THE USE OF A LARGE, HEAVY-LIQUID CHAMBER AT HIGH-ENERGY ACCELERATORS*

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I. Introduction

There have been many studies of possible uses of bubble chambers with high-energy machines. There are two such examples in SLAC Report No. 5, Summer 1962; SLAC-5E by Trilling and SLAC-5F by Chinowsky. Both reports conclude that it is essential to detect all particles in any interaction. In particular, π^0 mesons and gamma rays must be observed. The reason for this is simple; as the energy of the interactions becomes greater the effects make analysis more difficult. First, the measuring errors prevent the determination of missing masses. Hence, any reconstruction of an interaction will become impossible unless all products are identified; and secondly, the multiplicity increases so that identification of each particle becomes more tedious. Trilling's solution to this problem is a 4.5-meter hydrogen bubble chamber, the downstream two meters of which contain lead plates for the conversion of gamma rays. This, of course, restricts the conversion possibilities to a small cone in the forward direction. Chinowsky points out that this may be a serious limitation and suggests a hydrogen bubble chamber more or less surrounded by a lead-plate spark-chamber array to convert the gamma rays.

The emphasis on hydrogen chambers for low-energy machines ($\leq$ 10 Bev) is understandable, since the primary interaction is simple and the multiple scattering in hydrogen allows precise measurement analysis. However, it

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is possible that these advantages might disappear at high-energy machines ≥ 10 Bev and a large heavy-liquid chamber might, for a wide class of experiments, become competitive.

In fact, Trilling touched on this point in his paper.[1] In this he notes that heavy-liquid chambers will be useful for π^o detection and possible bubble counting for β measurements. Rau and Shutt[2] also point out the possible usefulness of heavy-liquid chambers. Their emphasis is on the gamma-ray detection possibilities of such a chamber.

The purpose of this report is to examine the usefulness of a heavy-liquid bubble chamber, with or without internal hydrogen target, at a high-energy accelerator (E ≥ 10 Bev).

II. Choice of Heavy-Liquid-Chamber Parameters

Table I contains a partial list of heavy liquids that have been successfully used in bubble-chamber work.

Table I

Liquid	Radiation Length	Density	Nuclear Mean Free Path
C_3H_8	109 cm	0.41 gm/cc	125 cm
CF_3Br	10.6 cm	1.5 gm/cc	56 cm
C_3F_8	27.5 cm	1.3 gm/cc	53 cm
30% C_3H_8 } 70% CH_3I	8.5 cm	1.06 gm	130 cm

1. G.H. Trilling, Lawrence Radiation Laboratory Internal Report UCID-1472, GHT-1, August 30, 1961.

2. R.R. Rau and R.P. Shutt, "The Use of Bubble and Cloud Chambers at Energies near 1000 Bev", Experimental Requirements for a 300 to 1000-Bev Accelerator and Design Study for a 300 to 1000-Bev Accelerator, p. 76, BNL 772, August, 1961 (revised December, 1962).

There are three important and well-known conclusions one can draw from this table.

The first is that the nuclear mean free paths of these liquids lie between one-half and two meters. The second is that by judicious choice of mixtures of these liquids (such as C_3H_8 and CH_3I) one can achieve a large range in radiation length. The third is that one can get mixtures that have densities of free hydrogen equal or greater than that of liquid hydrogen.

The fact that the nuclear mean free path in these liquids is of the order of 1.5 meters implies that the maximum path length available for momentum analysis is 1.5 meters. This also implies that the maximum production length available is 1.5 meters. Hence, in terms of production and analysis, the maximum usable chamber length is of the order of 3 meters. However, one must also consider flight paths for neutral particles such as Λ^o, K^o, Ξ^o, etc.

Consider the case of the Λ^o. If the machine energy is 800 Bev and the average multiplicity is 10, then the Λ^o will have a momentum of 80 Bev/c and a decay length of about 5 meters. If the multiplicity is less, then the energy of the Λ^o will be correspondingly larger and the decay length will be longer. For a Ξ^o, the decay length will be about 2.5 meters for a momentum of 80 Bev/c.

Ideally, of course, one would then make the chamber large enough to contain several decay lengths for the Λ^o at 80 Bev/c; one then has the possibility of capturing a large fraction of the produced Λ^o's. For obvious practical reasons, this is not too feasible. However, even if one had only one decay length for the size of the chamber, one would see a reasonable fraction of the Λ^o's decay in the chamber. Hence, in terms of seeing strange particles decay, a total length of 5 meters would seem to be enough to be useful even for machine energies of 800 Bev. In particular, if one uses the usual multiplicity of 10 for 800 Bev/c interactions, one should see a reasonable fraction of the decays of K_1^o, Λ^o, Σ^+, Σ^-, Ξ^o, Ξ^-, and, of course, their antiparticles.

An additional consequence of the 1.5-meter mean free path is that it is possible to consider identifying secondary particles by means of their interactions. If one chooses 5 meters as a length, the secondaries would have 2.5 mean free paths for interaction. Hence, identification of μ-mesons would be simple. In addition, the chance for a high-energy neutron to interact would be large and, due to its energy, the secondaries created will give a good indication of its direction.

By choosing a radiation length of 50 cm, one could have seven radiation lengths for conversion in the forward direction. The detection efficiency in the lateral directions would depend on the width and depth of the chamber.

To determine these parameters, one must look at the expected forward cone of the interactions. If one takes the incident momentum as 800 Bev/c, assuming a multiplicity of 10, this gives an average momentum of 80 Bev/c. Since the transverse momentum is about 0.5 Bev/c independent of momentum, the forward cone would have a half-angle of 0.06 radians. If the chamber is 5 meters long, the width (and depth) must be at least 0.06 meters, in order to contain the initial reaction. However, due to the fanning of secondaries in the magnetic field, a minimum size for this dimension (for purposes of round numbers) should be one meter. Hence, a reasonable proposal would be a chamber 5-meters long by one-meter wide and one-meter deep.

III. Properties of Proposed Five-Cubic-Meter Heavy-Liquid Chamber

A. Measurement Precision

In the papers quoted in SLAC-5 one can find equations for percentage errors for various measurements. For purposes of this study, the following formulas are used:

$$\left.\frac{\delta P}{P}\right|_c = \frac{57}{\beta H \sqrt{X_o L}} \qquad (1)$$

$$\left.\frac{\delta P}{P}\right|_s = 2.7 \times 10^4 \times \delta \frac{P}{HL^2} \qquad (2)$$

where

$\left.\frac{\delta P}{P}\right|_c$ is the fractional error in a momentum measurement due to coulomb multiple scattering,

$\left.\frac{\delta P}{P}\right|_s$ is the fractional error in a momentum measurement due to setting error,

H = magnetic field in kilogauss,

X_o = radiation length in centimeters,

L = track length in centimeters,

P = momentum in Bev/c,

δ = setting error in centimeters,

β = v/c.

Since one can use a path length of L = 150 cm, we can calculate $\frac{\delta P}{P}$ as a function of X_o and P.

Fig. 1 is the result of this calculation. For comparison, the calculation is shown both for X_o = 100 (propane) and X_o = 1000 (liquid hydrogen). As is to be expected, the accuracy of measurement beyond 50 Bev/c is determined essentially by the setting error δ.

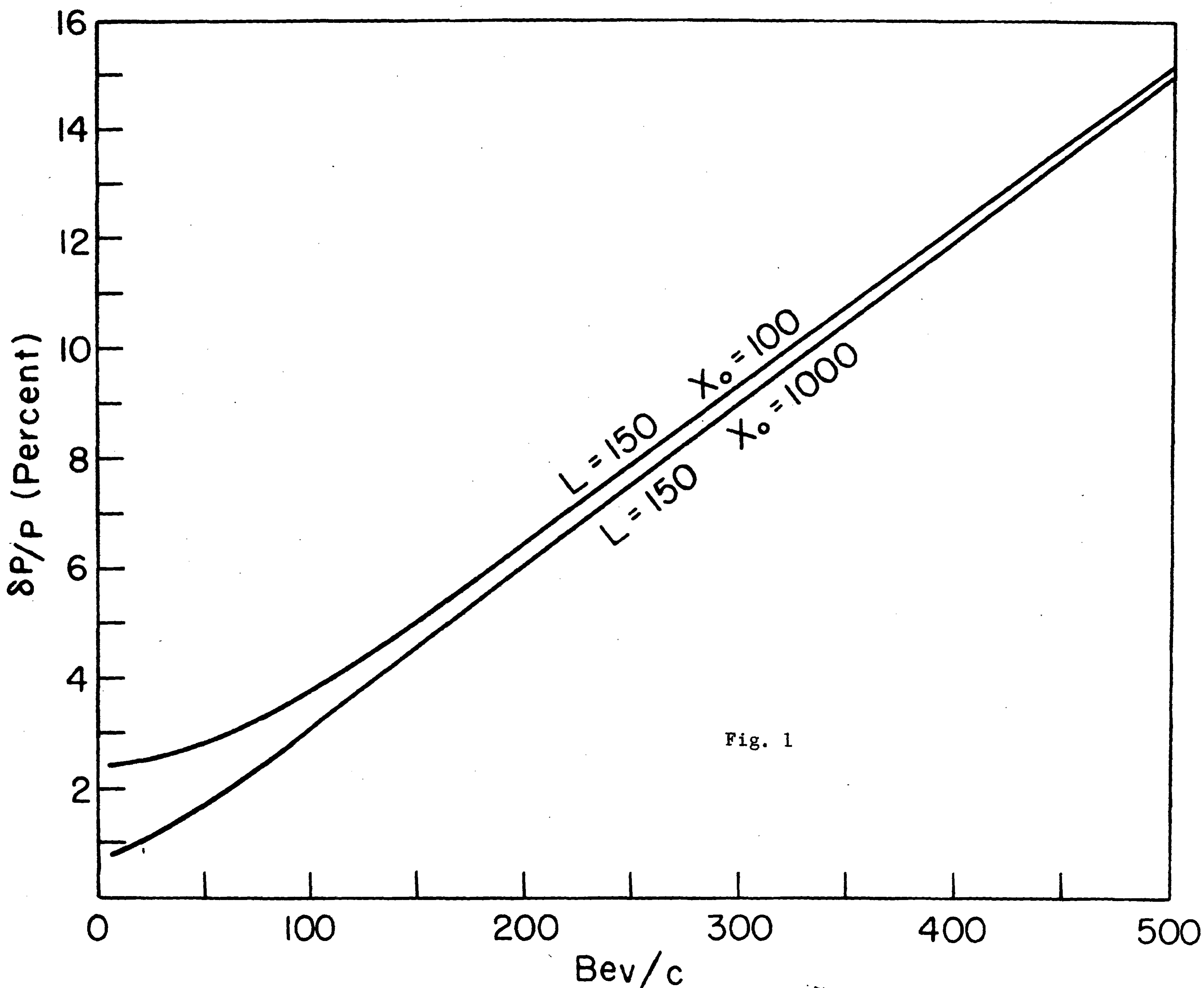
δP/p (Percent)
16
14
12
10
8
6
4
2
0
100
200
300
400
500
Bev/c
L = 150 X_0 = 100
L = 150 X_0 = 1000

Fig. 1

For this example in Fig. 1,

$$H = 20$$

$$L = 150$$

$$\delta = 0.005$$

$$\beta = 1$$

In fact, even in the 10 to 50 Bev/c region, the momentum measurements in propane are comparable to that of hydrogen.

If one assumes an incoming particle with 800 Bev/c one can ask how well can the momentum be balanced. Since the multiplicity will be 10 the total error in the longitudinal error will be:

$$\Delta P_L = \sqrt{\sum_{n=1}^{10} (\Delta P_i)^2} \cong \sqrt{10}\, \frac{\delta P}{P}\bigg|_s \times 80 = \pm\ 7.6 \text{ Bev/c} . \qquad (3)$$

This, of course, is a very large absolute error, although the percentage error is small.

The error in angle measurement is proportional to δ/L. Since one is in a magnetic field and has geometric reconstruction to contend with, $\Delta\theta$ can be approximated by $5\frac{\delta}{L}$. (Usually one uses three points for a curvature and then calculates the tangent, which results in the large numerical factor.)

Using this, one can calculate the transverse-momentum error

$$P_T = \Sigma P_{T_i} = \Sigma P_i \theta_i = 0$$

$$\Delta P_T = \sqrt{\Sigma (P_i)^2 (\Delta\theta_i)^2 + \Sigma \theta_i^2 (\Delta P_i)^2} = \sqrt{10} \sqrt{(P_i)^2 (\Delta\theta_i)^2 + \theta_i^2 (\Delta P_i)^2}$$

$$\theta_i \approx \frac{0.5}{P_i} = \frac{0.5}{80} = 6.2 \times 10^{-3}$$

$$P_i \approx 80$$

$$\Delta\theta_i \approx 5 \frac{0.005}{150} = 16.5 \times 10^{-5}$$

$$\Delta P_i = 2.4 \text{ Bev/c}$$

$$\Delta P_T \cong 4.8 \times 10^{-2} \text{ Bev/c} = \pm 48 \text{ Mev/c} \qquad (4)$$

Since the average transverse momentum of a particle is 500 Mev/c, it appears that, even at these high energies, the transverse-momentum test will yield information about missing neutrals.

Since a missing neutral is expected to carry away 80 Bev/c in longitudinal momentum and 0.5 Bev/c in transverse momentum, the above calculations indicate that in many cases one can determine that a neutral particle is missing.

This is the point, of course, where the utility of many nuclear mean free paths and gamma-ray conversion lengths becomes obvious. The size has been chosen so that all neutrals will be detected and their flight paths known.

Another question one can ask is how well can masses be determined? Under the same assumptions as before we can try to estimate this. Trilling (SLAC-5E) derives the same equations. We have

$$P + \frac{m^2}{2p} + M = \sum p_i + \sum \frac{m_i^2}{2p_i} \qquad (5)$$

$$P = \sum P_i \cos\theta_i \cong \sum P_i - \sum P_i \frac{\theta_i^2}{2} \qquad (6)$$

$$\rightarrow M + \frac{m^2}{2P} = \sum_1^{10} \left(\frac{m_1^2}{2P_i} + P_i \frac{\theta^2}{2} \right)$$

Let $$\Delta E = \Delta \left[\sum_1^{10} \left(\frac{m_1^2}{2P_i} + \frac{P_i \theta_i^2}{2} \right) - \frac{m^2}{2P} \right]$$

ΔE is the error in the kinematic balance due to measurement errors.

$$(\Delta E)^2 = \sum_1^{10} \left[[\Delta P_i] \left(\frac{\theta_i^2}{2} - \frac{m_i^2}{2P_i^2} \right) \right]^2 + \sum_1^{10} (P_i \theta_i \Delta\theta_i)^2 + \left(\frac{m^2}{2P^2} \right)^2 (\Delta P)^2 \qquad (7)$$

P = primary momentum = 800 Bev/c

ΔP = uncertainty in $P \sim 10^{-3}$ = 800 Mev

m = mass of incoming particle = 1 Bev

P_i = 80 Bev/c

$\Delta\theta_i$ = 1.6×10^{-4}

θ_i = 6.2×10^{-3}

ΔP_i = 2.4 Bev/c

m_i = mass of reaction particles $\sim$ 0.5 Bev

$$\Delta E = \left\{ 10 \left[(\Delta P_i)^2 \left(\frac{\theta_i^2}{2} - \frac{m_i^2}{2P_i^2} \right)^2 + (P_i \theta_i \Delta\theta_i)^2 \right] + \left(\frac{m^2}{2P^2} \right)^2 (\Delta P)^2 \right\}^{1/2}$$

$P_i \theta_i \sim 0.5$

$$\Delta E = \left\{ 10 \left[\left(\frac{\Delta P_i}{P_i} \right)^2_{set} \left(\frac{(0.5)^2 - m_i^2}{2P_i} \right)^2 + (0.5)^2 (\Delta\theta_i)^2 \right] + \left(\frac{m^2}{2P^2} \right)^2 (\Delta P)^2 \right\}^{1/2} \qquad (8)$$

The consequences of this formula are striking. The error in the kinematic balance is very small and indicates the same kinematical fitting techniques that are used at low energies will work equally well at multi-Bev energies for this interaction model.

As an example, consider an incident 800 Bev/c particle with multiplicity 10. The question is whether or not the kinematic unbalance due to mislabeling of particles will be larger than the kinematic unbalance due to measurement errors.

Using the formula, the kinematic unbalance due to measuring errors is $\pm\ 6.3 \times 10^{-4}$ Bev = ± 0.6 Mev.

The error due to a mislabeling of a mass m_i is just

$$\frac{2\ m_i \Delta m_i}{2\ P_i} = \frac{\Delta m}{P_i/m} \approx \frac{\Delta m_i}{\gamma_i}$$

interchanging a π with a K requires a Δm of 550-140 = 0.4 Bev

$$\gamma_i \cong 160$$

$$\frac{\Delta m}{\gamma} = 2.5 \text{ Mev} .$$

This result is due to the fact that we are testing the difference between longitudinal momentum and total energy. Due to the high correlations this quantity is very sensitive to the mass assignments.

This result is sufficiently striking as to cast doubt on its validity. In fact, Eq. (6) is probably in error. G. Parzen pointed out that, in fact, what is probably more nearly correct is that, as in bremsstrahlung, one particle comes out at the interaction with a momentum q equal to its mass, M, at a large angle θ_M. In this case he derives the following:

$$P_o = \Sigma P_i \cos\theta_i + q\cos\theta_M$$

$$0 = \Sigma P_i \sin\theta_i \cos\phi_i + q\sin\theta_M$$

$$q^2 = (P_o - \Sigma P_i \cos\theta_i)^2 + (\Sigma P_i \sin\theta_i)^2$$

$$\cong (P_o - \Sigma P_i + \frac{1}{2}\Sigma P_i \theta_i^{\,2})^2 + (\Sigma P_i\theta_i)^2$$

where $\theta_i \ll 1$.

We also have

$$E_o + M = \Sigma E_i + E_M$$

$$E_M - M = E_o - \Sigma E_i$$

$$= P_o + \frac{M^2}{2P} - \Sigma P_i - \Sigma \frac{m_i^{\,2}}{2P_i}$$

$$P_o - \Sigma P_i = E_m - M + \Sigma \frac{m_i^{\,2}}{2P_i} - \frac{M^2}{2P}$$

$$q^2 \cong (E_M - M + \Sigma \frac{m_i^{\,2}}{2P_i} - \frac{M^2}{2P} + \frac{1}{2}\Sigma P_i\theta_i^{\,2})^2 + (\Sigma P_i\theta_i)^2$$

$E_M - M \sim 0.4$ Bev for protons

0.08 Bev for pions.

For our example the following approximations hold

$$\frac{M^2}{2P} \ll E_M - M$$

$$\frac{10\ m_i^{\,2}}{2\ P_i} \le E_M - M$$

$$\frac{1}{2}\, 10\ P_i\theta_i^{\,2} \le E_M - M$$

Define $T_M = E_M - M$

Then $q^2 = (T_M)^2 + (\Sigma\, P_i\theta_i)^2$

$\rightarrow T_M{}^2 = (P_o - \Sigma\, P_i \cos\theta_i)^2$

$q^2 + M^2 = E_M{}^2$

$q^2 = E_M{}^2 - M^2 = (E_M + M)(E_m - M) = T_M(T_M + 2M)$

$\rightarrow T_M(T_M + 2M) = T_M{}^2 + (\Sigma\, P_i\theta_i)^2$

$$T_M = \frac{(\Sigma\, P_i\theta_i)^2}{2M}$$

T_m should be about 0.4 Bev if the reaction dynamics is similar to bremsstrahlung.

$q \sin\theta_M = -\sum_i P_i \sin\theta_i$

$q \cos\theta_M = P_o - \sum_i P_i \cos\theta_i$

$$\text{Tan}\,\theta_M = \frac{-\sum_i P_i \sin\theta_i}{P_o - \sum_i P_i \cos\theta_i} = \frac{-\sum_i P_i \sin\theta_i}{T_M}$$

$$= \frac{-\sum_i P_i \sin\theta_i\; 2M}{(\sum_i P_i\theta_i)^2}$$

$$= \frac{-2M}{\sum_i P_i\theta_i}$$

The maximum value of $\Sigma P_i\theta_i \leq 5$ Bev/c

$\left|\text{Tan}\,\theta_M\right| \geq \frac{2}{5} \geq 20$ degrees which is a rather large angle.

$E_o + M = \sum_i E_i + E_M$

in the high energy limit this is

$$P_o + \frac{M^2}{2P_o} + M = \sum_i \left[P_i + \frac{m_i{}^2}{2P_i}\right] + E_M$$

$P_o = \sum_i P_i \cos\theta_i + q \cos\theta_M$

This gives

$$\Sigma P_i \cos \theta_i + q \cos \theta_M + \frac{M^2}{2P_o} + M = \sum_i P_i + \frac{\Sigma M_i^2}{2P_i} + E_M .$$

Hence it follows

$$\sum_i P_i - \frac{1}{2} \sum_i P_i \theta_i^2 + q \cos \theta_M + \frac{M^2}{2P_o} + M = \sum_i P_i + \frac{\Sigma M_i^2}{2P_i} + E_M$$

or

$$M = \sum_i \left(\frac{M_i^2}{2P_i} + \frac{1}{2} P_i \theta_i^2\right) - \frac{M^2}{2P_o} + E_M - q \cos \theta_M$$

or

$$M = E + E_M - q \cos \theta_M$$

where E was the quantity considered when all angles were small. Hence the inclusion of one nonrelativistic particle has added an additional term:

$$H = E_M - q \cos \theta_M .$$

What is the error in H? Since E_M is considered nonrelativistic

$$H = \frac{q^2}{2M} + M - q \cos \theta_M$$

$$\Delta H = \left[\frac{q}{M} - \cos \theta_M\right] dQ$$

$$\Delta H \sim \frac{q}{M} dQ \sim dQ .$$

Now, if the particle M stops in our chamber (and hence a range can be measured), dQ ~ 1 Mev. This error, then, is about the same as the error on E. Therefore, in this case also (namely the stopping recoil case), one can hope to use kinematical fits at 800 Bev/c to identify particles.

It is almost certainly possible that both models used in this calculation are too simple to describe a real 800 Bev/c situation. However, it is unlikely that the results are wrong by an order of magnitude. Hence, it seems quite important to examine this problem in detail. The most obvious way is by use of constructed events and a high-speed computer to determine the kinematical fits.

B. Separation of Hydrogen Events and Carbon Events and the Use of a Hydrogen Target Inside the Chamber

The error calculated for the transverse-momentum measurements is probably correct. Hence, for carbon events which have a momentum unbalance larger than this, one can indeed expect to distinguish them from hydrogen events. It should not be unusual for a carbon fragment to take off 200 Mev/c in the transverse direction. Hence, most carbon events will identify themselves either in transverse momentum or charge nonconservation. However, even if one gets what appears to be a statistically-valid result, there will always be a question whether or not one really is seeing the effects of carbon.

In the case that all visible particles have a range greater than a few grams (which, at those high energies, should be the great majority of cases), then this doubt can be resolved at the expense of doing the experiment twice. The second time one uses a small (20 cm in diameter) liquid-hydrogen target inside the chamber. This is a well-known technique.[3] Since the type of event looked for is known, the question of seeing the origin is no longer crucial and one can now be certain the interaction is on hydrogen. Essentially the same precision holds in this case as in the previous case, as the incoming track and all outgoing tracks are seen.

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3. M. Chretien, D.R. Firth, R.K. Yamamoto, I.A. Pless and L. Rosenson, Nuclear Instruments and Methods 20, 120-124, 1963).

IV. Conclusions

A heavy-liquid chamber 1 meter by 1 meter by 5 meters in a magnetic field of 20,000 gauss has the following useful properties at a multi-Bev accelerator:

1) All neutral secondary particles (except very weakly-interacting ones such as neutrinos) will be detected.
2) In the case where all secondary particles are detected, it seems possible that kinematic-fit analysis will be valid up to at least 100 Bev and perhaps even higher.
3) By installing a small hydrogen target in the chamber, effects due to nuclear interactions can be determined.

As an example of current interest, a chamber of this size, filled with propane, would be invaluable with respect to contemplated neutrino physics at the AGS. Electrons, muons, pions and protons can be uniquely identified. In addition, range information and magnetic curvature would allow one to analyze the events with better certainty than in hydrogen. (Note: the density of hydrogen in this chamber is greater than that in a hydrogen chamber.)

Finally, it appears that momentum-measuring ability alone will not determine the usefulness of a bubble chamber at high energy. The resolution permitted by the distortions in the chamber and the optical distortions introduced by the photography is the critical factor. With five mean free paths in the chamber, the total number of tracks due to a 1000-Bev particle will be about 500, which is greater than the upper limit which can be resolved. On the other hand, with a 100-Bev particle one might expect less than 100 tracks. This is certainly within the present resolution limit.

The mechanical design of such a chamber and the associated optics will have to be done very carefully, pushing all technology to achieve the ultimate in resolution.