P-P ELASTIC SCATTERING WITH COLLIDING BEAMS

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This experiment has been considered quite thoroughly by B. de Raad.[1] L.W. Jones and G.K. O'Neill, both active proponents of colliding beams, have also thought about it in some detail. In the past few weeks I have examined the experiment, p-p elastic scattering in the diffraction region, to see if their enthusiasm is valid and to see if any additional problems arise. My conclusions are in general agreement with theirs: the experiment can be done with present techniques, with some phases easier and some more difficult than if performed with a single accelerator with the same center-of-mass energy.

Advantages In Using the Center-of-Mass System

Equivalent momentum transfers involve lower momentum and larger angles for the forward-scattered proton when working in the center-of-mass system than in the laboratory system, the factor being $(2\ p_{CM}/M_o)$, where p_{CM} is the momentum of one of the protons in the center-of-mass system and M_o is the proton rest mass $(c = 1)$. This factor can be seen quickly from the relativistic kinematics.

If, in the center-of-mass system, one proton has total energy E_{CM} and momentum p_{CM}, then (for forward-scattered protons)

1. B. de Raad, CERN Internal Report AR/Int. SG/62-12.

$$p_{lab} = \gamma_T (p_{CM} + \beta_T E_{CM}) ,$$

T being the center-of-mass to lab transformation. Now, since in extreme relativistic limits, $\beta_T \cong 1$ and $p_{CM} \cong E_{CM}$,

$$p_{lab} \cong \gamma_T \, 2p_{CM} .$$

Since you have to transform backward with the γ of the other proton to get back to the lab,

$$\gamma_T = \gamma_{\text{proton in CM}} \cong \frac{p_{CM}}{M_o} .$$

Therefore,

$$p_{lab} \cong \left(\frac{2p_{CM}}{M_o}\right) p_{CM} .$$

For p_{CM} = 25 Bev/c, the factor

$$\frac{2p_{CM}}{M_o} = \frac{2 \times 25}{.938} = 53.3$$

and p_{lab} = 1330 Bev/c. Since the transverse momentum is the same, the lab angle for the forward-scattered proton is decreased by the same factor,

$$\theta_{lab} = \frac{\theta_{CM}}{\frac{2p_{CM}}{M_o}} = \frac{\theta_{CM}}{53.3}$$

Thus an angle of 5 milliradians in the center-of-mass system corresponds to 0.1 milliradians in the laboratory system. So, when analyzing the forward-scattered particles, angle and momentum measurements of the same center-of-mass resolution are ~ 50 times easier in the center-of-mass system.

In another way of looking at it, as far as angular and momentum resolution are concerned, an elastic p-p experiment with total center-of-mass energy of 50 Bev performed in the center-of-mass system is only as difficult as the elastic scattering of 25-Bev protons in the laboratory, which has been successfully done.

Disadvantages of the Center-of-Mass System

There appear to be four major disadvantages:

1) The geometry of the experimental area is very cumbersome because you must work around the interaction region, limited by the quadrupoles and magnets of the machine, rather than at the end of an external beam in an unlimited space.

2) The interaction rate in colliding-beam experiments is equivalent to a beam from a single accelerator of $\sim 10^3$ to 10^5 particles/sec into a 1-meter length of a liquid-hydrogen target, i.e. you are down by a factor of $\sim 10^6$ from what you can do with an external proton beam; however, the rates are high enough to look at p-p elastic scattering in the diffraction region, as well as a large number of other experiments.

3) It is difficult to normalize the measured cross sections since the spatial distributions of the two colliding beams are unknown. These can be determined in an awkward manner by observing the background interactions with the residual gas.

4) Total-cross-section measurements are very difficult. If one considers a total-absorption measurement, then one can calculate that the beam lifetime from p-p events, in eight interaction regions with currents of 10 amperes/cm^2, is about 20 years. To reduce the gas-scattering lifetime to this value one would require a pressure of 2×10^{-12} torr, assuming the gas is nitrogen. One would probably have to arrive at the total cross section by addition of all types of reactions observed. To include the diffraction elastic scattering at small angles one would have to extrapolate to zero angle. It is not certain that one can check the "optical theorem".

R. Serber (Columbia): The total cross section itself might be interesting. Everybody assumes that the cross section is constant at high energy. There is very little evidence to support this.

E.H.S. Burhop (CERN): I'm not sure it's quite as difficult as you make out. If you use the ring method and decrease the size of your aperture and extrapolate down, then the main thing you will miss is the elastic diffraction aspect.

K.M. Terwilliger: Yes, you can do this extrapolating down to zero. There will be forward inelastic scattering too, with extra π^o 's coming off. It's a hard experiment, but you can probably do it with fair accuracy.

Beam Parameters

1. Momentum resolution

The momentum resolution of the beam can be estimated. For a single pulse assumed debunched at injection and at full energy and with no rf phase space mixing in between,

$$\frac{\Delta E_{\text{of the protons}}}{\beta} = \text{invariant} .$$

If one has an energy spread at injection (50 Mev) of 400 kev, then at full energy this spread becomes 0.4 Mev/.3 = 1.2 Mev, assuming that one handles the particles in an adiabatic manner. If one assumes a radius of the machine R = 150 meters and an AGS pulse of 5×10^{11} particles, (26 ma at full energy), then 40 such pulses are equivalent to 1 ampere of circulating beam. The corresponding energy spread will be ΔE = 50 Mev. Thus, at 25 Bev and for 40 stacked pulses,

$$\frac{\Delta p}{p} = \pm 0.1\% .$$

CERN often quotes $\Delta p/p$ about ± 1%. This, however, is for about 500 pulses.

2. Angular resolution

The particles in one pulse of the AGS at full energy have a vertical betatron-oscillation amplitude of $\Delta y \approx 0.25$ cm. If we limit ourselves, due to multiple scattering, to an amplitude of twice this height, i.e.

$$\Delta y = 0.5 \text{ cm} ,$$

then

$$\Delta y \approx \frac{2R\Delta\theta}{\nu}$$

or

$$\Delta\theta \approx 0.2 \text{ milliradians} ,$$

assuming R = 150 meters and $\nu = 8$. This angular spread will be similar for a number of stacks if we stack in energy space. Thus both the momentum and angular resolutions in the beam are quite good. If one stacked in betatron phase space instead of synchrotron phase space, one could achieve even smaller momentum spread at the cost of angular resolution.

3. Interaction rates

The interaction rates can be calculated for beams intersecting at an angle α, each having a maximum vertical amplitude of oscillation y (assuming a uniform vertical distribution)

$$\text{Interaction rate} = \frac{1.25\ I^2\sigma_{\text{millibarns}}}{2y\ \tan\frac{\alpha}{2}}$$

For I = 1 ampere, y = ½ cm, $\alpha = \frac{1}{4}(15^\circ)$, the reaction rate equals $10^{28}\sigma$ events per second (σ in cm^2). From cross sections given by A.E. Taylor,[2] extrapolated from CERN measurements at 25 Bev by assuming full shrinking (logarithmic or Pomeranchuk shrinking) of the diffraction peak

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2. A.E. Taylor, CERN Internal Report AR/Int. SG/62-12.

θ (in milliradians)	-t (momentum transfer) $(\mathrm{Bev/c})^2$	$\frac{d\sigma}{d\omega}$ cm^2/sterad
0	0	1.6×10^{-23}
30	.56	1×10^{-26}
40	1	8×10^{-30}

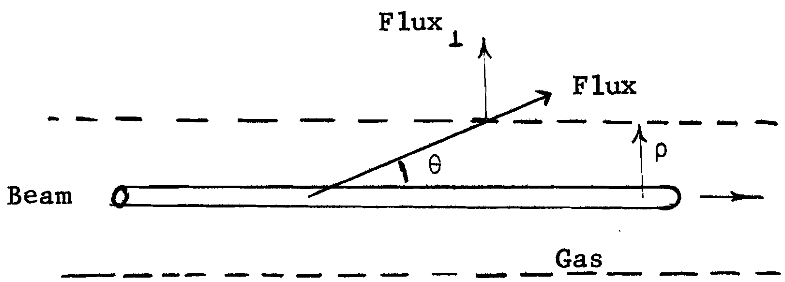
Thus, if one were to build a spectrometer with a solid angle $\Delta\omega = 10^{-4}$, then at $d\sigma/d\omega = 10^{-26}$ (-t = .56) you will get a rate $10^{28} \times 10^{-26} \times 10^{-4} = 10^{-2}$ per second. It is evident that one would require more solid angle and higher currents to explore appreciably larger values of -t.

R. Serber: On the other hand, if you assume that the cross section is just a function of momentum transfer, not of energy, you can take it directly from present experiments.

K.M. Terwilliger: Yes. The cross sections do not go down nearly as fast if there is no further shrinkage. For example, at θ = 30 milliradians, $d\sigma/d\omega = 10^{-25}$ cm^2/sterad and at 40 milliradians, $d\sigma/d\omega = 6 \times 10^{-27}$ cm^2/sterad, so if the peak doesn't shrink further, you can go to (-t) = 1 and do the experiment quite well. I have taken the most pessimistic case.

4. Background flux

The background flux due to nuclear proton-gas collisions can be easily estimated (assuming no shielding).



After a proton collision with a gas nucleus, secondary particles will go out with some average angle $\bar{\theta}$. The flux of secondary particles perpendicular to the beam will then be

$$\text{flux}_\perp \cong \text{flux (along secondaries)}\ \overline{\sin\theta}\ ,$$

so for small angles,

$$\text{flux} = \frac{\text{flux}_\perp}{\bar{\theta}} \text{ particles per cm}^2 \text{ per second.}$$

For elastic scattering $\bar{\theta} \approx 10^{-2}$ radians; for inelastic scattering the average transverse momentum is about the same as in elastic scattering ($\bar{p}_\perp \approx 300$ Mev/c) and the average longitudinal momentum ($\bar{p}_\parallel \sim p_{original}/m$), where m is the multiplicity of secondary particles in a collision. So, for elastic or inelastic collisions,

$$\bar{\theta} \sim .01m \text{ rad} .$$

Therefore, the secondary flux is

$$\text{flux}\ (\rho) = \frac{\text{flux}_{\perp}}{.01\ \text{m}} = \left[\frac{d}{dt}\left(\frac{dN}{dx}\right) \frac{m}{2\pi\rho} \right] \frac{1}{.01\ \text{m}} = \frac{\frac{d}{dt}\left(\frac{dN}{dx}\right)}{2\pi\rho(.01)}$$

independent of m. ρ is the perpendicular distance from the beam and $\frac{d}{dt}\left(\frac{dN}{dx}\right)$ is the number of interactions per centimeter per second in the beam. Assuming a pressure of 10^{-9} torr, the expression becomes

$$\text{flux} \sim \frac{3 \times 10^{3} I}{\rho\ \text{cm}} /\text{cm}^2/\text{sec} .$$

Thus, at 10 cm away and 1 ampere, you get 300 particles per cm^2 per second. Consider an annular region of width $\Delta\rho$ around the beam. The total flux through this region is, by integrating, $2 \times 10^4 I\Delta\rho$, i.e., for $I = 1$ ampere and $\Delta\rho = 100$ cm, you would get 2×10^6 particles per second through the region. Since spark chambers have a resolution time of $\sim$ 1 μsec, they could be made fairly large and only occasionally have an extra track.

Elastic Scattering Experiments

Fig. 1 shows a sketch of the interaction region of the proposed CERN colliding-beam machine, the region in which the experiments are to be

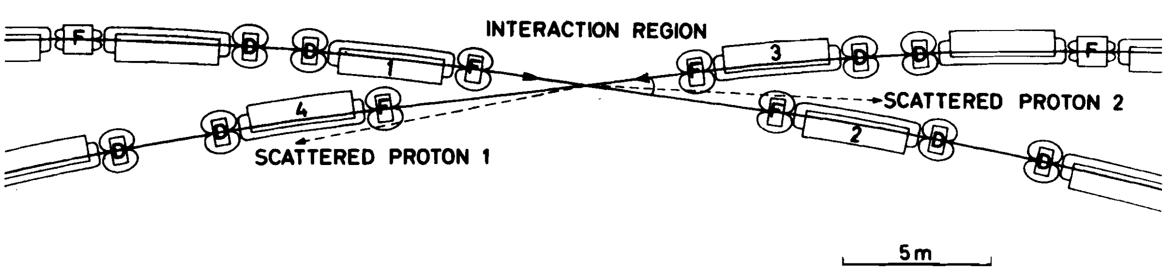


Fig. 1

performed. The distance between the interaction region and first quadrupole downstream is about 7 meters. Different experiments will require different vacuum-tank configurations in this region; the vacuum tanks must be removable units. The sketch shows particles scattered from each beam.

The elastic p-p experiments consist basically in analyzing the momentum and direction of both of these scattered particles. The experiments may be divided into three regions:

1) Diffraction-elastic region, $10 < \theta < 30$ milliradians

These angles are large enough to allow one to use spark chambers and the downstream quadrupoles and bending magnet as a momentum analyzer.

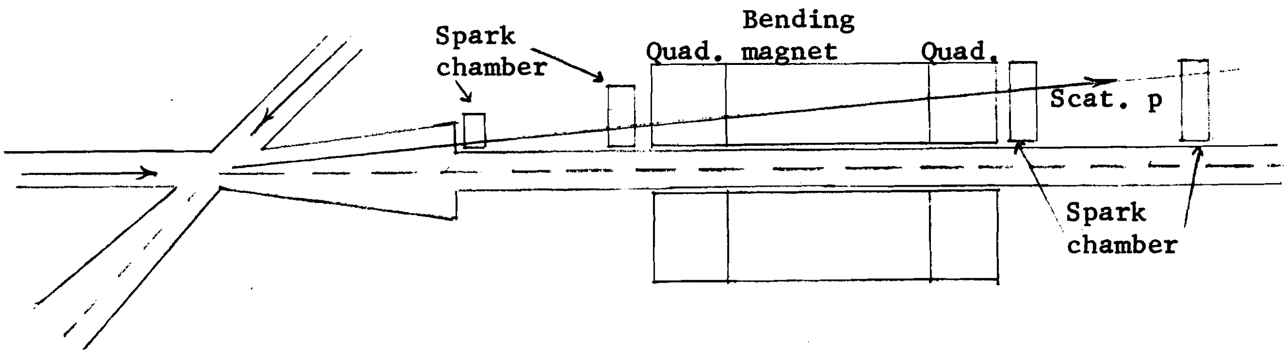
2) Coulomb interference region, $\theta < 10$ milliradians

These angles are probably too small for spark chambers and one must use small counters in order to get very close to the circulating beams.

3) "Large" angles, $\theta > 30$ milliradians

The cross sections at these large angles are low and one cannot use a spectrometer employing the downstream storage-ring magnets. One must use all the solid angle, with large spark chambers and an additional magnetic field.

Consider the intermediate-angle experiment (No. 1, $10 < \theta < 30$ milliradians). At 25 Bev the (momentum transfer)^2 is $0.06 < -t < 0.56$. The momentum resolution is determined by the position errors in the spark chambers, which are about 0.2 millimeters.



Since the spark chambers can be separated by 2 meters, angles may be determined to within $\Delta\theta \approx 0.1$ milliradian (limited there also by multiple scattering) and the momentum to within $\Delta p/p \approx 0.2\%$ (with a bending angle of 70 mrad). This resolution is consistent with that of the beam. The limitation on counting rate is imposed by the quadrupole and bending magnet which restrict the solid angle. Fig. 2 illustrates a quadrupole designed by de Raad to provide maximum solid angles for scattered beams. One can obtain solid angles of 4×10^{-5} steradians and thus counting rates (at $\theta = 30$ mrad) of about 4×10^{-3} per second with I = 1 ampere. This assumes a vertical angular acceptance of $\pm$ 7 milliradians and a radial acceptance of 3 milliradians (corresponding to $\Delta t = .1$). Thus one can get appreciable counting rates out to 30 milliradians. The resolution ($\Delta\theta \approx 0.1$ mrad and $\Delta p/p \approx 0.2\%$) is extremely good. The small value of $\Delta p/p$ alone is almost

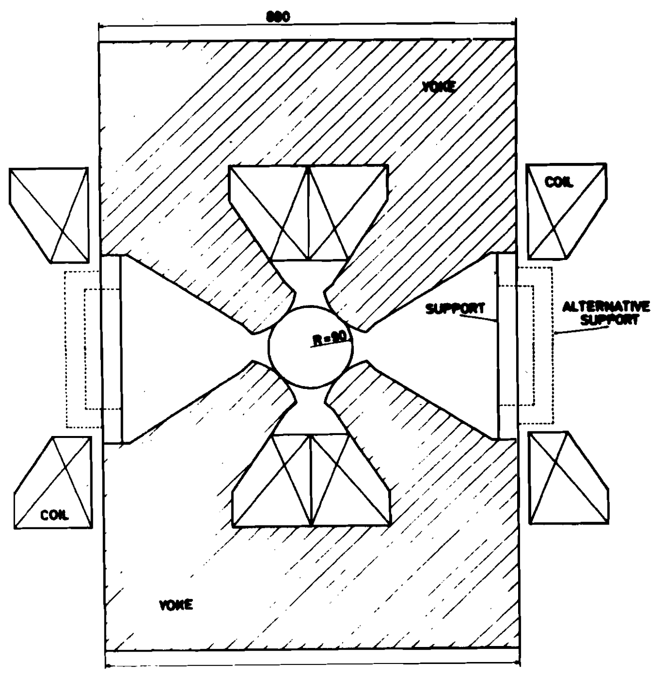


Fig. 2

sufficient to allow the detection of one extra pion at rest in the center-of-mass system. When one also includes angular resolution, the situation greatly improves. Consider the particular inelastic case

$$p + p \rightarrow p + N^{*}$$
$$\qquad\qquad\qquad \searrow p + \pi^{o}$$

where the N^{*} has a mass of 1.5 Bev, which can leave the π^{o} at rest in the lab. Even if the angular resolution is as poor as $\Delta\theta = 0.5$ milliradians, the fraction of N^{*} decay solid angle, for which this particular event could be confused with an elastic p-p event, is less than 10^{-3}. Thus there is essentially no background; like a bubble chamber, almost every inelastic event can be identified.

One can in fact perform this experiment with far less resolution. An example is the recent Yuan-Lindenbaum elastic p-p scattering experiment at 20 Bev.[3] In that case, $\Delta p/p$ of the incoming proton was about 1%. There was no momentum analysis of the scattered protons. The angular resolution of the forward-scattered proton, $\Delta\theta$, was $\pm$ 2 milliradians. With this resolution they carried the experiment to a $(-t = 1)$ and had there a 30% correction for background. Since this experiment worked well at 20 Bev, a similar colliding-beam experiment with 20 Bev in each beam should work as well. So the beam-momentum resolution can probably be relaxed to $\pm$ 1% and the beam current increased from 1 ampere to 10 amperes, increasing the rate of detectable events by a factor of 100.

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3. K.J. Foley, S.J. Lindenbaum, W.A. Love, S. Ozaki, J.J. Russell and L.C.L. Yuan, Phys. Rev. Letters 10, 376 (1963).

However, the beam-gas background must be considered again. For a pressure of 10^{-9} torr, giving a background flux = 3000 I/ρ per square cm per sec, a spark chamber subtending 4×10^{-5} steradians, of area 20 sq cm, located at ρ = 20 cm, would have a background flux of about 3×10^4 per second for I = 10 amperes, an acceptable rate. The accidental coincidence rate (spurious picture-taking rate) assuming a twofold coincidence, a 5 nanosecond resolution time and 10 amperes, would be $(3 \times 10^4)^2 \times (5 \times 10^{-9})$ or 5 per second. This rate could be reduced by the addition of more coincidence counters. These proton-gas background events will not have a common origin and so will not be confused with p-p events.

The large and small-angle elastic p-p experiments have been worked on by L.W. Jones.* Fig. 3 indicates an experimental arrangement for large-angle diffraction scattering, where one wishes to obtain the maximum solid angle, (an increase of a factor of 10 over the previous experiment where one is restricted to a small angle of azimuth). The magnetic field is created by conductors parallel to the beam and the particles are analyzed in this field. The momentum resolution is a few percent.

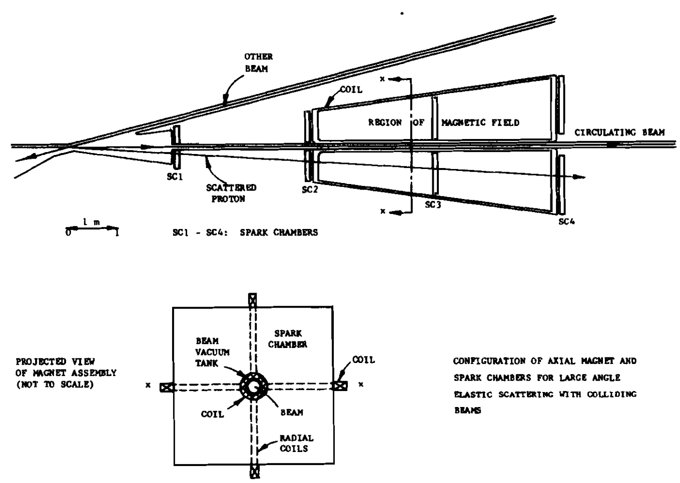

Fig. 3

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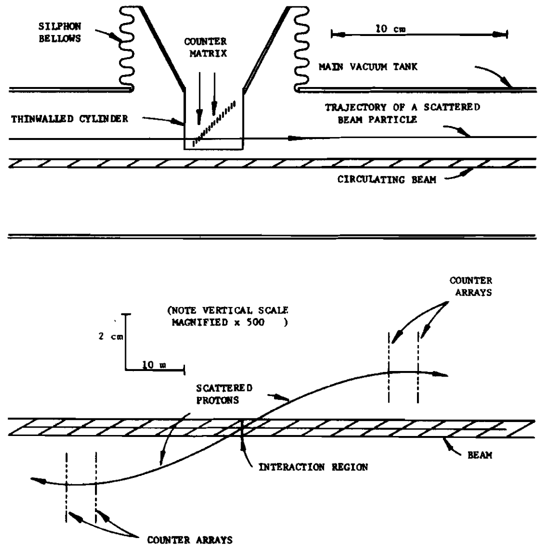
*Editor's Note: These large and small angle elastic scattering experiments are described in more detail in an appendix to L.W. Jones, "Physics To Be Done With Colliding Beams", p. 253 of this volume.

Fig. 4 illustrates a very small angle forward-scattering experiment



CONFIGURATION FOR STUDYING VERY SMALL ANGLE ELASTIC SCATTERING WITH COLLIDING BEAMS

Fig. 4

where spark chambers cannot be brought close enough to the beam. A series of solid-state counters 2 mm x 2 mm are used to investigate coulomb interference. If one can get within a centimeter of the beam (the background rates with such small counters are not excessive), one can investigate angles from 1 milliradian to about 10 milliradians. One would place these counters about a quarter betatron oscillation downstream from the interaction region, in order to increase the resolution.

Discussion

W.F. Baker (BNL): What are your opinions on high momentum transfer experiments? It is a very interesting experiment to do if the level of detection is high enough. What is your estimate of the upper limit of t?

K.M. Terwilliger: For $-t \geq 1$ with maximum shrinkage, the total cross section will be about 4×10^{-33} cm^2 and one would get 1 count per 7 hours, for I = 1 ampere. One can certainly work in this region; however, this is about the limit.

L.W. Jones (Michigan): If the diffraction peak doesn't shrink, you can measure out to -t = 4, corresponding to the above cross section.

E.D. Courant (BNL): The estimate of 1 ampere at an energy spread of Δp = 0.1% is somewhat conservative. It is based on the current AGS intensity and, by the time storage rings are built, it may increase to 3 or 10 amperes, thereby increasing counting rates by a factor of 10 or 100.