Dynamical intersecting branes

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(Hideo Kodama & K. Uzawa ; JHEP 03 (2006) 053)
(Pierre Binetruy, Misao Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)
(Kei-ichi Maeda, Nobuyoshi Ohta, K. Uzawa, JHEP 06 (2009) 051)

[1] Introduction

- Analysis of the early universe --- initial singularity, inflation
- Black Holes

⇒ Quantum gravity

- Candidate of quantum gravity --- String theory

☆ String theory requires the spacetime to be higher dimensional.

- We have to find a compactification.

☆ String theory gives D-brane.

- Warped compactification, Brane world

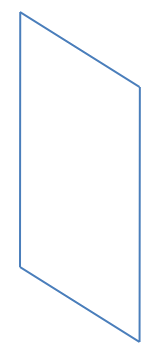
▪String theory, supergravity theory :
There are anti-symmetric tensor fields of higher rank.

▪(p+1)-form gauge field in D-dimensions:
There is (p+2)-form field strength.
⇒A p-dim charged objects couples to (p+1)-form gauge field.

p-(mem)brane

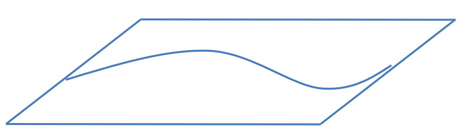
0-brane

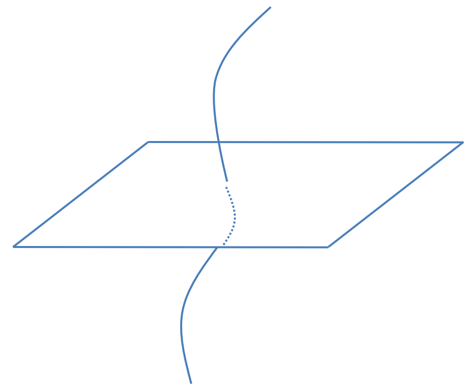
1-brane



2-brane

These higher dimensional objects (p-brane) intersect each other in D-dimensions: ⇒ intersecting branes





- Classical (brane) solution of SUGRA
  (G. Horowitz & A. Strominger; Nucl. Phys.B (1990) 197)

- Intersecting brane solution
  (R. Guven; Phys.Lett.B276 (1992) 49.)
  (E . Bergshoeff, et.al., Nucl.Phys.B494 (1997) 119)

- Classical solution : M-brane, D-brane
  → Time dependent solution

↓ **target**

- Compactification
- Analysis of the early universe
- BH in expanding universe

[2] Dynamical solutoin of p-brane system

(G.W. Gibbons, H. Lu, C.N. Pope Phys.Rev.Lett.94:131602,2005)
(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

Let us consider the case of an arbitrary p-brane background

$$\begin{aligned} S &= \frac{1}{2\kappa^2}\int\left(R*\mathbf{1}_D - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2}e^{-c\phi}F_{(p+2)}\wedge *F_{(p+2)}\right), \\ c^2 &= 4-\frac{2(p+1)(D-p-3)}{D-2}. \end{aligned}$$

The dynamical background of the p-brane can be written by

$$\begin{aligned} ds^2 &= h^{-(D-p-3)/(D-2)}q_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}u_{ij}dy^i dy^j, \\ e^\phi &= h^{-c/2}, \qquad h(x,y)=h_0(x)+h_1(y), \\ F_{(p+2)} &= d(h^{-1})\wedge\Omega(\mathrm{X}), \qquad \Omega(\mathrm{X})=\sqrt{-q}\,dx^0\wedge dx^1\wedge\cdots\wedge dx^p \end{aligned}$$

In the $c \neq 0$ case, the field equations are reduced to

$$\boxed{\begin{aligned} &R_{\mu\nu}(\mathrm{X})=0, \qquad R_{ij}(\mathrm{Y})=0, \\ &D_\mu D_\nu h_0 = 0, \qquad \triangle_{\mathrm{Y}} h_1 = 0 \end{aligned}}$$

	0	1	$\cdots$	p	$p+1$	$\cdots$	D
p-brane	○	○	○	○			

- Internal and external space are Ricci flat.

In the $c=0$ case, the field equations are reduced to

$$R_{\mu\nu}(\mathrm{X})=0, \qquad R_{ij}(\mathrm{Y})=\frac{b}{2}\lambda u_{ij}(\mathrm{Y}),$$
$$D_\mu D_\nu h_0=\lambda q_{\mu\nu}(\mathrm{X}), \qquad \triangle_{\mathrm{Y}} h_1=0.$$

◆ For example, in the case of $q_{\mu\nu}=\eta_{\mu\nu}, \quad u_{ij}=\delta_{ij}$
the solution is

$$(1)\ c\neq 0 \quad : \qquad h_0(x)=c_\mu x^\mu+\tilde{c}, \quad h_1(y)=\sum_l \frac{M_l}{|y^i-y^i_l|^{D-p-3}}$$

(G.W. Gibbons, H. Lu, C.N. Pope; Phys.Rev.Lett.94:131602,2005)

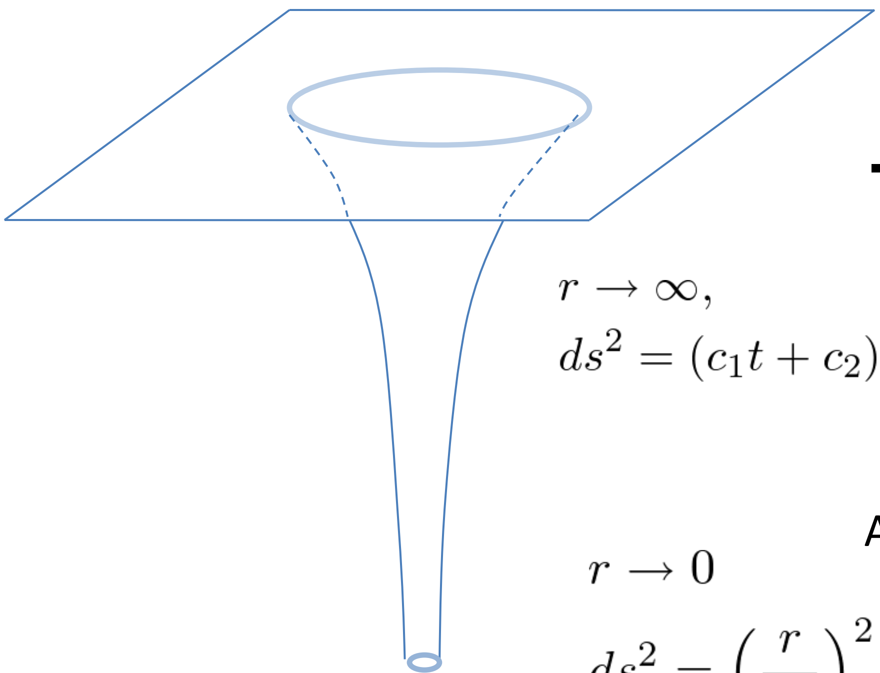
$$(2)\ c=0 \quad : \qquad h_0(x)=\frac{\lambda}{2}x^\mu x_\mu+c_\mu x^\mu+\tilde{c}, \quad h_1(y)=\sum_l \frac{M_l}{|y^i-y^i_l|^{D-p-3}}$$

$c_\mu, \quad \tilde{c}$: constant parameters

■ Dynamical solution of p-brane system

(G.W. Gibbons, H. Lu, C.N. Pope Phys.Rev.Lett.94 (2005) 131602)
(H. Kodama & K. Uzawa ; JHEP 07 (2005) 061)
(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

$$
\begin{aligned}
ds^2 &= h^{-(D-p-3)/(D-2)}\eta_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}(dr^2 + r^2 d\Omega^2_{D-p-2}),\\
h(t,r) &= c_1 t + c_2 + M r^{-D+p+3}, \quad F_{p+2} = d(h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p,\\
e^\phi &= h^{c/2}, \quad c^2 = 4 - 2(p+1)(D-p-3)(D-2)^{-1}
\end{aligned}
$$



▪10-dim D3-brane solution

$r \to \infty,$

$$ds^2 = (c_1 t + c_2)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (c_1 t + c_2)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$AdS_5 \times S^5$

$r \to 0$

$$ds^2 = \left(\frac{r}{M}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{M}{r}\right)^2 dr^2 + d\Omega_5^2$$

[3] Dynamical solution of intersecting p-brane system

(K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051)

★ Solution

$$ds^2 = -\prod_I h_I^a(t,y)dt^2 + \sum_{\mu=1}^{p}\prod_I h_I^{\delta_I^\mu/(D-2)}(t,y)\,(dx^\mu)^2\,(\mathrm{X}) + \prod_I h_I^b(t,y)u_{ij}(\mathrm{Y})dy^i dy^j$$

$$a = -\frac{D-p_I-3}{D-2}, \qquad b = \frac{p_I+1}{D-2}, \qquad \delta_I^\mu = \begin{cases} -(D-p_I-3) & \text{for} \quad \mu \parallel I \\ p_I+1 & \text{for} \quad \mu \perp I \end{cases}$$

$$e^\phi = \prod_I h_I^{\epsilon_I c_I/2}, \qquad c_I^2 = 4 - \frac{2(p_I+1)(D-p_I-3)}{D-2}, \qquad \epsilon_I = \pm 1,$$

$$F_{(p_I+2)} = d\left(h_I^{-1}\right)\wedge\Omega(\mathrm{X}_I), \qquad \Omega(\mathrm{X}_I) = dt\wedge dx^{p_1}\wedge\cdots\wedge dx^{p_I},$$

$$R_{ij}(\mathrm{Y}) = 0, \qquad h_I(t,y) = H_I(t) + K_I(y),$$

Above Equations can be satisfied it only if there is only one function h_I depending on both y^i and t .

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$
M2	$\circ$	$\circ$	$\circ$									$\surd$
M5	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					
	t	x^1	x^2	y^3	y^4	y^5	y^6	z^1	z^2	z^3	z^4	

☆We have the following intersections involving the two M-branes

$$\bar{p}+1=\frac{1}{9}\left(p_I+1\right)\left(p_{I'}+1\right), \qquad p_I \cap p_{I'}=\bar{p}$$

⇒ M2 ∩ M2=0, M5 ∩ M5=3, M2 ∩ M5=1

★ For Dp-brane,

$$\bar{p}=\frac{1}{2}(p_I+p_{I'}-4)$$

Above rules gives the following intersections :

⇒ Dp ∩ Dp=p-2, D(p-2) ∩ Dp=p-3, D(p-4) ∩ Dp=p-4

F1 ∩ F1=-1, F1 ∩ NS5=1, NS5 ∩ NS5=3

F1 ∩ Dp=0, Dp ∩ NS5=p-1, (p≦6)

★ Cosmology:

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$
M2	○	○	○									$\surd$
M5	○		○	○	○	○	○					
	t	x^1	x^2	y^3	y^4	y^5	y^6	z^1	z^2	z^3	z^4	

Let us consider the dynamics of 4-dim universe. To find an expanding universe, we have to smear and compactify the bulk space as well as the brane world volume (fixing our universe at some position in Z space).

	branes	dim(Z)	$s_{\tilde{I}}$ or $s_{\tilde{\not{I}}}$	$\beta_{\tilde{I}}$ or $\beta_{\tilde{\not{I}}}$	$\beta^{(1)}_{\tilde{I}}$ or $\beta^{(1)}_{\tilde{\not{I}}}$	$\beta^{(2)}_{\tilde{I}}$ or $\beta^{(2)}_{\tilde{\not{I}}}$	$\beta^{(3)}_{\tilde{I}}$ or $\beta^{(3)}_{\tilde{\not{I}}}$
case 1 ($\tilde{I}$ =M5)	M2-M5	4	$-1/3$	$-1/5$	0	$1/7$	$1/4$
	M5-M5	3	0	0	$1/7$	$1/4$	–
	M5-M5-M5	1	$-2/3$	$1/4$	–	–	–
	M2-M5-M5	3	0	0	$1/7$	$1/4$	–
	M2-M2-M5	3	0	0	$1/7$	$1/4$	–
	M2-M2-M5-M5	3	0	0	$1/7$	$1/4$	–
case 2 ($\tilde{I}$ =M2)	M2-M5	4	$-7/6$	$-1/5$	0	$1/7$	$1/4$
	M2-M5-M5	3	-1	0	$1/7$	$1/4$	–
	M2-M2-M5	3	-1	0	$1/7$	$1/4$	–
	M2-M2-M5-M5	3	-1	0	$1/7$	$1/4$	–

▪ 4-dimensional metric:

$$d\bar{s}_4^2 = -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{\beta_I} \delta_{ab} dx^a dx^b$$

☆ BH in expanding universe :

ex.) M2-M2-M5-M5-brane

	0	1	2	3	4	5	6	7	8	9	10
M2	○				○		○				
M2	○					○		○			
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			

Compactifying all brane volume, 4-dim metric is given by

$$ds_4^2 = -\left(h_2 H_2 H_5 H'_5\right)^{-1/2} d\tau^2 + a^2(\tau)\left(h_2 H_2 H_5 H'_5\right)^{1/2}\left(dr^2 + r^2 d\Omega_2^2\right), \quad \tau = \frac{4}{3} t_0 (t/t_0)^{3/4}$$

$$h_2 = 1 + \frac{M_2(\tau)}{r}, \quad a(\tau) \equiv \left(\frac{\tau}{\tau_0}\right)^{1/3}, \quad M_2 = \left(\frac{\tau}{\tau_0}\right)^{-4/3} Q_2, \quad \tau_0 \equiv \frac{4}{3} t_0$$

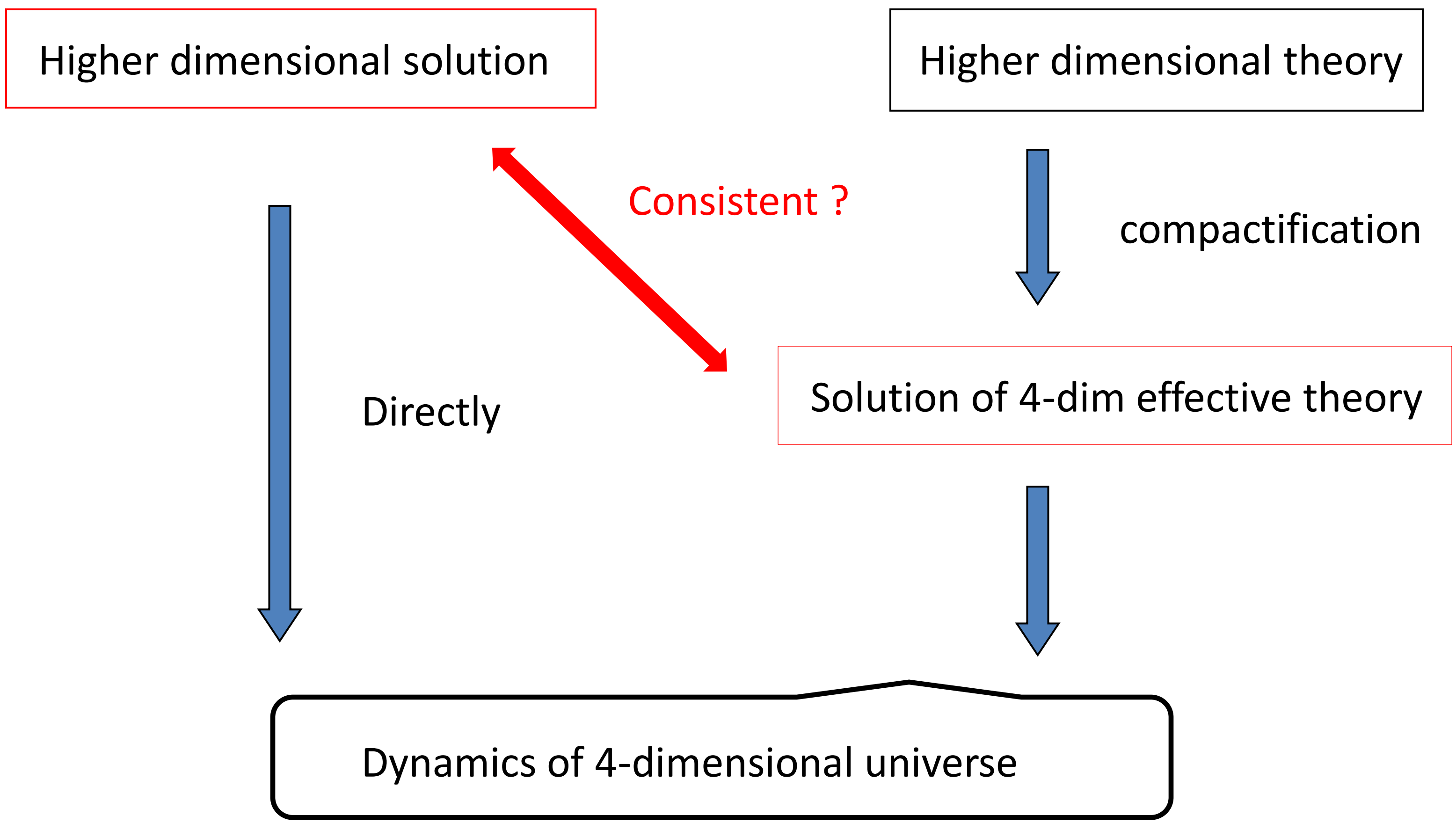
If r→∞, the line element becomes

$$ds^2 = -d\tau^2 + a^2(\tau)(dr^2 + r^2 d\Omega_2^2)$$

Hence, the solution approaches asymptotically FRW universe with scale factor. Since M2-M2-M5-M5-brane solution gives a BH, this is a BH in the expanding Universe.

[4] Lower dimensional effective theory

(H. Kodama & K. Uzawa ; JHEP 03 (2006) 053)
(Pierre Binetruy, Misao Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)
(K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051)



★(p+1)-dimensional effective theory with field strength

◇ D-dimensional model

	0	1	$\cdots$	p	$p+1$	$\cdots$	D
p-brane	○	○	○	○			

● Ansatz for background

$$
\begin{aligned}
ds^2 &= h^{-(D-p-3)/(D-2)}(x,y)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h^{(p+1)/(D-2)}(x,y)u_{ij}(\mathrm{Y})dy^i dy^j, \\
e^\phi &= h^{c/2}, \qquad h(x,y) = h_0(x) + h_1(y), \\
F_{(p+2)} &= d(h^{-1}) \wedge \Omega(\mathrm{X}_{p+1}), \qquad \Omega(\mathrm{X}_{p+1}) = \sqrt{-q}dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p
\end{aligned}
$$

- Internal space is Ricci flat (for c≠0) and Einstein space (for c=0).
- Gauge fields satisfy field equations.

◎ D-dimensional action

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int\left(R * \mathbf{1}_D - \frac{1}{2}d\phi \wedge *d\phi - \frac{1}{2}e^{-c\phi}F_{(p+2)} \wedge *F_{(p+2)}\right), \\
c^2 &= 4 - \frac{2(p+1)(D-p-3)}{D-2}.
\end{aligned}
$$

□ (p+1)-dimensional effective action (c≠0)

▪ Internal space is Ricci flat space

$$
\begin{aligned}
S &= \frac{1}{2\tilde{\kappa}^2}\int_{\mathrm{X}} H(x)R(\mathrm{X}) *_{\mathrm{X}} \mathbf{1}_{(p+1)}, \\
H(x) &= h_0(x) + \bar{c}; \quad \bar{c} := V^{-1}_{(D-p-1)}\int_{\mathrm{Y}} h_1 *_{\mathrm{Y}} \mathbf{1}_{(D-p-1)}, \\
\tilde{\kappa} &= \left[V_{(D-p-1)}\right]^{-1/2}\kappa, \qquad V_{(D-p-1)} = \int_{\mathrm{Y}} *\mathbf{1}_{(D-p-1)}
\end{aligned}
$$

★ Field equations ;

(p+1)-dimensional equation:

$$
\begin{aligned}
R_{\mu\nu}(\mathrm{X}) &= H^{-1}D_\mu D_\nu H, \\
\triangle_{\mathrm{X}} H &= 0
\end{aligned}
$$

D-dimensional equation :

$$
R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}) = 0,
$$
$$
h(x,y) = h_0(x) + h_1(y),
$$
$$
D_\mu D_\nu h_0 = 0, \quad \triangle_{\mathrm{Y}} h_1 = 0
$$

▪ Dynamical D3-brane (c=0):

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○						

$$ds^2 = h^{-1/2}(x,y)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h^{1/2}(x,y)u_{ij}(\mathrm{Y})dy^i dy^j,$$
$$F_5 = (1 \pm *)d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3,$$

★10-dimensional solution:

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}) = \lambda u_{ij}(\mathrm{Y}),$$
$$h(x,y) = h_0(x) + h_1(y),$$
$$D_\mu D_\nu h_0 = \lambda q_{\mu\nu}(\mathrm{X}), \quad \triangle_{\mathrm{Y}} h_1 = 0,$$

☆4-dimensional solution:

$$\begin{aligned} R_{\mu\nu}(\mathrm{X}) &= H^{-1}\left[D_\mu D_\nu H - \lambda q_{\mu\nu}(\mathrm{X})\right], \\ \triangle_{\mathrm{X}} H &= 4\lambda, \\ H(x) &= h_0(x) + V_6^{-1}\int_{\mathrm{Y}_6} h_1 *_{\mathrm{Y}} \mathbf{1}_{\mathrm{Y}}, \end{aligned}$$

The class of solutions obtained in the 4-dimensional effective theory are much larger than the higher-dimensional original theory. This is because the information of the internal space which gives constraints on the lower dimensions was lost after compactifying the internal space.

[5] Summary :

☆ We give some dynamical intersecting brane solutions in eleven-dimensional supergravity.

☆ We apply these solutions to cosmology and black holes. It is shown that these give FRW cosmological solutions. If we regard the bulk space as our universe, we may interpret them as black holes in the expanding universe.

★ The cosmological solutions we found have the property that they are genuinely higher-dimensional in the sense that one can never neglect the dependence on the coordinates of transverse space.

★ We also found lower-dimensional effective theories may give us some solutions which are inconsistent with the higher-dimensional Einstein equations.