The Nuclear Matrix Elements for Double Beta Decay

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OUTLINE

• Introduction
• Nuclear structure approaches
• Uncertainties in QRPA NME’s
• Conclusions and outlook
Neutrino oscillations $\Rightarrow$ Massive neutrinos

Solar neutrinos

Atmospheric neutrinos

Accelerator neutrinos

Reactor neutrinos
What is the nature of neutrinos?

<table>
<thead>
<tr>
<th>Quark mixing</th>
<th>Neutrino mixing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{CKM} =$</td>
<td>$U_{PMNS} =$</td>
</tr>
</tbody>
</table>
| \[
\begin{pmatrix}
0.98 & 0.22 & 0.003 \\
-0.22 & 0.97 & 0.04 \\
0.003 & -0.04 & 1.00
\end{pmatrix}
\] | \[
\begin{pmatrix}
0.83 & 0.55 & 0.05 \\
0.34 - 0.45 & 0.56 - 0.62 & 0.70 \\
0.34 - 0.45 & 0.55 - 0.62 & 0.70
\end{pmatrix}
\] |

Large off diagonal elements

Instruction for an extension of SM?

Dirac particle

Majorana particle

$\nu \neq \nu^C$

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$\nu = \nu^C = C \nu^T$
Double Beta Decay

\[ (A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e \]

\[ (T_{1/2}^{2\nu})^{-1} = G^{2\nu} |M_{GT}^{2\nu}|^2 \]

Observed for 10 isotopes: \(^{48}\text{Ca},^{76}\text{Ge},^{82}\text{Se},^{96}\text{Zr},^{100}\text{Mo},^{116}\text{Cd},^{128}\text{Te},^{130}\text{Te},^{150}\text{Nd},^{238}\text{U}\), \(T_{1/2} \approx 10^{18}-10^{24}\) years

\[ (A, Z) \rightarrow (A, Z + 2) + 2e^- \]

\[ (T_{1/2}^{0\nu})^{-1} = \eta^{LNV} G^{0\nu} |M^{0\nu}|^2 m_{\beta\beta} = \sum_i U_{ei}^2 m_i \]

SM forbidden, not observed yet: \(T_{1/2}^{0\nu} (^{76}\text{Ge}) > 10^{25}\) years

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\[ M^{2\nu}_{GT} = \sum_{m} < 0^+_f || \tau^+ \sigma || 1^+_m > < 1^+_m || \tau^+ \sigma || 0^+_i > \frac{E_m - E_i + \Delta}{E_m - E_i + \Delta} \]

2νββ-decay

\[ (T^{2\nu}_{1/2})^{-1} = G^{2\nu} |M^{2\nu}_{GT}|^2 \]

\[ \frac{M^{2\nu}_{GT}}{M^{2\nu}_{GT}} \]

Continuum states

Gamow-Teller transitions

1+

GT resonance

QRPA RQRPA

shell model

virtual states

low lying states

SSD hypothesis

(A, Z)

(A, Z+1)

(A, Z+2)

(A, Z+3)

(A, Z+4)

\( (T^{2\nu}_{1/2})^{-1} \) difference: factor \( \sim 8 \)
The $0\nu\beta\beta$-decay NME (light $\nu$ exchange mech.)

The $0\nu\beta\beta$-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z) |M^{0\nu}|^2 |\langle m_{\beta\beta} \rangle|^2,$$

NME = sum of Fermi, Gamow-Teller and tensor contributions

$$M^{0\nu} = \left( \frac{g_A}{1.25} \right)^2 \langle f \mid -\frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu} \mid i \rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2 R} \int_0^\infty f_K(q r_{12}) \frac{h_K(q^2)q dq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(q r_{12}) = j_0(q r_{12}), \quad f_T(q r_{12}) = -j_2(q r_{12})$$

Induced pseudoscalar coupling (pion exchange)

Form-factors: finite nucleon size

$$h_F = g_V^2(q^2)$$

$$h_{GT} = g_A^2 \left[ 1 - \frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} + \frac{1}{3} \left( \frac{q^2}{q^2 + m_\pi^2} \right)^2 \right]$$

$$h_T = g_A^2 \left[ \frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} - \frac{1}{3} \left( \frac{q^2}{q^2 + m_\pi^2} \right)^2 \right]$$

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f} \sum_{J \text{ pnp' n'}} (-1)^{j_n+j_{p'}+J+J} \sqrt{2J+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{p'} & j_{n'} & J \end{array} \right\} J^\pi = \begin{array}{c} 0^+, 1^+, 2^+ \ldots \\ 0^-, 1^-, 2^- \ldots \end{array}$$

Jastrow f. s.r.c.

$$\langle p(1), \mathbf{p}(2) : J \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2) : J \rangle$$

$$\times \langle 0_f^+ \parallel [c_p^+ \tilde{c}_{n'}]_{J} \parallel J^\pi k_f \rangle \langle J^\pi k_f \mid J^\pi k_i \rangle \langle J^\pi k_f \parallel [c_p^+ \tilde{c}_n]_{J} \parallel 0^+_f \rangle$$
Nuclear structure approaches

\[ H \Psi = E \Psi \]

We can not solve the full problem in the complete

Systematical study of the $0 \nu \beta \beta$-decay NME

Projected mean field (Vampir)

Shell model:

QRPA, RQRPA: About 10 papers 1987→ 2006

Other approaches:

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Shell Model

- Define a valence space
- Derive an effective interaction $H \Psi = E \Psi \rightarrow H_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}}$
- Build and diagonalize Hamiltonian matrix $(10^{10})$
- Transition operator $<\Psi_{\text{eff}} | O_{\text{eff}} | \Psi_{\text{eff}}>$
- Some phenomenological input needed
  - energy of states, systematics of B(E2) and GT transitions (quenching f.)

$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$

$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$

$^{40}\text{Ca}$

$^{56}\text{Ni}$

Small calculations

76Se$_{42}$ in the valence
6 protons and 14 neutrons

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Comparison of $M^{0\nu}$ of Rodin et al. (RQRPA) and Nowacki et al. (SM, private comm., preliminary 2004) and older published (Caurier et al. 1996)

$0\nu\beta\beta$-decay

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>RQRPA</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>2.3-2.4</td>
<td>1.6</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>1.9-2.1</td>
<td>1.7</td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td>0.3-0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>1.1-1.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>1.2-1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>1.3</td>
<td>2.0 (1.0)</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>0.6-1.0</td>
<td>1.6 (0.6)</td>
</tr>
</tbody>
</table>

Except for $^{100}\text{Mo}$ the agreement between these very different calculations is reasonably good. Note that the SM calculations include the reduction caused by the s.r.c. and induced currents.

SM results

$2\nu\beta\beta$-decay

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$T_{1/2}(\text{th.})[\text{y}]$</th>
<th>$T_{1/2}(\text{exp.})[\text{y}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>$3.7 \times 10^{19}$</td>
<td>$4.2 \times 10^{19}$</td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>$1.2 \times 10^{21}$</td>
<td>$1.4 \times 10^{21}$</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>$3.4 \times 10^{19}$</td>
<td>$9.0 \times 10^{19}$</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$4.1 \times 10^{20}$</td>
<td>$6.1 \times 10^{20}$</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>$6.1 \times 10^{20}$</td>
<td>$8.1 \times 10^{20}$</td>
</tr>
</tbody>
</table>

Strasbourg group, Nowacki, IDEA meeting, Heidelberg, 2004
QRPA-like approaches
QRPA, RQRPA, SRQRPA

Particle number condition

i) Uncorrelated BCS ground state
\[ Z = \langle \text{BCS} | Z | \text{BCS} \rangle \]
\[ N = \langle \text{BCS} | N | \text{BCS} \rangle \]

QRPA

RQRPA

ii) Correlated RPA ground state
\[ Z = \langle \text{RPA} | Z | \text{RPA} \rangle \]
\[ N = \langle \text{RPA} | N | \text{RPA} \rangle \]

SRQRPA

Pauli exclusion principle

i) violated (QBA)
\[ [A, A^+] = \langle \text{BCS} | [A, A^+] | \text{BCS} \rangle \]

QRPA

ii) Partially restored (RQBA)
\[ [A, A^+] = \langle \text{RPA} | [A, A^+] | \text{RPA} \rangle \]

RQRPA

SRQRPA

Complex numerical procedure

BCS and QRPA equations are coupled
QRPA
2νββ-decay NME

\[ H = H_0 + g_{ph} H_{ph} + g_{pp} H_{pp} \]

\[ ^{76}\text{Ge} \rightarrow ^{76}\text{Se} \]

Collapse of the QRPA
21 l.m.s. 12 l.m.c
Uncertainties in $0\nu\beta\beta$–decay NME?

Bahcall, Murayama, Pena-Garay,

Please no!
Do not put different NME calculations at the same level (democratic approach)

Civitarese, Suhonen,

This suggest an uncertainty of NME as much as factor 5

<table>
<thead>
<tr>
<th>System</th>
<th>$G_{1}^{(0v)} \times 10^{14}$</th>
<th>N.M.E.</th>
<th>N.M.E. (this work)</th>
<th>$\langle m_{\nu} \rangle_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>6.43</td>
<td>1.08–2.38</td>
<td>3.33</td>
<td>8.70–19.0</td>
</tr>
<tr>
<td>$^{76}$Ge</td>
<td>0.63</td>
<td>2.98–4.33</td>
<td>3.44</td>
<td>0.30–0.43</td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>2.73</td>
<td>2.53–3.98</td>
<td>3.55</td>
<td>4.73–7.44</td>
</tr>
<tr>
<td>$^{96}$Zr</td>
<td>5.70</td>
<td>2.74</td>
<td>2.97</td>
<td>19.1–24.7</td>
</tr>
<tr>
<td>$^{100}$Mo</td>
<td>4.57</td>
<td>0.77–4.67</td>
<td>3.75</td>
<td>2.18–13.2</td>
</tr>
<tr>
<td>$^{116}$Cd</td>
<td>4.68</td>
<td>1.09–3.46</td>
<td>3.75</td>
<td>2.37–8.18</td>
</tr>
<tr>
<td>$^{128}$Te</td>
<td>0.16</td>
<td>2.51–4.58</td>
<td>3.49</td>
<td>9.51–17.4</td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>4.14</td>
<td>2.10–3.59</td>
<td>3.49</td>
<td>1.87–3.20</td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>4.37</td>
<td>1.61–1.90</td>
<td>4.64</td>
<td>0.79–2.29</td>
</tr>
</tbody>
</table>

Is it really so bad?!
The $0\nu\beta\beta$-decay NME: $g_{pp}$ fixed to $2\nu\beta\beta$-decay

Each point: (3 basis sets) x (3 forces) = 9 values

By adjusting of $g_{pp}$ to $2\nu\beta\beta$-decay half-life the dependence of the $0\nu\beta\beta$-decay NME on other things that are not a priori fixed is essentially removed

Neutrinoless Double Beta Decay Nuclear Matrix Elements

Light Majorana Neutrino Exchange Mechanism

Recommemded $0\nu\beta\beta$-decay NME (April 2005)

List of reasons, why QRPA-like $0\nu\beta\beta$–decay NME are different (13 reasons)

Quasiparticle mean field fixing of pp, nn (pn) pairing

Many-body approximations
QRPA, RQRPA, SRQRPA

Choice of NN interaction
Schem., realistic (Bonn, Paris …)

the closure approximation

p-h interaction ($g_{ph} \approx 1$)
fixed to GT resonance

The size of model space

p-p interaction ($g_{pp}$)
fixed to $\beta$ or $\beta\beta$–decay resonance, or $g_{pp} = 1$

two-nucleon s.r.c. ($\sim 50\%$) has to be considered

finite size of nucleon ($\sim 10\%$) form factors

h.o.t. of nucleon curr. ($\sim 30\%$)
Induced PS, weak magnetism

the overlap factor
the BCS overlap

the axial-vector coupling
$g_A = 1.0$ or 1.25

Nuclear shape
Spherical, not deformed yet

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A comparison with Muto (\(^{76}\text{Ge}\)) results
(the same procedure of fixing \(g_{\text{pp}}\))


\(^{76}\text{Ge}\), small model space (3-4 \(\hbar\omega\)), dependence on size of model space not studied, \(g_{\text{pp}}\) fixed to 2\(\nu\)\(\beta\)\(\beta\)-decay half-life, h.o.t of nucleon current not considered

\[
M^{0\nu} = 4.59 \text{ (QRPA)} \quad 3.88 \text{ (RQRPA)} \quad Muto
\]
\[
2.68 \quad 2.41 \quad \text{Rodin et al}
\]

- Without higher order terms of nucleon current \(\Rightarrow\) reduction of 30%
- \(r_0=1.2\ \text{fm (Muto)}, 1.1\ \text{fm (Rodin et al.)}\) \(\Rightarrow\) reduction of 10%

\[
M^{0\nu} = 0.6 \times 4.59 = 2.76 \Leftrightarrow 2.68 \text{ (Rodin at al.)} \quad \text{(QRPA)}
\]
\[
0.6 \times 3.88 = 2.34 \Leftrightarrow 2.41 \text{ (Rodin et al.)} \quad \text{(RQRPA)}
\]

There is a good agreement!
A comparison with Stoica, Klapdor-Kleingrothaus (\(^{76}\text{Ge}\)) results (the same procedure of fixing \(g_{pp}\))


\(^{76}\text{Ge}\), s.m.s (3-4 \(\hbar \omega\)) and l.m.s (2-4 \(\hbar \omega\)), \(g_{pp}\) fixed to 2\(\nu\)\(\beta\beta\)-decay half-life, h.o.t of nucleon current not considered

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(M^0\nu) (x 0.6) =</td>
<td>(M^0\nu) =</td>
</tr>
<tr>
<td>2.67 (QRPA) s.m.s</td>
<td>2.68 (QRPA) s.m.s</td>
</tr>
<tr>
<td>1.03 l.m.s.</td>
<td>2.62 l.m.s.</td>
</tr>
<tr>
<td>2.24 (RQRPA) s.m.s</td>
<td>2.41 (RQRPA) s.m.s</td>
</tr>
<tr>
<td>1.14 l.m.s.</td>
<td>2.32 l.m.s.</td>
</tr>
</tbody>
</table>

s.m.s. = 3-4 \(\hbar \omega\) l.m.s. = 2-4 \(\hbar \omega\)  
(+ 0d\(_{3/2}\), 0d\(_{5/2}\), 1s\(_{1/2}\))

This should have impact on \(\beta\)-strength distribution?!  

Negligible dependence on the size of model space

There is a strong disagreement
A comparison with Civitarese & Suhonen results

Aunola, Suhonen, NPA 643, 207 (1998); Civitarese, Suhonen, NPA 729, 867 (2003)

No short range correlations, no higher order terms of nucleon currents,
$G_{pp}$ fixed to single $\beta$-decay observables
Two-nucleon short range correlations: a question of physics

There is no double counting in QRPA
- QRPA violates Pauli exclusion principle
- $1/(0.2 \text{ fm}) \sim 1 \text{ GeV}$
Suppression of higher multipolarities due to
- two-nucleon short-range correlations
- induced pseudoscalar coupling


Many multipoles contribute. Most of them with exceptions have the same sign.
The importance of transition through higher-lying states of \((A,Z+1)\) nucleus

Shortcoming of fixing \(g_{pp}\) to the g.s. single \(\beta\)-decay observables
Negative $M^{2\nu}$ is disfavored:

- The QRPA is designed to describe small amplitude exc. around the mean field minim.
- Disagreement with systematic study of single $\beta$-decay
  - Homma et al., PRC 54, 2972 (1996)
- The lowest $\beta^+/EC$ transition $(A,Z+1)\rightarrow(A,Z)$ too strong
- If Pauli exclusion principle fully taken into account negative $M^{2\nu}$ appears for too large value of $g_{pp}$
  - Šimkovic et al., PRC 61, 044319 (2000)

\[ (T_{1/2}^{2\nu})^{-1} = G^{2\nu} |M_{GT}^{2\nu}|^2 \Rightarrow M^{2\nu} > 0 \text{ or } M^{2\nu} < 0 \]

However, for single $\beta$-decay this problem was ignored

- we used to fixed $g_{pp}$
  - Civitarese, Suhonen, NPA 761, 313 (2005)
- used both values to fix $g_{pp}$

\[ M^{2\nu-\text{exp}} > 0 \text{ or } M^{2\nu-\text{exp}} < 0 \]
The outliers predict wrong 2νββ half-life. The matrix elements of SM and Rodin et al. are quite close.
**SRQRPA results**

\( g_{pp} \) fixed to 2\( \nu \beta \beta \)-half life (Beneš, F.Š., Faessler, to be submitted)

- Pairing fixed to exp. pairing gaps (constant pairing considered)
- BCS overlap factor taken into account (factor \( \sim 0.8 \))
- Uncertainties of 2\( \nu \beta \beta \)-decay half-lives not considered

\( ^{48}\text{Ca}, ^{116}\text{Sn}, ^{130}\text{Xe} \) need careful treatment of Pairing \( \Rightarrow \) Lipkin-Nogami BCS
Nuclear deformation

\[ \beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Zr_c^2} \]

Exp. I (nuclear reorientation method)
Exp. II (based on measured E2 trans.)
Theor. I (Rel. mean field theory)
Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

Till now, in the QRPA-like calculations of the \( 0\nu\beta\beta \)-decay NME spherical symmetry was assumed

The effect of deformation on NME has to be considered

<table>
<thead>
<tr>
<th>Nucl.</th>
<th>Exp. I</th>
<th>Exp. II</th>
<th>Theor. I</th>
<th>Theor. II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{48}\text{Ca})</td>
<td>0.00</td>
<td>0.101</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{48}\text{Ti})</td>
<td>+0.17</td>
<td>0.269</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{76}\text{Ge})</td>
<td>+0.09</td>
<td>0.26</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>(^{76}\text{Se})</td>
<td>+0.16</td>
<td>0.31</td>
<td>-0.24</td>
<td>-0.24</td>
</tr>
<tr>
<td>(^{82}\text{Se})</td>
<td>+0.10</td>
<td>0.19</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>(^{82}\text{Kr})</td>
<td></td>
<td>0.20</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>(^{96}\text{Zr})</td>
<td></td>
<td>0.081</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>(^{96}\text{Mo})</td>
<td>+0.07</td>
<td>0.17</td>
<td>0.17</td>
<td>0.08</td>
</tr>
<tr>
<td>(^{100}\text{Mo})</td>
<td>+0.14</td>
<td>0.23</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>(^{100}\text{Ru})</td>
<td>+0.14</td>
<td>0.22</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>(^{116}\text{Cd})</td>
<td>-0.11</td>
<td>0.19</td>
<td>-0.26</td>
<td>-0.24</td>
</tr>
<tr>
<td>(^{116}\text{Sn})</td>
<td>+0.04</td>
<td>0.11</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{128}\text{Te})</td>
<td>+0.01</td>
<td>0.14</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{128}\text{Xe})</td>
<td></td>
<td>0.18</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>(^{130}\text{Te})</td>
<td>+0.03</td>
<td>0.12</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{130}\text{Xe})</td>
<td></td>
<td>0.17</td>
<td>0.13</td>
<td>-0.11</td>
</tr>
<tr>
<td>(^{136}\text{Xe})</td>
<td></td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{136}\text{Ba})</td>
<td></td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(^{150}\text{Nd})</td>
<td>+0.37</td>
<td>0.28</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>(^{150}\text{Sm})</td>
<td>+0.23</td>
<td>0.19</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>
New Suppression Mechanism of the DBD NME

The suppression of the NME depends on relative deformation of initial and final nuclei
F.Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the $2\nu\beta\beta$-decay NME within deformed QRPA
Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F.Š.,
Which $0\nu\beta\beta$-decay NME to consider

If the authors do not specify what choices they made, and do not discuss the dependence of their result on the particular choice they made, their result should not be taken on the same footing as those where points are carefully explained!

**Recommendation 2005**

The most carefully calculated QRPA/RQRPA $0\nu\beta\beta$–decay NME are:


However, further progress is needed

Even, it might be that the true NME is not within the determined “variance”

Problems of deformation, overlap matrix, …have to be studied
Inverted Hierarchy

Normal Hierarchy

\[ \sqrt{\Delta m^2_{31}} \leq |m_{32}| \leq \sqrt{\Delta m^2_{32}} \]

\[ \sin^2 \theta_{12} \leq \sin^2 \theta_{13} \leq \Delta m^2_{32} \]

Goals for $0^{\nu}\beta\beta$-experiments

Bilenky, Faessler, Gutsche, F. Š., PRD 72, 053015 (2005)
A product of $0\nu\beta\beta$--decay NME

Can you imagine $0\nu\beta\beta$--decay of 4 nuclei (e.g. $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{130}\text{Te}$, $^{136}\text{Xe}$) is observed ...

Physical quantity of interest

$$M_{\text{aver}}^{0\nu} = \left( \prod_{k=1}^{k_{\text{max}}} |M^{0\nu}(k)| \right)^{-1/k_{\text{max}}}$$

$$|m_{\beta\beta}|_{\text{aver.}} = \left( \prod_{k=1}^{k_{\text{max}}} |M^{0\nu}(k)| \right)^2 \prod_{k=1}^{k_{\text{max}}} T_{1/2}^{0\nu}(k) \prod_{k=1}^{k_{\text{max}}} G^{0\nu}(k)^{-1/(2k_{\text{max}})}$$

uncertainty

$$|m_{\beta\beta}(k)| = \frac{1}{|M^{0\nu}(k)| \sqrt{T_{1/2}^{0\nu}(k) G^{0\nu}(k)}}$$

$$\delta(k) = |m_{\beta\beta}\text{ aver.} - |m_{\beta\beta}(k)|$$

Distinguishing different mechanisms
- light neutrino exchange
- heavy neutrino exchange
- pion-exchange mech.

Observation of the $0\nu\beta\beta$--decay of at least 3-4 different nuclei is needed

6/12/2006
Fedor Simkovic
Conclusions

• The sources of uncertainties reexamined. The differences between QRPA-like results understood, the recommended values of NME presented

• SRQRPA results are close to QRPA and RQRPA results, if \( g_{pp} \) is fixed to \( 2\nu\beta\beta \) half-life and pairing to mass differences

\[ \Rightarrow \text{There is a convergence of QRPA-like results} \]

• The story about NME not finished yet. Study of further effects (deformation, overlap factor) and cross-check with other approaches required.

\[ 0\nu\beta\beta \text{-decay NME} \]

- Absolute \( \nu \) mass scale
- Neutrino mass pattern
- CP-violating Majorana phases
- Distinguishing \( 0\nu\beta\beta \)-mechanisms

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Outlook
There will be a progress

Shell model: increased computer power, larger m.s., better effective interactions, new groups (T. Otsuka – Tokyo U.)

MCSM: unsolved problems?
H. Nakada (NNR05), Alhassid, Bertsch, Liu, Fang

QRPA: further progress expected → effects of deformation, overlap factor

Cross-check with charge-changing reactions, muon capture, neutrino-scattering data needed

Moore’s law of 0nbb-decay
Elliott, Vogel,

Inverted hierarchy
There is still some time