Theory and phemomenology of neutrino oscillations

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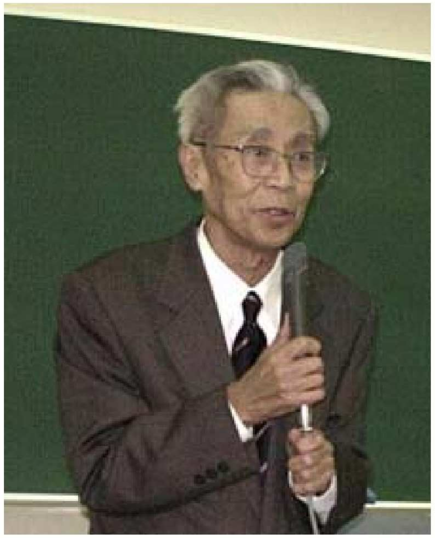
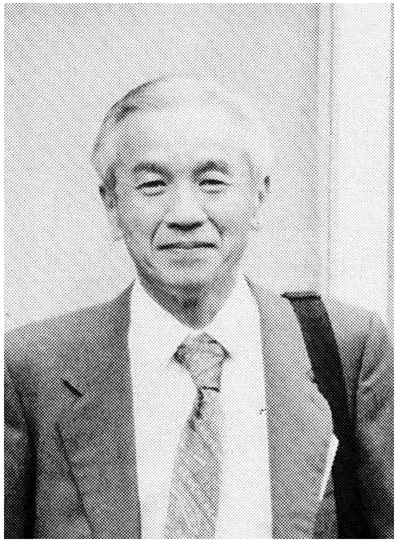
B. Pontecorvo
1913 - 1993

S. Sakata
1911 – 1970



Z. Maki
1929 – 2005



M. Nakagawa
1932 – 2001

I. Theory

Leptonic mixing

For $m_\nu \neq 0$ weak eigenstate neutrinos ν_e, ν_μ, ν_τ do not coincide with mass eigenstate neutrinos ν_1, ν_2, ν_3

Diagonalization of leptonic mass matrices:

$$e_L \to V_L\, e_L\,, \qquad \nu_L \to U_L\, \nu_L\, \ldots \qquad \Rightarrow$$

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma_\mu\, V_l^\dagger U_L\, \nu_L)\, W^\mu + \text{diag. mass terms}$$

Leptonic mixing matrix: $U = V_l^\dagger U_L$

$$\diamondsuit \qquad |\nu_a^{\text{fl}}\rangle = \sum_i U_{ai}^*\, |\nu_i^{\text{mass}}\rangle$$

Oscillation probability in vacuum

For relativistic neutrinos: $E \simeq p + \frac{m^2}{2p}, \quad L \simeq t,$

♢ $$P(\nu_a \to \nu_b; L) = \left| \sum_i U_{bi}\, e^{-i\frac{m_i^2}{2p}L}\, U_{ai}^* \right|^2$$

– standard oscillation formula. For 2-flavor oscillations (good first approximation in many cases):

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

♢ $$P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}\,L\right)$$

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- Dirac + Majorana mass terms

  $\bar{\nu}_L m_D N_R \; + \; \bar{\nu}_L m_L (\nu_L)^c \; + \; \bar{N}_R M (N_R)^c + h.c.$:
  - ◇ active - active oscillations $\nu_{aL} \leftrightarrow \nu_{bL}$
  - ◇ active - sterile oscillations $\nu_{aL} \leftrightarrow (N_{bR})^c \equiv (N_b^c)_L$
  - ◇ Neutrinos are Majorana particles

Modes of ν oscillations – contd.

Would observation of active - sterile ν oscillations mean that neutrinos are Majorana particles?

– Not necessarily!

In principle one can have active - sterile oscillations with only Dirac - type mass terms at the expense of introducing additional species of sterile neutrinos with opposite L

Neutrino oscillations in matter

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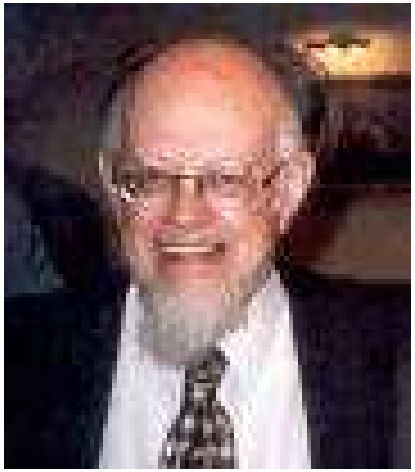
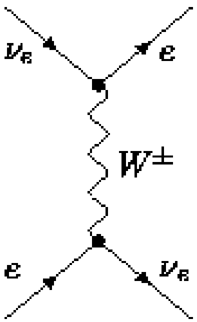
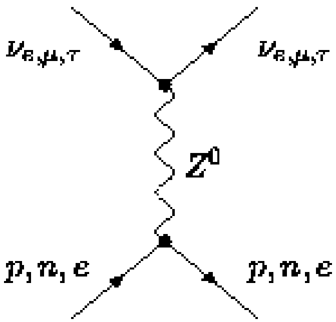


Neutrino oscillations in matter

Coherent forward scattering on the particles in matter





$$V_e^{\mathrm{CC}} \equiv V = \sqrt{2}\, G_F\, N_e$$

2f neutrino evolution equation:

◊ $$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Mixing in matter

- $$\sin^2 2\theta_m = \frac{\sin^2 2\theta \cdot (\frac{\Delta m^2}{2E})^2}{[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e]^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}$$

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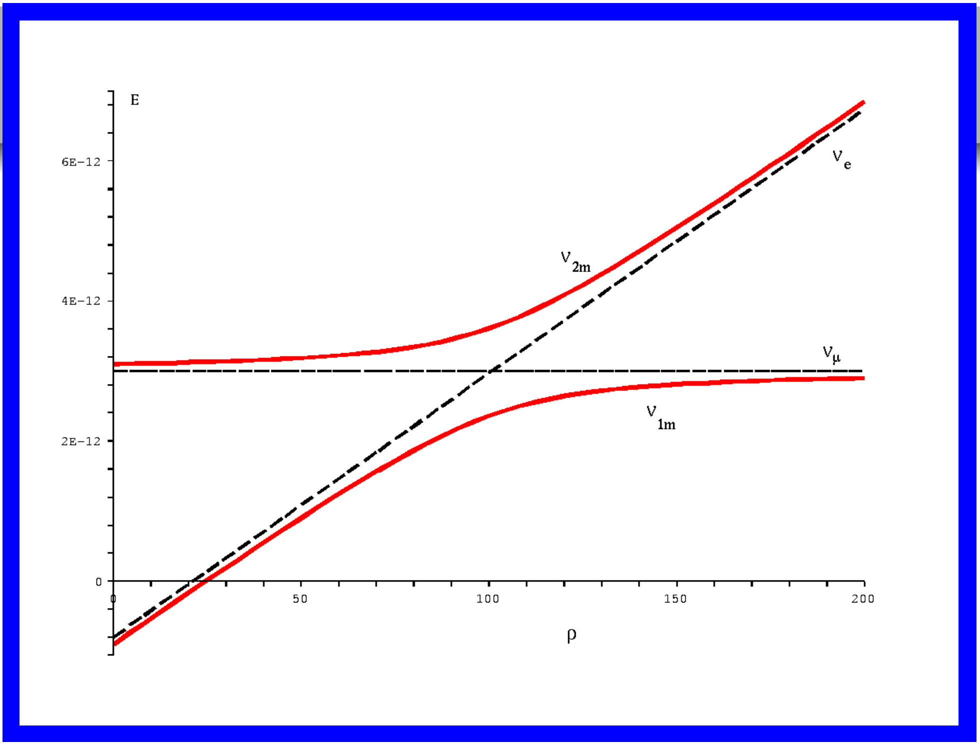
$$N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90°$$

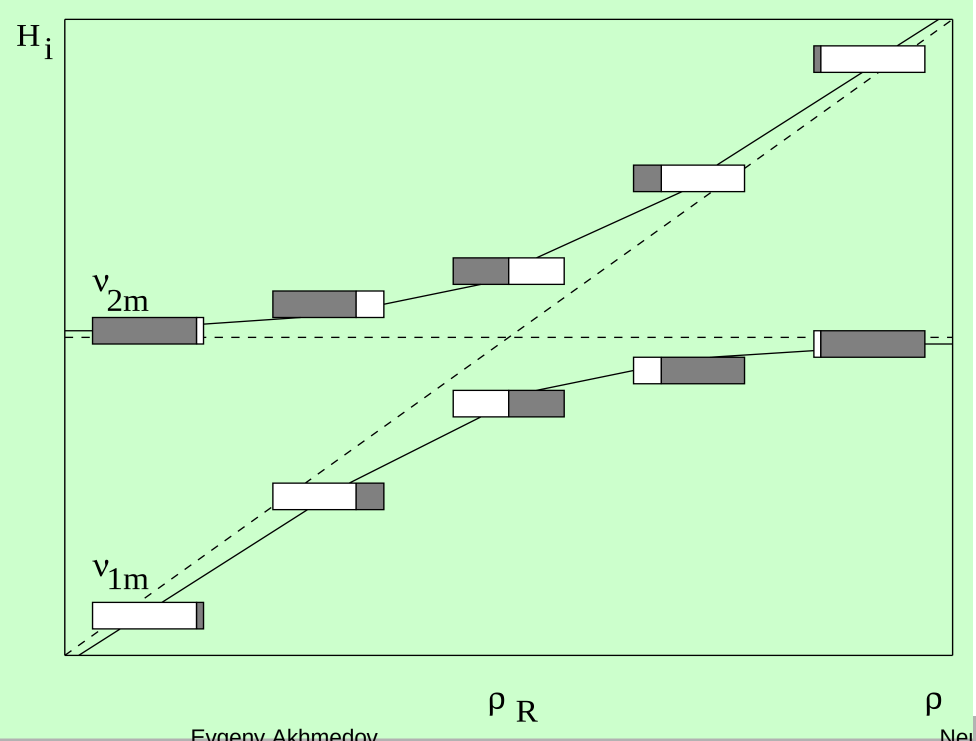
$$N_e = (N_e)_{\rm res}: \quad \theta_m = 45°$$

$$N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$$

$|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – eigenstates of H in matter (matter eigenstates)

Adiabatic flavour conversion





Adiabaticity: slow density change along the neutrino path

$$\frac{\sin^2 2\theta}{\cos 2\theta}\,\frac{\Delta m^2}{2E}\,L_\rho \gg 1$$

L_ρ – electron density scale hight:

$$L_\rho = \left|\frac{1}{N_e}\,\frac{dN_e}{dx}\right|^{-1}$$

2f conversion probability

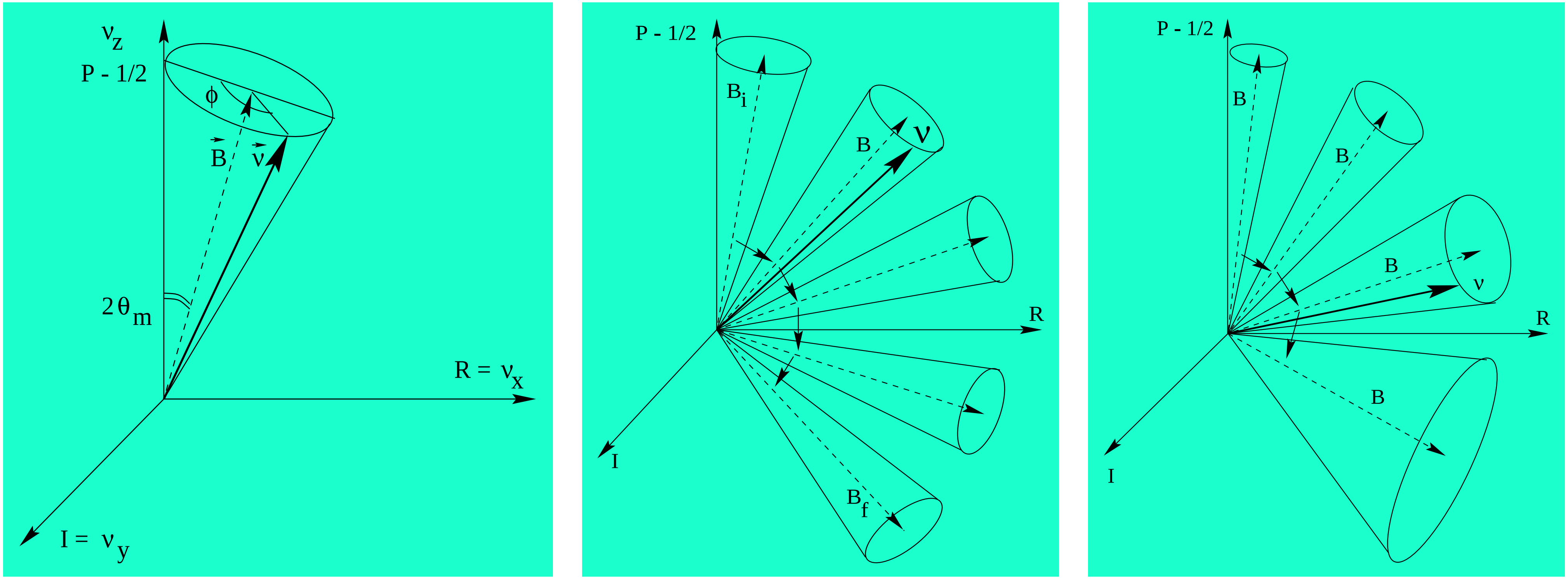
Simple and useful formula for 2f conversion probability averaged over production/detection positions (or small energy intervals) (Parke, 1986):

$$\diamond \qquad \boxed{\overline{P}_{\rm tr} = \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f \,(1 - 2P')}$$

θ_i, θ_f – mixing angles in matter in the initial and final points,
P' – hopping probability.

$$P' : \qquad \begin{cases} \ll 1 & \text{in adiab. regime} \\ \sin^2(\theta_i - \theta_f) & \text{in extreme non} - \text{adiab. regime} \end{cases}$$

Analogy: Spin precession in a magnetic field

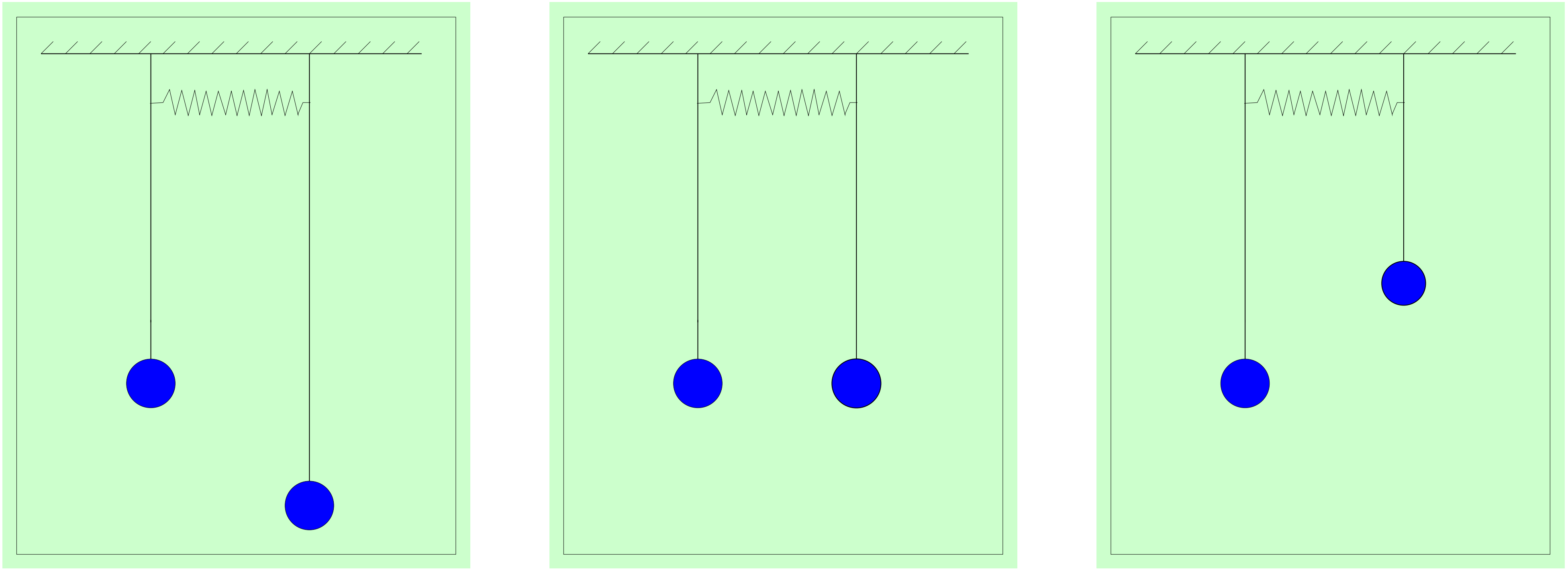


$$\frac{d\vec{S}}{dt} = 2(\vec{B} \times \vec{S})$$

$$\vec{S} = \{\mathrm{Re}(\nu_e^* \nu_\mu)\,,\ \mathrm{Im}(\nu_e^* \nu_\mu)\,,\ \nu_e^* \nu_e - 1/2\}$$

$$\vec{B} = \{(\Delta m^2/4E)\sin 2\theta_m\,,\ \ 0\,,\ \ V/2 - (\Delta m^2/4E)\cos 2\theta_m\}$$

Analogy: Two coupled pendula



Mechanical model of the MSW effect

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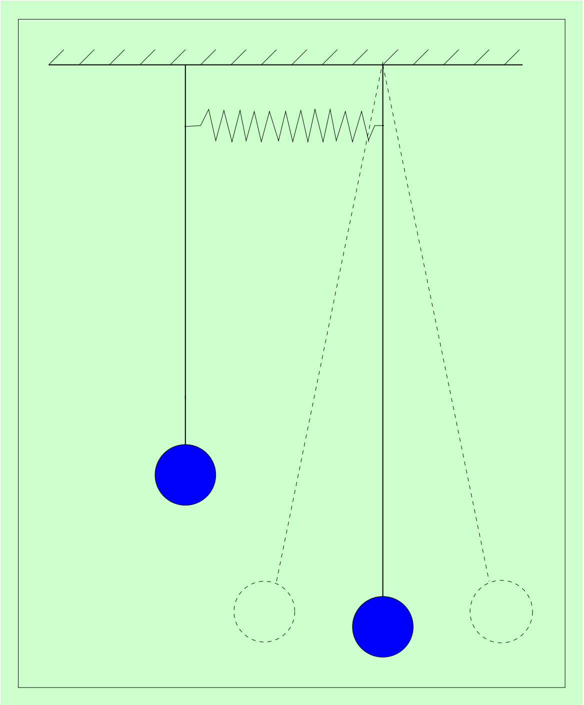

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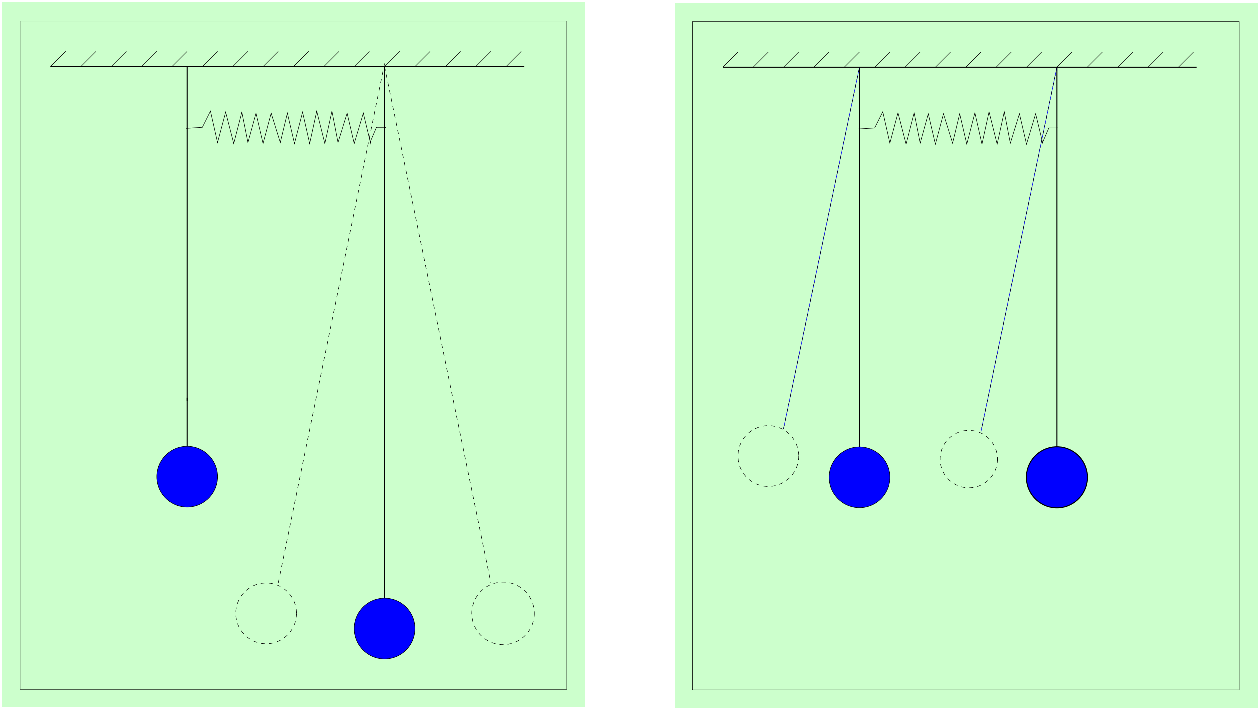

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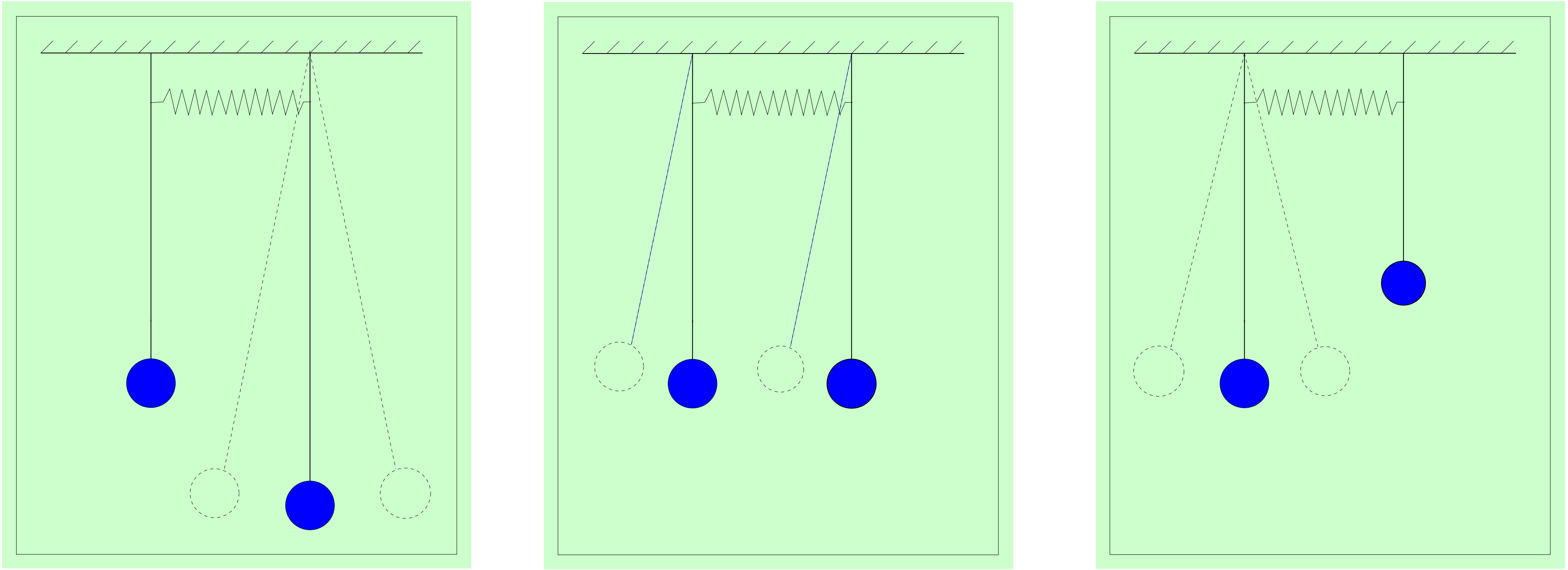

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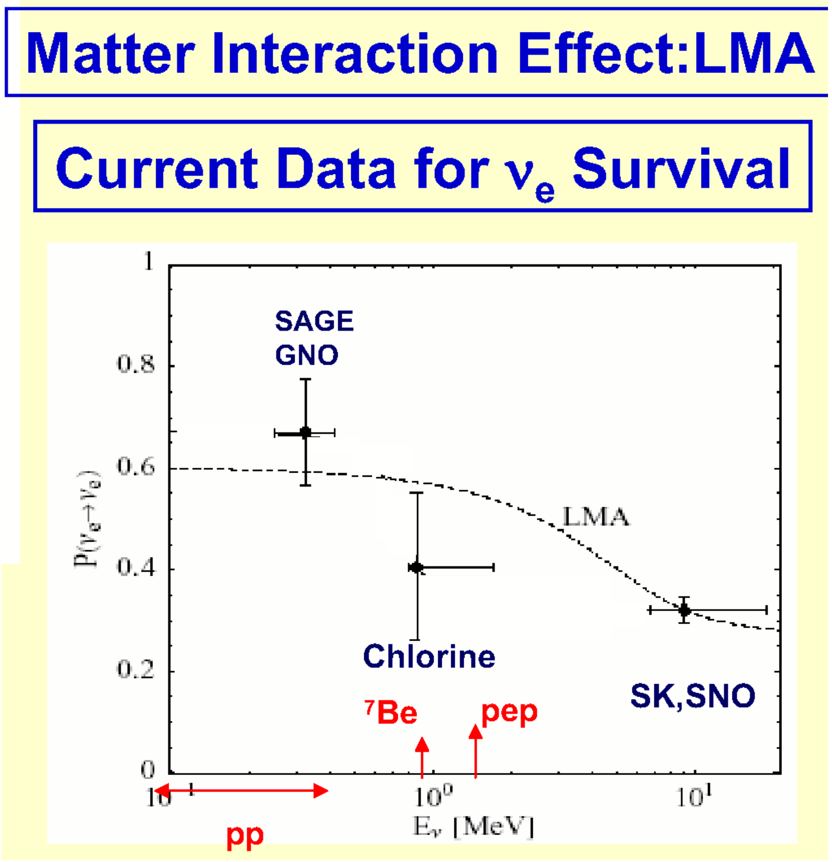
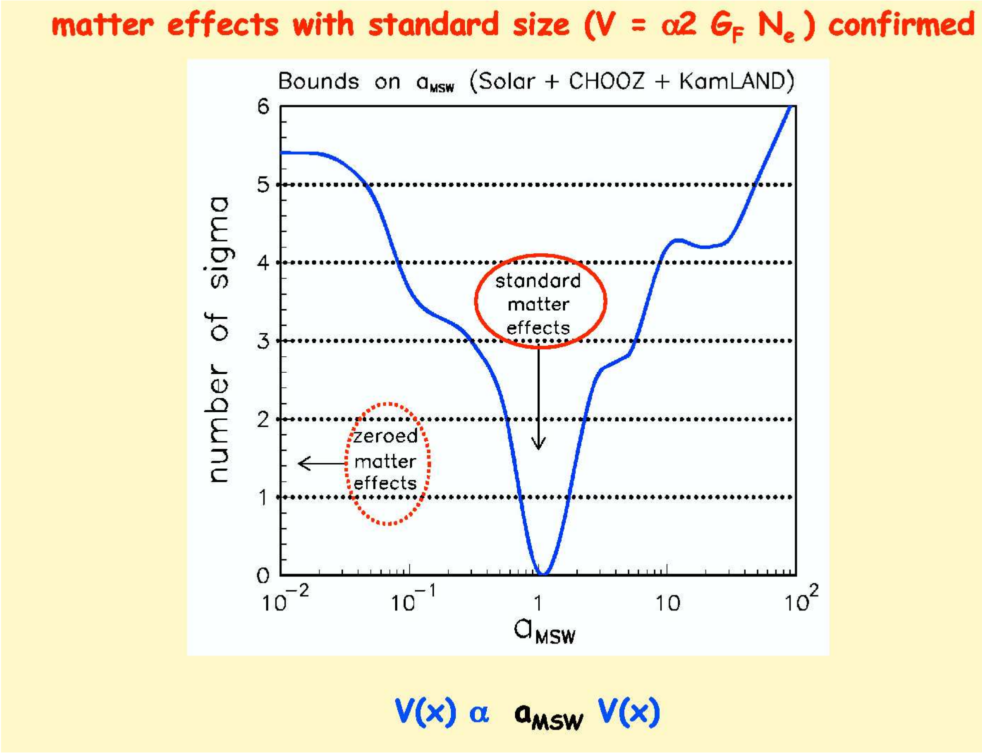
Evidence for the MSW effect





$V(x) \Rightarrow a_{\rm MSW} V(x)$; $a_{\rm MSW} = 1$ strongly favoured

(Fogli et al. 2003, 2004; Fogli & Lisi 2004)

More on MSW effect: talk of A. Friedland

II. Phenomenology

3ν vs $N_\nu \geq 4$ oscillation schemes

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LSND: most likely would require ≥ 1 light sterile neutrinos ν_s

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But: even if the LSND result is not confirmed, this would not exclude the possibility of light sterile neutrinos and $\nu_{\rm a} \leftrightarrow \nu_{\rm s}$ oscillations – an intriguing possibility with implications to particle physics, astrophysics and cosmology

More on sterile neutrinos: talk of A. Kusenko

3f neutrino mixing and oscillations

For 3 neutrino species: mixing matrix $\tilde{U}$ depends on θ_{12}, θ_{23}, θ_{13}, $\delta_{\rm CP}$, $\sigma_{1,2}$. Majorana-type $\not{C}\not{P}$ phases can be factored out in the mixing matrix:

$$\tilde{U} = U K\,, \qquad K = {\rm diag}(1\,, e^{i\sigma_1}\,, e^{i\sigma_2})$$

$\Rightarrow$ Majorana-type phases do not affect neutrino oscillations.

The relevant part of the mixing matrix:

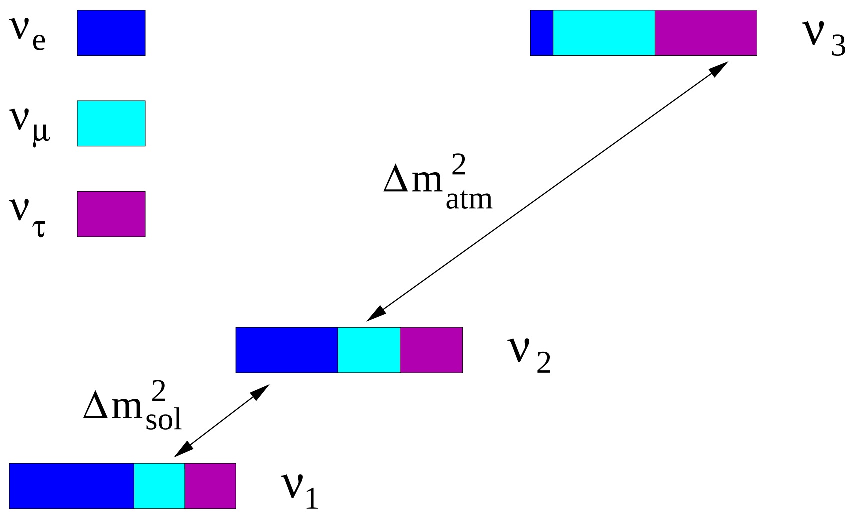
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= O_{23}\,(\Gamma_\delta\, O_{13}\, \Gamma_\delta^\dagger)\, O_{12}\,, \qquad \Gamma_\delta \equiv {\rm diag}(1\,,\ 1\,,\ e^{i\delta_{\rm CP}})$$

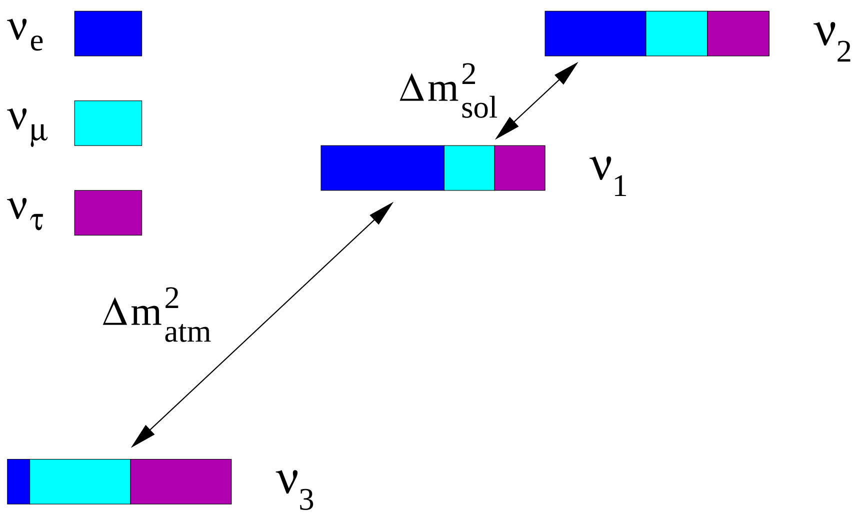
Leptonic mixing – contd.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Normal hierarchy:



Inverted hierarchy:



2f and effective 2f approximations

2f description: A good 1st approximation in most cases.
Reasons:

- Hierarchy of Δm^2: $\Delta m^2_{\rm sol} \ll \Delta m^2_{\rm atm}$
- Smalness of $|U_{e3}|$.

Exceptions: $P(\nu_\mu \leftrightarrow \nu_\tau)$, $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$ when oscillations due to the solar frequency ($\sim \Delta m^2_{\rm sol}$) are not frozen.

In any case, coorections due to 3-flavorness can reach $\sim 10\%$
– cannot be ignored at present

Also: a number of pure 3f effects exist $\Rightarrow$

$\diamondsuit$ 3f analyses are a must !

Effective 2f approximations

For oscillations driven by $\Delta m^2_{\rm sol}$ ν_3 essentially decouples. Still a "memory" of ν_3 through unitarity $\Rightarrow$ powers of c_{13}. Examples:

Survival probability of solar ν_e (Lim, 1987)

(the same for reactor $\bar{\nu}_e$ in KamLAND) :

$$\diamondsuit \quad P(\nu_e \to \nu_e) \;\simeq\; c_{13}^4 P_{2ee}(\Delta m^2_{21},\ \theta_{12},\ c_{13}^2 V) \;+\; s_{13}^4\ ,$$

3f effects for Day-Night effect for solar ν_e :

While $P_D(\nu_e) \propto c_{13}^4$,

$$P_N(\nu_e) - P_D(\nu_e) \propto c_{13}^6$$

(Blennow, Ohlsson & Snellman, 2004; E.A., Tortola & Valle, 2004)

Deviations from 2f results: $(1 - c_{13}^4) \leq 0.1, \quad (1 - c_{13}^6) \leq 0.13$

Reactor $\bar{\nu}_e$ oscillations

$\bar{\nu}_e$ survival probability:

$$\diamondsuit \quad \boxed{P_{\bar{e}\bar{e}} \simeq 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right) - c_{13}^4 \sin^2 2\theta_{12} \cdot \sin^2\left(\frac{\Delta m_{21}^2}{4E} L\right)}$$

- CHOOZ, Palo Verde, Double CHOOZ, ... ($L \lesssim 1$ km)

$$\overline{E} \sim 4\ \text{MeV}\,; \qquad \frac{\Delta m_{31}^2}{4E} L \sim 1\,; \qquad \frac{\Delta m_{21}^2}{4E} L \ll 1$$

One mass scale dominance (2f) approximation:

$$\diamond \quad P(\bar{\nu}_e \to \bar{\nu}_e; L) = 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E}\, L\right)$$

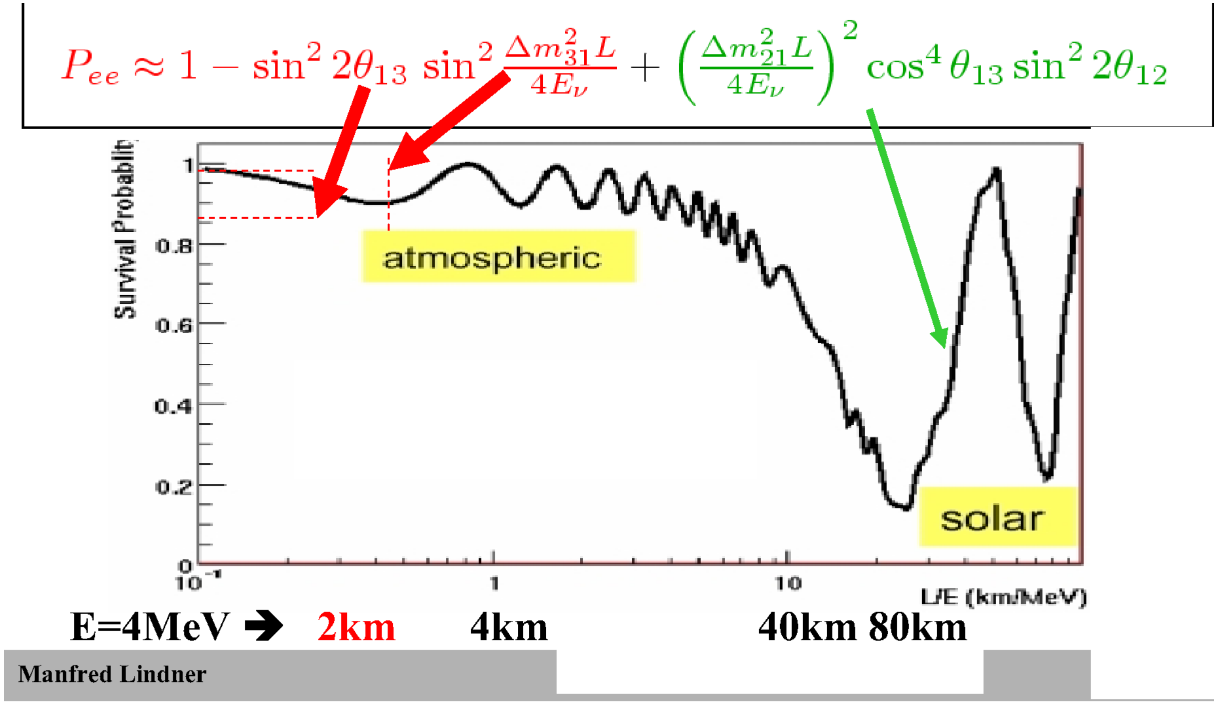
(Note: Term $\sim \sin^2 2\theta_{12}$ cannot be neglected if $\theta_{13} \lesssim 0.03$, which is about the reach of currently discussed future reactor experiments)

Reactor $\bar{\nu}_e$ oscillations – contd.

- KamLAND ($\bar{L} \simeq 170$ km): $\frac{\Delta m^2_{21}}{4E} L \gtrsim 1$; $\frac{\Delta m^2_{31}}{4E} L \gg 1$

  ◇ $P(\bar{\nu}_e \to \bar{\nu}_e) \simeq c^4_{13} P_{2\bar{e}\bar{e}}(\Delta m^2_{21}, \theta_{12})$

N.B.: Matter effects a few % – can be comparable with effects of $\theta_{13} \neq 0$!



Genuine 3f effects

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in vacuum

- $\not{C}\not{P}$: $\quad P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$
- $\not{T}$: $\quad P(\nu_a \to \nu_b) \neq P(\nu_b \to \nu_a)$

CPT invariance: $\diamond\ P(\nu_a \to \nu_b) \to P(\bar{\nu}_b \to \bar{\nu}_a)$

$\not{C}\not{P} \Leftrightarrow \not{T}$ – *consequence of CPT*

Measures of $\not{C}\not{P}$ and $\not{T}$ – probability differences:

$$\Delta P_{ab}^{\rm CP} \equiv P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)$$

$$\Delta P_{ab}^{\rm T} \equiv P(\nu_a \to \nu_b) - P(\nu_b \to \nu_a)$$

From CPT:

$\diamond \quad \Delta P_{ab}^{\rm CP} = \Delta P_{ab}^{\rm T}\,; \qquad \Delta P_{aa}^{\rm CP} = 0$

3f case

One Dirac-type phase $\delta_{\rm CP}$ $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

◇ $\Delta P_{e\mu}^{\rm CP} = \Delta P_{\mu\tau}^{\rm CP} = \Delta P_{\tau e}^{\rm CP} \equiv \Delta P$

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$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP} \times \left[\sin\left(\frac{\Delta m_{12}^2}{2E}\,L\right) + \sin\left(\frac{\Delta m_{23}^2}{2E}\,L\right) + \sin\left(\frac{\Delta m_{31}^2}{2E}\,L\right)\right]$$

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$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$

$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}\,L\right)\right]$$

Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or 90°
- $\delta_{\rm CP} = 0$ or 180°
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

3f case

One Dirac-type phase $\delta_{\rm CP}$ $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

$$\diamond \quad \Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}\,L\right)\right]$$

Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or 90°
- $\delta_{\rm CP} = 0$ or 180°
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

See talk of O. Mena

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in matter

Normal matter [(# of particles) $\neq$ (# of anti-particles)]:

The very presence of matter violates C, CP and CPT

$\Rightarrow$ Fake (extrinsic) $\not{C}\not{P}$. Exists even in 2f case. May complicate study of fundamental (intrinsic) $\not{C}\not{P}$

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Induced $\not{T}$: absent when either $U_{e3} = 0$ or $\Delta m^2_{\rm sol} = 0$ (2f limits)

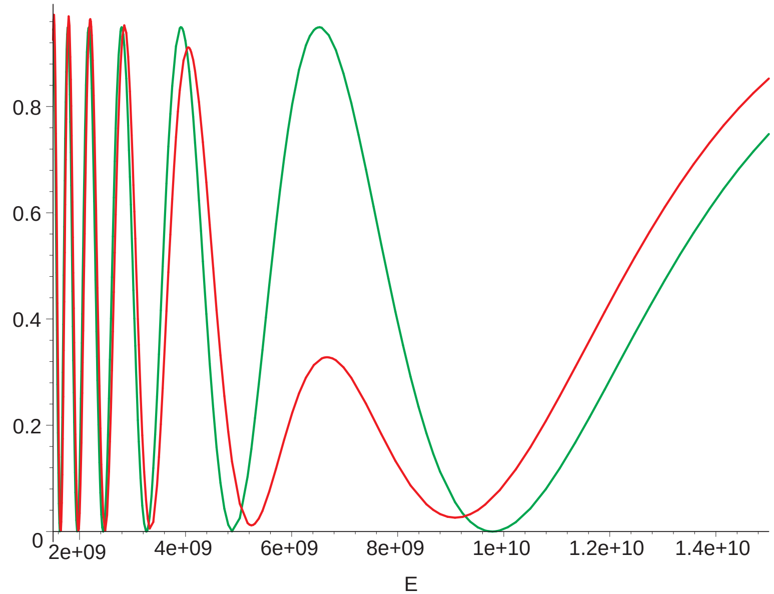
$\Rightarrow$ Doubly suppressed by both these small parameters

– effects in terrestrial experiments are small

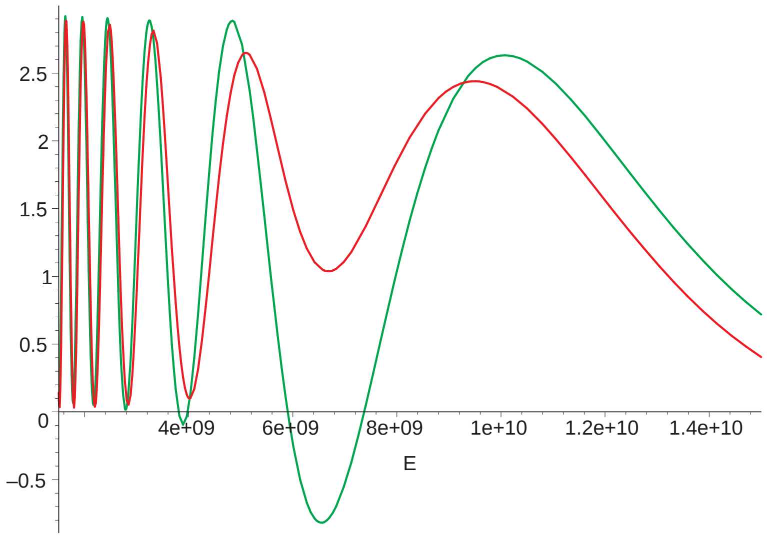
Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

In 2f approximation: no matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations $[V(\nu_\mu) = V(\nu_\tau)$ modulo tiny rad. corrections].

Not true in the full 3f framework! (E.A., 2002; Gandhi et al., 2004)



$P_{\mu\tau}$



Oscillated flux of atm. ν_μ

$\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.026$, $\theta_{23} = \pi/4$, $\Delta m^2_{21} = 0$, $L = 9400$ km

Red curves – w/ matter effects, green curves – w/o matter effects on $P_{\mu\tau}$

Another possible matter effect

Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves

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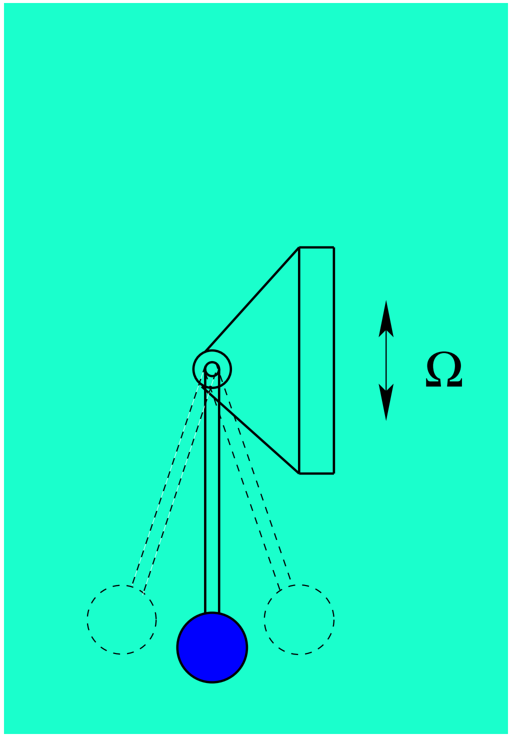

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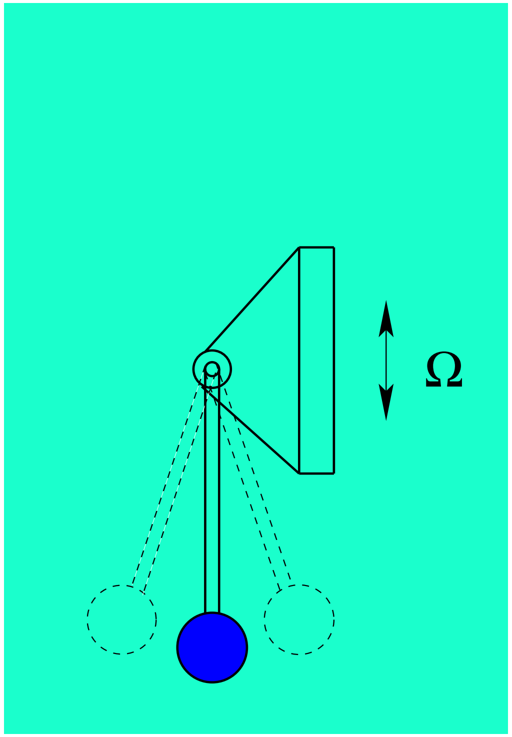

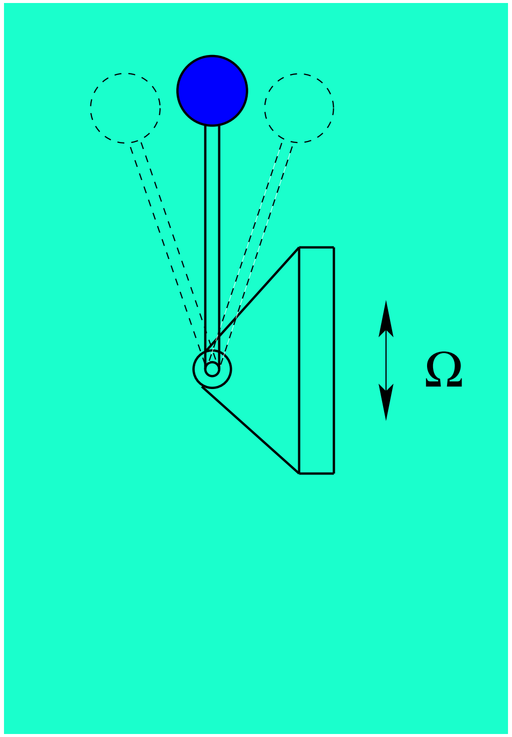

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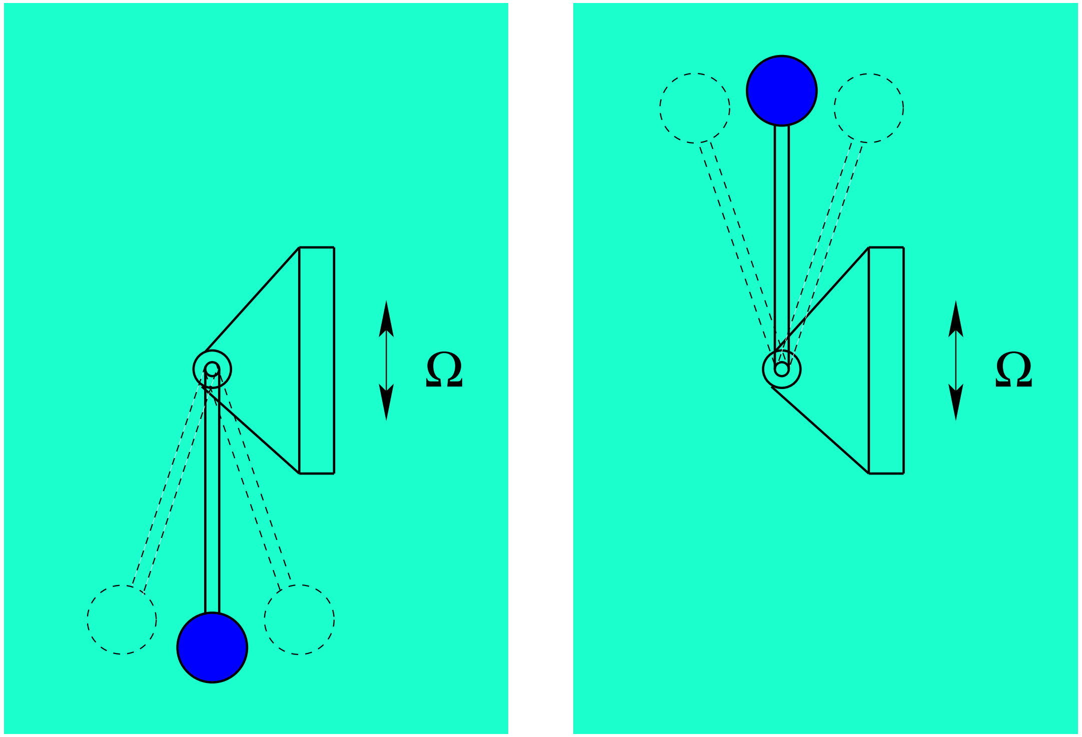


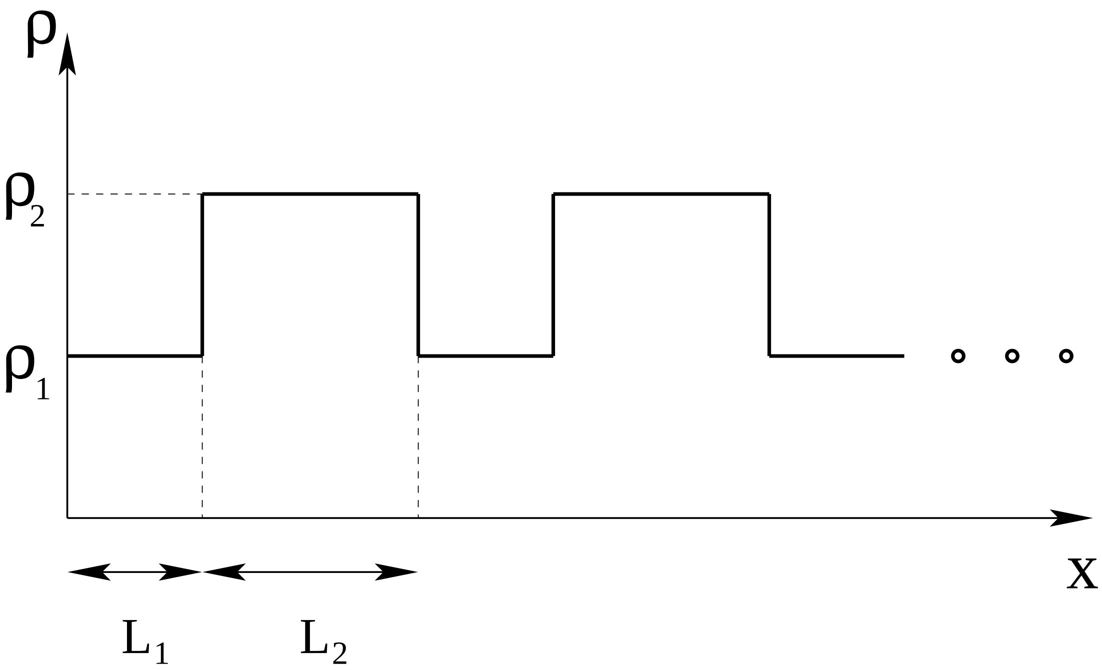
For small-ampl. osc.:

$$\Omega_{\text{res}} = \frac{2\omega}{n}$$

$n = 1, 2, 3...$

Different from MSW eff. – no level crossing !

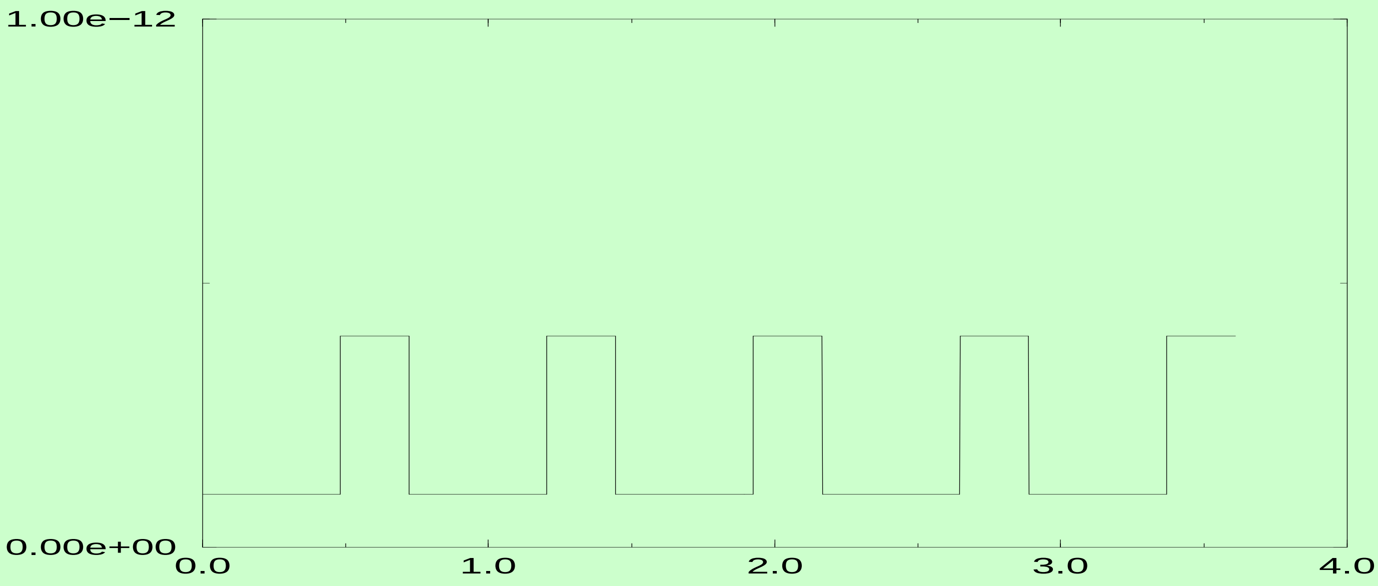
An example admitting an exact analytic solution – “castle wall” density profile (E.A., 1987, 1998):

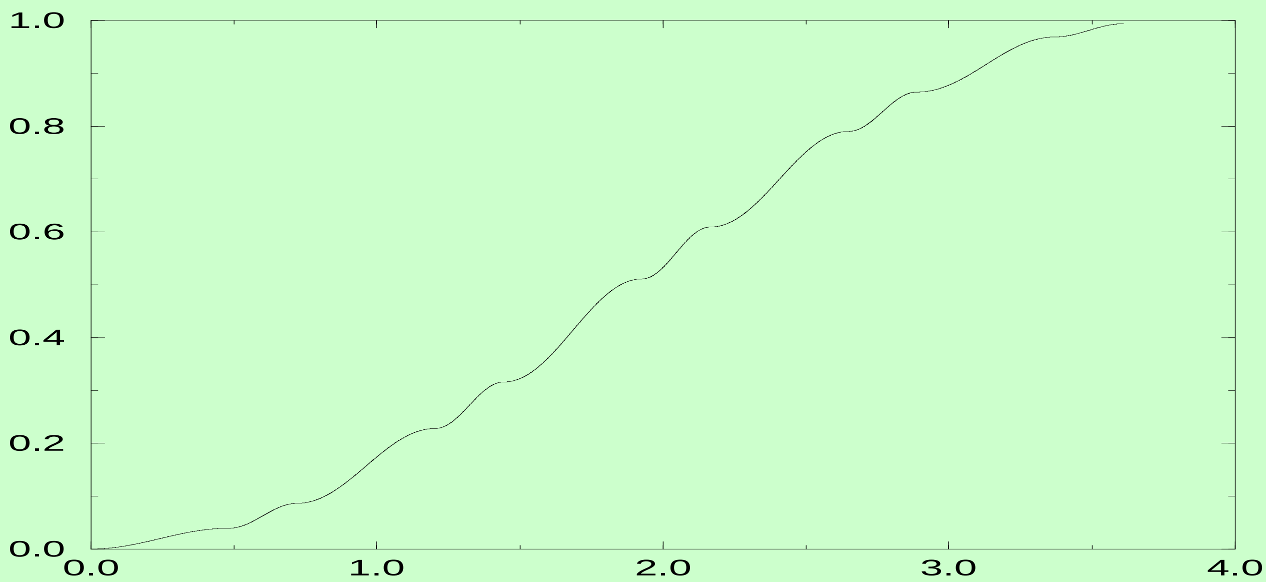


Resonance condition:

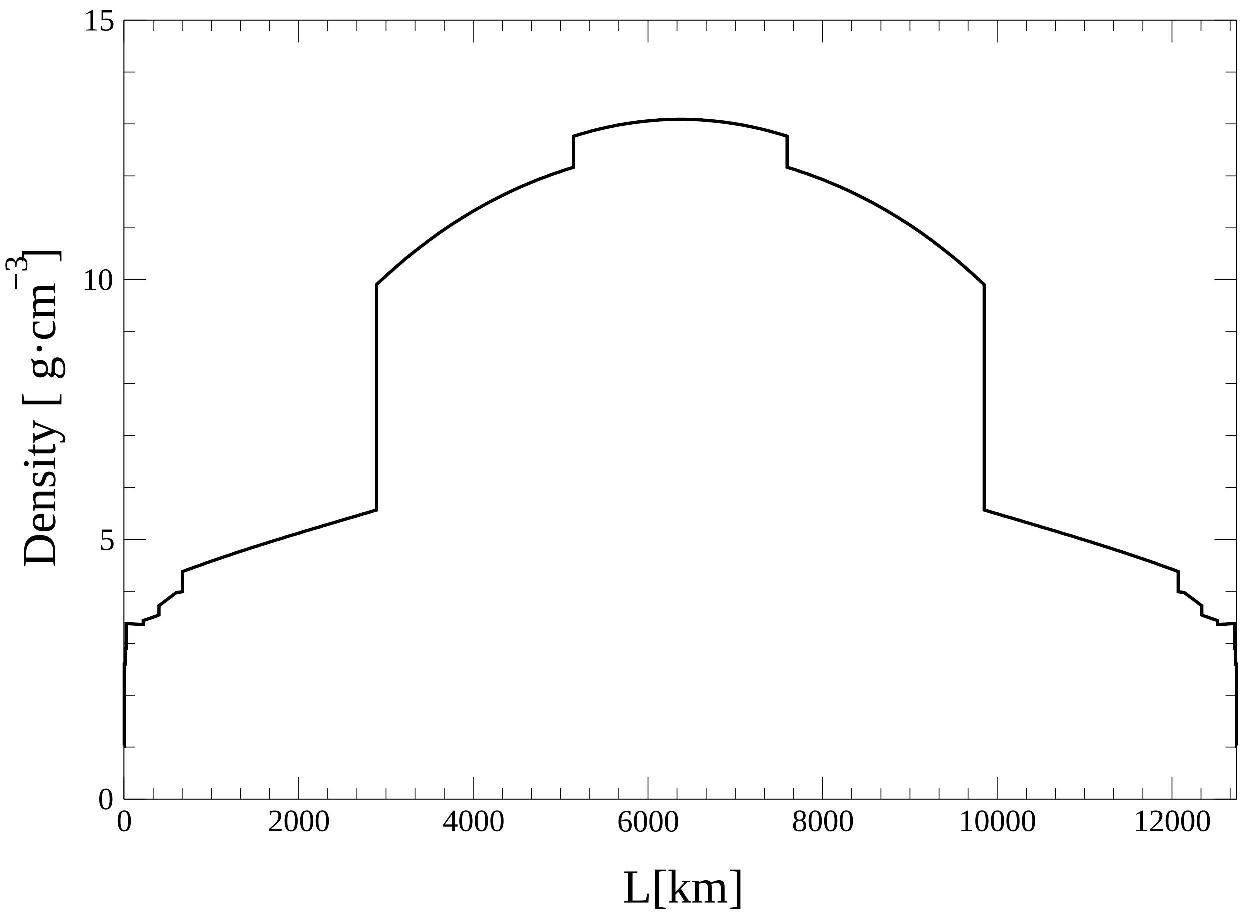
$$X_3 \equiv -(\sin\phi_1 \cos\phi_2 \cos 2\theta_{1m} + \cos\phi_1 \sin\phi_2 \cos 2\theta_{2m}) = 0$$

$\phi_{1,2}$ – oscillation phases acquired in layers 1, 2


1.00e−12
0.00e+00
0.0
1.0
2.0
3.0
4.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0

Earth's density profile (PREM model) :



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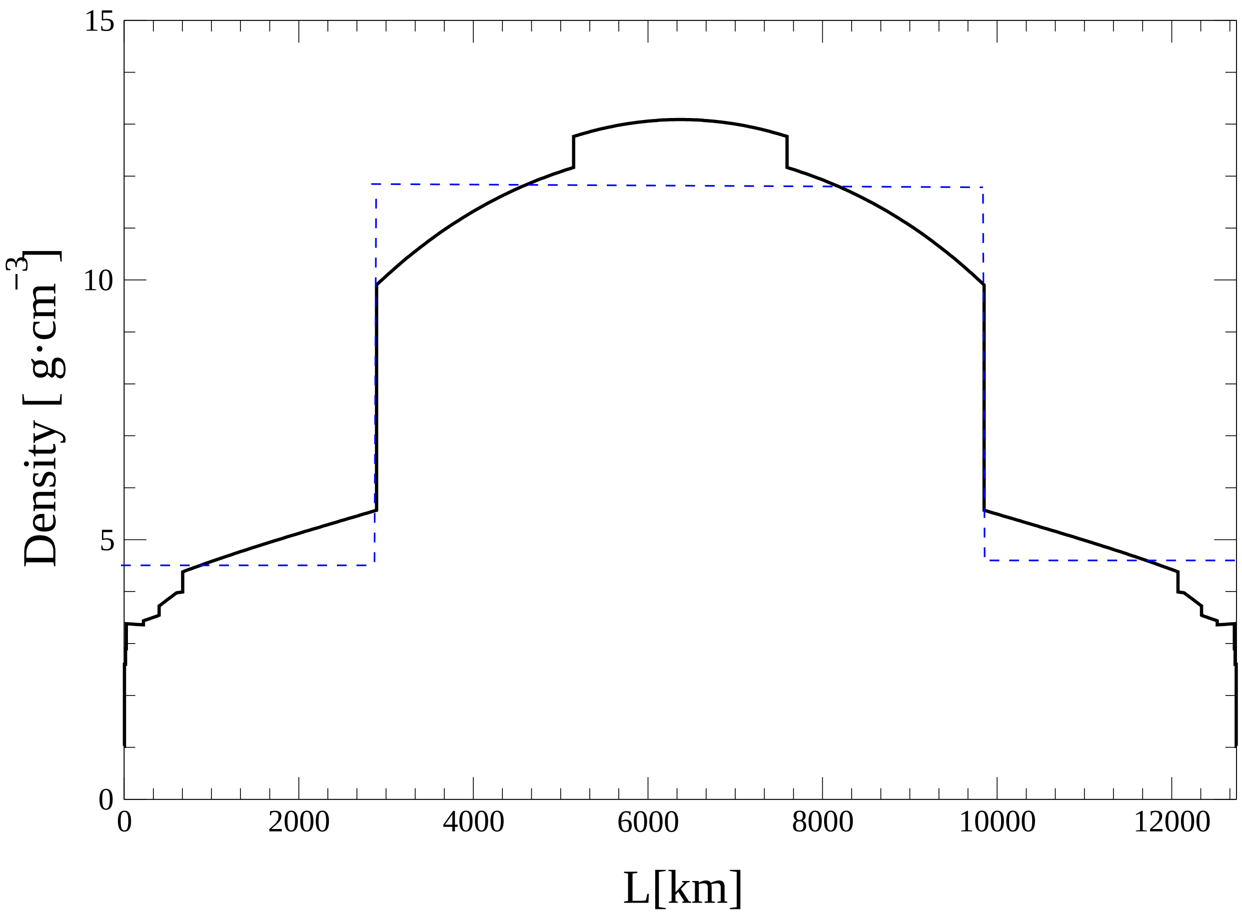
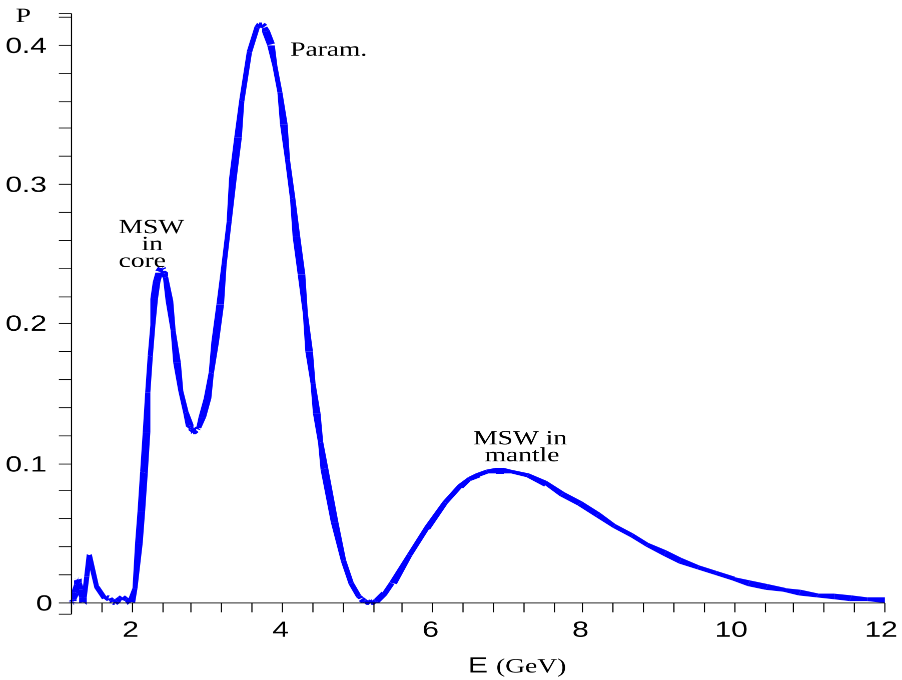
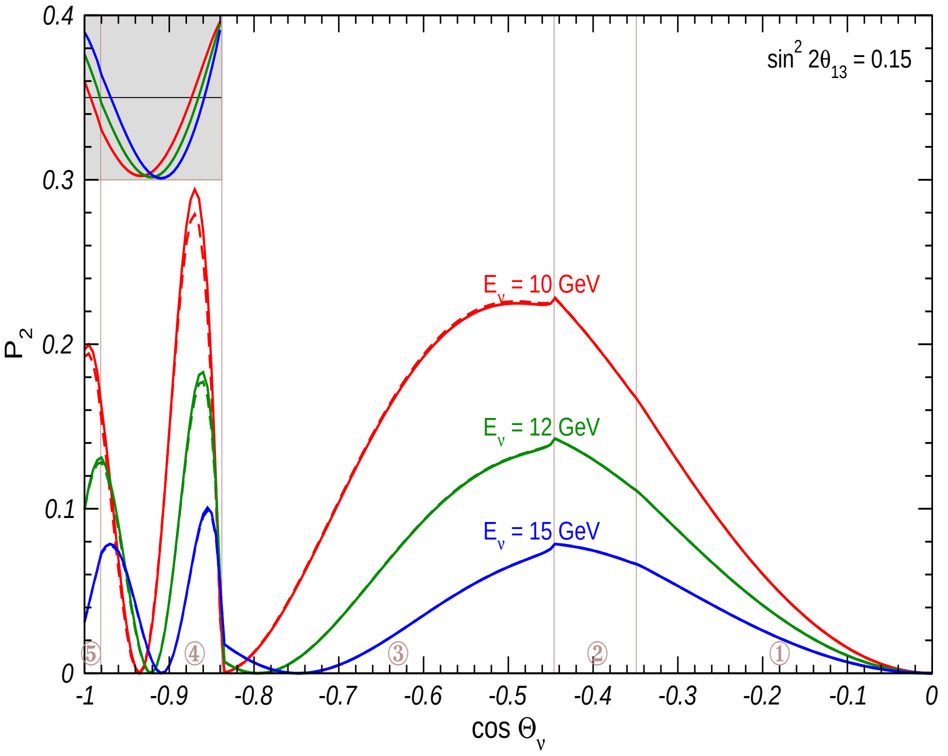


Param. res. condition: $(l_{osc})_{matt} \simeq l_{density\ mod.}$

Fulfilled for $\nu_e \leftrightarrow \nu_{\mu.\tau}$ oscillations of core-crossing ν's in the Earth for a wide range of energies and zenith angles !





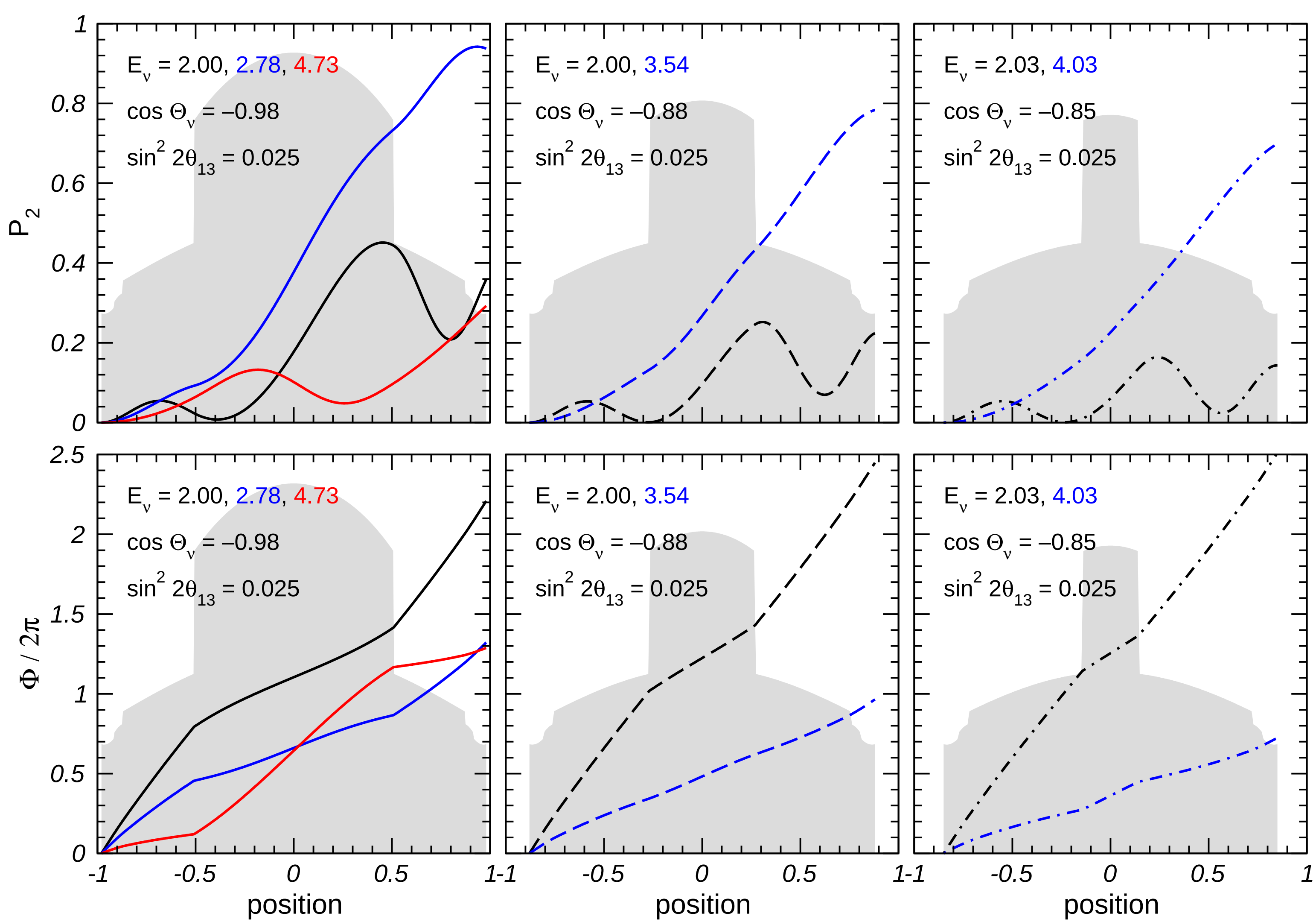
Intermed. energies

$\cos\Theta = -0.89$ $\sin^2 2\theta_{13} = 0.01$

(Liu, Smirnov, 1998; Petcov, 1998; EA 1998)

High energies, $\cos\Theta$ - dependence

(EA, Maltoni & Smirnov, 2005)



◇ Parametric resonance of ν oscillations in the Earth: can be observed in future atmospheric or accelerator experiments if θ_{13} is not much below its current upper limit

Some recent developments

Oscillations of low-E neutrinos in matter

Equivalently: Oscillations in low-density matter $(V \ll \frac{\Delta m^2}{2E})$.

Matter effects small – can be considered in perturbation theory.

Implications: oscillations of solar and SN neutrinos in the Earth.

In 3f framework

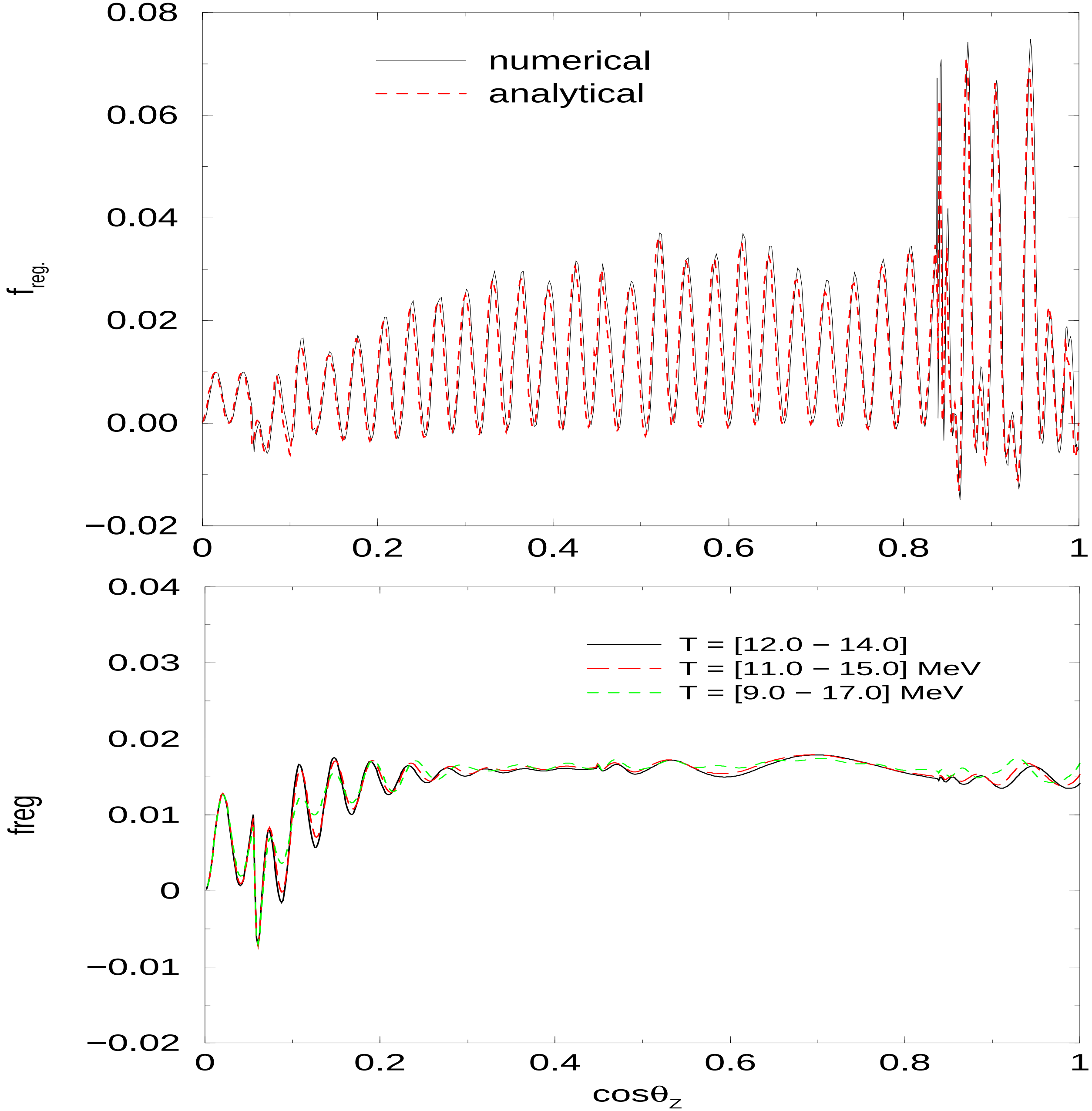
$$P_{2e}^{\oplus} - P_{2e}^{(0)} = \frac{1}{2}\, c_{13}^4 \,\sin^2 2\theta_{12} \int_0^L dx\, V(x) \sin\left[2 \int_x^L \omega(x')\, dx'\right]$$

where

◇ $\omega(x) = \sqrt{[\cos 2\theta_{12}\,\delta - c_{13}^2 V(x)/2]^2 + \delta^2 \sin^2 2\theta_{12}}\,, \qquad \delta = \dfrac{\Delta m_{21}^2}{4E}$

2f case ($\theta_{13} = 0$): de Holanda, Liao & Smirnov, 2004; Ioannisian & Smirnov, 2004; 3f case: E.A., Tórtola & Valle, 2004

Attenuation effect



Perfect energy resolution

Finite energy resolution: effects of density variation far from detector suppressed. Attenuation length d:

$$d \simeq l_{\rm osc} \frac{E}{\Delta E}$$

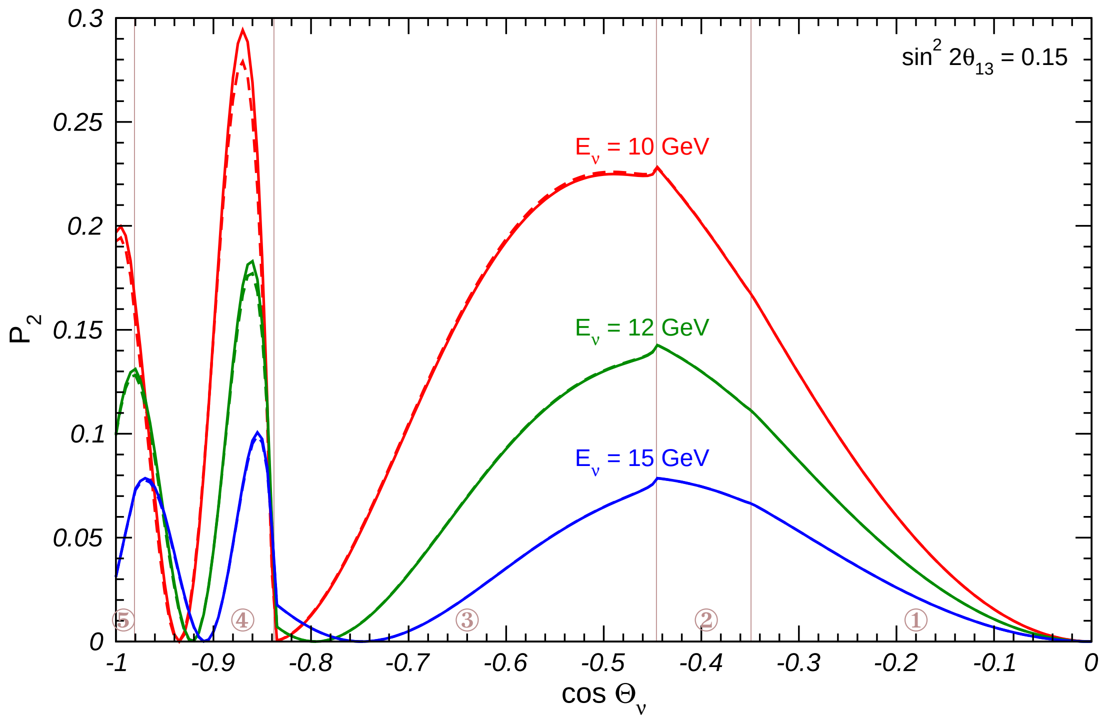
(de Holanda et al., 2004)

Oscillations above the MSW resonance

Equivalently: oscillations in dense matter $(V > \delta \equiv \frac{\Delta m^2}{4E})$

Oscillation probability in matter of arbitrary density profile:

$$P = \delta^2 \sin^2 2\theta \left| \int_0^L dx e^{-2i\phi(x)} \right|^2, \quad \phi(x) = \int_0^x dx' \omega(x') - \text{adiab.phase}$$



E.A., Maltoni & Smirnov, 2005

Unsettled issues?

A number of issues in ν oscillation theory still being debated

- Equal energies or equal momenta?

- Evolution in space or in time?

  Claim: evolution in time is never observed.

- Is wave packet description necessary?

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- Is wave packet description necessary?

  – Yes, if one wants to rigorously justify the standard oscillation probability formula. Once done, can be forgotten unless the issues of coherence become important.

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A number of subtle issues of oscillation theory remain unsettled (e.g., rigorous wave packet treatment, limits of applicability of standard formula, oscillations of non-relativistic neutrinos, ...). At present, this is (rightfully) of little concern for practitioners.

Future tasks

- Search for best strategies for measuring neutrino parameters
- Study of subleading effects and effects of non-standard neutrino interactions
- Study of the domains of applicability and limitations of the current theoretical framework

Future experimental results may bring some new surprises and pose more challenging problems !

Backup slides

General properties of P_{ab}

3 flavours $\Rightarrow 3 \times 3 = 9$ probabilities

$$P_{ab} = P(\nu_a \to \nu_b),$$

plus 9 probabilities for antineutrinos $P_{\bar{a}\bar{b}}$.

Unitarity conditions (probability conservation):

$$\sum_b P_{ab} = \sum_a P_{ab} = 1 \qquad (a, b = e, \mu, \tau)$$

5 indep. conditions $\Rightarrow 9 - 5 = 4$ indep. probabilities left.

Additional symmetry: the matrix of matter-induced potentials $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23} $\Rightarrow$ additional relations between probabilities.

Dependence on θ_{23} and # of indep. P_{ab}

Define

$$\tilde{P}_{ab} = P_{ab}(s_{23}^2 \leftrightarrow c_{23}^2,\, \sin 2\theta_{23} \to -\sin 2\theta_{23})$$

(e.g., $\theta_{23} \to \theta_{23} + \pi/2$). Then

$$P_{e\tau} = \tilde{P}_{e\mu} \quad P_{\tau\mu} = \tilde{P}_{\mu\tau} \quad P_{\tau\tau} = \tilde{P}_{\mu\mu}$$

2 out of 3 conditions are independent $\Rightarrow$ $4 - 2 = 2$ indep. probabilities (e.g., $P_{e\mu}$ and $P_{\mu\tau}$) $\Rightarrow$

◊ *All 9 neutrino ocillation probabilities can be expressed through just two!* (E.A., Johansson, Ohlsson, Lindner & Schwetz, 2004)

$$P_{\bar{a}\bar{b}} = P_{ab}(\delta_{\rm CP} \to -\delta_{\rm CP},\, V \to -V) \quad \Rightarrow$$

◊ *All 18 ν and $\bar{\nu}$ probab. can be expressed through just two*

General dependence on $\delta_{\rm CP}$

Another use of essentially the same symmetry: rotate by

$$O'_{23} = O_{23} \times \mathrm{diag}(1, 1, e^{i\delta_{\rm CP}})$$

From commutativity of $\mathrm{diag}(V(t), 0, 0)$ with O'_{23} $\Rightarrow$
General dependence of probabilities on $\delta_{\rm CP}$:

$$P_{e\mu} = A_{e\mu} \cos\delta_{\rm CP} + B_{e\mu} \sin\delta_{\rm CP} + C_{e\mu}$$

$$P_{\mu\tau} = A_{\mu\tau} \cos\delta_{\rm CP} + B_{\mu\tau} \sin\delta_{\rm CP} + C_{\mu\tau}$$

$$+ D_{\mu\tau} \cos 2\delta_{\rm CP} + E_{\mu\tau} \sin 2\delta_{\rm CP}$$

(Yokomakura, Kimura & Takamura, 2002)

3f effects in atm. ν oscillations

◇ $\Delta m^2_{21} \to 0$ (E.A., Dighe, Lipari & Smirnov, 1998) :

$$\frac{F_e - F_e^0}{F_e^0} = P_2(\Delta m^2_{31},\, \theta_{13}, V_{\rm CC}) \cdot (r\, s^2_{23} - 1)$$

◇ $s_{13} \to 0$ (Peres & Smirnov, 1999) :

$$\frac{F_e - F_e^0}{F_e^0} = P_2(\Delta m^2_{21},\, \theta_{12}, V_{\rm CC}) \cdot (r\, c^2_{23} - 1)$$

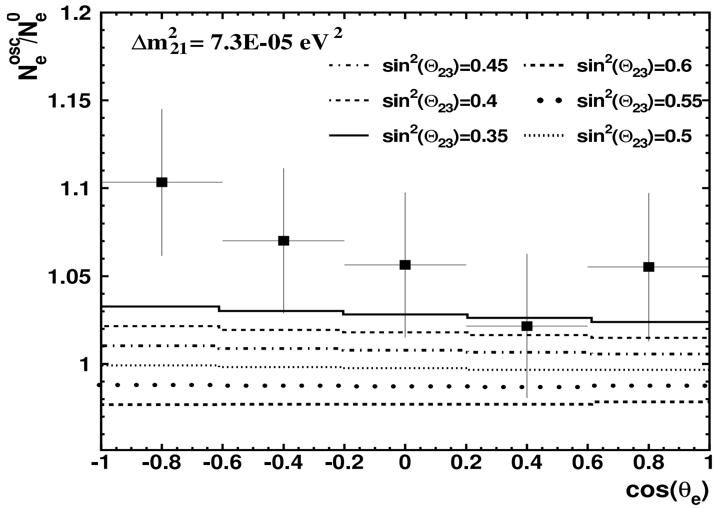
At low energies $r \equiv F_\mu^0/F_e^0 \simeq 2$; also $s^2_{23} \simeq c^2_{23} \simeq 1/2$ –
a conspiracy to hide oscillation effects on e-like events!
Reason: a peculiar flavour composition of the atmospheric ν flux.

(Because of $\theta_{23} \simeq 45°$, $P_{e\mu} \simeq P_{e\tau}$; but the original ν_μ flux is ~ 2 times larger than ν_e flux $\Rightarrow$ compensation of transitions from and to ν_e state).

Breaking the conspiracy – 3f effects

$$\frac{F_e - F_e^0}{F_e^0} \simeq P_2(\Delta m_{31}^2,\, \theta_{13}) \cdot (r\, s_{23}^2 - 1) + P_2(\Delta m_{21}^2,\, \theta_{12}) \cdot (r\, c_{23}^2 - 1) - 2 s_{13}\, s_{23}\, c_{23}\, r\, \mathrm{Re}(\tilde{A}_{ee}^*\, \tilde{A}_{\mu e})$$

Interference term not suppressed by the flavour composition of the $\nu_{\rm atm}$ flux; may be (partly) responsible for observed excess of upward-going sub-GeV e-like events



Interf. term may not be sufficient to fully explain the excess of low-E e-like events – a hint of $\theta_{23} \neq 45°$? (Peres & Smirnov, 2004)

Evolution in the rotated basis

Evolution matrix $S(t, t_0)$: $\nu(t) = S(t, t_0)\,\nu(t_0)$. Satisfies

$$\diamondsuit \quad i\frac{d}{dt}S(t, t_0) = H\,S(t, t_0) \qquad \text{with} \quad S(t_0, t_0) = \mathbb{1}\,.$$

$$H = (O_{23}\,\Gamma_\delta\,O_{13}\,\Gamma_\delta^\dagger\,O_{12}\,)\,\mathrm{diag}(0,\,\delta,\,\Delta)\,(O_{12}^T\,\Gamma_\delta O_{13}^T\,\Gamma_\delta^\dagger\,O_{23}^T) + \mathrm{diag}(V(t),\,0,\,0)$$

$$= (O_{23}\,\Gamma_\delta\,O_{13}\;O_{12}\,)\,\mathrm{diag}(0,\,\delta,\,\Delta)\,(O_{12}^T\,O_{13}^T\,\Gamma_\delta^\dagger\,O_{23}^T) + \mathrm{diag}(V(t),\,0,\,0)$$

where

$$\delta \equiv \frac{\Delta m_{21}^2}{2E}\,, \qquad \Delta \equiv \frac{\Delta m_{31}^2}{2E}$$

Oscillation probabilities:

$$P_{ab} = |S_{ba}|^2$$

Define

$$O_{23}' = O_{23}\,\Gamma_\delta$$

The matrix $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23}' $\Rightarrow$ go to the rotated basis

Evolution in the rotated basis – contd.

$$\nu \;=\; O'_{23}\,\nu'\,, \qquad \text{or} \qquad S(t,\,t_0) \;=\; O'_{23}\,S'(t,\,t_0)\,{O'_{23}}^{\dagger}\,,$$

In the rotated basis $H' \;=\; O'_{23}\,H\,{O'_{23}}^{\dagger}$. Explicitly:

$$H'(t) = \begin{pmatrix} s_{12}^2 c_{13}^2 \delta + s_{13}^2 \Delta + V(t) & s_{12}c_{12}c_{13}\delta & s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) \\ s_{12}c_{12}c_{13}\delta & c_{12}^2\delta & -s_{12}c_{12}s_{13}\delta \\ s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) & -s_{12}c_{12}s_{13}\delta & c_{13}^2\Delta + s_{12}^2 s_{13}^2\delta \end{pmatrix}$$

Dependence on θ_{23} and $\delta_{\rm CP}$ can be obtained in the general case by rotating back to the original flavour basis. Also: easy to apply PT approximations

- If $\frac{\Delta m_{21}^2}{2E}L \ll 1$ – neglect $\delta \;=\; \frac{\Delta m_{21}^2}{2E}$
- If θ_{13} is very small – neglect s_{13}

or use expansion in these small parameters