CERN STUDIES ON PROTON STORAGE RINGS

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I. History

Devices for colliding-beam experiments were first proposed in the United States. In 1956, D.W. Kerst[1] described a system composed of two intersecting fixed-field accelerators, and A.M. Sessler and K.R. Symon[2] developed the theory of accumulation of high currents by radiofrequency "stacking" of many accelerator pulses. In the same year, the suggestion of storing particles for colliding-beam experiments in separate storage rings was put forward by G.K. O'Neill.[3] A proposal by MURA to build a 15-Gev two-way FFAG proton accelerator did not materialize. Electron colliding-beam experiments, on the other hand, appeared both more interesting and more feasible at that time. A storage-ring system for colliding-beam experiments with 500-Mev electrons has been devised and built at Stanford University (jointly with Princeton University).[4] A group at Frascati constructed a single storage ring for 200 to 250-Mev electron-plus-positron experiments.[5,6] Both devices have reached the stage of

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1. D.W. Kerst, CERN Symposium on High-Energy Accelerators and Pion Physics, p. 36, June 1956.
2. A.M. Sessler and K.R. Symon, ibid., p. 44.
3. G.K. O'Neill, ibid., p. 64.
4. G.K. O'Neill, International Conference on High-Energy Accelerators and Instrumentation, p. 125, CERN, Sept. 1959.
5. C. Bernardini, G.F. Corazza, G. Ghigo, and B. Touschek, Nuovo Cimento 18, 1293 (1960).
6. B.F. Touschek, p. 171 of this volume.

producing information on colliding-beam techniques. After the first papers from MURA and Princeton had become known at CERN, colliding beams appeared to us an interesting technique to develop, offering a possible way to very high energies in the future. We tried to get familiar with the ideas developed elsewhere, and we studied designs for a suitable pilot model accelerator. For a time, an FFAG two-way electron accelerator similar to the MURA model, but pushed to higher energy and modified to make colliding-beam experiments practicable, was studied in great detail,[7] but abandoned at the end of 1959 when this design was found to become too complex and too expensive or even infeasible at or above 100 Mev, and had also become of rather marginal interest for colliding-beam experiments after the Stanford 500-Mev electron storage rings had got under way in 1959.

Furthermore, the performance of the CERN PS led us to conclude that a storage-ring system joint to the PS for colliding-beam experiments with 25-Gev protons should be given serious consideration. With regard to the nonconventional features of such a project (beam stacking and ultra-high vacuum) it was desirable to have a pilot device suited to convince ourselves and other people of the feasibility of the crucial steps involved with a minimum of complexity. We managed to obtain the support for the construction of a storage-ring model for electrons of low enough energy so that stacking conditions holding for protons can be simulated. A brief description of this model, which is expected to run soon, will be given below.

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7. M. Barbier, F.A. Ferger, E. Fischer, P.T. Kirstein, G.L. Munday, M. Morpurgo, M.J. Pentz, A. Schoch, A. Susini and N. Vogt-Nilsen, International Conference on High-Energy Accelerators and Instrumentation, p. 100, CERN, Sept. 1959.

A preliminary study we had made in 1960[8] of a 25-Gev proton storage ring project provoked fairly animated discussions, and strong division of opinion as to its value became apparent. A conference on very-high-energy physics organized by L. van Hove at CERN in 1961[9] tended towards a negative verdict. In the spring of 1962 we tried to convene a small working party to discuss, in more detail, type and feasibility of experiments with proton storage rings. We enjoyed the cooperation of several non-CERN physicists in this working party, and I would like to mention particularly that substantial contributions to its progress were made by L.W. Jones and G.K. O'Neill. The working party, whose conclusions have been collected in a CERN Accelerator Research Division internal report,[10] surveyed possible significant experiments and produced tentative designs for some of them.

There are no basically new aspects to add to the conclusions arrived at one year ago. Work done at CERN on proton storage rings during the past year concerned mainly a more detailed study of the general design ideas that had evolved from the working party discussions, and is due mainly to B. de Raad and W. Schnell.[11,12]

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8. H.G. Hereward, K. Johnsen, A. Schoch, and C.J. Zilverschoon, Proc. 1961 International Conference on High-Energy Accelerators, p.265.

9. International Conference on Theoretical Aspects of Very-High-Energy Phenomena, 1961 (proceedings published as CERN Report 61-22.

10. CERN Study Group on High-Energy Projects, Experimental Use of Proton Storage Rings, AR/Int. SG/62-11.

11. B. de Raad, "Some Aspects of Concentric Storage Rings", AR/Int.SG/62-6. "Shielding for Concentric Storage Rings", AR/Int. SG/62-10/Rev. "Design of Some Experiments with Proton Storage Rings", AR/Int.SG/62-12. "Concentric Storage Rings for the CERN PS", AR/Int. SG/63-30.

12. W. Schnell, "Radiofrequency System for 25-Gev Proton Storage Rings", AR/Int. SG/62-6.

On the scientific policy level, on the other hand, a committee of European physicists, with E. Amaldi as Chairman and E.H.S. Burhop as Secretary, worked out recommendations for a coherent program for high-energy facilities in Europe.[13] These recommendations include a "summit" program for design and construction of both storage rings for the CERN PS and a very-high-energy proton synchrotron in the 300-Gev range. The recommendations have been submitted to the Scientific Policy Committee advising the CERN Council, whose reaction is at present still open.

II. Basic Parameters of Colliding Beams: Interaction Rates

Much of what follows has been said many times before and is recalled here for convenient reference.

The equivalent lab energy E_1 (total) of a particle with mass M_1 colliding with a particle of mass M_2 at rest is related to the total energy $(\overset{*}{E}_1 + \overset{*}{E}_2)$ in the center-of-mass system by

$$(\overset{*}{E}_1 + \overset{*}{E}_2)^2 = M_1^{\,2} + M_2^{\,2} + 2M_2E_1 .$$

If the particles are highly relativistic and have equal masses $M_2 = M_1 = M$,

$$E_1 \simeq \frac{(\overset{*}{E}_1 + \overset{*}{E}_2)^2}{2M} = \frac{2\,\overset{*}{E}_1{}^2}{M} .$$

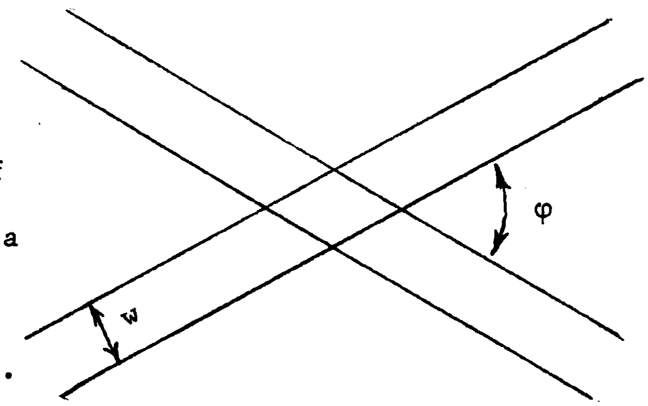
For colliding protons of $\overset{*}{E}_1 = \overset{*}{E}_2 = 25$ Gev, $E_1 \simeq 1300$ Gev.

13. Report of the Working Party on the European High Energy Accelerator Program, CERN FA/WP/23/Rev. 3, June 12, 1963.

The crucial quantity for a colliding-beam experiment is the interaction rate. In a practical proton storage-ring design, the beams would form flat ribbons of radial width w and height h, intersecting at an angle φ of a few degrees in the orbital plane (see sketch). The interaction rate is then (for $\varphi \ll 1$)

$$\text{Interaction rate} = \frac{2\, c\, \sigma\, N^2}{h\, \varphi}$$

(particle velocity $\simeq$ velocity of light = c, σ = cross section of a specified event, N = number of particles per cm length of beam).



For a given line density N, the interaction rate turns out to be independent of the width of the beam. The circulating current for line density $N\ \mathrm{cm}^{-1}$ is

$$I = 4.8 \times 10^{-9}\, N \text{ (ampere)} .$$

Typical figures for contemplated designs are:

$$N = \text{a few times } 10^9\ (\mathrm{cm}^{-1}),\ h = 1\ \mathrm{cm},\ \varphi = 0.3\ (\sim 17^\circ) .$$

Taking $N = 10^9\ \mathrm{cm}^{-1}$ ($\sim$ 5 amp) as a normalizing value, we have

$$\text{Interaction rate} = 2 \times 10^{29}\ \sigma_{\mathrm{cm}^2}\ (\mathrm{sec}^{-1})$$

increasing with N as N^2. A cross section of $10^{-30}\ \mathrm{cm}^2$ could, therefore, be detected with a rate of 0.2 events/second. Comparing with a beam passing through 1 meter of liquid-hydrogen target, it would require 0.5×10^5 particles per second to produce the same interaction rate.

The line density of a PS pulse (debunched) is, with present average performance,

$$N = \frac{6 \times 10^{11}}{2\pi R} = 10^7 \text{ cm}^{-1} \ , \ (R = \text{mean radius of PS})$$

thus, hundreds of pulses have to be accumulated in the storage rings in order to obtain the above rates.

III. Filling Process

The number of pulses that can be stacked in a proton storage ring is limited by phase-space conservation. It is at most equal to the phase-space volume of the storage ring divided by the phase-space volume of one pulse of injected beam. This is a less favorable situation than in the case of electron storage rings, where damping of betatron and phase oscillations invalidates Liouville's theorem and can be used to make the phase space occupied by the beam shrink.

The phase space available is transverse (betatron oscillations) and longitudinal (azimuthal angular motion); ideally, these modes are decoupled.

In a PS- or AGS-like device, whose parameters turn out to be of appropriate order of magnitude also for storage rings, the number of pulses that can be accommodated in betatron phase space is

$$\text{Number of pulses (transverse phase space)} = \frac{\pi \frac{\nu_1 a_1^2}{R} \frac{\nu_2 a_2^2}{R}}{\pi \frac{\nu_1 \nu_2 b^4}{R^2}} = \left(\frac{a_1 a_2}{b^2} \right)^2 \simeq 100 \ ,$$

where a_1, a_2 are available radial and vertical half apertures, b is the radius of the beam cross section (maximum betatron amplitude), ν_1, ν_2 are the betatron tunes, R = mean radius of the storage ring. Assuming

$\nu_1 \simeq \nu_2$; $a_1 = 10$ b ; $a_2 = b$ (i.e., filling only one layer radially) one finds $\simeq$ 100 pulses.

In angular, or angular momentum phase space, we can accommodate:

$$\text{Number of pulses (in momentum space)} = \frac{\Delta p}{\delta p} = \frac{1}{\alpha}\frac{\Delta R}{R}\frac{P}{\delta p} \simeq 100\ \Delta R_{cm}$$

where $\Delta P = \frac{\Delta R}{\alpha R}\, p$ is the momentum range of closed orbits that can sit on a radial interval ΔR, δp is the momentum spread of one injected pulse after debunching, and α is here the momentum compaction factor. δp is essentially determined by the phase stable momentum width at injection into the PS, since phase space seems to be fairly well conserved during acceleration in the PS. Thus

$$\frac{\delta p}{p} \simeq 5 \times 10^{-5} \text{ (debunched) at 25 Gev in the PS,}$$

and assuming

$$\alpha^{-1} \simeq \nu_1^{\,2} \simeq 75\ , \quad R \simeq 140 \text{ m} \quad \text{for the storage rings,}$$

the above figure of $\sim$ 100 pulses per cm of radial stacking space results. A stacking space of 5 to 6 cm should be feasible, which permits storage of a total of $\sim$ 500 pulses. A more accurate estimate of this number will have to make allowance for the orbit wiggles induced by momentum deviation. As far as the number of PS pulses which can be stacked are concerned, stacking in longitudinal momentum space appears superior. In this process, however, the full stack has an appreciable width in momentum of 2 to 2.5%.

In the practical process of stacking in momentum space, injected pulses are shifted from the injection orbit to the storage orbit by rf acceleration or deceleration. The use of rf permits selective action on the injected pulse without affecting the beam already stored.

When the rf bucket carrying a new pulse approaches the stack, a certain amount of nonadiabatic perturbation must of course be expected, and the question arises whether the process can work without undue dilution of phase-space density. Semianalytic estimates by H.G. Hereward,[14] and a computer study of the rf stacking process by D.A. Swenson[15] from MURA (carried out while he spent a year at CERN) resulted in encouraging figures for stacking efficiency with respect to the Liouvillian limit, even for a fairly simple kind of rf program, in which each pulse is dropped at the same momentum, under the condition that the number of pulses stacked is large.

Recently, J.P. Blewett[16] suggested a process for stacking in betatron phase space in order to avoid the build-up of momentum spread, which will often be undesirable. One should keep in mind, however, that in this way the momentum definition is still limited considering that momentum is a vector, whose angular spread is increased by stacking in betatron phase space, in lieu of the increase of spread of absolute value in the case of momentum stacking. Since furthermore, as we have seen, the maximum number of pulses is likely to be inferior with betatron space stacking, we should really compare momentum stacking under conditions of restriction to a comparable number of pulses, which would permit keeping momentum spread down to more favorable values. Before definitive conclusions on the optimum process of filling can be drawn, a more detailed study of the various methods will certainly be necessary.

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14. H.G. Hereward, CERN Internal Report PS/Int. AR/60-33.
15. D.A. Swenson, Proc. 1961 International Conference on High-Energy Accelerators, p. 187.
16. J.P. Blewett, p. 414 of this volume.

IV. Particle Density Limits

Several factors impose limits on the particle density in phase space and physical space.

The line density achieved at present in the CPS is, as already mentioned in Section II, 10^7 cm^{-1} within less than ± 0.5 cm of betatron amplitude and $\frac{\delta p}{p} = \pm 2 \times 10^{-5}$ (after adiabatic debunching of the beam; the bunching factor at 25 Gev is > 10).

The momentum width is essentially determined by the phase stable momentum range at injection into the PS. Improvements appear possible by adopting an adiabatic trapping process at injection into the PS. A potential factor of > 10 increase in beam density per given momentum interval had been estimated by F.E. Mills (see reference 10). It would then be possible to stack more than 10 times the number of pulses in the available momentum interval in the storage rings.

The observed beam width due to betatron oscillations is essentially as expected from adiabatic change of the emittance of the injector linac output ($\sim 5 \times 10^{-4}$ cm.rad in one plane at 50 Mev). The linac emittance, in turn, is not much larger than the theoretical minimum which would be expected from ion temperature in the ion source. Improvement of particle density in transverse phase space is, therefore, not a trivial possibility.

Apart from limitations due to source and injector, there are limitations due to space-charge forces on the beams in the PS and the storage rings.

An estimate of space-charge limits is usually made by calculating the particle density which causes the tolerable maximum shift $\delta\nu$ of the betatron oscillation tune. The line density N of a beam of cylindrical cross section with diameter 2b, in a machine of radius R, causing a shift $\delta\nu$ is, if wall effects are neglected and no neutralizing ions are present,

$$N = \frac{M c^2}{e^2} \frac{b^2}{R^2} \nu |\delta\nu| \beta^2 \gamma^3$$

(e and M particle charge and mass, $\beta = \frac{v}{c}$, $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$). Since

$$A = \frac{\nu b^2}{FR} \text{ (F = orbit wiggle factor)}$$

is the emittance of the beam, this can also be written

$$N = \frac{M c^2}{e} \frac{F}{R} A |\delta\nu| \beta^2\gamma^3 .$$

Assuming that the beam injected at 50 Mev into the CPS is allowed to swell to a maximum radius of $b = 3$ cm without hitting obstacles, the "acceptance" of the PS is

$$A \simeq 45 \times 10^{-4} \text{ cm.rad} \quad \text{(about 10 times actual linac emittance)}$$

and taking $\delta\nu = 0.25$, we find

$$N = 0.66 \times 10^{16} \frac{1.3}{10^4} 45 \times 10^{-4} \times 0.25 \times (0.3)^2(1.05)^3 \simeq 1 \times 10^8 \text{ cm}^{-1}.$$

An unbunched beam of this line density would correspond to

$$2 \pi R N \simeq 6 \times 10^{12} \text{ protons/turn.}$$

With a bunching ratio $B(< 1)$, we have only B times this figure, i.e., perhaps $2 \ldots 3 \times 10^{12}$ protons per turn, which is about three times the value so far reached.

This calculated line density limit is modified by neutralization of the beam (due to ionization of residual gas) and by the influence of the vacuum-chamber walls (image forces). Both these influences tend to destroy the balance between electrostatic and electromagnetic space-charge forces in a relativistic beam. For a non-neutralized beam of diameter $2b$ inside a flat vacuum chamber with top and bottom walls a distance a apart and poor electric conductivity, $\beta^2\gamma^3$ in the above formula has to be replaced by

$$\frac{\beta^2\gamma^3}{\left|1 \pm (0.4\,\frac{2b}{a})\,\gamma^2\right|},$$

and for a neutralized beam by

$$\frac{\beta^2\gamma^3}{\left|1 - \left[\beta \mp (0.4\,\frac{2b}{a})(1 - B)\right]\gamma^2\right|},$$

(with whichever sign yields the larger value of the denominator). Neither of these modifications has much effect at nonrelativistic injection energies, for a bunched beam ($B < 1$) at any rate. At relativistic energies in the storage rings we have

$$0.4\,\frac{2b}{a}\,\gamma^2 \gg 1 \text{ and } B\gamma^2 \gg 1 ,$$

so that the expressions replacing $\beta^2\gamma^3$ become

$$\frac{\beta^2\,\gamma}{0.4\,\frac{2b}{a}} \quad \text{and} \quad \beta^2\gamma ,$$

respectively for the non-neutralized and the neutralized unbunched beam.

Thus, the partial or complete cancellation of the electrostatic part of the space-charge forces by either image charges or neutralizing ions causes a substantial reduction of the line density limit which would be expected from the formula at the beginning of this section. Using the proton storage ring parameters (and b = 0.5 cm) we find for the neutralized beam only

$$N = \frac{Mc^2}{e^2}\,\frac{b^2}{R^2}\,\nu|\delta\nu|\beta^2\gamma = 0.66 \times 10^{16}\,\frac{0.25}{(1.4)^2 \times 10^8}\,8.75 \times \frac{1}{4} \times 25 \simeq 5 \times 10^8\ \mathrm{cm}^{-1},$$

or 2.5 amperes. Clearing-out of the neutralizing charges would increase the line density limit by a factor of

$$(0.4\,\frac{2b}{a})^{-1} \simeq (0.4\,\frac{1}{4})^{-1} \simeq 10\ .$$

If one wants to take account of the fact that a full stack in a storage ring has the shape of a flat ribbon of width w and height $h \ll w$ rather than of a cylinder, $\frac{b^2}{R^2}$ should be replaced by $\frac{hw}{2\pi}$. Taking $h \simeq 2b$ (= width of an injected turn), a space-charge limit results which is higher by a factor $\frac{w}{\pi b}$. It is, therefore, reasonable to believe that a non-neutralized line density of 10^{10} cm^{-1}, or 50 amperes, would be compatible with space-charge effects.

In intersecting storage rings, another effect upsetting the balance between electrostatic and electromagnetic space-charge forces is due to the intersecting parts of the beams, where the magnetic forces cancel. The line density causing the shift $\delta\nu$ by this is

$$N = \frac{2\,\pi\,R}{2\,\ell}\,\frac{M\,c^2}{e^2}\,\frac{b^2}{R^2}\,\nu\,|\,\delta\nu\,|\,\beta^2\gamma\ ,$$

where ℓ is the total length of the intersections. Since

$$\frac{2\,\pi\,R}{2\,\ell} \simeq \frac{\pi \times 1.4 \times 10^4}{1.5 \times 10^2} = 300 \;,$$

we find about 300 times the space-charge limit for a neutralized beam, i.e., $N \simeq 1.5 \times 10^{11}$. The intersection effect, which severely restricts currents in the electron storage rings[17] because of the large value of the lost factor γ^2, is therefore not important for protons.

Finally, we have to examine the danger of "negative mass instability" of a stored beam. Applying the theory developed by C.E. Nielsen, A.M.Sessler and K.R. Symon,[18] a beam above transition energy is stable, if its energy spread

$$\frac{\Delta E}{E} > \left[\frac{4}{\alpha\,\gamma^3} \frac{e^2}{M\,c^2} N \left(1 + 2 \ln \frac{2a}{\pi b}\right) \right]^{\frac{1}{2}} ,$$

where α is the momentum compaction factor, and the beam of width $2b$ moves in a flat vacuum chamber of height a ; the formula is written for the relativistic limit $E \gg Mc^2$. We find

$$\begin{aligned} \text{for } N &= 10^7 \text{ cm}^{-1} & \frac{\Delta E}{E} &> 6 \times 10^{-6} \\ &\ 10^{10} \text{ cm}^{-1} & &> 2.2 \times 10^{-4} \;, \end{aligned}$$

and conclude that, under the conditions of interest here, there is always more energy spread than required for longitudinal stability.

Whether the transverse beam instability due to interaction with the vacuum chamber (recently discovered at Stanford and at MURA) can cause trouble in proton storage rings, has still to be investigated.

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17. F. Amman and D. Ritson, Proc. 1961 International Conference on High-Energy Accelerators, p. 471.

18. C.E. Nielsen, A.M. Sessler and K.R. Symon, International Conference on High-Energy Accelerators and Instrumentation, p. 239, CERN, Sept. 1959.

V. Lifetime of Stored Beams

A number of phenomena limit the lifetime of stored beams or affect their characteristics in the course of time.

1. Collisions with residual gas

Dominant are nuclear interactions. The reciprocal mean life of a circulating proton due to these is

$$\tau^{-1} = c\,\sigma\,n = 3 \times 10^{10} \times 40 \times 10^{-27} \times n = 1.2 \times 10^{-15}\ n\ (\text{sec}^{-1})$$

where the mean nucleon density is

$$n = 3.5 \times 10^{16} \times A \times p_{torr}\ \text{cm}^{-3}$$

with A = number of nucleons per molecule, p_{torr} = pressure of residual gas in torr. Assuming air

$$A = 28 \quad \text{and} \quad p_{torr} = 10^{-9}\ \text{torr}$$

gives

$$\tau^{-1} = 1.2 \times 10^{-6}\ \text{sec}^{-1}\ ,$$

i.e., a comfortable lifetime of $\sim$ 10 days.

This assumes that every nuclear collision leads to loss of the proton. In fact, only $\sim 10^{-3}$ of the nuclear encounters are elastic with small enough angles for the protons to remain inside the beam.

Remarks:

a) $\sigma_{nucleus} \simeq \sigma_{p\text{-}nucleon} \times A^{2/3}$ instead of proportional to A.

b) The average A in an ultra-high vacuum system is generally smaller than 28 (see Section VIII below).

c) A vacuum of 10^{-10} torr or better should be aimed at for reasons of background in colliding-beam experiments (see Section VII).

For these reasons, the single collision lifetime given above is underestimated.

2. Multiple scattering

The angular divergence of beam particles due to betatron oscillations is within

$$\frac{F\,\nu\,b}{R} \simeq \frac{1.5 \times 9 \times 0.5}{1.4 \times 10^4} \lesssim 0.5 \times 10^{-3} \text{ radian}$$

(F = orbit wiggle factor). Multiple, small-angle scattering makes the r.m.s. betatron oscillation amplitude $\sqrt{\langle x^2 \rangle}$ grow according to

$$\frac{d\langle x^2 \rangle}{dt} = \frac{\langle \theta^2 \rangle}{\tau} \frac{R^2}{\nu^2} = c\,\sigma\,n\,\langle \theta^2 \rangle_{n.m.f.p.} \frac{R^2}{\nu^2}$$

with $\langle \theta^2 \rangle_{n.m.f.p.}$ the mean square scattering angle per nuclear mean free path, and τ the mean nuclear collision time of Section V.1.

Small-angle scattering is dominated by coulomb scattering: at 25 Gev, the differential cross section for coulomb scattering exceeds the differential cross section for nuclear scattering for $\theta \lesssim 2$ mrad. For air of 10^{-9} torr and 25-Gev protons

$$\sqrt{\langle \theta^2 \rangle}_{n.m.f.p.} \simeq 0.4 \times 10^{-3} ,$$

giving

$$\frac{d\langle x^2 \rangle}{dt} = 1.2 \times 10^{-6} \, (0.4 \times 10^{-3})^2 \, \frac{(1.4)^2 \times 10^8}{75} \simeq 0.5 \times 10^{-6} \, \frac{cm^2}{sec} .$$

Within one nuclear collision time, the beam spreads by the order of 1 cm in r.m.s. betatron oscillation amplitude due to multiple small-angle scattering.

3. The "Touschek" effect

Dr. Touschek has explained (see reference 6) how coulomb interaction between particles of the same beam leads to beam loss in electron storage rings which can become fatal at high beam intensities. Looking at two particles whose trajectories intersect at a small angle 2ϵ (depending on amplitude and phase of the respective betatron oscillations) in their

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center-of-mass system, the momentum components are

perpendicular to the CM motion $\overset{*}{p}_{\perp} = p_{\perp} = \epsilon p$

parallel to the CM motion $\overset{*}{p}_{\parallel} = \frac{\delta p}{\gamma_o} = \frac{1}{\gamma_o} \frac{\delta p}{p} p$

where γ_o is the Lorentz factor of the CM motion, and δp the difference of the particle momenta in the laboratory. In the proton storage rings

$$\overset{*}{p}_{\perp} = \epsilon p \lesssim 0.5 \times 10^{-3} p$$

$$\overset{*}{p}_{\parallel} = \frac{5}{25} \times 10^{-3} p = 0.2 \times 10^{-3} p$$

for 0.5 mrad maximum angle due to betatron oscillations, and $\frac{\delta p}{p} = 5 \times 10^{-3}$, which is the momentum spread corresponding to a radial range of stacked orbits of

$$\Delta R = \alpha \frac{\delta p}{p} R \simeq \frac{5}{75} \times 10^{-3} \times 1.4 \times 10^{4} \simeq 1 \text{ cm} .$$

In the center-of-mass system of this beam, the momenta are not as dominantly transversal as in the case of electron storage rings; they are in fact not far from equipartition between transverse and longitudinal motion. What coulomb scattering can do is increase momentum spread in the beam by a factor of about 2.5, or, conversely, increase betatron amplitudes slightly by scattering from the longitudinal into the transverse direction.

The reciprocal time for the above anisotropic momentum distribution to become isotropic by coulomb scattering is of the order

$$\tau^{-1} = 2\pi \frac{e^2}{Mc^2} cn \frac{(Mc)^3}{\Delta p^2 p_{\perp max}} = 2\pi \frac{e^2}{Mc^2} cn \frac{1}{\gamma^3 (\frac{\Delta p}{p})^2 (\frac{p_{\perp max}}{p})}$$

$$= \frac{2\pi \times (1.5)^2 \, 10^{-32} \times 3 \times 10^{10}}{25^3 \times 5^2 \times 10^{-6} \, 0.5 \times 10^{-3}} n = 2.1 \times 10^{-17} n \; (\mathrm{sec}^{-1})$$

where τ is the time for $p_\perp \leq p_{\perp max}$ to be scattered into a momentum departure $\delta p > \Delta p$, n is the number of particles per cm^3, $\frac{p_{\perp max}}{p} = 0.5 \times 10^{-3}$ has been taken as above, and $\frac{\Delta p}{p} = 5 \times 10^{-3}$ has been assumed. For the particle densities considered of $n \simeq 10^9$ protons/cm^3,

$$\tau^{-1} \simeq 2 \times 10^{-8} \; \mathrm{sec}^{-1} ,$$

i.e., this is a slower process than gas scattering and does, therefore, not seem to be disturbing for protons.

The formula for τ^{-1} used is the one derived by Touschek and his collaborators[19] applying to electron storage rings. In view of the lesser importance of the Touschek effect in the proton case, we may use it here, for the purpose of estimate, although several modifications would have to be made in order to adapt it properly.

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19. C. Bernardini, G.F. Corazza, G. di Giugno, G. Ghigo, J. Haissinski, P. Marin, R. Querzoli, and B. Touschek, "Lifetime and Beam Size in a Storage Ring", Phys. Rev. Letters 10, 407 (1963).

4. Energy loss

a) By ionization and excitation of residual gas, a relativistic proton loses energy at the rate:

$$-\frac{dE}{dt} = c\,\frac{dE}{ds} = c\rho \times \frac{dE}{d(\rho s)} = 3 \times 10^{10} \times \frac{1.2 \times 10^{-3-9}}{760} \times 2 \left(\frac{\text{Mev}}{\text{sec}}\right)$$

$$\simeq 1 \times 10^{-4}\ \frac{\text{Mev}}{\text{sec}},$$

assuming air at 10^{-9} torr.

In 10^{6} sec the loss is 100 Mev, which is about 4×10^{-3} of the initial energy. Considering that straggling causes a spread of the same order in addition, the beam just about conserves the necessary definition during its lifetime.

b) By orbit radiation on a radius r of curvature, a proton (or electron) suffers an energy loss of

$$E = \frac{4\pi}{3}\, e^{2}\, \frac{\gamma^{4}}{r} = 0.6 \times 10^{-6}\ \frac{25^{4}}{0.7 \times 10^{4}} = 3.3 \times 10^{-5}\ \text{ev/turn}\ .$$

In 10^{6} sec, we have 3×10^{11} turns, and thus a total loss of 10^{7} ev = 10 Mev, which is of no importance.

5. Orbit perturbations by field fluctuations

Proton storage rings differ from electron storage rings in that there is no damping of betatron oscillations. Even small perturbations must, therefore, be carefully examined as to their effect on beam life or beam quality.

Ripple and noise of the magnetic field move the beam about, essentially adiabatically if the field variation is slow in comparison with the betatron frequency. High-frequency field noise would, however, cause the random build-up of betatron oscillation amplitudes proportional to $t^{\frac{1}{2}}$.

For an estimation of orders of magnitude, we write an equation of motion for betatron oscillations in the form:

$$\ddot{x} + (2\pi f_o)^2 x = - R\dot{\theta}^2 \frac{\delta B}{B} = F(t) ,$$

where $f_o = \frac{\nu\dot{\theta}}{2\pi}$ is the number of betatron oscillations per second and $\delta B(t)$ and $F(t)$ are random functions representing noise.

The statistical average of the square of the amplitude

$$\hat{x}^2 = x^2 + \frac{\dot{x}^2}{(2\pi f_o)^2}$$

then grows according to

$$\frac{d\,\overline{x^2}}{dt} = \frac{G(f_o)}{(2\pi f_o)^2} ;$$

here $G(f)$ is the spectral density of the noise, defined in such a way that

$$2 \int_0^\infty G(f)df = \langle F^2(t) \rangle ,$$

the time average of $F^2(t)$.

The minimum noise, which is always there, is thermal noise. The thermal field noise is easily figured out, if we remember that it must be the black body radiation at the ambient temperature. Since, however, the familiar formula (Planck's law) for the spectral density of black body radiation is derived under the assumption that the dimensions of the body are large in comparison with the wavelengths of interest, the mode counting has to be modified for our present purpose. The betatron oscillation frequencies are in the range of one to a few Mc/s, and in this range a vacuum chamber, or even a synchrotron magnet, permits only one-dimensional wave propagation. Under this assumption, the mean square of a component of the noise field is found to be

$$\langle \delta B^2 \rangle = \frac{8\pi kT}{cS} \int df ,$$

where k = Boltzmann's constant, T = temperature in degrees Kelvin, and S the cross section of the vacuum chamber, or, more generally speaking, of the boundary of the system cut perpendicular to its major elongation.

With this result we find a growth rate

$$\frac{d\overline{\hat{x}^2}}{dt} = \frac{G(f_o)}{(2\pi f_o)^2} = \frac{R^2 \dot{\theta}^2 4\pi kT}{\nu^2 \dot{\theta}^2 B^2 cS} = \frac{4\pi ckT}{\nu^2 B^2 S} \approx 2 \times 10^{-14} \frac{cm^2}{sec}$$

where the figure holds for: $T = 300^oK$, $\nu^2 = 75$, $B = 10^4$ gauss, $S = 100\ cm^2$. It shows that the effect of thermal noise is completely negligible within the lifetime limits we have found before.

Knock-out by the electric field components of thermal noise is of the same magnitude.

It is probable that there are more important contributions to field noise from other origins. In particular there may be "plasma noise" due to space charge or current fluctuations seen by a particle in the surrounding beam. The spectral density at the betatron oscillation frequency would have to be about 10^4 times the thermal density (in amplitude) in order to compete with gas scattering. This would require a "plasma temperature" of $\sim 3 \times 10^{10\,o}K$, i.e., a few Mev or $\sim 10^{-4}$ of 25 Gev, which is in the range to be expected. In substance, excitation of oscillations by this type of noise forms a limiting case of Touschek scattering, namely the contribution due to collective or long-range interactions of beam particles.

Another mechanism of particle loss to be examined is build-up of betatron oscillations by nonlinear resonances. This can occur if the (small amplitude) betatron wave numbers ν_1 and ν_2 are close to the "resonance lines" defined in the (ν_1, ν_2)-plane by

$$n_1 \nu_1 + n_2 \nu_2 = k ,$$

(with n_1, n_2, k integers) and if a k-th harmonic of the azimuthal distribution of the radial derivative of order $|n_1| + |n_2| - 1$ of the magnetic guide field B is present to act as a driving force for the resonance. Normally, care has been taken that such driving forces are not inherent in the design; then they appear only as a result of imperfections of the magnetic guide field.

The width of the region around the resonance line, within which the oscillations are unstable, is related to the strength of the perturbation and increases with the amplitude of oscillation in the case of nonlinear resonances ($|n_1| + |n_2| > 2$). The growth of amplitude is, however, in general limited by nonlinearities in the focusing forces, moving the betatron frequencies out of tune as the amplitudes change. The result is that the amplitudes "beat" between a minimum and a maximum value. Therefore, nonlinear resonances are normally not dangerous under conditions of stationary parameters. They can, however, become dangerous if parameters vary in time in such a way that the working point in the (ν_1, ν_2)-plane sweeps repeatedly across a resonance line. Such repeated crossing of resonance lines might occur as a result of ripple of the magnetic field.

The behavior of particle motion when a resonance is crossed is very complex. Particles end up partly with increased, partly with reduced amplitudes, depending on initial conditions. In the limit of slow crossing certain regions of betatron oscillation phase space can be carried to larger amplitudes by the oscillation frequency getting locked in synchronism with the frequency of perturbation; the change of frequency with amplitude then compensates the variation of the small amplitude frequency caused by varying parameters. (This is a process analogous to the rf acceleration of particles inside the phase-stable "bucket" in the energy-rf phase trajectory diagram.) After reverse slow crossing the particles will be brought back to occupy their initial phase space, except for such particles which have been spilled out by departures from adiabaticity.

In the opposite limit of fast crossing, changes of amplitude are non-adiabatic and due to the short interval of time of synchronism between particle oscillation and perturbation. Since sign and magnitude of the change depend on the phase of oscillation at the instant of synchronism and so will be distributed more or less at random, the effect of a large number of repeated crossings will resemble a random build-up process. We try to estimate whether this process can cause significant beam loss in proton storage rings by taking the example of a third-order radial resonance (whose effect is likely to be most pronounced).

The change of amplitude from $\hat{x}_1$ to $\hat{x}_2$ due to one rapid crossing of a nonlinear resonance can be calculated approximately using the methods of "stationary phase". For a third-order resonance, one finds[20]

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20. A. Schoch, "Theory of Linear and Nonlinear Perturbations of Betatron Oscillations in Alternating-Gradient Synchrotrons", CERN Report 57-21, Section 14.

$$\frac{1}{\hat{x}_1} - \frac{1}{\hat{x}_2} = \left(\frac{2\pi}{3|\nu'|}\right)^{\frac{1}{2}} \frac{1}{8\nu}\left[\frac{dn}{dr} + \tilde{c}\,\frac{d^2n}{dr^2}\right]_k \sin\left(3\Psi_o + \frac{\pi}{4}\right) \equiv \epsilon \ .$$

Here, $\frac{dn}{dr}$ and $\frac{d^2n}{dr^2}$ are radial derivatives of the magnetic field index n; $\tilde{c}(\theta)$ is the closed orbit distortion (if any). The suffix k indicates that the k-th Fourier harmonic of the quantity in square brackets, assumed to vary with θ, is to be taken. $[\ ]_k$ represents the driving force of the third order resonance, given by the appropriate harmonic of the sextupole part of the focusing field as seen on the particle orbit. $-\Psi_o$ is the betatron oscillation phase at the instant of synchronism with the perturbation. $\nu' = \frac{d\nu}{d\theta}$ measures the speed of crossing. If it is due to a variation of the magnetic field B, it is given by

$$\nu' = \frac{d\nu}{d\theta} = \frac{d\nu}{dB}\,\frac{dB}{dt}\,\frac{1}{\dot{\theta}} \ ,$$

$\dot{\theta}$ being the angular frequency of revolution of the particle.

Since, in a large number of crossings, one will hit all values of the phase Ψ_o at random, the mean square change per crossing will be

$$\overline{\left(\frac{1}{\hat{x}_1} - \frac{1}{\hat{x}_2}\right)^2} = \overline{\epsilon^2} = \left|\frac{\pi}{3\nu'}\right| \left(\frac{\left[\frac{dn}{dr} + \tilde{c}\,\frac{d^2n}{dr^2}\right]_k}{8\nu}\right)^2$$

Starting from amplitude $\hat{x}_o$, the expectation value $\hat{x}_N$ after N crossings follows, therefore, from

$$\frac{1}{\hat{x}_o^{\,2}} - \frac{1}{\hat{x}_N^{\,2}} = N\,\overline{\epsilon^2}$$

$\hat{x}_N$ reaches infinity after

$$N = \frac{1}{\hat{x}_o^2 \overline{\epsilon^2}} = \left|\frac{3\nu}{\pi}\right| \left(\frac{8\nu}{\left[\hat{x}_o \frac{dn}{dr} + \ldots\right]_k} \right)^2$$

crossings. (Growth to infinity in a finite time is a feature characteristic of nonlinear perturbations rising with amplitude according to a power law.)

We estimate the number N using CPS parameters and field errors. Assuming the dependence of ν on B holding for a linear AGS, we have first

$$\frac{d\nu}{dB} B = - \frac{d\nu}{dp} p = \nu \simeq 6$$

Further, we assume field ripple with amplitude $\frac{\tilde{B}}{B} = 10^{-4}$ and frequency $\frac{\omega}{2\pi} \simeq 0.5 \times 10^3 \ S^{-1}$. Thus

$$\nu' = \frac{d\nu}{dB} \frac{dB}{dt} \frac{1}{\dot{\theta}} = \nu \frac{\omega}{\dot{\theta}} \frac{\tilde{B}}{B} = 6 \times \frac{0.5 \times 10^3}{0.5 \times 10^6} 10^{-4} = 6 \times 10^{-7} .$$

Finally, from the measurements made on CPS magnet blocks before assembly, one can derive a guess for the driving force[21]

$$\left[\left(\frac{dn}{dr} + \tilde{c} \frac{d^2n}{dr^2}\right)\right]_k \simeq 10^{-4} \ \text{cm}^{-1} .$$

Taking everything together, we find

$$N = \frac{1.5 \times 10^4}{\hat{x}_o}$$

happening in a time

$$\tau = N \frac{2\pi}{\omega} \simeq \frac{30}{\hat{x}_o} \ \text{sec}^*$$

21. A. Schoch, "Theory of Linear and Nonlinear Perturbations of Betatron Oscillations in Alternating-Gradient Synchrotrons, CERN Report 57-21 Section 15.

*A similar figure had been indicated by A.A. Kolomenski in a discussion at CERN.

This is frighteningly fast loss. We must remember, however, that it affects only that fraction of beam particles whose ν-values are such that they are swept across a resonance by field ripple. The swing in ν is

$$\frac{\delta\nu}{\nu} = \frac{\tilde{B}}{B} = 10^{-4}$$

under the assumptions made. Therefore a fraction of beam filling a momentum range

$$\frac{\delta p}{p} = \frac{\delta\nu}{\nu} = 10^{-4}$$

is cut out by the resonance, which would correspond to a few pulses of the stack in 25-Gev storage rings.

Higher order resonances take longer times for loss, but by factors 10 to 100 only for orders 4 and 5.

The effect of nonlinear resonances is, therefore, to produce gaps in the momentum distribution of a stored beam resembling the gaps in the ring of Saturn. But even before particles get definitively lost, the beam is spoiled by the halo of particles kicked out by this process.

There is no hope of improving enough on field imperfections to remove damage by crossing of resonances, since we talk of lifetimes orders of magnitude longer than what we have just found. It helps, however, to improve on field ripple, and to make the ν-values less dependent on orbit position by suitable design of the magnetic field. In this way, the ν-swing is reduced, and with it the fraction of beam affected.

In the first line, one will of course attempt at a good enough control of the working point ν_1, ν_2 to keep the whole stack clear from perturbing resonances.

We should remember again that the self-field of a stacked beam will become more influential than the errors of the external magnetic field as the stacked current becomes high.

It is worth pointing out here that evidence of strong influence of nonlinear resonances on beam life has been found in the MURA 50-Mev electron accelerator (private communication).

VI. Design Problems

1. General layout

We became converted at CERN to the idea of concentric storage rings with several beam intersections. Fig. 1 reflects present thinking on layout with respect to the CPS.

The mean radius of the rings has crept up to 150 m by now, the maximum possible on the available site extension. The large mean radius is due to the large proportion of straight sections.

Both rings are placed in one tunnel. Experimental areas of moderate size, forming extensions of the ring tunnel around the eight interaction regions, are believed to be adequate for collding-beam experiments. One large experimental hall for 25-Gev experiments of conventional type is suggested at present.

2. Choice of parameters and geometry

It is tempting to ask whether one could not use much stronger focusing and correspondingly smaller aperture than in the CPS, in particular since high momentum compaction and small beam size are desirable. The rf stacking process, however, requires that the transition energy of the storage rings be outside the energy range to be used. This restricts the permissible

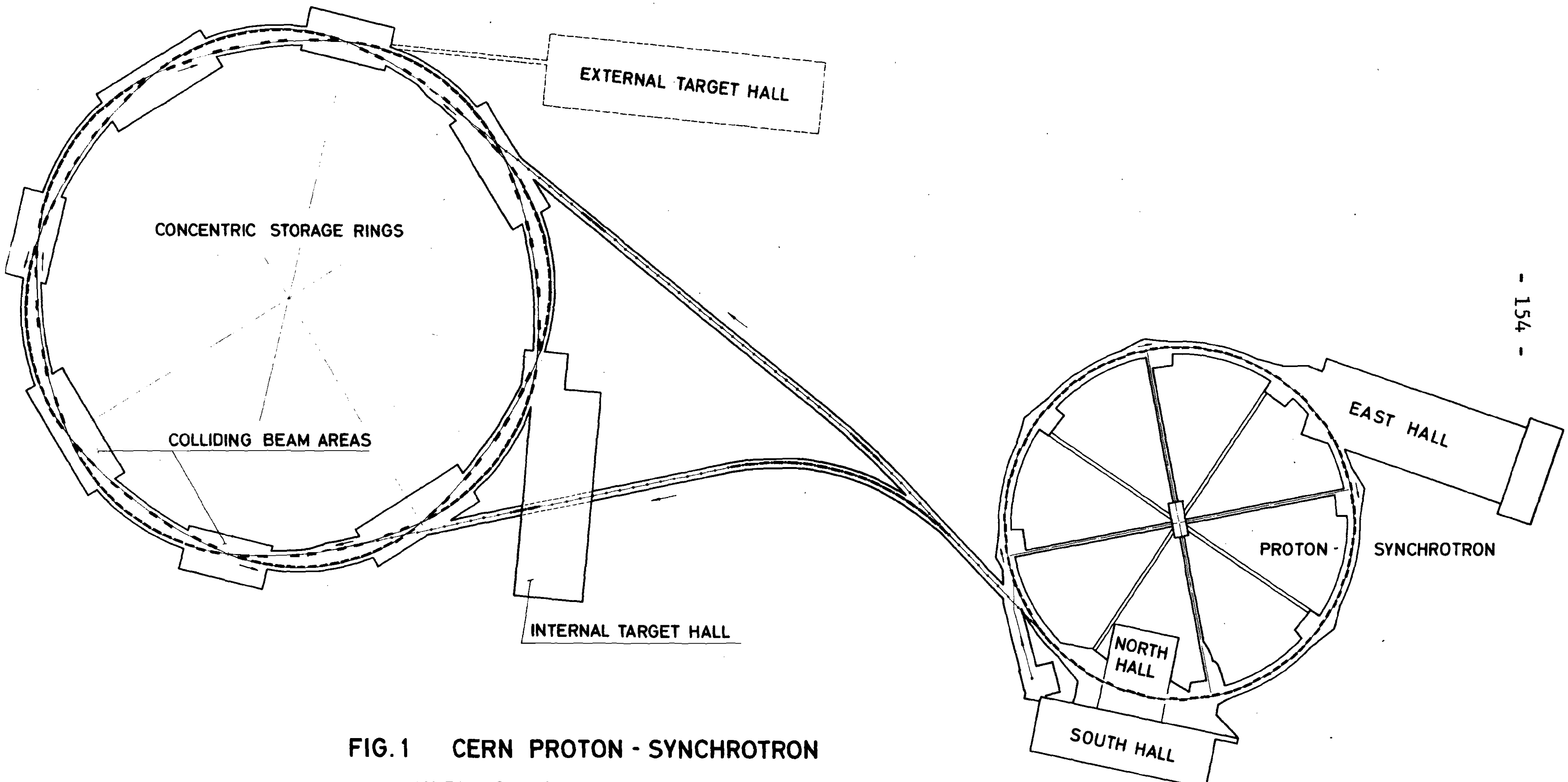

FIG. 1 CERN PROTON - SYNCHROTRON WITH CONCENTRIC STORAGE RINGS

ν-values to either $\nu < \frac{E_{min}}{Mc^2}$ or $\nu > \frac{E_{max}}{Mc^2}$, or for CPS storage rings to $\nu \lesssim 10$ or $\nu \gtrsim 25$. Thus a very big step would have to be made in focusing strength in order to go for smaller aperture. The very high-gradient magnets then required look difficult. Also the ultra-high vacuum problems would not be eased by too small an aperture. Therefore, only devices resembling the CPS and AGS in general parameters have been studied in detail.

The choice of ν-values is linked with the choice of geometry. The magnet lattice of concentric storage rings has strongly pronounced super-periods (their number S being equal to half the number N of intersections) because of the long straight intersections, and also because of different density of magnets on the outer and inner arcs. Wide stopbands have therefore to be expected for ν = multiple of $\frac{S}{2}$. If one excludes working regions which are either adjacent to these stopbands, or contain a nonlinear resonance line of order 3 or 4 which would be excited by a multiple of S, the choice of ν and N becomes very restricted. In fact, requiring ν to be in the range of 6 to 9, one is left with N = 8 intersections and ν = 7-1/4 or 8-3/4.

Regarding the question whether ν_z and ν_r should be approximately equal or not, the arguments (high momentum compaction and small vertical beam size) are in favor of making either as large as possible. $\nu_z \simeq \nu_r$ is therefore a good compromise, since raising one of the values is in practice only possible at the expense of the other.

In proton storage rings, a wider range of momentum has to be accommodated simultaneously than in a normal AGS. In order to minimize the wiggles of off-center orbits, long straight sections should be placed mid-F or mid-D. Beam height in the intersection regions is minimized by arranging the latter to be in mid-F (radially focusing). Nonlinear field corrections are necessary, to reduce the dependence of ν_r and ν_z on momentum to acceptable limits over the desired region of the vacuum chamber.

Regarding the magnet, our thinking moved from AG magnets to a design where bending and focusing is obtained by separated elements (magnets with uniform field and quadrupole lenses) and recently back again to AG magnets, plus special units around the intersection regions. The arguments in favor of AG magnets are essentially economy and available space.

An optimization study carried out by B. de Raad (see reference 11) along these lines resulted in the following list of main parameters (subject to modifications in the course of further work*). The straight sections around the intersections, 13 m long in this design, could be stretched to 16 m if a slight increase of the orbit wiggle factor is accepted.

*The figures given here are the latest available at the time of writing (August 1963), differing somewhat from those given in the talk at Brookhaven in June 1963, mainly by longer straight sections.

Main Parameters of Concentric Storage Rings for the CPS

Maximum energy	28 Gev
Peak field on central orbit	12 kgauss
Magnetic radius	79 m
Mean radius	150 m
Number of intersections	8
Angle of intersection	15°
Number of magnet periods	48
$\nu_r = \nu_z =$	8.75
Length of long straight sections	13 m
δr for $\frac{\delta p}{p} = 1\%$	mid F 2.3 cm mid D 1.6 cm
Maximum wiggle factor $(\frac{\nu}{R}\beta)^{\frac{1}{2}}$, ($\beta$ = Courant-Snyder function)	radial 1.42 vertical 1.50
Aperture of vacuum chamber	radial 150 mm vertical 50 mm

Allowing for tolerances on imperfections in beam transfer from the PS to the storage rings, the beam width and height of one pulse after transfer at 25 Gev are estimated by de Raad to be close to 2 cm and 1 cm respectively.

The radial extension due to the momentum spread of a full stack can be reduced in the interaction regions by a suitable quadrupole perturbation of the focusing, as was first suggested by Terwilliger.[22] In a ring with separated bending and focusing this would be particularly easy to introduce; it is, however, also feasible in the AG magnet rings by means of appropriately placed quadrupoles.

22. K.M. Terwilliger, International Conference on High-Energy Accelerators and Instrumentation, p. 53, CERN, Sept. 1959.

3. Radiofrequency system

It is of advantage to transfer the bunched PS beam and to inject it into preformed rf buckets in the storage rings. The rf system of the storage rings works on the same frequency as that of the PS and is locked to the latter. The harmonic number will be higher in the storage rings because of their larger radius, and the radii have to be in a rational ratio.

The requirements on the rf system have been studied by W. Schnell (see reference 12). I briefly review his principal conclusions:

At the instant a beam pulse is deposited after displacement by rf modulation from the injection orbit to the storage orbit, the rf buckets must fit the bunches tightly in order to avoid dilution of particle density in phase space. The time required for doing the full displacement to the top of the stack with a tightly fitting bucket is, however, too long with regard to the period between consecutive PS cycles, except for $\sin \varphi_s$ (φ_s = stable phase angle) close to unity. $\sin \varphi_s$ too close to 1 is undesirable because (i) the tolerances on the rf program become too stringent, (ii) computations (see reference 15) indicate that the stacking efficiency is less favorable. With $\sin \varphi_s \simeq 0.5$, the rf program has to be made up of three steps: The PS-bunches are injected into large rf buckets moving them most of the way to the stack in a convenient time T_1 ; the phase area of a bucket is much larger than that of a bunch during this step. Thereafter, a time T_2 is required to reduce the bucket area to fit the bunch before approaching the stack; this has to be done adiabatically in order to avoid deterioration of stacking efficiency. The tightly-held bunch is dropped at the bottom of the stack, but some time T_3 is necessary to move through the diluted tail of the stack and to readjust for energy deviations due to rf program errors. With an rf voltage of 20 kv and $\sin \varphi_s = 0.5$ as a reasonable compromise, W. Schnell finds for the contemplated storage ring parameters, certain assumptions on tolerances, and E = 25 Gev

$$T_1 + T_2 + T_3 \simeq 0.3 + 0.4 + 0.8 = 1.5 \text{ sec} ,$$

leaving still a good reserve, since one PS cycle takes 3 sec at 25 Gev. Below 25 Gev, the stacking time first decreases with E, to rise again if transition energy is approached. Stacking would, however, be possible as close as 5% to transition energy.

The effect of rf noise on the stacking process is a serious concern. Fortunately, the frequency modulation required covers a very narrow band. In order to confine phase blow-up by rf noise to less than 10% of the half-length of the bucket, a tolerance of 0.14 cycles/sec r.m.s. frequency deviation within 100 cycles/sec of bandwidth results. Measurements in progress for checking whether this noise tolerance can be met are so far encouraging. In addition, it might be possible to use automatic phase-lock of the rf system to the injected turn, despite the presence of the stored beam, at least during part of the stacking cycle.

4. Beam transfer from the PS to the storage rings

The question, whether the beam can be extracted from the PS and channeled to the storage rings without unreasonable modifications has been examined in a preliminary way, with a positive conclusion.

Problems of fast kicker magnets shall only be mentioned. Progress in this field justifies reasonable optimism.

5. Vacuum system

The vacuum system will have to be designed for 10^{-10} torr or better, in the colliding-beam region at least. This requirement emerges more strongly from the evaluation of background reactions due to residual gas than from considerations of beam life. We are presently collecting information on the design problems of such a vacuum system from the CERN storage-ring model described below. Despite the technological difficulties encountered, here again experience justifies the hope that the required performance can be achieved.

VII. Design of Experiments with Colliding Proton Beams

The ideas that have emerged from the Working Party on Experimental Use of Proton Storage Rings,[10,23] were developed further in 1962/63 by B. de Raad and E.H.S. Burhop. Since this subject is taken up in more detail at the present Summer Study,[24] only a brief summary of conclusions arrived at shall be given here, in particular as far as they reflect on the storage-ring design.

The experimental problems of p-p elastic scattering have been given more attention than others until now, since in the beginning they looked more difficult.

1. p-p elastic scattering

Detectable events are in all probability confined to angles below 50 mrad (especially if the diffraction peak shrinks further with increasing energy). Protons must therefore be detected very close to the primary beams, hence the importance of long, unobstructed straight sections. Discrimination against other events can be obtained by observing coincidences

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23. L.W. Jones and B. de Raad, "Experimental Utilization of Proton Storage Rings", Proc. 1962 Conference on Instrumentation for High-Energy Physics, p. 477.

24. K.M. Terwilliger, p. 238, L.W. Jones, p. 253, and E.H.S. Burhop, p. 277 of this volume.

of particles emerging in opposite directions from the interaction region and measuring their angles and momenta by means of a suitable arrangement of spark chambers around the vacuum chamber, using the deflection in the storage-ring magnets adjacent to the interaction region. In order to do the momentum analysis in a proper way, magnet units on both sides of a beam intersection can be replaced by a combination of a radially extended bending magnet B and quadrupole lenses Q as shown in Fig. 2. B. de Raad proposed a special design of quadrupole with unobstructed median plane (Fig. 3). (Note that the straight sections are divided asymmetrically by the intersection point. For this reason, those four of the intersections where the particles move, after crossing into the longer part of the straight section, are more useful than the other four.)

In Fig. 4, trajectories of elastically scattered protons are drawn, with indications of the location of spark chambers (SC) and triggering counters (S) with respect to the quadrupoles. Measurements are preferably done on protons scattered radially (x) at angles greater than 10 mrad, and vertically (z) at angles below 10 mrad (provided the gap height of the bending magnets is adequate). The estimated precision is ± 0.25 mrad in angle and ± 0.75% in momentum. For comparison, the definition of the primary beams is about ± 0.3 mrad in angle and ± 1.25% in momentum for a full stack; by extrapolation of the trajectories of both scattered protons it may be possible to locate the vertex of the event in the stack with enough precision to warrant a momentum definition of ± 0.3%. The vacuum tank around the interaction region has to be suitably shaped, with thin exit windows for secondary particles. Multiple scattering in beryllium windows and in the surrounding air remains within the above uncertainties.

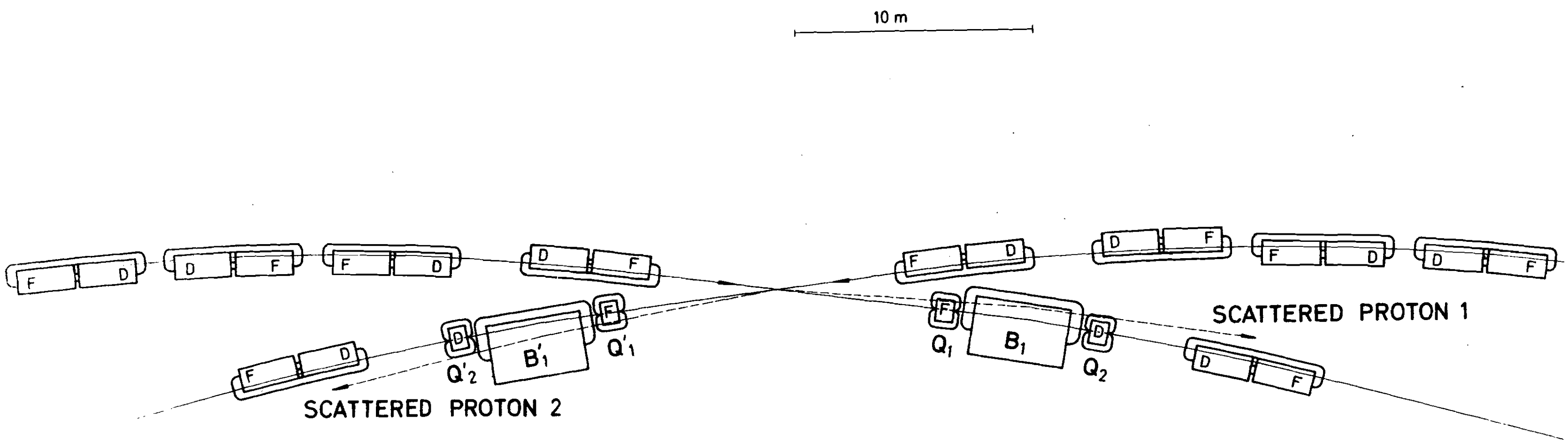

Fig. 2 - Intersecting storage rings with special magnet sections.

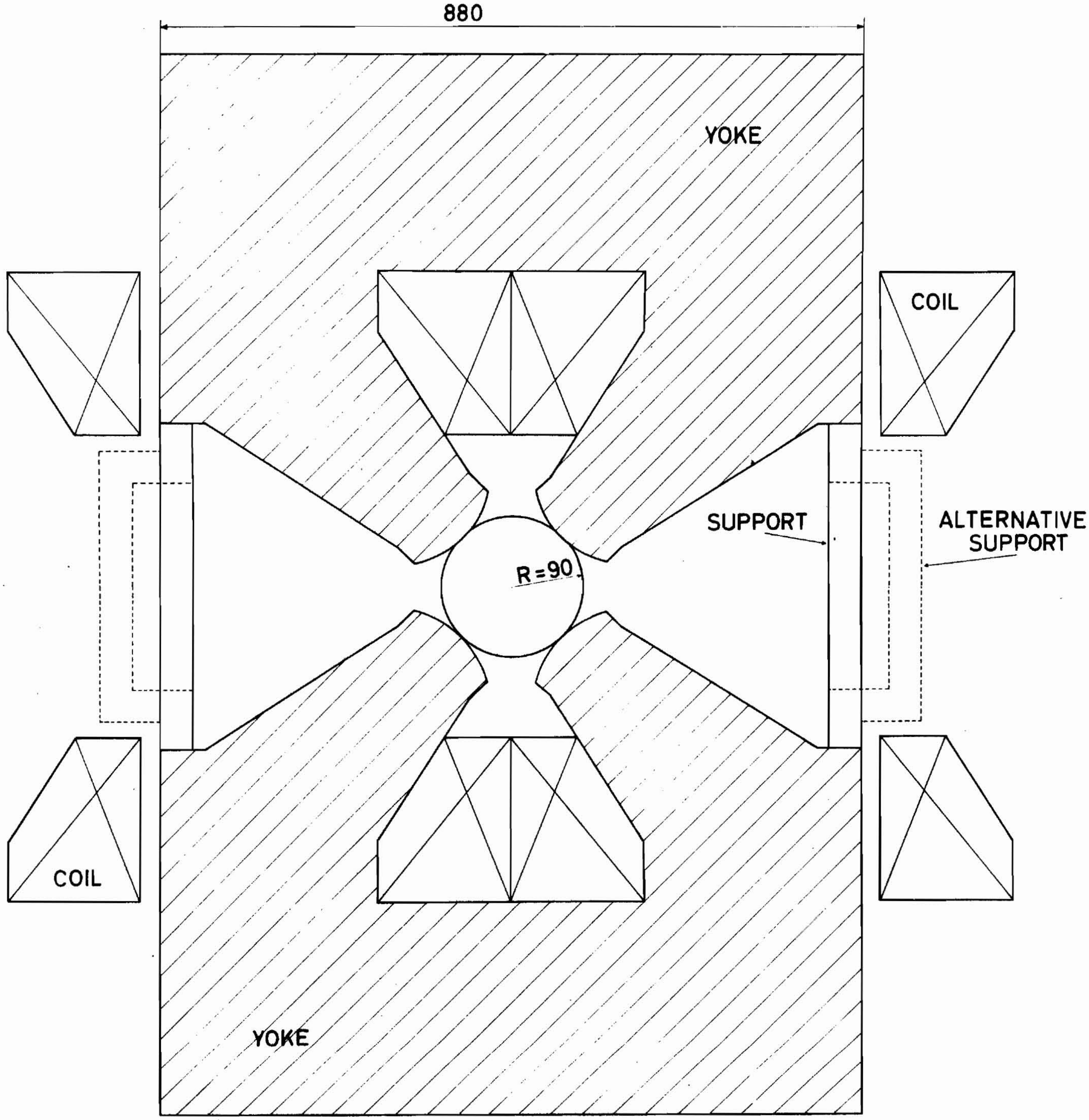

Fig. 3 - Quadrupole with open median plane.

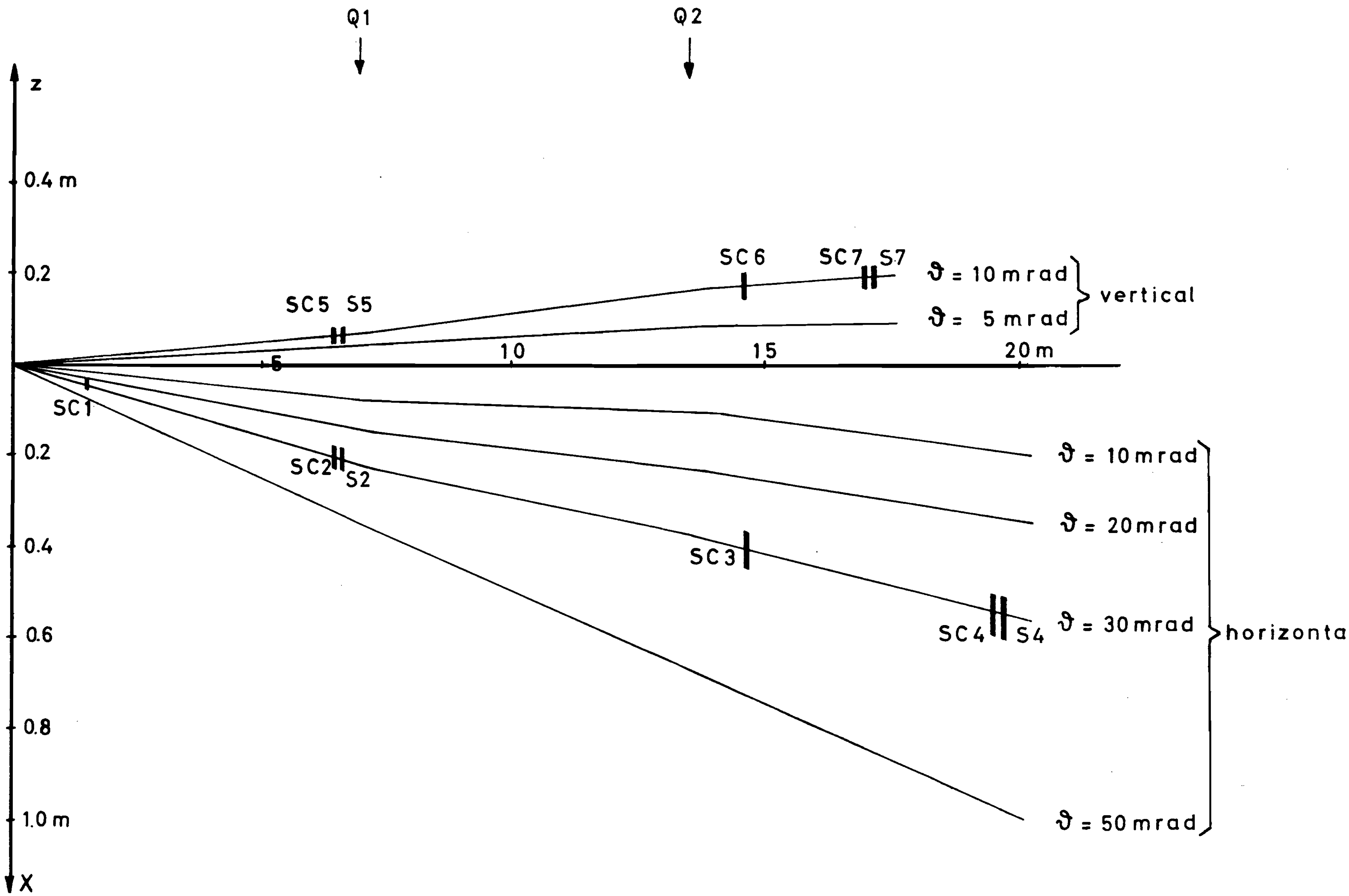

Fig. 4 - Trajectories of elastically scattered protons and location of spark chambers.

A typical counting rate for elastic p-p events, expected with 20 ampere beam current within a solid angle defined by 30 ± 1 mrad horizontally, ± 7 mrad vertically, is one event per second. Protons scattered elastically from residual gas are difficult to distinguish from colliding-beam events, since they show a very similar angular distribution and hardly detectable energy loss. At a residual-gas pressure of 10^{-10} torr, the same solid angle at 30 mrad intercepts a background of this kind which about equals the rate of colliding-beam events (nucleon density in the gas about 1/10 of nucleon density in the beam; at the same time smaller cross section of colliding-beam scattering because of shrinkage of diffraction peak). There is more background of inelastically-produced secondaries from beam-gas reactions. At 10^{-10} torr and 20 amperes of beam, the total background flux through an annular surface 1 cm wide and surrounding the beam is 10^4 to 10^5 particles per second (independent of the radius of the annulus). With coincidence selection of colliding-beam events, the influence of the background is found to become negligible and the proportion of spurious tracks in spark chambers reasonably low.

2. Inelastic processes

For nearly elastic processes, in which one or both of the colliding protons are excited, the same set-up could in principle be used which has been described for the measurement of elastic scattering. The identification of the presently-known excited isobars by the momenta of the scattered protons would, however, be marginal with the accuracies of measurement assumed above. B. de Raad has shown (see reference 11) that detection and measurement in addition of a decay pion in coincidence permits the determination of isobar masses with a fairly good resolution in this case.

The bulk of inelastic processes is represented by events in which many secondary particles are produced, not necessarily at small angles. For these, detectors covering the full solid angle 4π are required. It will be necessary to analyze the individual events as fully as possible. The feasibility of a magnetic field in the intersection region permitting the momentum analysis of secondary particles and the necessary compensation of the effect on the primary beams has been studied in a preliminary way. Arrays of spark chambers are envisaged as detectors. Bubble chambers in their present form have a very low duty factor, and do not provide sufficient possibilities of reducing the interference by background.

3. Normalization of cross sections and measurement of total cross sections

The determination of absolute cross sections requires the knowledge of the density distribution in the interpenetrating parts of the colliding beams. Apart from a destructive method, in which the density distribution is measured by moving a beam stopper slowly across the beam, no more appealing method has been suggested to my knowledge.

For the measurement of total cross sections, the transmission method is not applicable here. A method worked out (see reference 11) proposes to intercept all colliding-beam events by a counter arrangement practically enclosing the interaction region, with minimum size holes for letting the beams pass. Colliding-beam events are discriminated against background by coincidence and directional Cerenkov counters. The effect of the holes must be estimated by extrapolation to a zero size hole; this extrapolation is not an unreasonable procedure for angles below $\sim$ 5 mrad.

VIII. The CERN Electron Storage Ring Model

The purpose of this model is to permit (i) the experimental demonstration of beam stacking with satisfactory efficiency under the restrictions of Liouville's theorem, (ii) the development of ultra-high vacuum technology, and (iii) the study of beam handling and space-charge problems.

The general design had been directed by K. Johnsen and C.J. Zilverschoon; development and construction was in the hands of a group whose members are F.A. Ferger, E. Fischer, E. Jones, P.T. Kirstein, H. Koziol, and which is headed by M.J. Pentz. Operation is about to start.

The model, whose layout is shown in Fig. 5, is a strong-focusing device, with separated bending magnets and quadrupole lenses for reasons of simplicity and flexibility. The circumference is 24 m, half of which is in straight sections. The relatively large circumference of the model has been chosen in order to stay in the range of manageable electronics (revolution frequency 12 Mc/s), and in order to gain experience on a substantial size ultra-high vacuum system.

To start with, experiments will be done with 2-Mev electrons. At this energy, orbit radiation is negligible so that the electrons can simulate the behavior of protons in rf stacking. Single-turn pulses are injected from a Van de Graaff accelerator into the ring by means of a pulsed delay-line inflector. The injection orbit is close to the inner edge of the vacuum chamber. The beam can be displaced radially by modulated radiofrequency, or also by means of a betatron core; the latter mainly for diagnostic purposes. Current transformers, remotely controlled scraping targets, and a bunch pickup electrode serve for beam observation.

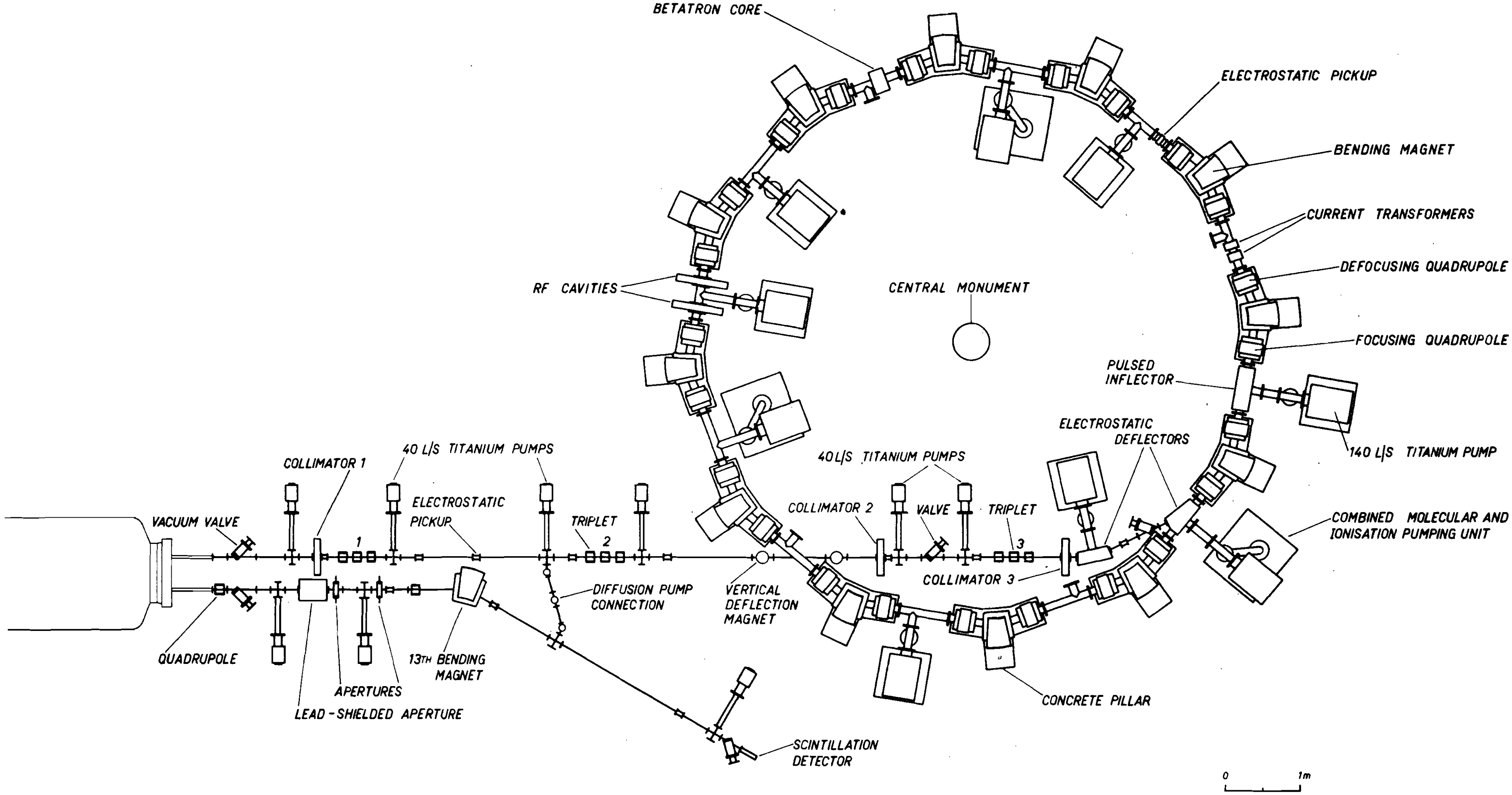

Fig. 5 - Layout of CERN storage-ring model.

The quadrupoles are adjusted for a betatron oscillation wave number $\nu_r = \nu_z = 2.75$. Orbits covering a momentum range of ± 6% can be accommodated under these focusing conditions. Variation of ν_r and ν_z with momentum is reduced by sextupole shims fixed to the quadrupole lenses.

The range of ν-values accessible in principle by adjustment of the quadrupole excitation extends from ~2 to ~6.

The injection system is designed to inject a beam of 10^{-5} cm.rad emittance, obtained by stopping down the Van de Graaff beam by collimators, which should correspond to about 1 mm beam width in the storage ring. A second accelerator tube in the Van de Graaff tank provides a second electron beam for monitoring momentum with a magnetic spectrometer system (see Fig. 5). A momentum definition of the injected beam of $\pm 1 \times 10^{-4}$ is aimed at. Fifty pulses of ~ 1 ma each satisfying these specifications can be injected per second. A total of up to 50 pulses are expected to be stacked.

The lifetime of 2-Mev electrons at 10^{-9} torr should be about 7 seconds, which would be sufficient for the stacking experiments.

From the point of view of big proton storage rings, the ultra-high vacuum system of the model is perhaps its most interesting feature. The vacuum chamber has elliptical cross section inside the magnets, with dimensions 10×4 cm^2, and circular cross section of 10 cm diameter in the straight sections. It is made of vacuum-molten stainless steel, inner walls electrolytically polished, and contains about 150 flange connections with gold wire seals. It can be baked in situ by surface heaters; magnets and lenses are protected by cooled cooper jackets. Pumping is effected by 8 Vac-ion sputter pumps of 140 l/s pumping speed each. Three molecular pumps are used for prepumping and during bake-out.

Outgassing and leak rate determine the pressure that can be maintained. It was found that outgassing after long enough bake-out at 300°C drops to the same level as after shorter bake-out at 400°C, with the advantage of keeping the leak rate provoked by baking smaller. Electropolishing of the inner surface reduced the outgassing rate by a factor 3 to 10.

In a 3-m section of chamber a pressure of 10^{-10} torr had been obtained. In the complete ring, the pressure is still limited by leak trouble to between 10^{-9} and 10^{-8} torr. There is confidence that, after improvements in the design of some of the larger flanges, a pressure of close to 10^{-10} torr can be achieved. Leak hunting (and repair) seems to be the major trouble.

The gas composition has been analyzed by means of an omegatron mass spectrometer. At 10^{-8} torr, hydrogen and carbon monoxide are preponderant, accounting together for 80% of the total pressure.

Studies on further improvements of vacuum by cryogenic pumping have been started.

In many respects, the storage-ring model may be as difficult to get working as a big storage ring, because of the low magnetic fields involved and the requirements on voltage stability and pulse-to-pulse reproducibility of the injector; difficulties which are not relevant in the same way for a large device. The experience collected on ultra-high vacuum technology has, on the other hand, already proved invaluable.