NEUTRINO 2006

Santa Fe, June13-19 2006

The Nuclear Matrix Elements for Double Beta Decay

Fedor Šimkovic

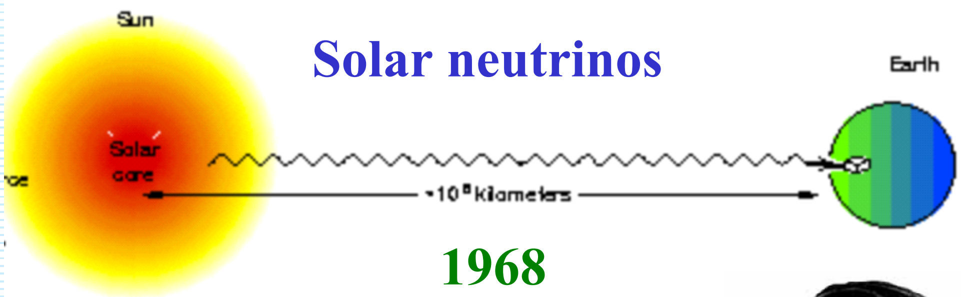
Comenius University, Bratislava

OUTLINE

- **Introduction**
- **Nuclear structure approaches**
- **Uncertainties in QRPA NME's**
- **Conclusions and outlook**

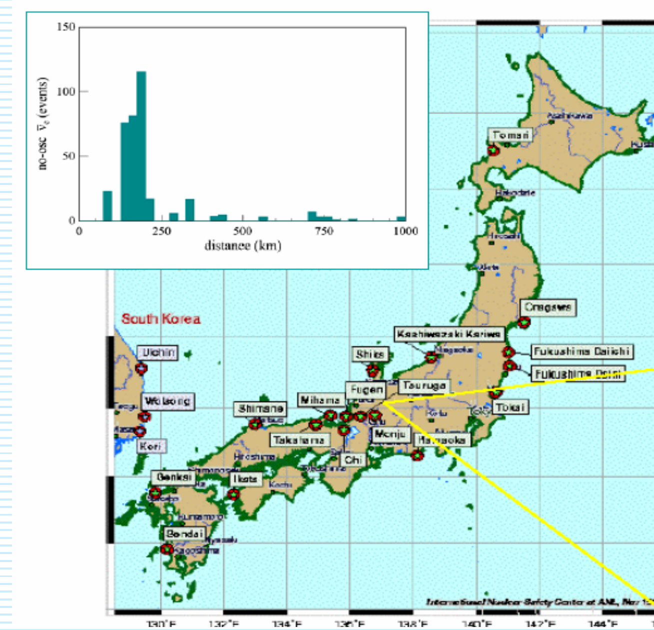
Neutrino oscillations ⇒ Massive neutrinos

Solar neutrinos

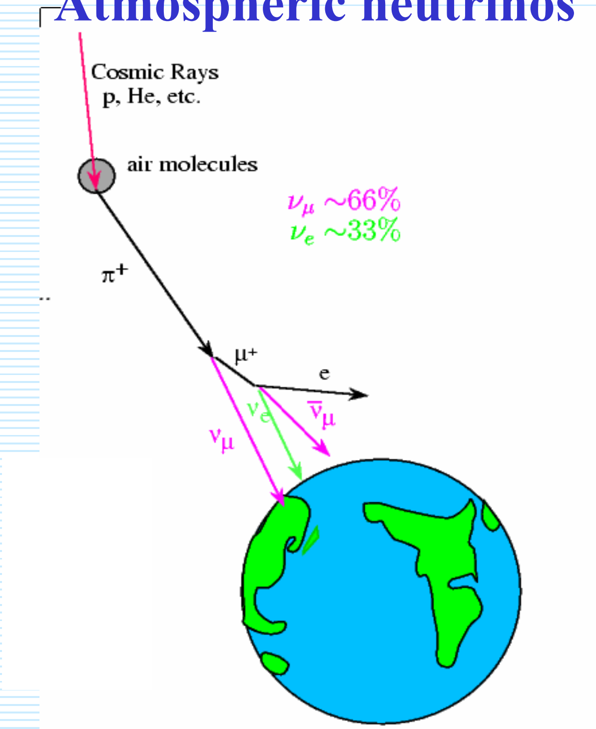


1968

Reactor neutrinos

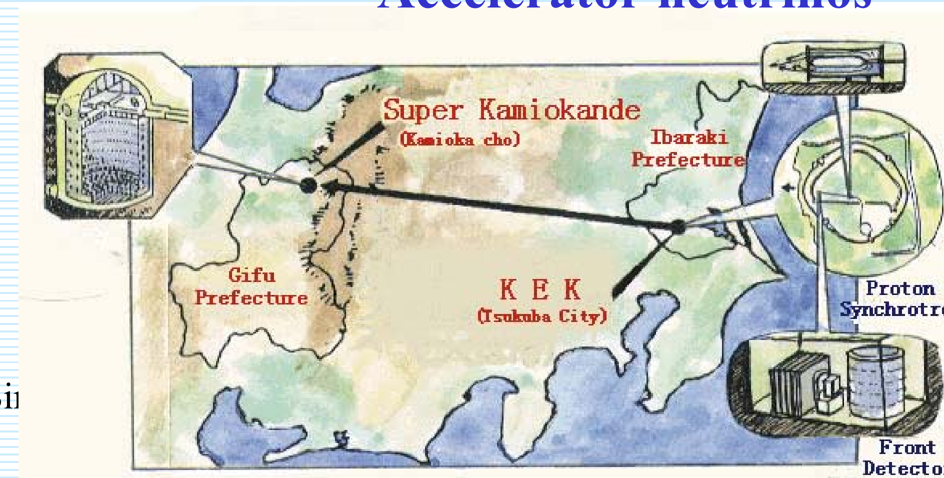


Atmospheric neutrinos



1957

Accelerator neutrinos



What is the nature of neutrinos?

Quark mixing

$$U_{CKM} = \begin{pmatrix} 0.98 & 0.22 & 0.003 \\ -0.22 & 0.97 & 0.04 \\ 0.003 & -0.04 & 1.00 \end{pmatrix}$$

Neutrino mixing

$$U_{PMNS} = \begin{pmatrix} 0.83 & 0.55 & 0.05 \\ 0.34-0.45 & 0.56-0.62 & 0.70 \\ 0.34-0.45 & 0.55-0.62 & 0.70 \end{pmatrix}$$

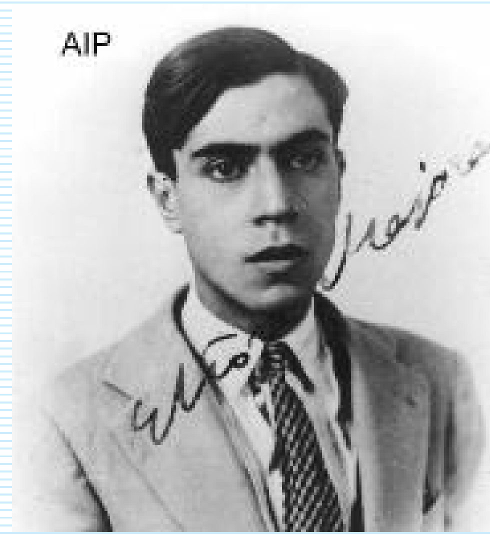
Large off diagonal elements

Instruction for an extension of SM?

Dirac particle

$$\nu$$

Majorana particle

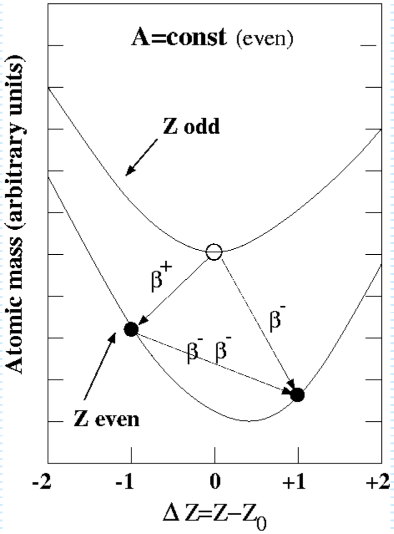


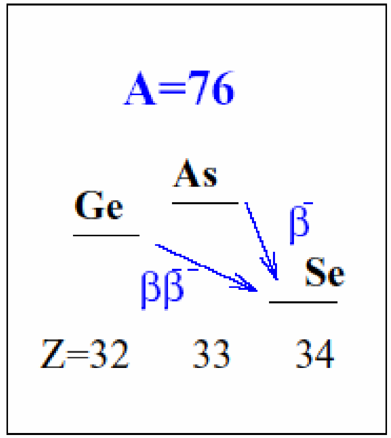
$$\nu \neq \nu^c$$

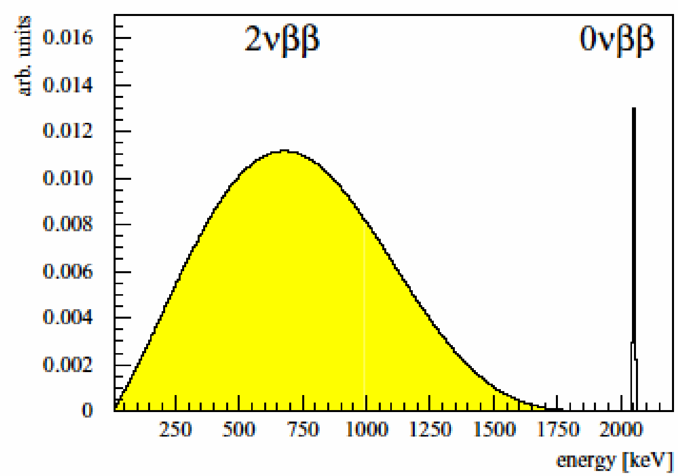
Fedor Simkovic

$$\nu = \nu^c = C\overline{\nu}^T$$

Double Beta Decay







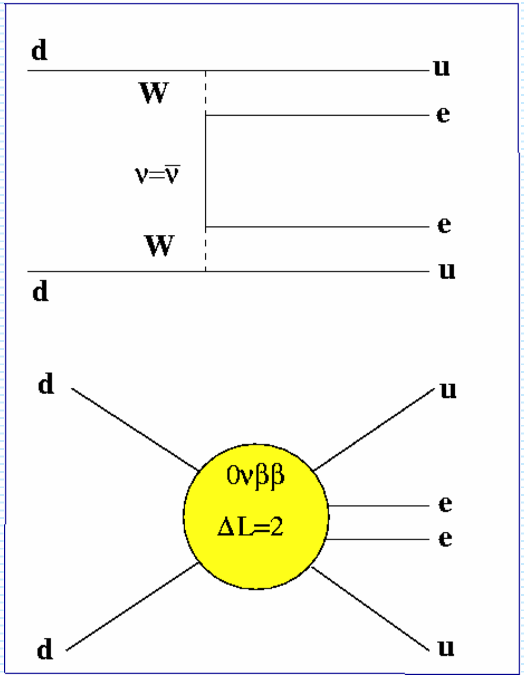
$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

$$\left(T^{2\nu}_{1/2}\right)^{-1} = G^{2\nu} \left| M^{2\nu}_{GT} \right|^2$$

Observed for 10 isotopes: **^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr,^{100}Mo. ^{116}Cd. ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U,** **$T_{1/2} \approx 10^{18}$-10^{24} years**

$$(A,Z) \rightarrow (A,Z+2) + 2e^-$$

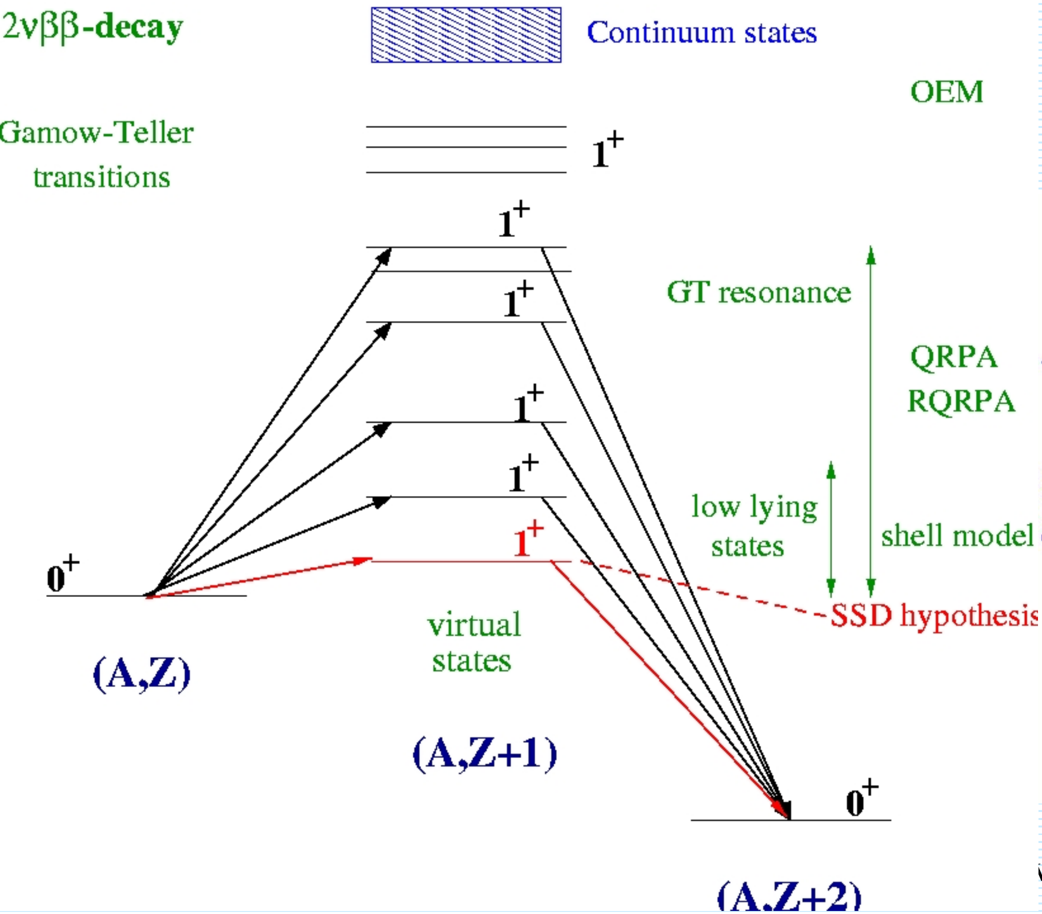
$$\left(T^{0\nu}_{1/2}\right)^{-1} = \eta^{LNV} \, G^{0\nu} \, \left| M^{0\nu} \right|^2 \qquad m_{\beta\beta} = \sum_i U^2_{ei} m_i$$



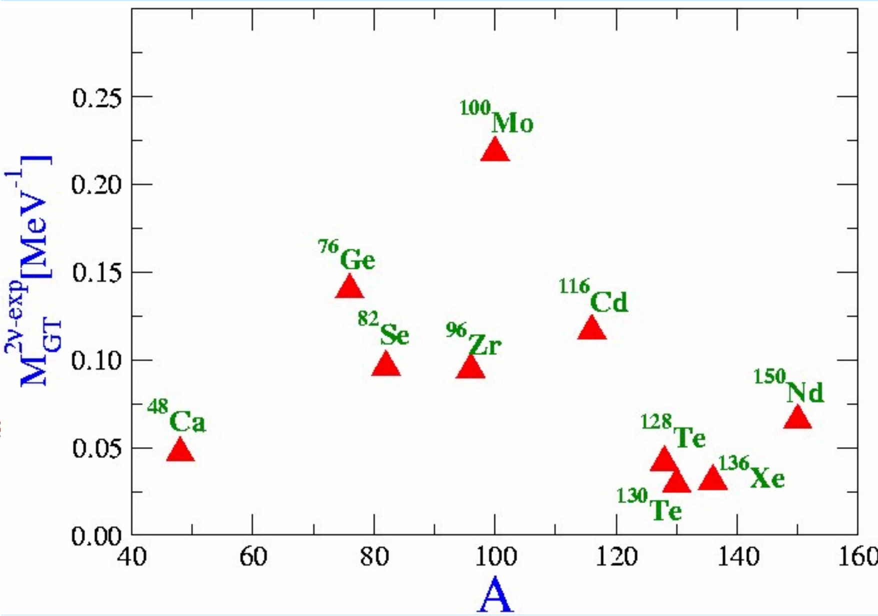
SM forbidden ,not observed yet: $T_{1/2}$ (^{76}Ge)>10^{25} years

2νββ-decay nuclear matrix elements

$$M^{2\nu}_{GT} = \sum_m \frac{< 0^+_f||\tau^+\sigma||1^+_m >< 1^+_m||\tau^+\sigma||0^+_i >}{E_m - E_i + \Delta}$$



$$(T^{2\nu}_{1/2})^{-1} = G^{2\nu}|M^{2\nu}_{GT}|^2$$



difference: factor ~ 8

The $0\nu\beta\beta$-decay NME (light ν exchange mech.)

The $0\nu\beta\beta$-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \; ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f| -\frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2) q dq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors: finite nucleon size

$$\begin{aligned} h_F &= g_V^2(q^2) \\ h_{GT} &= g_A^2\left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \\ h_T &= g_A^2\left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \end{aligned}$$

Induced pseudoscalar coupling (pion exchange)

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$
$$\times \langle 0_f^+ ||[c_{p'}^+ \tilde{c}_{n'}]_J|| J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f ||[c_p^+ \tilde{c}_n]_J|| 0_i^+\rangle$$

Jastrow f. s.r.c.

J^π = $0^+, 1^+, 2^+ \ldots$ $0^-, 1^-, 2^- \ldots$

Nuclear structure approaches

$$H\,\Psi = E\,\Psi$$

We can not solve the full problem in the complete

Systematical study of the 0νββ-decay NME

Projected mean field (Vampir)

- **Tomoda, Faessler, Schmid, Grummer, PLB 157, 4 (1985)**

Shell model:
- **Haxton, Stephensson, Prog. Part. Nucl. Phys. 12, 409(1984)**
- **Caurier, Nowacki, Poves, Retamosa, PRL 77, 1954 (1996)**

QRPA, RQRPA: About 10 papers 1987→ 2006

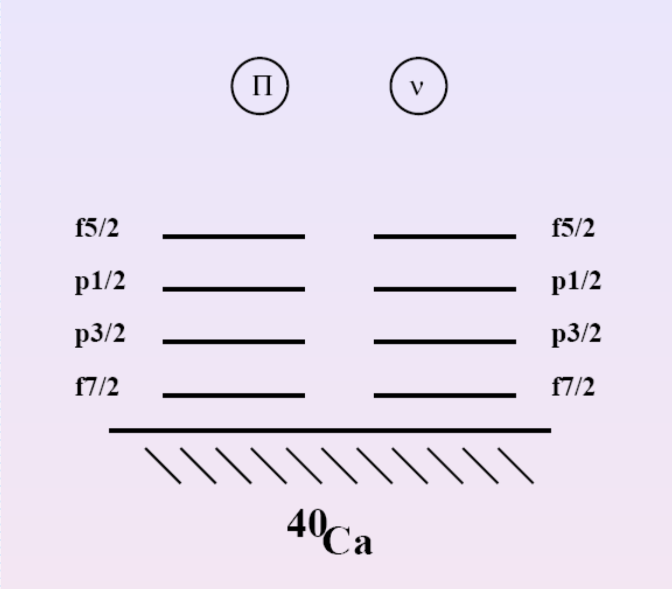
Other approaches:

Shell Model Monte Carlo (1996), Operator Expansion Method (1988-1994)...

Shell Model

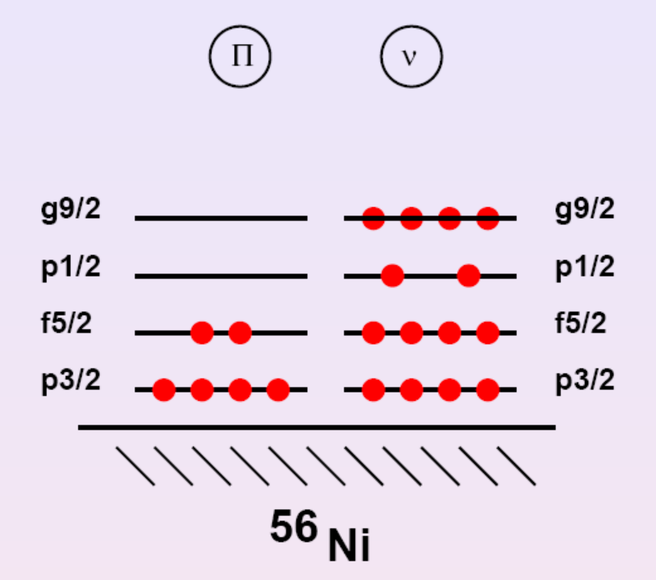
- Define a valence space
- Derive an effective interaction $H\,\Psi = E\,\Psi \rightarrow H_{eff}\,\Psi_{eff} = E\,\Psi_{eff}$
- Build and diagonalize Hamiltonian matrix (10^{10})
- Transition operator $<\Psi_{eff}\,|\,O_{eff}\,|\,\Psi_{eff}>$
- Some phenomenological input needed
  energy of states, systematics of B(E2) and GT transitions (quenching f.)

$^{48}Ca \rightarrow {}^{48}Ti$



Small calculations

$^{76}Ge \rightarrow {}^{76}Se$



$^{76}Se_{42}$ in the valence
6 protons and 14 neutrons

SM results

$2\nu\beta\beta$–decay

Isotope	$T_{1/2}$(th.)[y]	$T_{1/2}$(exp.)[y]
^{48}Ca	$3.7\ 10^{19}$	$4.2\ 10^{19}$
^{76}Ge	$1.2\ 10^{21}$	$1.4\ 10^{21}$
^{82}Se	$3.4\ 10^{19}$	$9.0\ 10^{19}$
^{130}Te	$4.\ 10^{20}$	$6.1\ 10^{20}$
^{136}Xe	$6.\ 10^{20}$	$8.1\ 10^{20}$

Strasbourg group, Nowacki,
IDEA meeting, Heidelberg, 2004

Comparison of $M^{0\nu}$ of Rodin et al. (RQRPA) and Nowacki et al. (SM, private comm., preliminary 2004) and older published (Caurier et al. 1996)

$0\nu\beta\beta$-decay

Nucleus	RQRPA	SM
^{76}Ge	2.3-2.4	1.6
^{82}Se	1.9-2.1	1.7
^{96}Zr	0.3-0.4	0.4
^{100}Mo	1.1-1.2	0.3
^{116}Cd	1.2-1.4	1.9
^{130}Te	1.3	2.0 (1.0)
^{136}Xe	0.6-1.0	1.6 (0.6)

Except for ^{100}Mo the agreement between these very different calculations is reasonably good. Note that the SM calculations include the reduction caused by the s.r.c. and induced currents.

QRPA-like approaches
QRPA, RQRPA, SRQRPA

Particle number condition

i) Uncorrelated BCS ground state

$$Z=\langle BCS|Z|BCS\rangle$$

$$N=\langle BCS|N|BCS\rangle$$

QRPA **RQRPA**

ii) Correlated RPA ground state

$$Z=\langle RPA|Z|RPA\rangle$$

$$N=\langle RPA|N|RPA\rangle$$

SRQRPA

Pauli exclusion principle

i) violated (QBA)

$$[A,A^{+}] =\langle BCS|[A,A^{+}]|BCS\rangle$$

QRPA

ii) Partially restored (RQBA)

$$[A,A^{+}] =\langle RPA|[A,A^{+}]|RPA\rangle$$

RQRPA **SRQRPA**

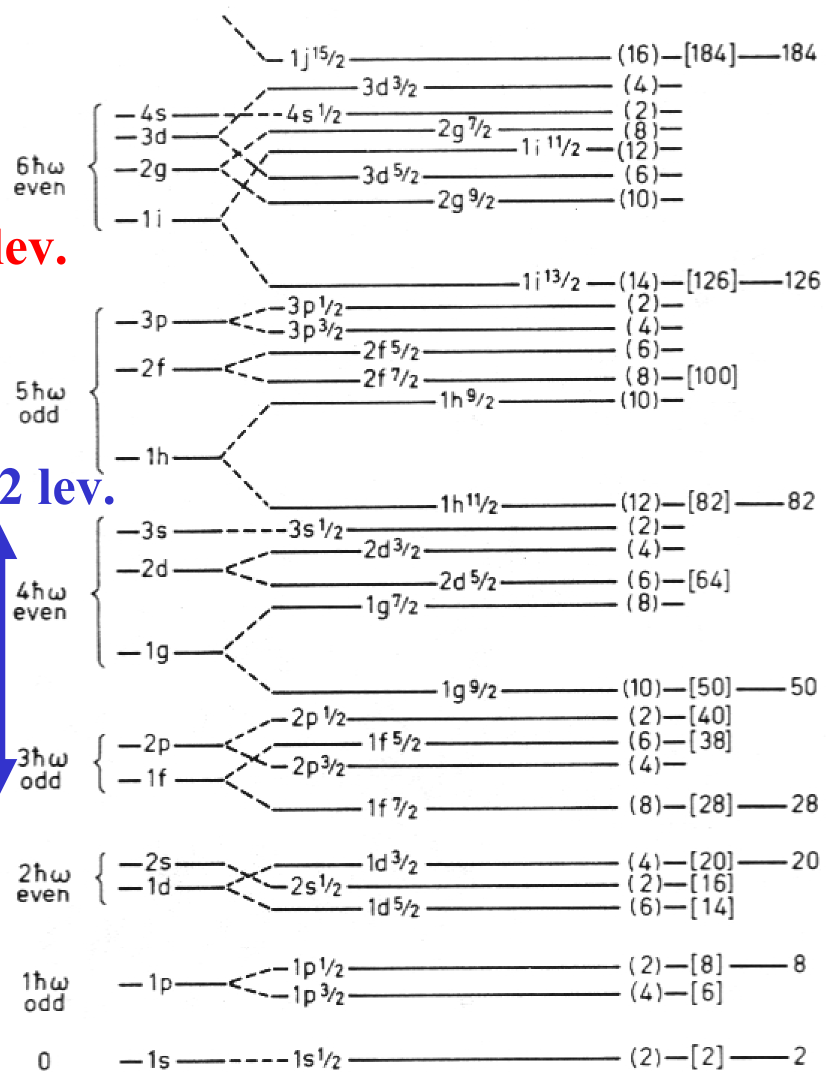
Complex numerical procedure
BCS and QRPA equations are coupled

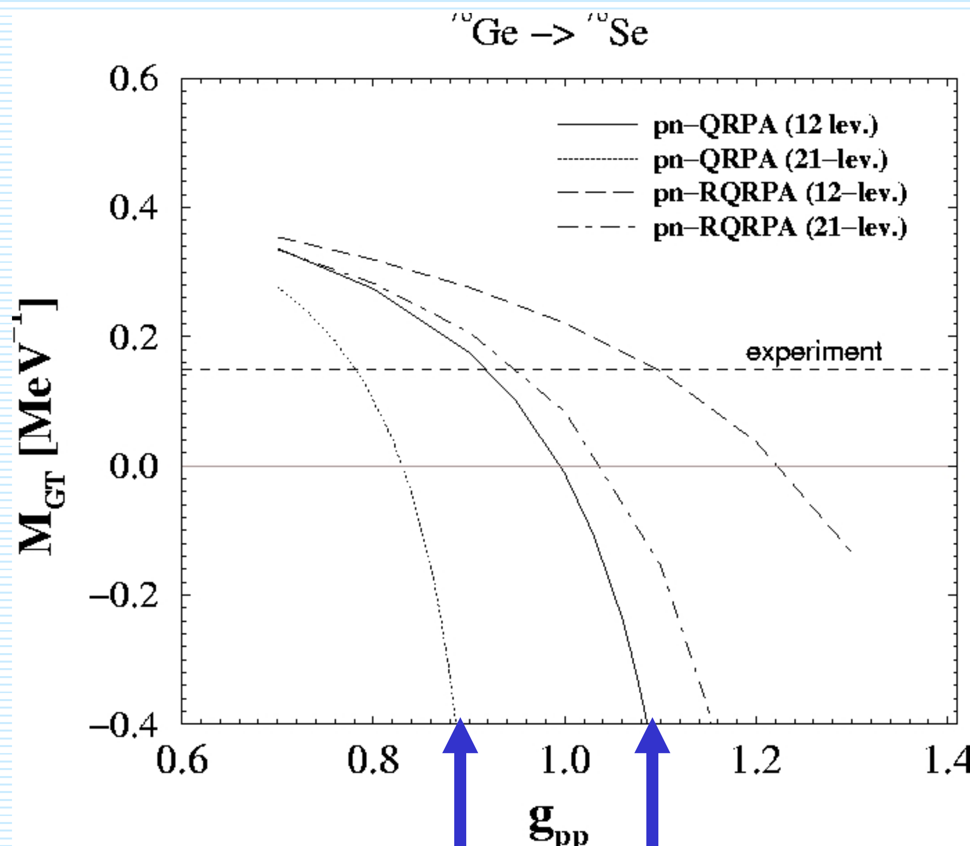
QRPA
2νββ-decay NME

$$H = H_0 + g_{ph}\, H_{ph} + g_{pp}\, H_{pp}$$

21 lev.

12 lev.

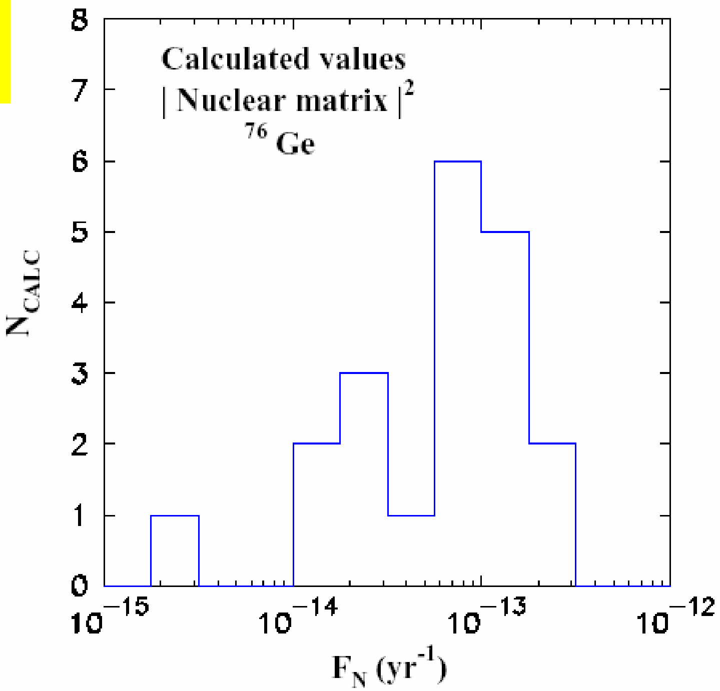




Collapse of the QRPA
21 l.m.s. 12 l.m.c

Uncertainties in 0νββ–decay NME?

Bahcall, Murayama, Pena-Garay,
Phys. Rev. D 70, 033012 (2004)



Please no!
Do not put different NME calculations at the same level (democratic approach)

This suggest an uncertainty of NME as much as factor 5

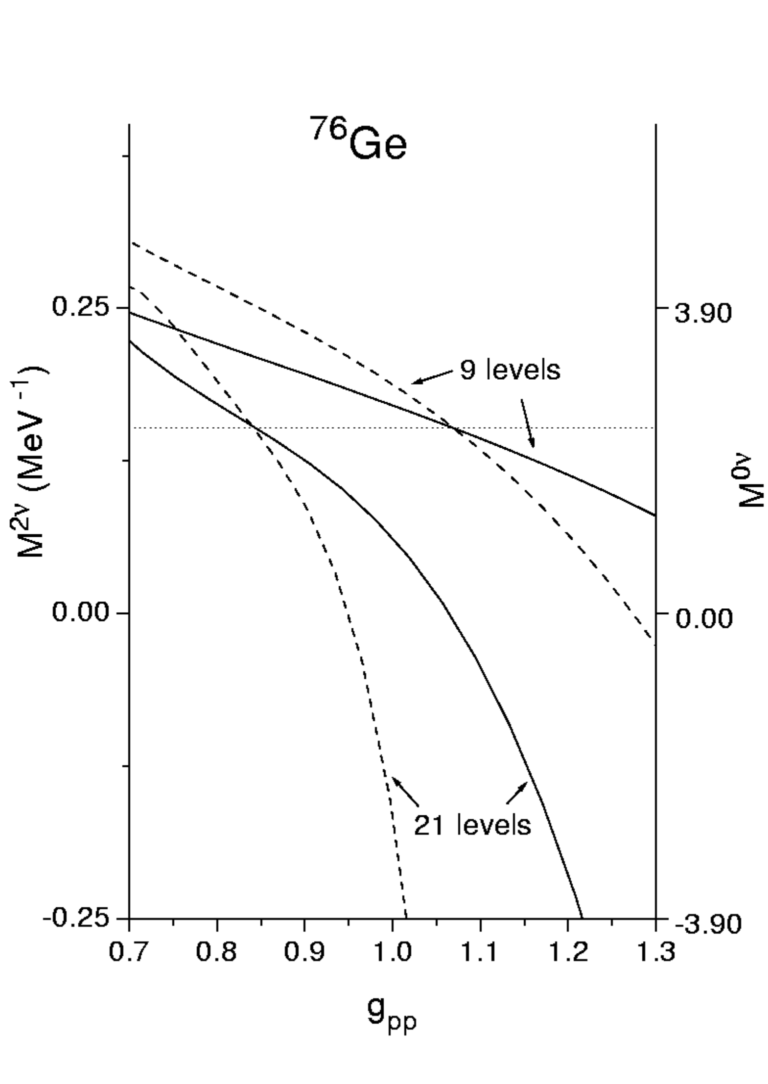
Civitarese, Suhonen,
Nucl. Phys. A 729, 867 (2003)

System	$G_1^{(0\nu)} \times 10^{14}$	N.M.E.	N.M.E. (this work)	$\langle m_\nu \rangle$max
^{48}Ca	6.43	1.08–2.38		8.70–19.0
^{76}Ge	0.63	2.98–4.33	3.33	0.30–0.43
^{82}Se	2.73	2.53–3.98	3.44	4.73–7.44
^{96}Zr	5.70	2.74	3.55	19.1–24.7
^{100}Mo	4.57	0.77–4.67	2.97	2.18–13.2
^{116}Cd	4.68	1.09–3.46	3.75	2.37–8.18
^{128}Te	0.16	2.51–4.58		9.51–17.4
^{130}Te	4.14	2.10–3.59	3.49	1.87–3.20
^{136}Xe	4.37	1.61–1.90	4.64	0.79–2.29

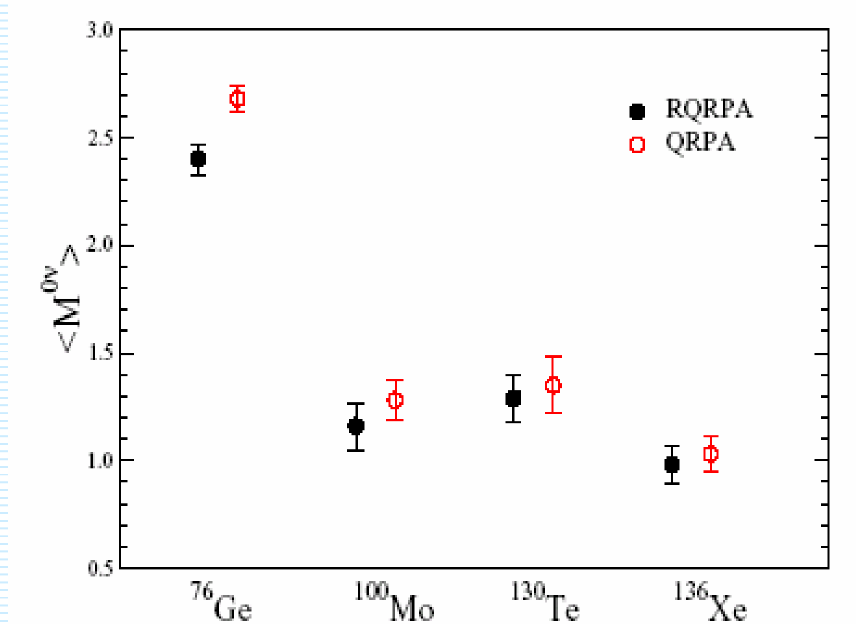
Is it really so bad?!

The 0νββ-decay NME: g_{pp} fixed to 2νββ-decay

Each point: (3 basis sets) x (3 forces) = 9 values



By adjusting of g_{pp} to 2νββ-decay half-life the dependence of the 0νββ-decay NME on other things that are not a priori fixed is essentially removed

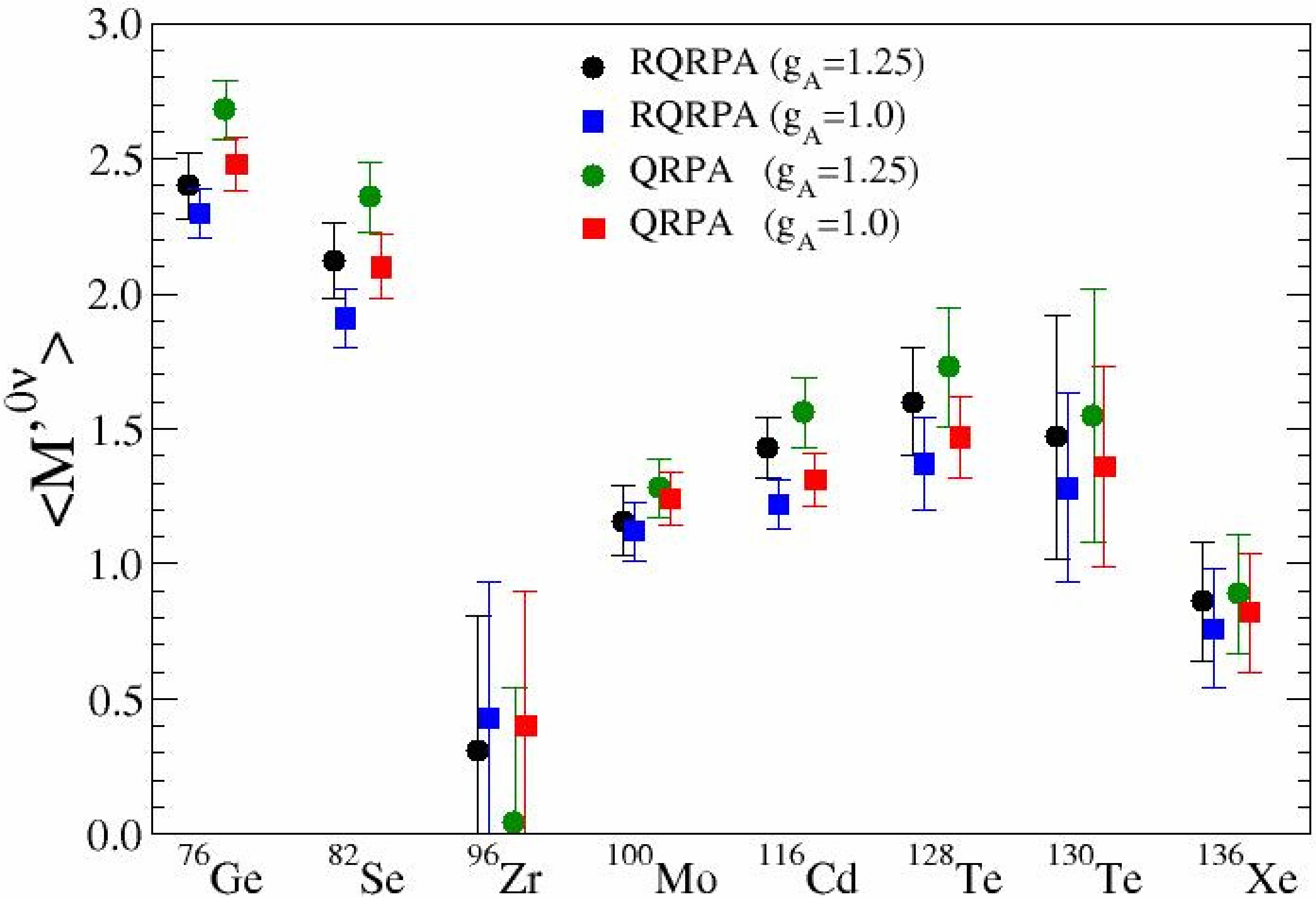


Rodin, Faessler, F.Š.,Vogel,
F **Phys. Rev. C 68, 044302 (2003)**

Neutrinoless Double Beta Decay Nuclear Matrix Elements

Recommended 0νββ−decay NME (April 2005)

Light Majorana Neutrino Exchange Mechanism



V.Rodin, A. Faessler, F. Š., P. Vogel, NPA 766 (2006) 107

List of reasons, why QRPA-like $0\nu\beta\beta$–decay NME are different (13 reasons)

Quasiparticle mean field
fixing of pp,nn (pn) pairing

Many-body approximations
QRPA, RQRPA, SRQRPA

Choice of NN interaction
Schem., realistic (Bonn, Paris …

the closure approximation

p-h interaction ($g_{ph} \cong 1$)
fixed to GT resonance

The size of model space

p-p interaction (g_{pp})
fixed to β or $\beta\beta$–decay resonance,
or $g_{pp}=1$

two-nucleon s.r.c. (~ 50%)
has to be considered

finite size of nucleon (~10%)
form factors

h.o.t. of nucleon curr. (~30%)
Induced PS, weak magnetism

the overlap factor
the BCS overlap

the axial-vector coupling
g_A=1.0 or 1.25

Nuclear shape
Spherical, not deformed yet

A comparison with Muto (^{76}Ge) results
(the same procedure of fixing g_{pp})

Muto, Phys. Lett. B 391, 243 (1997)

^{76}Ge, small model space (3-4 ħω), dependence on size of model space not studied, g_{pp} fixed to 2νββ-decay half-life, h.o.t of nucleon current not considered

$M^{0\nu}$ = 4.59 (QRPA) 3.88 (RQRPA) Muto
2.68 2.41 Rodin et al

- Without higher order terms of nucleon current => reduction of 30%
- r_0=1.2 fm (Muto), 1.1 fm (Rodin et al.) => reduction of 10%

$M^{0\nu}$ = 0.6 x 4.59 = 2.76 ⇔ 2.68 (Rodin at al.) (QRPA)
0.6 x 3.88 = 2.34 ⇔ 2.41 (Rodin et al.) (RQRPA)

There is a good agreement!

A comparison with Stoica, Klapdor-Kleingrothaus (^{76}Ge) results

(the same procedure of fixing g_{pp})

Stoica, Klapdor-Kleingrothaus, Phys.Rev. C 63, 064304 (2001), Nucl. Phys. A 694, 269 (2001)

^{76}Ge, s.m.s (3-4 ħω) and l.m.s (2-4 ħw), g_{pp} fixed to 2νββ-decay half-life, h.o.t of nucleon current not considered

Stoica Klapdor-Kleingrothaus (2001)

$M^{0\nu}$ (x 0.6) =	2.67 (QRPA)	s.m.s
	1.03	l.m.s.
	2.24 (RQRPA)	s.m.s
	1.14	l.m.s.

s.m.s. = 3-4 hω l.m.s. = 2-4 hω
(+ $0d_{3/2}$, $0d_{5/2}$, $1s_{1/2}$)

This should have impact on β-strength distribution?!

Rodin et al. (2003)

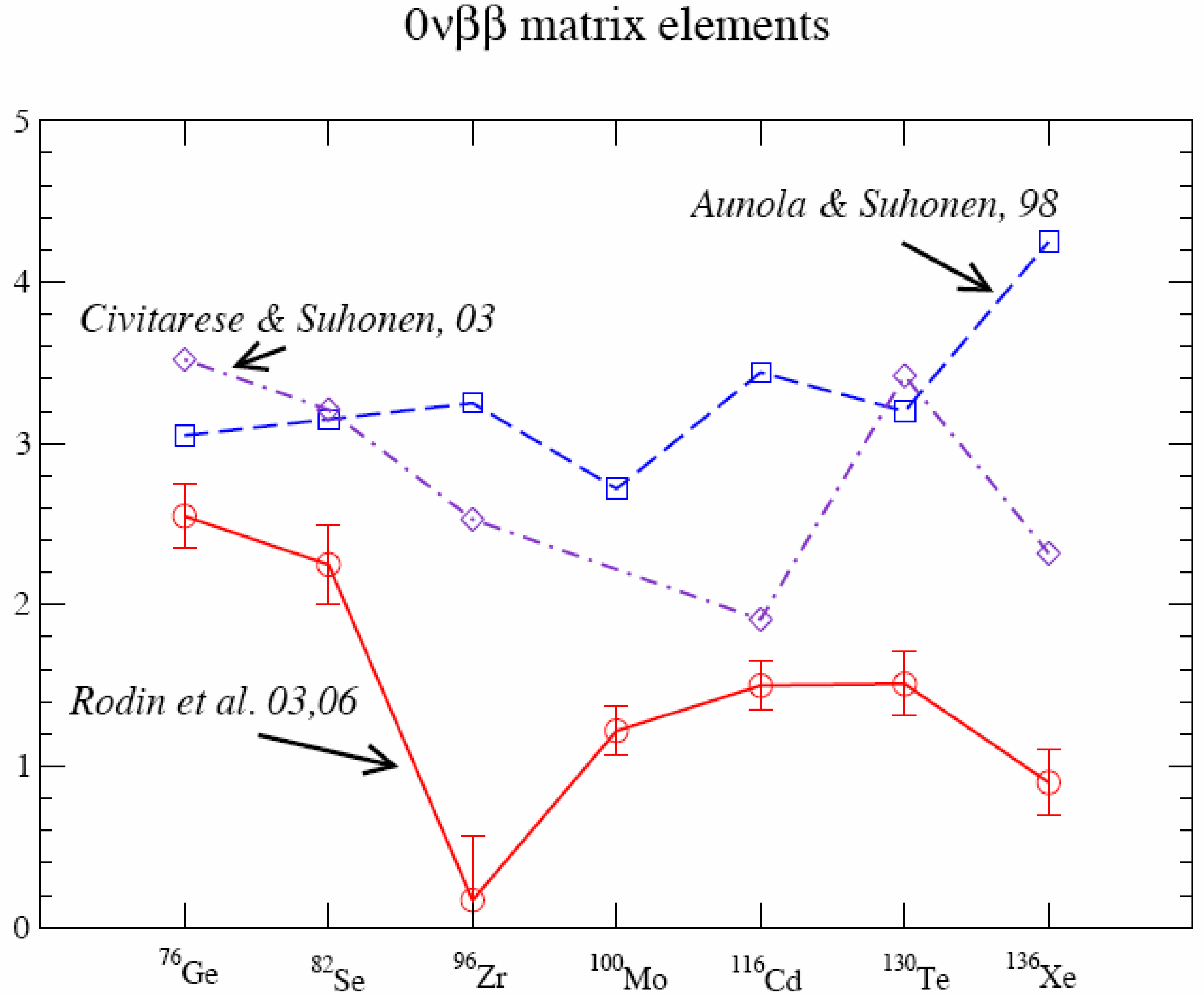
$M^{0\nu}$ =	2.68 (QRPA)	s.m.s
	2.62	l.m.s.
	2.41 (RQRPA)	s.m.s
	2.32	l.m.s.

s.m.s. = 3-4 hω l.m.s. = 0-5 hω

Negligible dependence on the size of model space

There is a strong disagreement

A comparison with Civitarese & Suhonen results

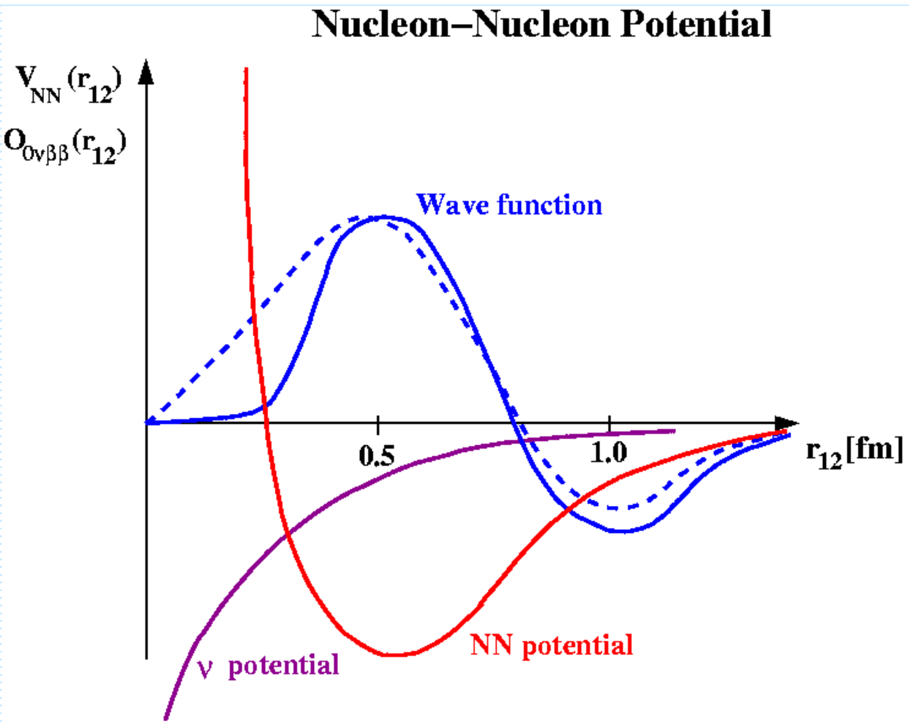


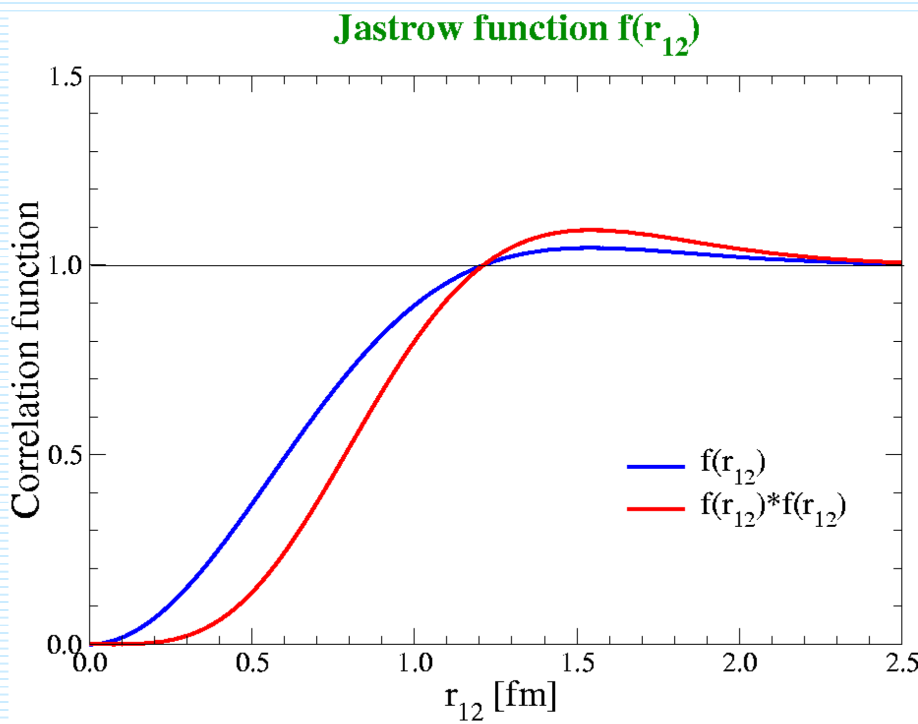
Aunola, Suhonen, NPA 643, 207 (1998); Civitarese, Suhonen, NPA 729, 867 (2003)

No short range correlations, no higher order terms of nucleon currents, G_{pp} fixed to single β-decay observables

Two-nucleon short range correlations: a question of physics

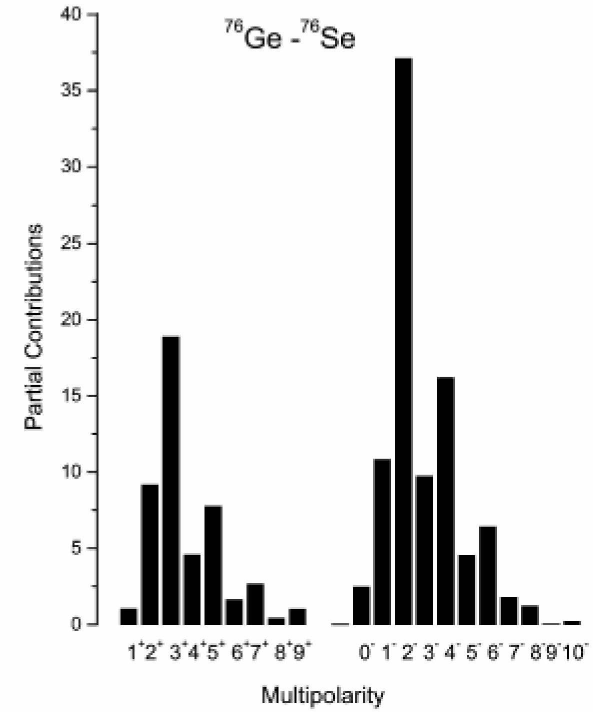
There is no double counting in QRPA
- QRPA violates Pauli exclusion principle
- 1/(0.2 fm) ~ 1 GeV



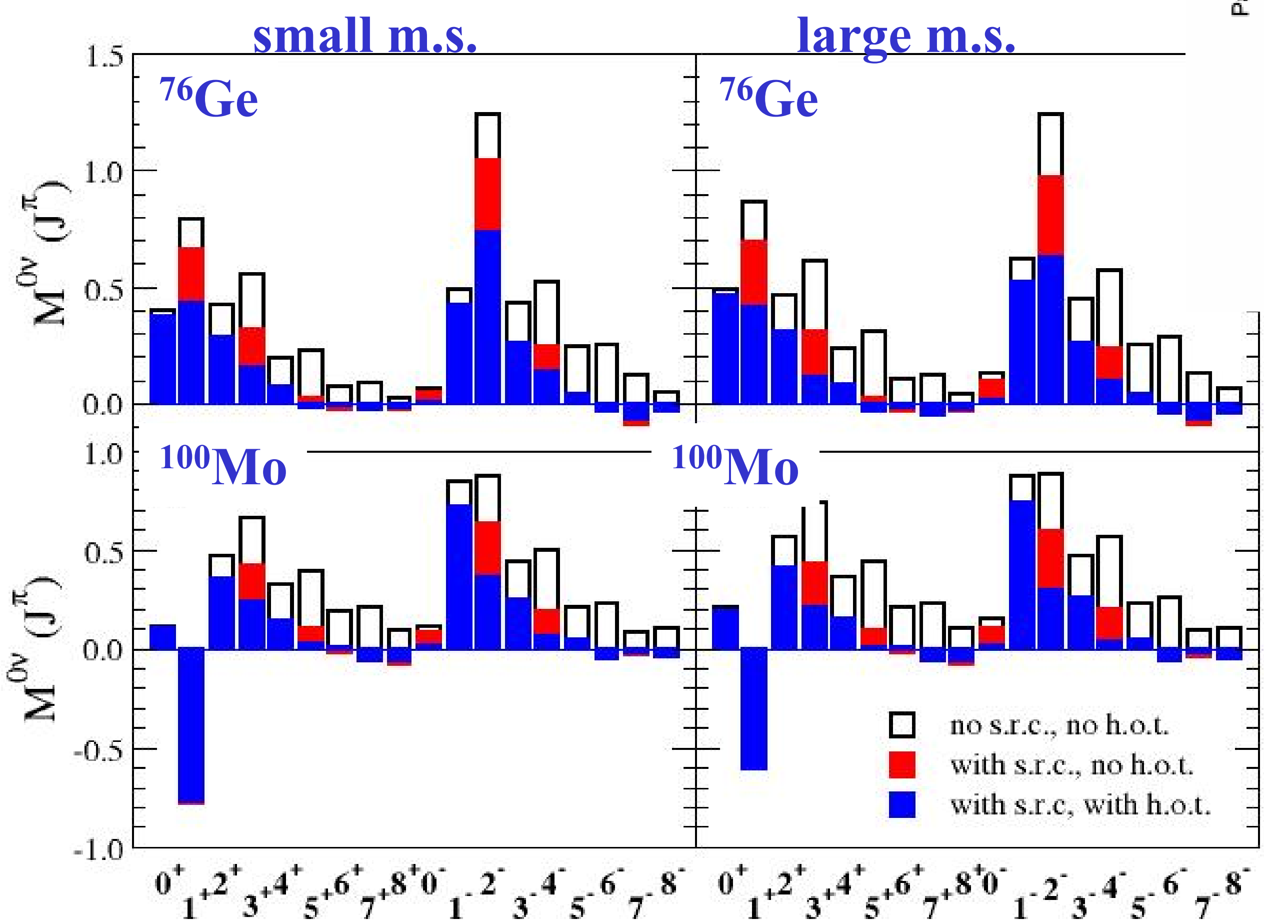


Suppression of higher multipolarities due to

- **two-nucleon short-range correlations**
- **induced pseudoscalar coupling**



V.Rodin, A. Faessler, F. Š., P. Vogel, NPA 766 (2006) 107

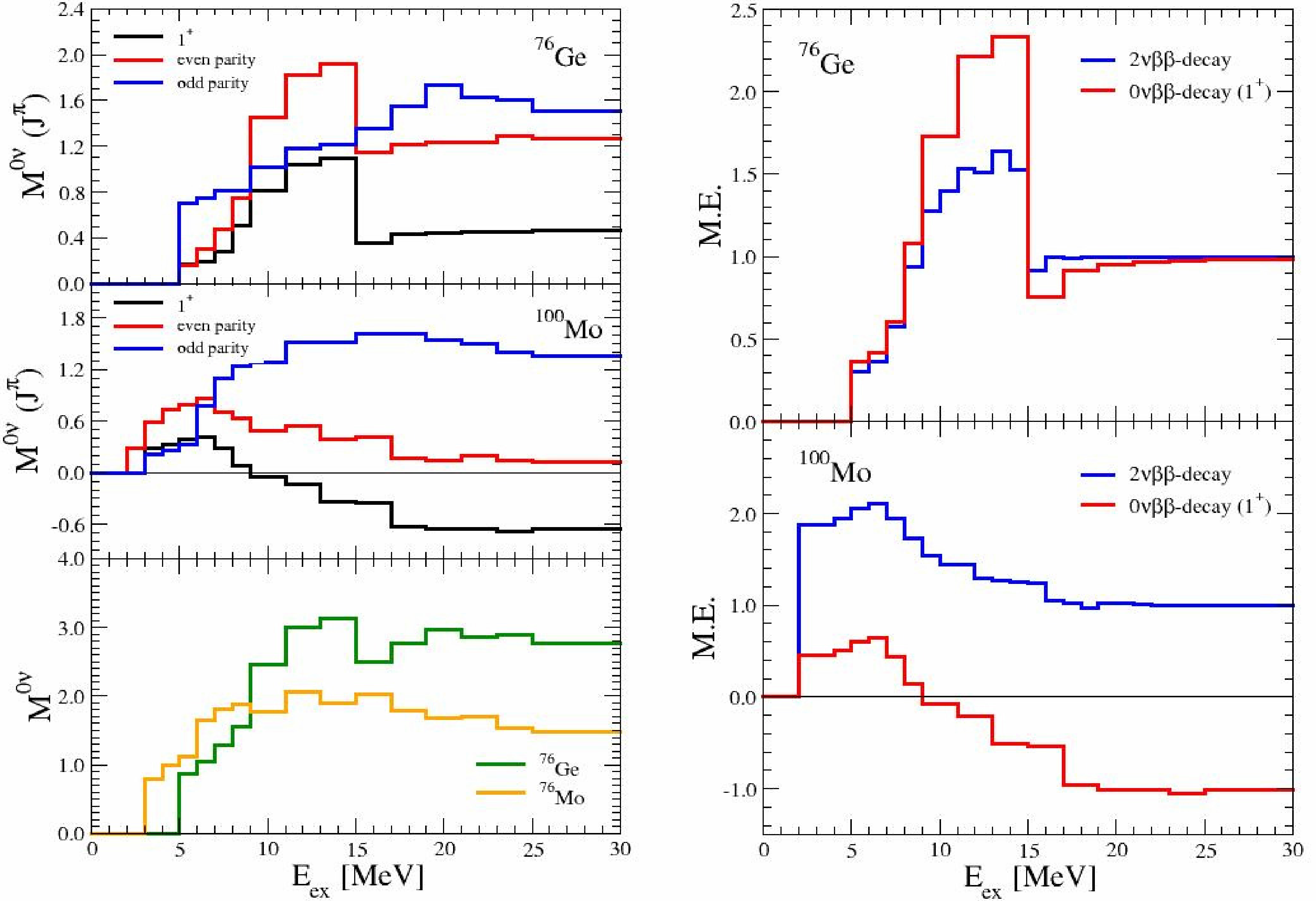


Civitarese, Suhonen, PLB 626, 80 (2005)

- **no two-nucleon s.r.c**
- **no induced pseudoscalar coupling**
- **Overlap factor?**
- **nucleon formfactors?**

Many multipoles contribute. Most of them with exceptions have the same sign.

The importance of transition through higher-lying states of (A,Z+1) nucleus



Shortcoming of fixing g_{pp} to the g.s. single β-decay observables

Negative $M^{2\nu}$ is disfavored

$$(T^{2\nu}_{1/2})^{-1} = G^{2\nu}|M^{2\nu}_{GT}|^2 \Rightarrow M^{2\nu} > 0 \quad \text{or} \quad M^{2\nu} < 0$$

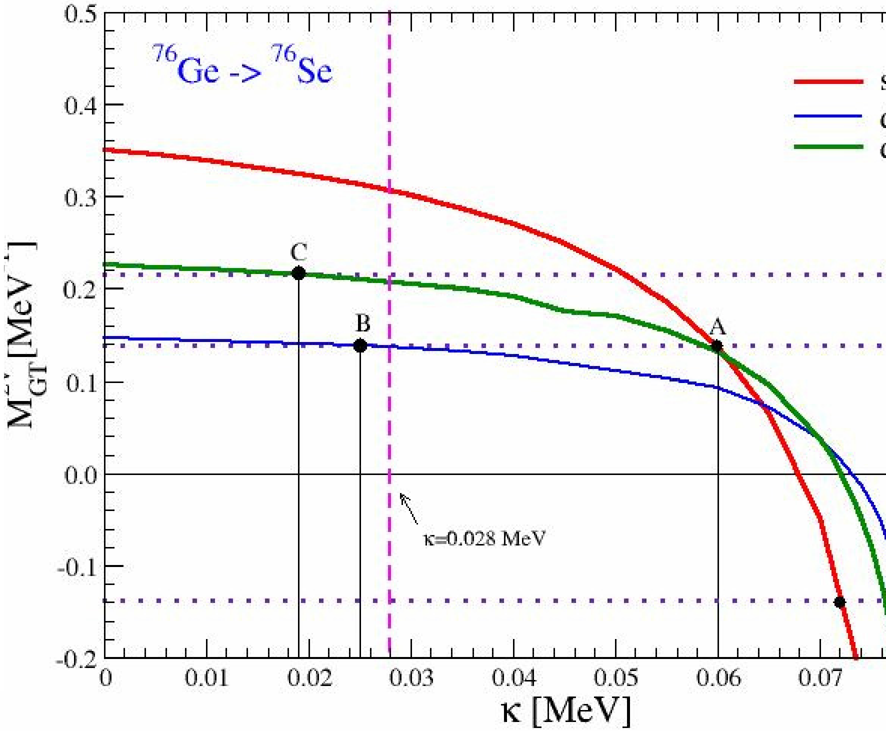
we used to fixed g_{pp}

Civitarese, Suhonen, NPA 761, 313 (2005) used both values to fix g_{pp}

However, for single β-decay this problem was ignored

Negative $M^{2\nu}$ is disfavored:

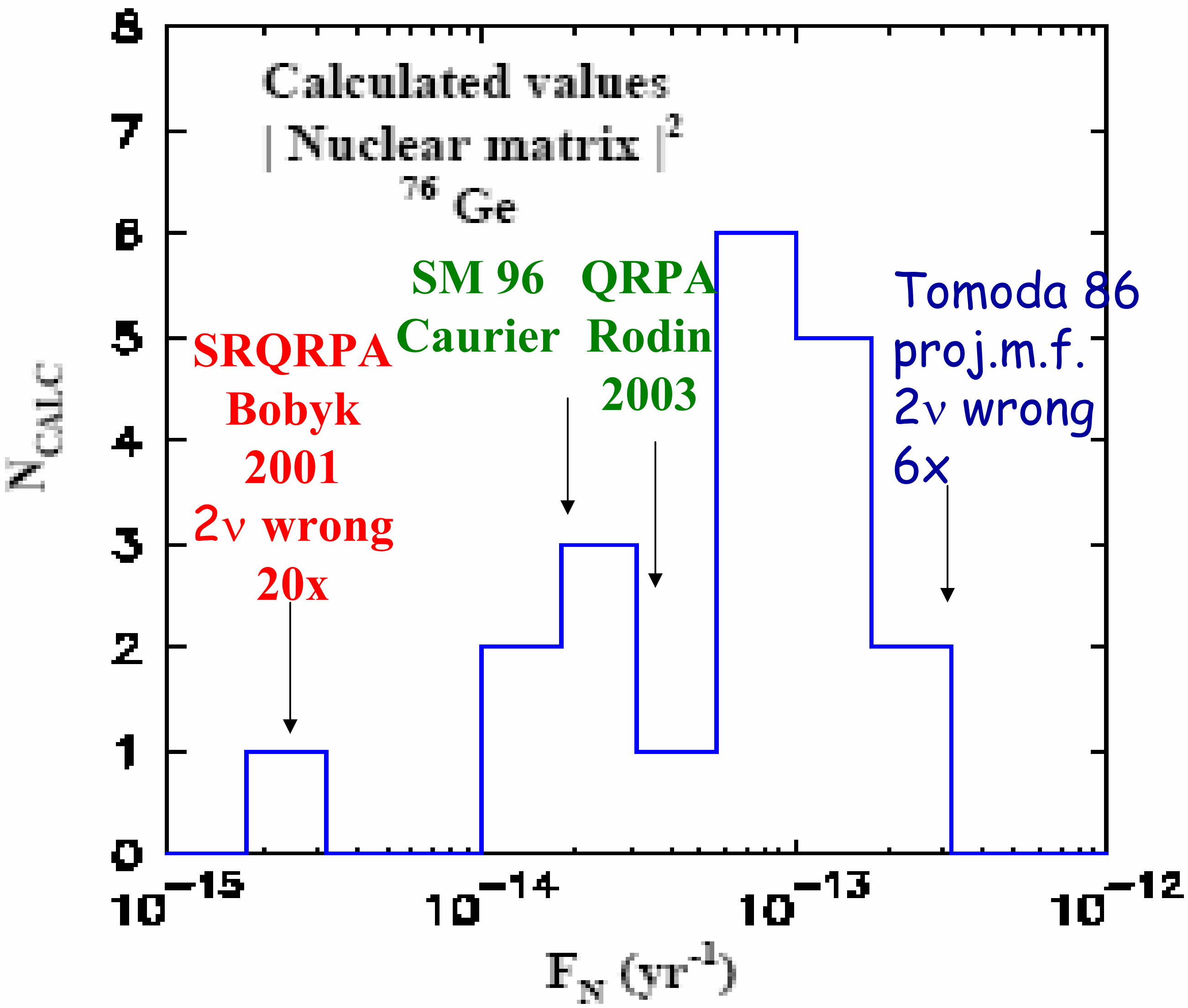
- The QRPA is designed to describe small amplitude exc. around the mean field minim.
- disagreement with systematic study of single β-decay
  Homma et al., PRC 54, 2972 (1996)
- The lowest β^+/EC transition (A,Z+1)→(A,Z) too strong
- If Pauli exclusion principle fully taken into account negative $M^{2\nu}$ appears for too large value of g_{pp}
  Šimkovic et al., PRC 61, 044319 (2000)



$M^{2\nu\text{-exp}} > 0$

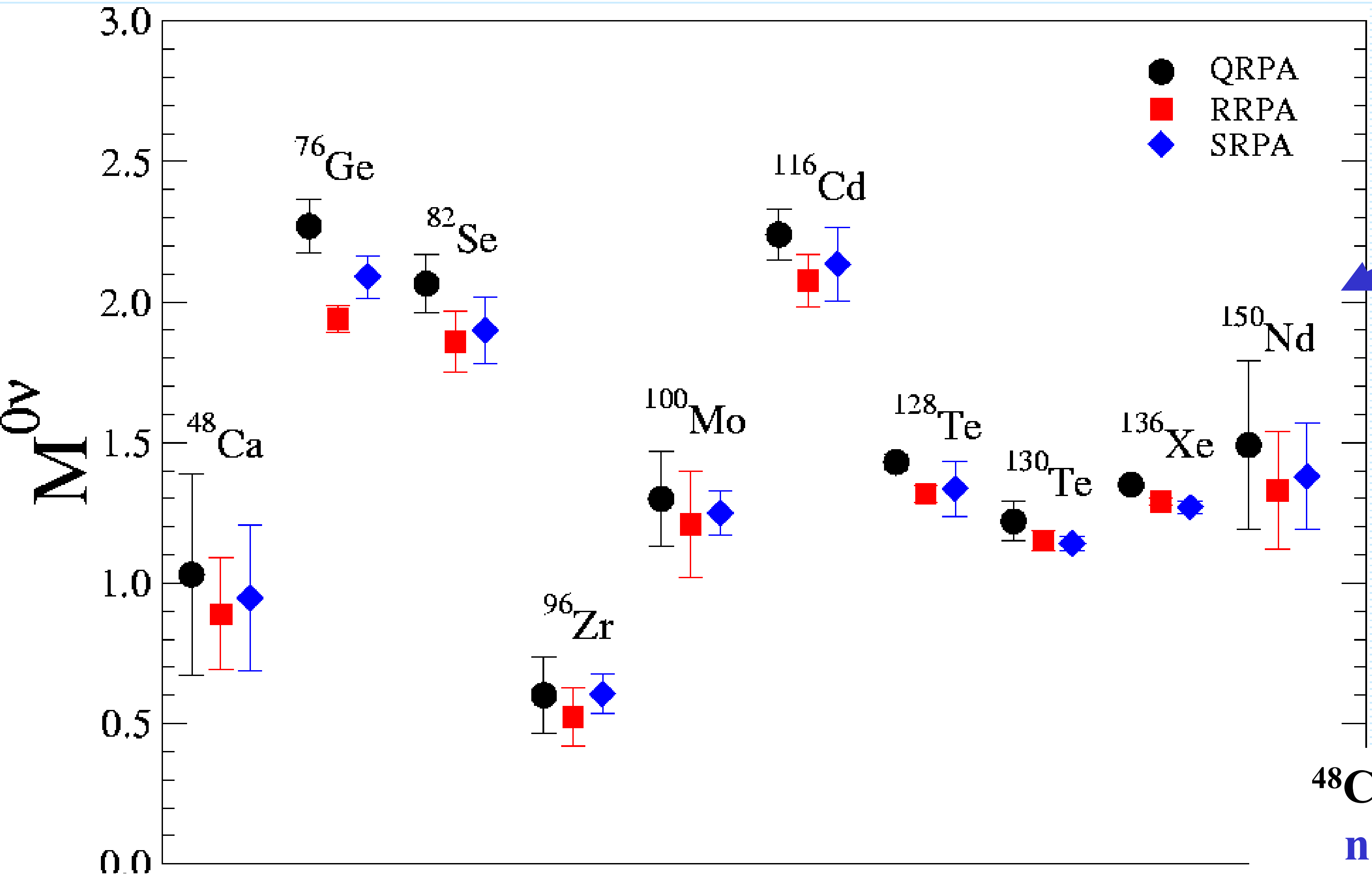
$M^{2\nu\text{-exp}} < 0$

The outliers predict wrong $2\nu\beta\beta$ halflife. The matrix elements of SM and Rodin et al. are quite close.



SRQRPA results

g_{pp} fixed to $2\nu\beta\beta$-half life (Beneš, F.Š., Faessler, to be submitted)



strongly deformed

^{48}Ca, ^{116}Sn, ^{130}Xe need careful treatment of Pairing ⇒ Lipkin-Nogami BCS

- Pairing fixed to exp. pairing gaps (constant pairing considered)
- BCS overlap factor taken into account (factor ~ 0.8)
- uncertainties of $2\nu\beta\beta$-decay half-lives not considered

Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

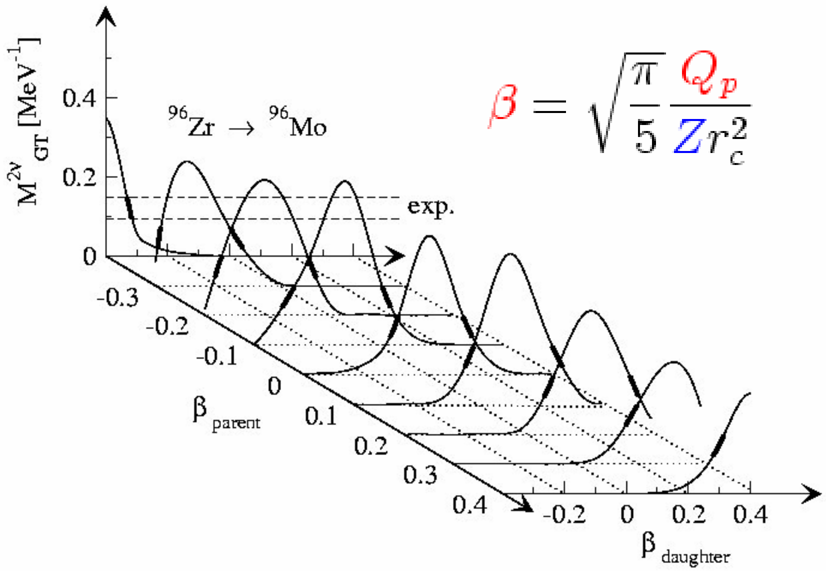
Exp. I **(nuclear reorientation method)**
Exp.II **(based on measured E2 trans.)**
Theor. I **(Rel. mean field theory)**
Theor. II **(Microsc.-Macrosc. Model of Moeller and Nix)**

Till now, in the QRPA-like calculations of the 0νββ-decay NME spherical symetry was assumed

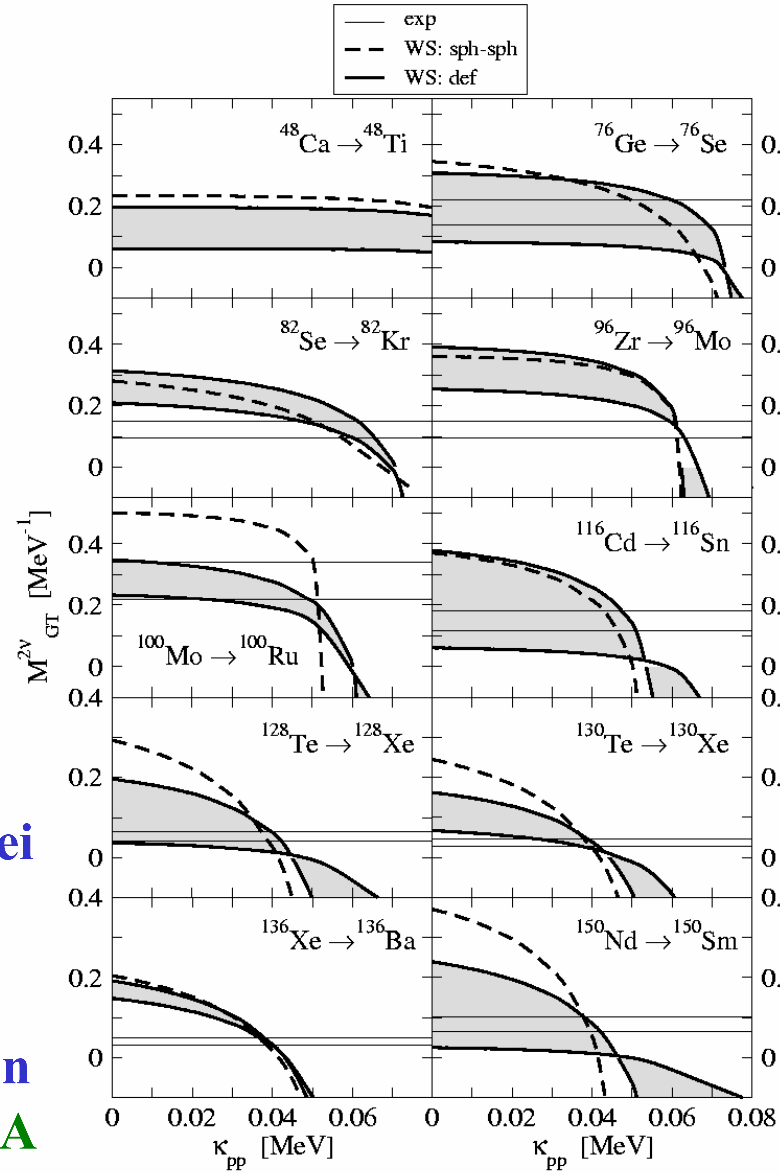
The effect of deformation on NME has to be considered

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
^{48}Ca	0.00	0.101	0.00	0.00
^{48}Ti	+0.17	0.269	-0.01	0.00
^{76}Ge	+0.09	0.26	0.16	0.14
^{76}Se	+0.16	0.31	-0.24	-0.24
^{82}Se	+0.10	0.19	0.13	0.15
^{82}Kr		0.20	0.12	0.07
^{96}Zr		0.081	0.22	0.22
^{96}Mo	+0.07	0.17	0.17	0.08
^{100}Mo	+0.14	0.23	0.25	0.24
^{100}Ru	+0.14	0.22	0.19	0.16
^{116}Cd	+0.11	0.19	-0.26	-0.24
^{116}Sn	+0.04	0.11	0.00	0.00
^{128}Te	+0.01	0.14	-0.00	0.00
^{128}Xe		0.18	0.16	0.14
^{130}Te	+0.03	0.12	0.03	0.00
^{130}Xe		0.17	0.13	-0.11
^{136}Xe		0.09	0.00	0.00
^{136}Ba		0.12	0.00	0.00
^{150}Nd	+0.37	0.28	0.22	0.24
^{150}Sm	+0.23	0.19	0.18	0.21

New Suppression Mechanism of the DBD NME



$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Zr_c^2}$$



The suppression of the NME depends on relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez,Sarriguren, Moya,Pacearescu, Faessler, F.Š.,
Phys. Rev. C 70 (2004) 321

Which 0νββ-decay NME to consider

If the authors do not specify what choices they made, and do not discuss the dependence of their result on the particular choice they made, their result should not be taken on the same footing as those where points are carefully explained!

Recommendation 2005

The most carefully calculated QRPA/RQRPA 0νββ–decay NME are:

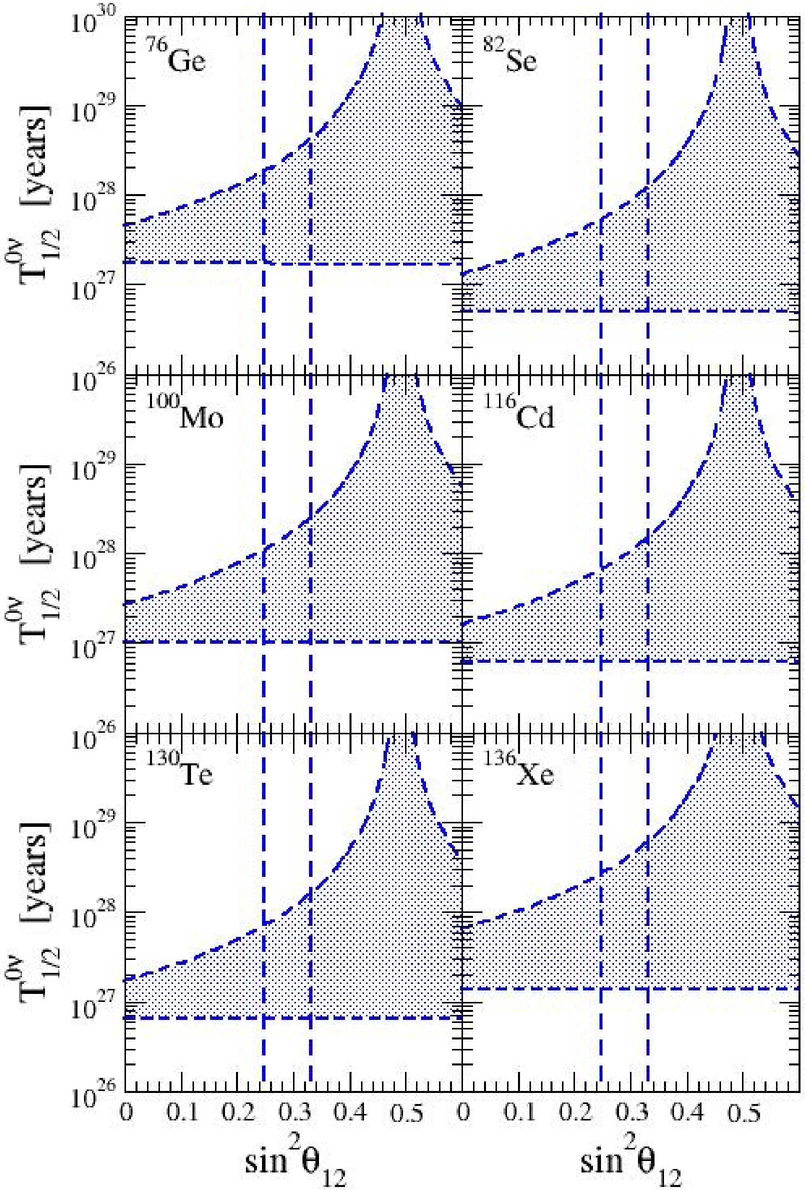
V.Rodin, A. Faessler, F. Š., P. Vogel, NPA 766 (2006) 107

However, further progress is needed

Even, it might be that the true NME is not within the determined "variance"

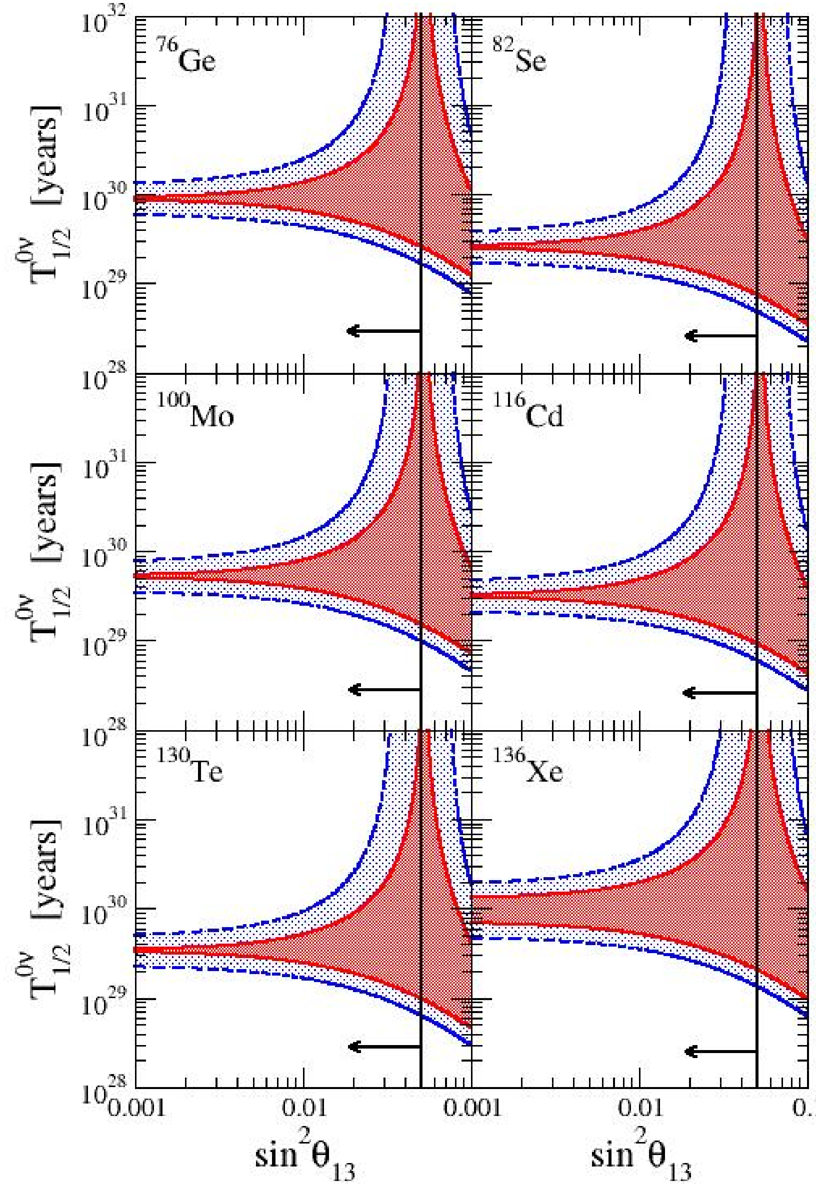
Problems of deformation, overlap matrix, …have to be studied

$$\sqrt{|\Delta m^2_{13}|}\cos 2\theta_{12} \le |m_{\beta\beta}| \le \sqrt{|\Delta m^2_{13}|}$$



Inverted hierarchy

$$|m_{\beta\beta}| \le \sin^2\theta_{12}\sqrt{\Delta m^2_{12}} + \sin^2_{13}\sqrt{\Delta m^2_{23}}$$



Normal hierarchy

Bilenky, Faessler, Gutsche, F.Š., PRD 72, 053015 (2005)

10^{27}-10^{28}years **Goals for 0νββ-experiments** **10^{29}-10^{30}years**

A product of 0νββ–decay NME

Can you imagine 0νββ–decay of 4 nuclei (e.g. ^{76}Ge, ^{82}Se, ^{130}Te, ^{136}Xe) is observed …

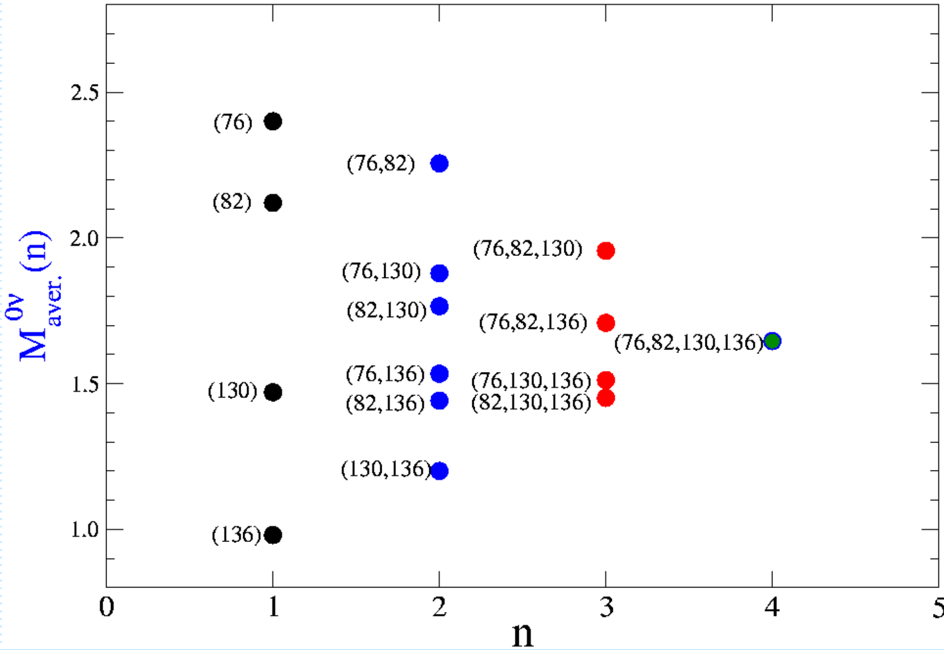
Physical quantity of interest $M^{0\nu}_{aver} = \left(\Pi_{k=1}^{kmax} |M^{0\nu}(k)|\right)^{-1/kmax}$

$$|m_{\beta\beta}|_{aver.} = \left(\Pi_{k=1}^{kmax} |M^{0\nu}(k)|^2 \; \Pi_{k=1}^{kmax} T^{0\nu}_{1/2}(k) \; \Pi_{k=1}^{kmax} G^{0\nu}(k)\right)^{-1/(2\; kmax)}$$

uncertainty

$$|m_{\beta\beta}(k)| = \frac{1}{|M^{0\nu}(k)|\sqrt{T^{0\nu}_{1/2}(k)\; G^{0\nu}(k)}}$$

$$\delta(k) = |m_{\beta\beta}|_{aver.} - |m_{\beta\beta}(k)|$$



Distinguishing different mechanisms

- **light neutrino exchange**
- **heavy neutrino exchange**
- **pion-exchange mech.**

Observation of the 0νββ–decay of at least 3-4 different nuclei is needed

Conclusions

- The sources of uncertainties reexamined. The differences between QRPA-like results understood, the recommended values of NME presented
- SRQRPA results are close to QRPA and RQRPA results, if g_{pp} is fixed to $2\nu\beta\beta$ half-life and pairing to mass differences

⇒ There is a convergence of QRPA-like results

- The story about NME not finished yet. Study of further effects (deformation, overlap factor) and cross-check with other approaches required.

$0\nu\beta\beta$-decay NME → Absolute ν mass scale
Neutrino mass pattern
CP-violating Majorana phases
Distinguishing $0\nu\beta\beta$-mechanisms

Outlook

There will be a progress

Moore's law of 0nbb-decay

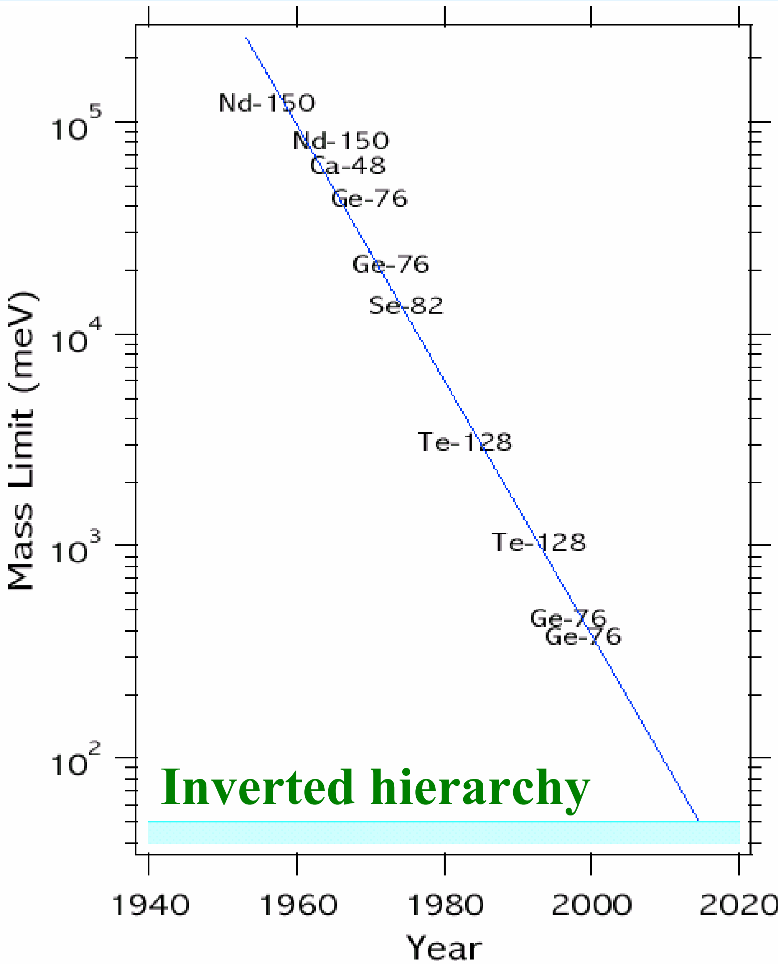
Elliott, Vogel, Ann.Rev.Nucl.Part.Sci. 52 (2002)115

Shell model: increased computer power, larger m.s., better effective interactios, new groups (T. Otsuka – Tokyo U.)

MCSM: unsolved problems? H. Nakada (NNR05), Alhassid, Bertsch, Liu, Fang

QRPA: further progress expected → effects of deformation, overlap factor

Cross-check with charge-changing reactions, muon capture, neutrino-scattering data needed



There is still some time