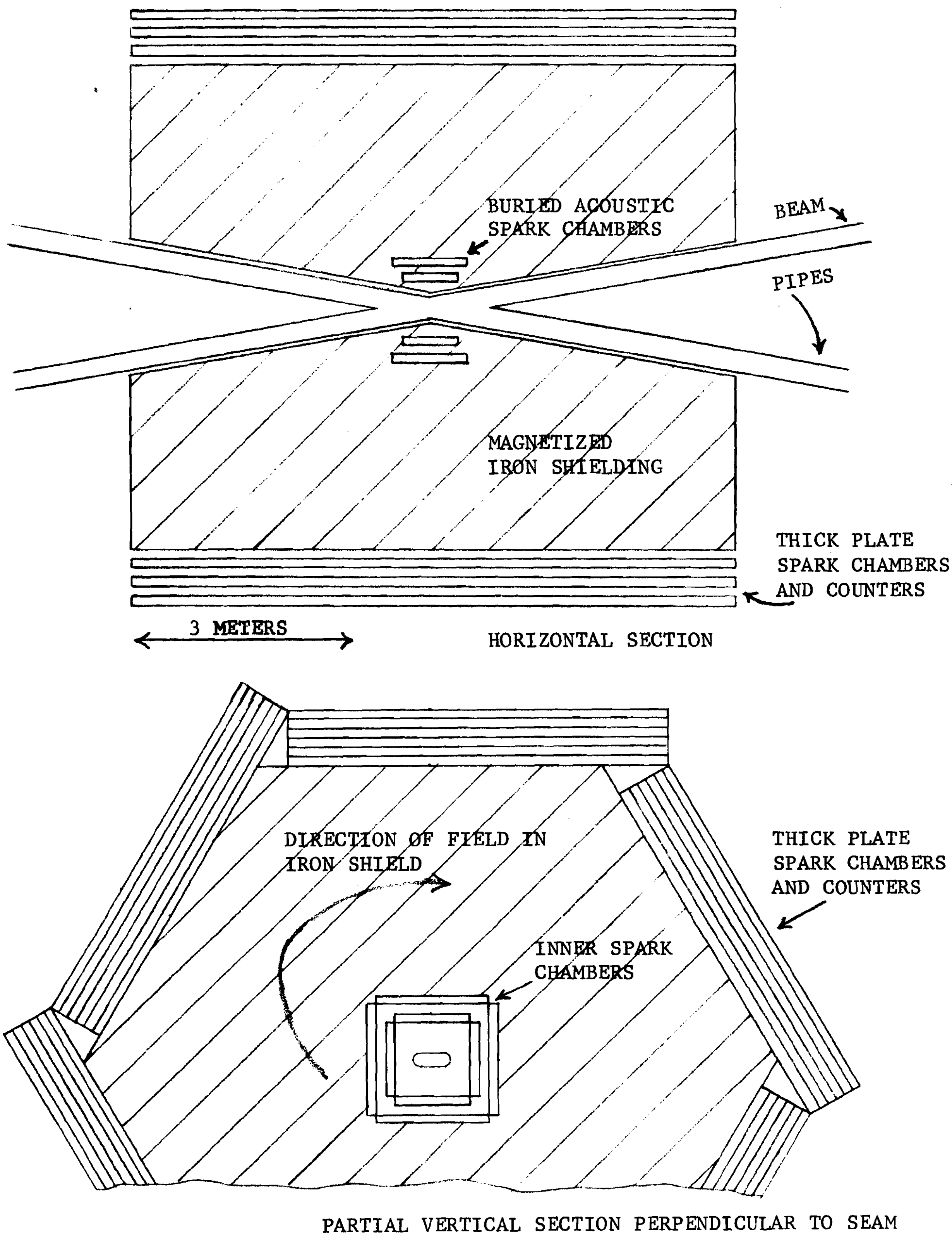
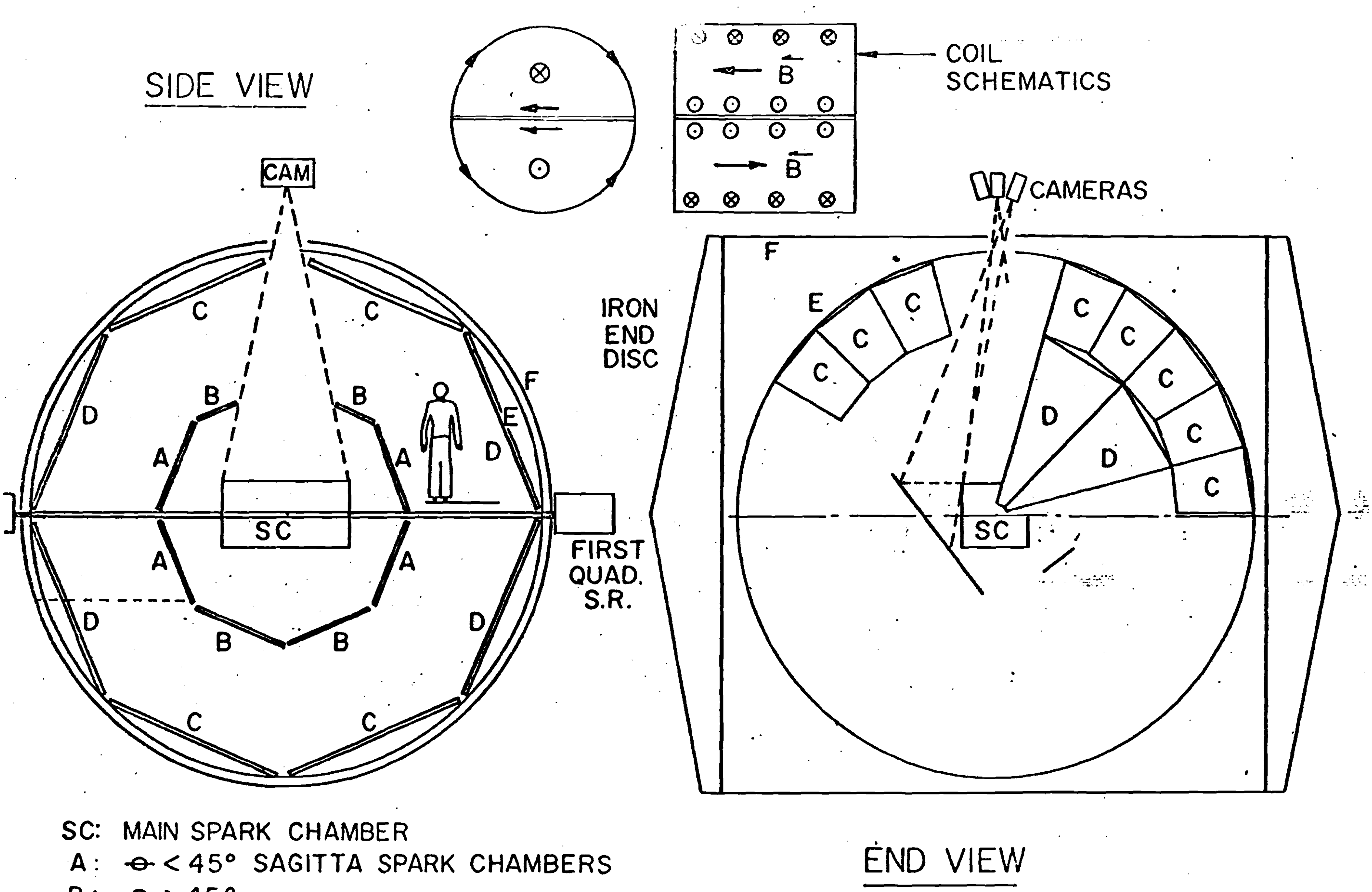


Fig. 2 - Configuration for colliding-beam neutrino experiment

iron of the shield, the muon momentum would be determined to about 20% (in view of the multiple scattering of the muons) so that intermediate bosons should be detectable against a background of muons by the peaking of their momentum spectrum and a different characteristic angular distribution. Trigger rates and backgrounds from beam-gas interactions do not appear to present a serious problem.

4π Solid-Angle Experiments

If a detector can be built which is sensitive to particles emerging, from an interaction region, over a solid angle of 4π steradians, then one can collect events at a large rate. As an example, assume 7×10^{11} particles in the beam (one CERN pulse) and rf beam bunching to compress the beam into bunches totaling about 1/20 of the circumference. Also assume the pressure is sufficiently low that, due to multiple scattering over a few hours, the beam height does not exceed ½ cm. Then there will be about 25 beam-beam interactions per second and only about six background collisions per cm in the same time. One could trigger spark chambers 25 times per second and collect data at a rapid rate. One of the geometries proposed, the one by G.K. O'Neill in Fig. 3, involves surrounding the entire interaction region with a large magnet. The sketch shows a magnet 8 meters in diameter producing a horizontal field perpendicular to the circulating field and having iron end disks to terminate the field. The conductors are so arranged to produce zero field at the circulating beam. A large spark chamber is placed directly around this interaction region to determine the point of origin and trajectory of the particles, and scintillation counters are arranged in a large sphere just inside the



SC: MAIN SPARK CHAMBER
A: $\theta < 45°$ SAGITTA SPARK CHAMBERS
B: $\theta > 45°$ " " "
C: $\theta > 45°$ END-POINT SPARK CAMBERS
D: $\theta < 45°$ " " " "
E: SPHERE OF TIME-OF-FLIGHT COUNTERS
F: ANALYSING MAGNET COILS

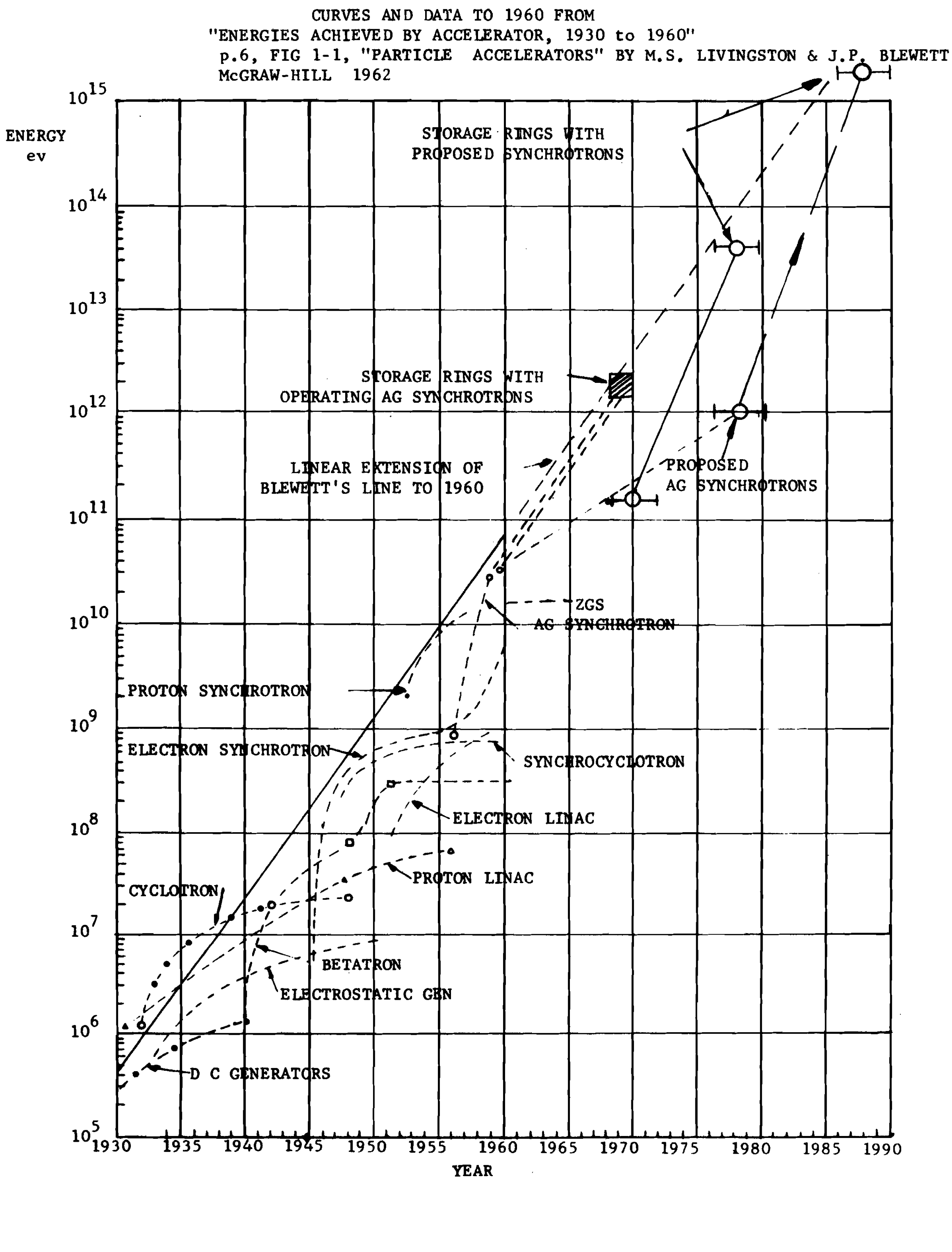
Fig. 3 - Geometry proposed by G.K. O'Neill for 4π experiment

magnet to make time-of-flight measurements and to distinguish π mesons from K mesons up to a few Bev/c. One difficulty, however, is that one cannot detect particles deflected through such a small angle that they remain in the beam pipe. Since we are circulating only one pulse or stack, we could replace the vacuum chamber with a pipe of very small diameter (a few cm in the reaction region) and hence reduce this problem.

B. de Raad and L. Resegotti at CERN have designed other arrangements which are similar, in that they involve a large magnet and many spark chambers.

If one could reduce the pressure in the beam pipe to 10^{-10} torr, then it would be feasible to place a 10-meter-long hydrogen bubble chamber around the interaction region. If its sensitive time was 1 millisecond, then in the 10 meters of length one would have one background event per expansion and a beam-beam interaction once every 40 pictures. If one were willing to replace the hydrogen in the chamber with a heavier liquid or add metal plates, one could detect π^0's. As with 100 to 1000-Bev single-beam accelerators, these events will be hard to analyze because of the large number of secondaries produced.

In conclusion one can see, from an extrapolation to the year 1990 of the J.P. Blewett graph (Fig. 4), that storage rings must be built if the progress in high-energy physics is to stay "on schedule".


CURVES AND DATA TO 1960 FROM
"ENERGIES ACHIEVED BY ACCELERATOR, 1930 to 1960"
p.6, FIG 1-1, "PARTICLE ACCELERATORS" BY M.S. LIVINGSTON & J.P. BLEWETT
McGRAW-HILL 1962
ENERGY
ev
STORAGE RINGS WITH
PROPOSED SYNCHROTRONS
STORAGE RINGS WITH
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PROTON SYNCHROTRON
ELECTRON SYNCHROTRON
SYNCHROCYCLOTRON
ELECTRON LINAC
PROTON LINAC
CYCLOTRON
BETATRON
ELECTROSTATIC GEN
D C GENERATORS
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Fig. 4

APPENDIX A

ELASTIC SCATTERING

I. Very Small Angle p-p Elastic Scattering

The purpose of this experiment would be to explore elastic scattering in the angular interval where electromagnetic and strong interactions should interfere. It would be interesting also in order to look for a possible real part to the scattering amplitude and to study the t dependence of diffraction scattering at small values of t (invariant four-momentum transfer squared). The range of angles studied could be 0.5 to 2.5 milliradians; coulomb and nuclear scattering amplitudes are expected to be comparable at about 1.5 mrad. For σ_{total} = 40 mb, $(d\sigma/d\Omega)\ 0^{o} = 23 \times 10^{-24}$ cm^2/sterad. We consider the experiment to consist of four matrices of solid-state counters or scintillation counters each 2 X 2 mm and displaced on either side of the intersection region by a quarter betatron wavelength (Fig. 5). Each matrix would be mounted in a thin-walled stainless-steel canister on a silphon bellows made to extend into the vacuum tank so that it could be brought to within 1 cm of the center of the circulating beam and permit measurements out to the full aperture of the tank, which would be 4 or 5 cm from the beam axis. From linear orbit theory the displacement at $\lambda/4$ would be uniquely related to scattering angles in the interaction region within uncertainties due to the size of the counters, and the position and angle spread of the circulating beams (betatron phase space). The average angle θ would be related to the displacement of a scattered particle by

$$\theta = \chi/\beta\bar{\lambda}$$

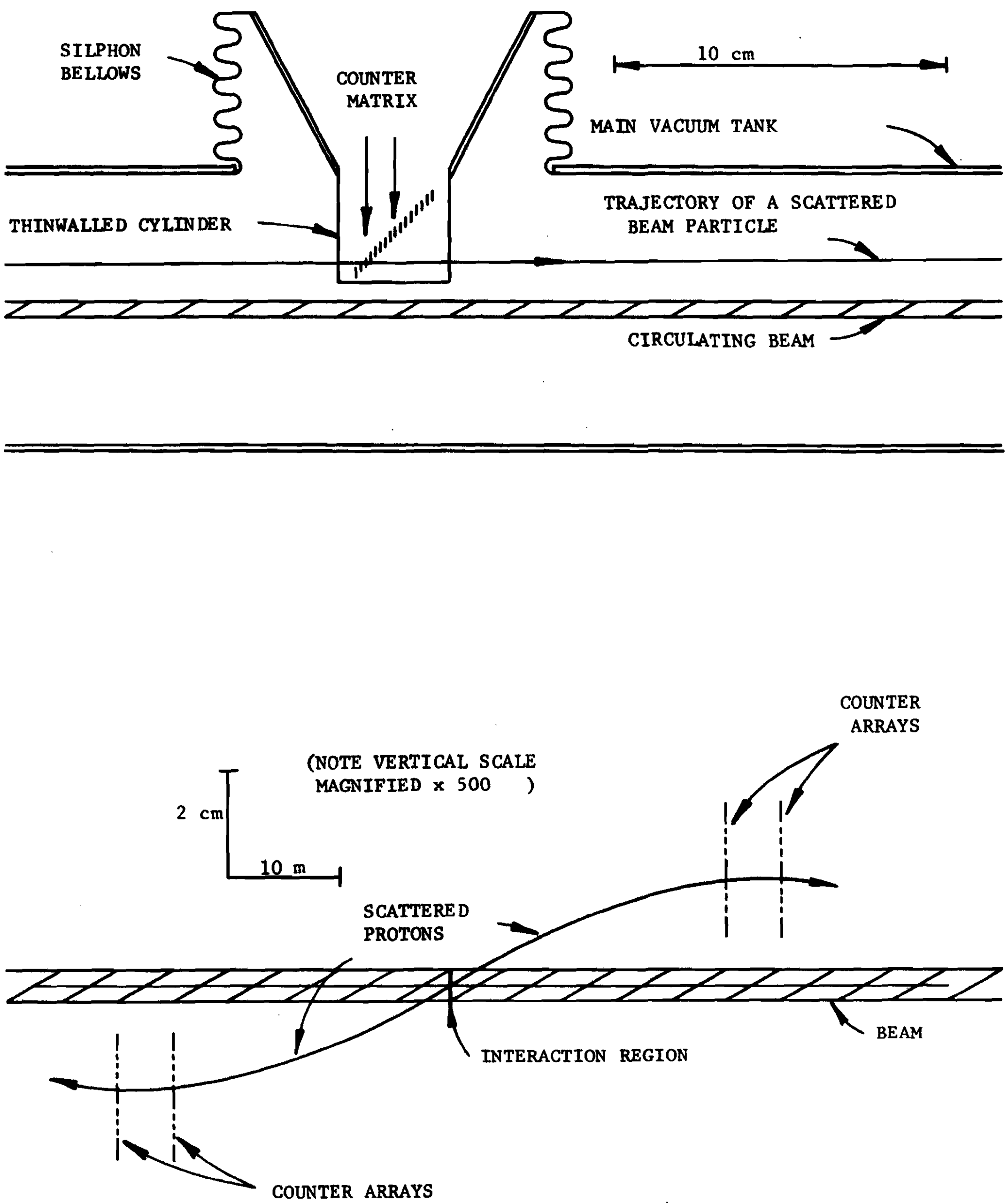

Fig. 5 - Configuration for studying very small angle elastic scattering with colliding beams

where β is a factor between 1 and 2 depending on detailed orbit parameters. Taking β as unity gives the values of χ and θ mentioned above. The solid angle subtended from the intersection region by each 2×2 mm^2 counter 25 to 30 m from the intersection region is 10^{-8} sterad. [$\delta\Omega = \delta x \delta y / \lambda\!\!\!^{-2}$ due to betatron focusing, not $(\lambda/4)^2$] Thus for a one-ampere stacked beam, the rate through one counter is

$$R = 10^{28} \times \frac{1}{0.5} \times 23\times10^{-24} \times 10^{-8} = 5\times10^{-3}/\text{sec}.$$

The stacked beam of one ampere will have a spread due to betatron oscillations of ±0.25 cm and 0.25 cm due to energy spread, or dimensions 5×7.5 mm^2. The beam would not grow significantly larger than this for several hours due to multiple scattering from residual gas at 10^{-9} mm Hg (χ_{rms} = 0.2 cm after 2×10^4 seconds), so that this experiment could be most effectively done by replenishing the storage rings every 6 hours or so with 40 stacks from the AGS, requiring 4 minutes of AGS time. The background singles rate would be

$$P = 2.5\times10^3 \times 4\times10^{-2} = 100/\text{sec}$$

so that a two-fold coincidence with 5 nsec time resolution would have an accidentals rate of 5×10^{-5} sec^{-1} or 1% of the events rates. In practice, it would be safer to use a two-fold coincidence in each leg, with a second counter matrix (of somewhat larger counter elements, such that the first matrix defines the angles) several meters beyond the first. The average background events contribute particles of a fraction of the beam momentum at 10-100 mrad so that the singles in the two matrices would rarely be correlated. Hence the background from residual gas would be entirely negligible. The angle and momentum resolution of the scattering would be determined again by beam and counter size and by orbit theory. The scattering angles would be known to ±0.13 mrad, but the momentum would be uncertain by 100 to 200 Mev/c (due to the momentum compaction in the beam). With the extremely small momentum transfers (down to 15 Mev/c) and large cross sections here, there seems little reason to believe that contamination of inelastic events would pose a major problem.

Considering each counter matrix to be a 5 × 10 array of 2 mm counters (2 cm^2 over-all) there would be one elastic event recorded every 4 seconds, or about 20,000 per day.

Similar counter systems in re-entrant wells closer to the target region could explore the angular intervals from 2 to 8 milliradians (in steps of, say, 2 mrad) to join smoothly onto the range covered by external spark-chamber arrays. Such counters closer to the beam-beam intersection would ideally be smaller, work with a smaller circulating beam, etc., to preserve comparable momentum and angular resolution and rates. Our attention above has been concentrated on studying the smallest angles which might be reached, and it appears that as the angles studied become larger the problems become simpler.

II. Elastic Scattering at Large Angles

In order to study elastic scattering at large angles, e.g., where $d\sigma/d\Omega = 10^{-32}$ cm^2/sterad, it is necessary to subtend as much azimuthal angle as possible, and to maintain large solid-angle coverage with maximum angular resolution. In order to do this, a spark-chamber system could be employed, with four chambers on each side of the interaction region. Each chamber would be 2.5 meters from its neighbors, with the closest 2 meters from "target". Each spark chamber would be subdivided into 4 subchambers, each in one quadrant about the beam axis. Running along the beam pipes from 5 to 10 meters downstream from the "target" would be a current cylinder carrying 10^5 ampere turns (at 10^3 amperes/cm^2) with an inner diameter of 10 cm and an outer diameter of 20 cm. This current would be paraxial with the beam, producing no field on the stacked beam, and would be closed in four loops returning at a radius of 1 meter from the beam pipe (Fig. 6). This configuration then produces a magnetic field of $2 \times 10^4/r$ gauss directed about the beam axis, terminated by the axial currents at r = 5 cm and by the return currents at 1 m. The first two spark chambers along each beam then determine angles to about 10^{-4} radians and the target vertex to about 3 mm^3, and the magnetic field with the spark-chamber system defines momentum to 0.1r% (where r is the average displacement of the track from the beam axis in the magnet). Thus the radius of curvature in the magnet for a 30-Bev/c particle is 50 r meters, and the deflection of a particle after 5 meters of field is 25/r cm. In order to study elastic scattering of 10^{-32} cm^2/sterad, Baker et al. report using momentum resolution of 7% and angular resolution of 1^o, so that the above figures appear adequate.

The elastic scattering appears to obey a law such as

$$d\sigma/dt = (d\sigma/dt)_{t=0} \; e^{At}$$

where $A \cong 12$ at the highest energies studied.

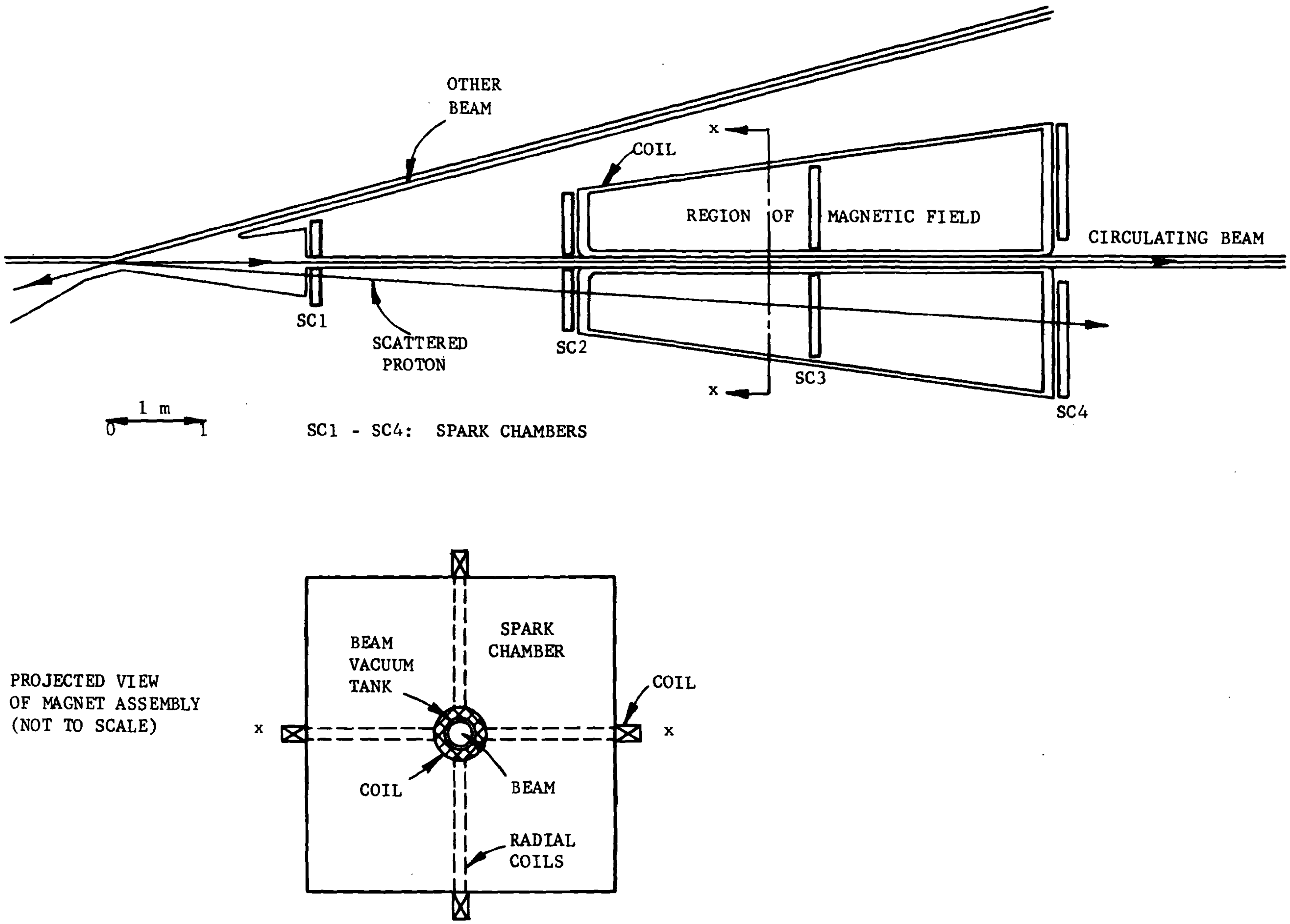

Fig. 6 - Configuration of axial magnet and spark chambers for large-angle elastic scattering with colliding beams

If we detect $|t| \geq |t|_{min}$,

$$\int_{t_{min}}^{-\infty} \frac{d\sigma}{dt}\, dt = \frac{1}{A}\left(\frac{d\sigma}{dt}\right)_{t=0} e^{At_{min}}$$

For $t_{min} = -1\ (\text{Bev}/c)^2$, and $\sigma_{\substack{total \\ elastic}} = 8 \times 10^{-27}\ \text{cm}^2$

$$\int_{-1}^{-\infty} \frac{d\sigma}{dt}\, dt = 3 \times 10^{-30}\ \text{cm}^2$$

so that the experiment should have a reasonable rate at 10^{-30} cm^2 and detect reactions down below 10^{-32} cm^2. We consider a 1-ampere circulating beam again, now bunched with rf to 1/4 the azimuth, f = 1/4. This gives an interaction rate

$$R = 10^{28}\ I^2\ \sigma/\Delta y\ f\ .$$

For $\sigma = 3 \times 10^{-30}$, $y = 0.5$, the rate is one event in 4 seconds or 1000 per hour, 2×10^4 per day. The background is less than one extraneous track in each meter-square spark chamber per event. The trigger counter system could be as selective as the effort warrants, by subdividing the counter sectors, etc. One type of trigger counter might employ a lead glass Cerenkov counter to require > 10 Bev of energy of relativistic particles in several nuclear mean free paths of Cerenkov radiator. This would be a "pion-shower detector". However, a simple scintillation counter system would be entirely adequate to keep the spurious trigger rate to about one per second. Thus an initial counter annulus in four sectors, just before the first spark chamber, of radius $r_1 = 5$ cm, $r_2 = 20$ cm would have a singles rate of 250 kc, and a system of two such counter systems in coincidence with two other arrays downstream in each scattered "beam" would hold the accidental rate to about one per second.

The minimum angle subtended by the apparatus would be 25 mrad (from the diameter of the vacuum pipe inside the spark chambers at 2 meters from the target region) corresponding to momentum transfers of $|t| \cong 0.5$ (Bev/c)^2. The largest angles are limited only by the maximum size of the magnet coils and the requisite momentum resolution, and for these figures would be 100 mrad or about 8 (Bev/c)^2. The accepted ϕ angle would be close to 2π, lacking only the four radial cuts where the magnet currents leave the vacuum pipe.

In summary, the rates would be 1000 per hour for an integrated cross section of 3×10^{-30} cm^2, or 10^5/week of running. In that time 30 events would of course be seen where $\sigma = 10^{-33}$ cm^2, and could be separated from inelastic background according to present experience in p-p scattering at the AGS, making use of the resolution available in this experiment.

APPENDIX B

TABLE OF PARAMETERS FOR COLLIDING BEAMS WITH THE 30-BEV AGS

Storage Rings

r	=	150 m average radius
$2\pi r$	=	940 m circumference
f	=	3.2×10^5 sec^{-1} revolution frequency of protons
I_1	=	2.6×10^{-2} amperes/pulse, corresponding to:
N_1	=	5×10^{11} protons/pulse injected into the storage ring from the AGS
ν	=	8.75 number of betatron wavelengths per revolution
$\lambda\!\!\!^{-}$	=	17 m betatron wavelengths/2π
$\Delta p/p$	=	75 $\Delta r/r$; (momentum-compaction factor)$^{-1}$ $\times$ $\Delta r/r$
Δp	=	750 Mev for $\Delta r = 5$ cm, $p_o = 30$ Bev/c
Δp_1	=	1.2 Mev momentum spread (debunched) of one AGS pulse
I	$\simeq$	16 amperes from 600 AGS pulses stacked in $\Delta r = 5$ cm, corresponding to:
N	$\simeq$	3×10^{14} protons circulating in each beam
l	=	15 m straight-section length available in colliding-beam region
α	=	$15^o = 0.26$ radians beam-crossing angle
Δy	=	0.5 cm vertical height of beam from AGS
θ_{max}	=	$\pm 10^{-4}$ radians angular spread of stacked beam due to betatron oscillations

Interaction Rates

$$R = \begin{cases} 30\, N^2 \sigma / f\, \Delta y \\ 10^{28} I^2 \sigma / f\, \Delta y \end{cases}$$

interactions per target region per second, where f is the fraction of the circumference occupied by circulating beam if bunched by rf

$$R \cong 400\, I^2 \quad \text{for } \sigma = 40 \text{ mb}, \ \Delta y = f = 1$$

TABLE OF PARAMETERS FOR COLLIDING BEAMS WITH THE 30-Bev AGS (Contd.)

Background Rates

$p = 10^{-9}$ mm Hg

$\sigma_N = 380 \times 10^{-27}$ cm^2 proton-nitrogen total cross section

$n = 7 \times 10^7$ atoms of diatomic gas/cm^3 at 10^{-9} mm Hg

$P = \begin{cases} 8.6 \times 10^{-12}\, N \\ 1.6 \times 10^{2}\, I \end{cases}$ beam-gas interactions per cm per sec

$\Phi_\perp = 16\, P/\rho$ flux of particles from beam-gas interactions per cm^2 per sec through an area normal to the beam axis

$\Phi_\perp = 2.6 \times 10^3\, I/\rho$ ρ cm from the beam (unshielded)

$\int_{\rho_1}^{\rho_2} 2\pi\rho\, \Phi_\perp\, d\rho = 1.6 \times 10^4\, (\rho_2 - \rho_1) I$ flux of background particles through an annulus of inner and outer radii ρ_1 and ρ_2

$\tau = 1.2 \times 10^6$ sec mean lifetime of beam against nuclear interactions with residual gas

$\frac{d\langle x^2 \rangle}{dt} = 2 \times 10^{-6}$ cm^2/sec growth rate of average size of beam due to multiple coulomb scattering