

Models of Neutrino Masses: A Brief Overview

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Why is neutrino mass physics important?

☞ Two major unresolved puzzles in particle physics

➤ (i) MASS PUZZLE

Why and How $\langle H \rangle \neq 0$?

(Higgs boson, SUSY, Extra Dimension etc.)

➤ FLAVOR PUZZLE

Why 3 generations ? Mixings among quarks and leptons;
Origin of CP violation, strong CP etc.

(ν - mass physics, B-physics, Rare processes- $d_{n,e,\mu}^e$,
 $(\mu, \tau) \rightarrow (e, \mu) + \gamma$ etc will throw light on this.)

➤ Both vital to unravel the nature of new physics

$m_\nu \neq 0$ and Flavor Physics for Leptons



- $m_\nu = 0$ in the standard model; $\rightarrow e, \mu, \tau$ are “mass locked” (no mixings) and therefore Leptons are “FLAVOR STERILE” !
- Once $m_\nu \neq 0$, leptons develop a full flavor physics.
- One may hope that in the true theory, quark and lepton flavor physics may be related (as in GUT theories) or it may reveal new symmetries.

Plan of the talk



- DIRAC vrs MAJORANA NEUTRINO
- SEESAW PARADIGM AND ITS TESTS
 - Lepton flavor violation, leptogenesis etc
- UNDERSTANDING LARGE MIXINGS:
 - Bottom up approach: LEPTONIC SYMMETRIES;
 - Top-down: SO(10) GUT MODELS AND TESTS.
- CONCLUSION.

Dirac vrs Majorana Neutrinos

☞ Dirac neutrino needs a global symmetry !

- For a fermion field, Lorentz invariance allows two kinds of mass terms: $\bar{\psi}\psi$ and $\psi^T C^{-1}\psi$.
- Under $\psi \rightarrow e^{i\alpha}\psi$ transformation ($U(1)$ symmetry), the first is invariant whereas the second is not. **the first mass term is Dirac mass and second is Majorana.**
- $e, \mu, q..$ must be Dirac fermions because they have $Q_e \neq 0$ (the $U(1)$ symmetry is $U(1)_{em}$).
- **since $Q(\nu) = 0$, $U(1)_{em}$ symmetry does not force ν to be Dirac. So it can be Dirac type i.e. $\nu \neq \bar{\nu}$ if there is exact lepton number sym. Otherwise Majorana i.e. $\nu = \bar{\nu}$.**
- Important point: 3 Majorana ν 's imply 3 ν degrees of freedom; whereas 3 Dirac ν 's imply six i.e. 3 active plus 3 sterile.

How to tell if ν is Dirac or Majorana

☞ • Observation of $\beta\beta_{0\nu}$ decay implies Majorana ν , regardless of whether the neutrino mass dominates the process or heavy particles do;


• Need more than one expt to tell if it is Dirac:

e.g. No $\beta\beta_{0\nu}$ signal down to $m_\nu = 20$ meV **plus** $\Delta m_{31}^2 < 0$ from Long Base Line expts. and/or evidence for m_ν in KATRIN expt. would mean neutrino is Dirac (or Pseudo-Dirac which means predominantly Dirac)

Theoretical Challenges Posed by Neutrinos



1. Why $m_\nu \ll m_{u,d,e}$?
2. Why are neutrino mixings so **much larger** than quark mixings ? Is there CP violation for neutrinos ?
3. How do neutrinos fit into the big picture that relates to other particle physics issues ?
4. Are there new types of neutrinos (i.e. sterile neutrinos, ν_s)
See Kusenko's talk

 **New physics depends on whether neutrinos are Majorana or Dirac. This talk assumes them to be Majorana type and presents some highlights of what we may have learned!!**

Seesaw: The Dominant Paradigm:

☞ Add ν_R to the standard model

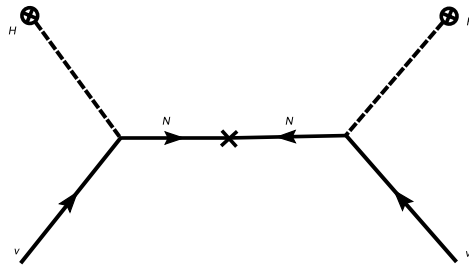


Figure 1: Seesaw diagram

☞ $\rightarrow \mathcal{M}_\nu \simeq -\frac{h_\nu^2 v^2}{M_R}$ (Type I).

This implies $m_{\nu_i} \ll m_{u,d,e\dots}$.

Seesaw mechanism Leads to Majorana neutrinos.

Ultralightness of Dirac ν 's

☞ **Models not as simple e.g. loop models, extra dim. models etc.**

Involve new physics not related to supersymmetry.

Seesaw and the Big Picture



- Scale of the RH neutrino mass:

$M_{R,max} \simeq \frac{m_t^2}{\sqrt{\Delta m_A^2}} \simeq 10^{14} - 10^{15} \text{ GeV}$: M_R close to the conventional SUSY GUT scale !! neutrinos provide a glimpse of high scale physics.

- $M_R \ll M_{Pl}$: implies new symmetry of Nature: $B - L$
- N_R restores quark-lepton symmetry and leads to theories which conserve parity at high energies (e.g. left-right symmetric models based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$);
- N_R provides a simple way to understand the origin of matter via leptogenesis: gives some information on lightest RH neutrino mass in some simple cases i.e. $M_1 \geq 10^8 - 10^9 \text{ GeV}$.

Testing seesaw with Lepton Flavor Violation

Borzumati and Masiero;...; review: Masiero, Vives and Vempati.....

☞ **Large neutrino mixings can induce significant $\tau \rightarrow \mu + \gamma$ and $\mu \rightarrow e + \gamma$. In std model (without SUSY, $B(\mu \rightarrow e + \gamma) \sim 10^{-50}$).**

With Seesaw + supersymmetry \rightarrow , superpartners remember high scale effects; large lepton mixings, radiatively inducing large 23 and 12 slepton mixing, and hence significant $B(\tau \rightarrow \mu + \gamma)$ and $B(\mu \rightarrow e + \gamma)$.

Predictions depend on supersymmetry parameters e.g. $m_{1/2}, m_0, A, \tan\beta$ and μ as well as on seesaw scale M_R and the Dirac coupling Y_ν .

GUT models can fix M_R and Y_ν leading to definite predictions for LFV effects(see later). Typical expectations are between $10^{-11} - 10^{-14}$ for $B(\mu \rightarrow e + \gamma)$ and upto 10^{-7} for $B(\tau \rightarrow \mu + \gamma)$

Falsifying seesaw

☞ **Simple seesaw models can be falsified if:**

- neutrino is a Dirac particle or
- there is evidence for extra dimensions around a TeV scale or
- Discovery of a Z' at LHC in the TeV scale

Large Lepton Mixings?

☞ How we understand this, is likely to throw light on the general flavor puzzle for quarks as well since it may reveal new flavor symmetries.

☞ **QUARK MIXINGS:** $\theta_{13}^q \simeq .22^\circ$; $\theta_{23}^q \simeq 2.2^\circ$ and $\theta_{12}^q \simeq 13^\circ$

Lepton mixings: SOLAR: $\theta_{12} \sim 34^\circ$

ATMOS: $\theta_{23} \simeq 45^\circ$

REACTOR: $\theta_{13} \leq 13^\circ$

Why so different- especially if quarks and leptons are to be unified at some scale ?

Seesaw by itself cannot explain flavor structure !!

☞ **Two issues of great importance:**

- **New symmetries of leptons;**
- **Understanding the relation between leptons and quarks.**

How to proceed ?

Two Approaches:

1. Bottom-Up:

Search for leptonic symmetries, mass matrix textures etc.

2. Top-Down: Grand unified theories:

Bottom-Up: Searching for symmetries of leptons

☞ ●: Maximal θ_{23} , zero θ_{13} and $\mu - \tau$ Symmetry

➤ mass matrix:

$$\text{➤ } \mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} d\epsilon^n & b\epsilon & a\epsilon \\ b\epsilon & 1 + \epsilon & 1 \\ a\epsilon & 1 & 1 + c\epsilon \end{pmatrix}; n \geq 1.$$

$a = b$ and $c = 1 \rightarrow \theta_{23} = \pi/4$ and $\theta_{13} = 0$;

The mass matrix has $\mu - \tau$ symmetry.

➤ Breaking leads to $\theta_{13} \simeq \frac{\Delta m_{\odot}^2}{\Delta m_A^2} \simeq 0.04$ correlated to $\theta_{23} - \pi/4$.

➤ $\mu - \tau$ symmetric limit has 4 parameters in the CP conserving case.

➤ $m_\mu \neq m_\tau$ not a problem since ν_α and l_α can get mass from different sources.

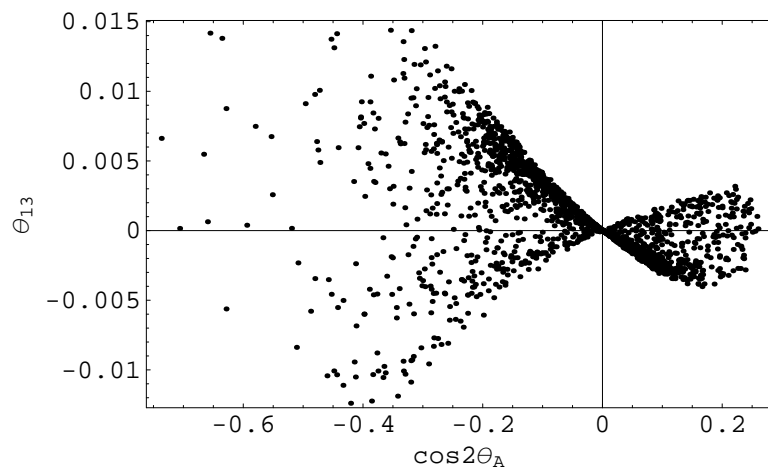


Figure 2: **Departure from $\mu - \tau$ symmetry and correlation between θ_{13} and θ_A**

RNM(2004); de Gouvea; Lam; Grimus, Lavoura, Joshipura, Kaneko, Tanimoto(2004); Kitabayashi and Yasue; RNM, Rodejohann; Nasri, Yu (2005)...

☞ Neutrino telescope tests of approximate $\mu - \tau$ symmetry

$$\phi_e : \phi_\mu : \phi_\tau = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta).$$

Xing (06)

• Tri-bi-maximal mixing and Higher symmetries

☞ Remarkable feature of neutrino mixing:

$$U_{PMNS} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\sqrt{1}}{\sqrt{6}} & \frac{\sqrt{1}}{\sqrt{3}} & \frac{\sqrt{1}}{\sqrt{2}} \\ \frac{\sqrt{1}}{\sqrt{36}} & -\frac{\sqrt{1}}{\sqrt{3}} & \frac{\sqrt{1}}{\sqrt{2}} \end{pmatrix}$$

Wolfenstein (1978) Harrison, Perkins, Scott (2002); Xing (2002)

☞ An indication of higher symmetry ?

Mass matrix has only 3 parameters

$$\mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} \epsilon & b\epsilon & b\epsilon \\ b\epsilon & 1 + \epsilon & b\epsilon - 1 \\ b\epsilon & b\epsilon - 1 & 1 + \epsilon \end{pmatrix}$$

Ma; Babu, He; Altarelli, Ferruglio; Adhikary, et al; Volkas, Low, He, Keum; King, Ross, M-Varzeilas; Grimus, Lavoura, Rodejohann, Pfltinger; Jora, Nasri, Schecter; Haba, Watanabe

An S_3 embedding of $\mu - \tau$ symmetry

☞ A simple observation: tbm follows from the observation that the above mass matrix has the following group theoretic structure

Caravaglios, Morrisi (06); Nasri, Yu, RNM (06))

☞
$$M_\nu = M_0(S_3) + M_1(Z_{2,\mu-\tau})$$

If tbm is confirmed, the leptonic symmetry could be a broken permutation symmetry S_3 among three generations; this will be a significant piece of information about flavor !!

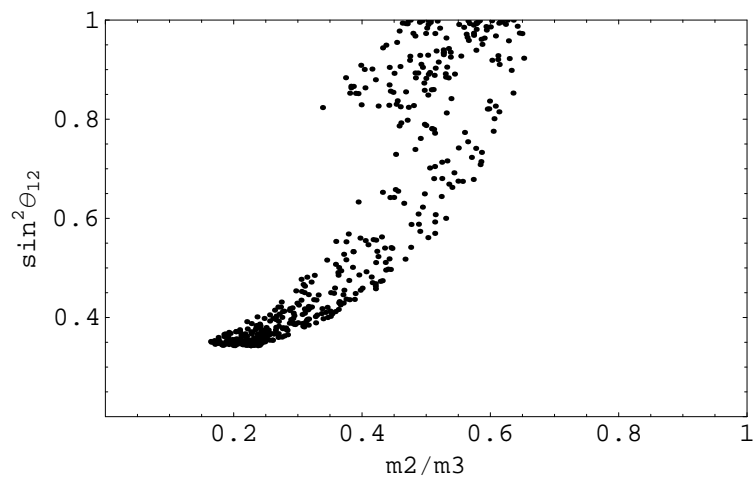


Figure 3: $\sin^2 \theta_{12}$ off by 5σ or more

**Degenerate neutrinos incompatible with tribi-
maximal in a seesaw model!**

Understand large mixings without symmetries

☞ **Are large mixings a random phenomena ?-Anarchy approach: Predicts large θ_{13} .**

Hall, Murayama, Weiner; de Gouvea, Murayama; Altarelli, Feruglio, Masina; Haba, Murayama,...
see Espinoza for a contrary point of view.

☞ **Large mixings purely as a consequence of radiative renormalization group extrapolation-for degenerate neutrinos. Requires $m_0 \geq 0.2$ eV. Will be tested soon in $\beta\beta_{0\nu}$ experiments.**

Parida, Rajasekaran, RNM.

☞ **Use of singular seesaw:**

Goldman, Stevenson, McKeller, Garbutt (04)-poster #94.

Top-down approach to neutrino mass:

☞ Generic starting point is a Quark-Lepton Unified theory at superheavy scale.

Any hint of Q-L connection ?

☞ • $M_{seesaw} \sim M_{GUT}$

Perhaps a signal of Q-L grand unification:

• $\theta_{12}^\nu + \theta_{12}^q \simeq 45^\circ$; $\theta_{23}^\nu + \theta_{23}^q \simeq 45^\circ$; (Called (QLC))

Smirnov; Raidal; Minakata and Smirnov

☞ **QLC** requires specific product of mixings i.e. $U_{PMNS} = U_{bm} U_{CKM}^\dagger$: models $U_{PMNS} = U_{CKM}^\dagger U_{bm}$.

☞ • $\sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}} \sim 0.22 \simeq \theta_{Cabibbo}$

Top-down approach:

☞ Quarks and leptons are unified at some scale.
How to shield quark mixings from lepton mixings ?

☞ Theoretical ideas

• **New flavor structure for RH neutrinos:**

$$\mathcal{M}_\nu \simeq -M_D^T M_R^{-1} M_D$$

• **Type II seesaw:**

In Left-right and SO(10) models,

$$\mathcal{M}_\nu \simeq M_R \frac{v_{wk}^2}{v_R^2} - M_D^T M_R^{-1} M_D$$

☞ • **Double Seesaw with extra singlet fermions.**

$$\mathcal{M}_\nu \simeq m_D M^{-1} \mu M^{-1} m_D.$$

ν -mixings dictated by singlet fermion texture μ

m_ν and Grand unification

☞ M_R close to M_U

- raises the hope that seesaw scale and GUT scale have common origin
- Perhaps neutrino masses and mixings can be predicted due to higher symmetry of GUT theories;
- Unified approach to quark and lepton flavor structure.

SO(10) SUSY GUT -Just right



- unification of all **16** fermions of one generation
- $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$ fits into **16** dim. rep of SO(10);
- Contains the N_R needed for seesaw automatically
- Breaking SO(10) down
- Contains B-L symmetry whose breaking essential to get seesaw formula
- How one breaks B-L leads to different physics:

Breaking B-L

☞ Breaking B-L: 16 Higgs vrs 126 Higgs

- (a) **16-** Higgs breaking \rightarrow no dark matter without additional assumptions
- (b) **126** Higgs breaking B-L leads automatically to dark matter: no additional symmetry needed

Minimal SUSY SO(10) with 126

Babu, RNM (92); Bajc, Senjanovic, Vissani (2002); Goh, RNM, Ng (03)

☞ Gives a sum rule for neutrino mass at M_U

$\mathcal{M}_\nu = 10^{-11}(M_d - M_l)$ Note that

$$M_{d,l} = m_{b,\tau} \begin{pmatrix} \sim \lambda^4 & \sim \lambda^3 & \sim \lambda^3 \\ \sim \lambda^3 & \sim \lambda^2 & \sim \lambda^2 \\ \sim \lambda^4 & \sim \lambda^2 & 1 \end{pmatrix}$$

$\lambda =$ Cabibbo angle.

since at GUT scale $m_b \simeq m_\tau + O(\lambda^2)$

$$\mathcal{M}_\nu = m_b c \lambda^2 \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & 1 + \lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

• Gives large θ_{23} and θ_{12} ; measurable θ_{13}

• $\sqrt{\frac{\Delta m_{\odot}^2}{\Delta m_A^2}} \sim \lambda$

λ describes both Quark and Lepton flavors.

Including CP violation

☞ CP violation in the minimal model

Goh, et al (03); Bertolini, Malinsky and Schwetz (06); Babu and Macesanu (05).

☞ CP violation with 120 Higgs.

Note: $16 \otimes 16 = 10 \oplus 120 \oplus 126$.

Uses the complete minimal set-solves SUSY CP problem etc.

Dutta, Mimura and RNM. (04,05)

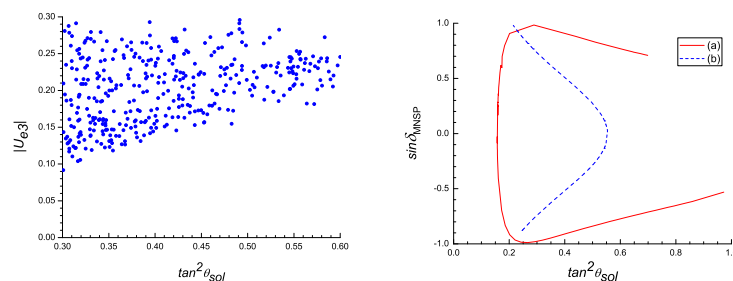


Figure 4: Prediction for θ_{13} and Dirac phase in **120** model

Predictions for lepton flavor violation in SO(10) with 126

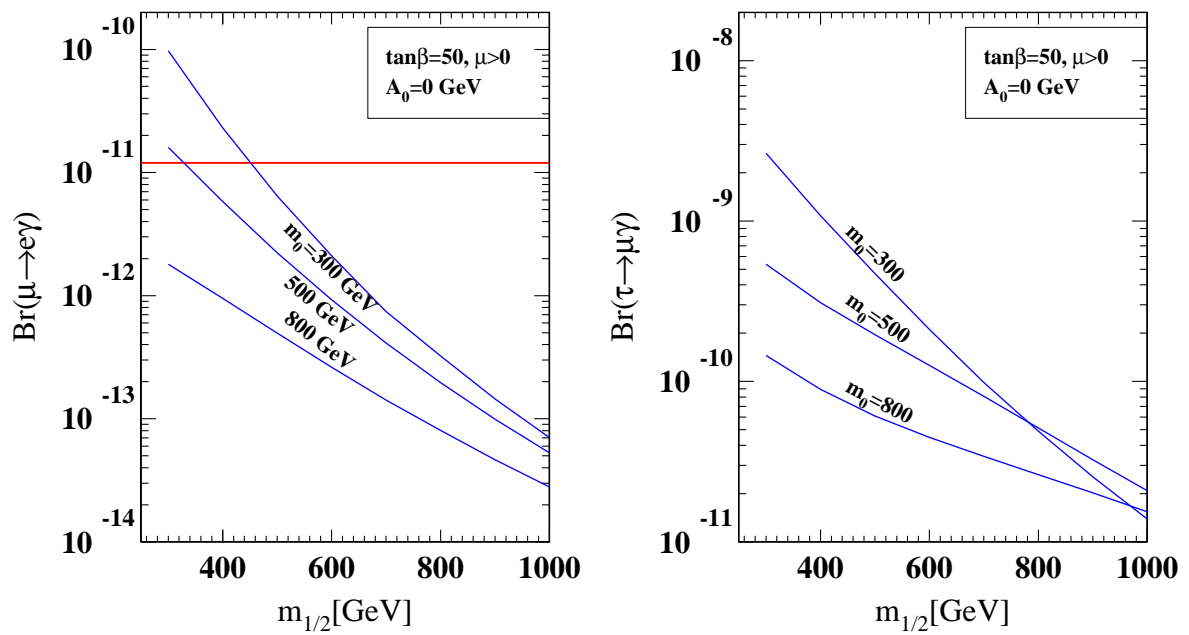


Figure 5: Typical predictions for $B(\mu \rightarrow e + \gamma)$ and $\tau \rightarrow \mu + \gamma$ in some SO(10) models. Present limit: (Los Alamos MEGA: $B \leq 1.2 \times 10^{-11}$; MEG (PSI) goal: 10^{-14})

SO(10) with 16-Higgs

☞ Many more couplings are allowed;

Large mixings come from the charged lepton mixings (lopsided) since $U_{PMNS} = U_l^\dagger U_\nu$.

Typically predicts large $B(\mu \rightarrow e + \gamma)$.

Albright, Barr (99); Ji, Li, RNM (05)

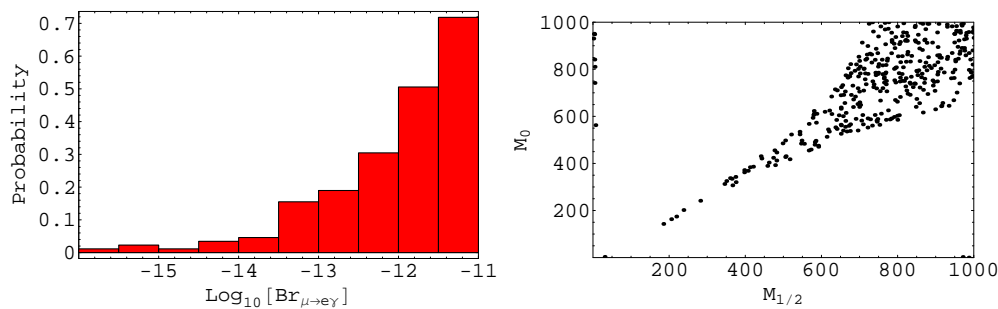


Figure 6: (A) gives the probability of $B(\mu \rightarrow e + \gamma)$ in the parameter space of the model.; (B) is the scatter plot of the parameter space satisfying current bound. (Y. Li)

Comparative survey of SO(10) models

☞ θ_{13} , n_B/n_γ and $M_{R,\text{lightest}}$

	BPW	JLM	AB	DMM
η_B	$\leq 10^{-9}$	5×10^{-10}	2.6×10^{-10}	$\leq 10^{-9}$
$\sin^2 2\theta_{13}$	≤ 0.1	0.12	0.0008	0.014 – 0.048
$M_1(\text{GeV})$	10^{10}	10^{13}	5.4×10^8	10^{13}

BPW= Babu, Pati and Wilczek model with 16-Higgs

w/Ji, Li, Nasri, Zhang 2006

Status and Outlook for the GUT approach to flavor

☞ We now have simple grand unified frameworks that correlate quark and lepton flavor structure making testable predictions !!

Given their disparate flavor structure, it was not obvious that this could be done.

Next step would be to discover flavor symmetries that would make this flavor structure follow naturally and make more testable predictions.

Possible such symmetries are S_4 , A_4 or $SU(3)$, D_5 etc.

Ross, Velasco-Sevilla; Hagedorn, Lindner, RNM; Hagedorn, Lindner, Plentinger;...

Some tests of the key theory ideas:

Generic Seesaw	$\beta\beta_{0\nu}$
SUSY GUT Seesaw	$\mu, \tau \rightarrow e + \gamma$ normal hierarchy
Type II seesaw	Quasi-deg neutrinos
Leptogenesis	low energy CP phase $d_{e,\mu}$
B-L breaking	Possible $N - \bar{N}$ Osc.
Generic SO(10)	Normal hierarchy Proton decay $\tau_p \leq 10^{36}$ yrs 10^{34} yr in some models

Testing leptonic symmetries:

$\mu - \tau$ exchange sym.	$\theta_{13} \leq 0.04$
QLC	$\theta_{13} \geq 0.08$
$\sim L_e - L_\mu - L_\tau$	Inverted hierarchy (LBL, $\beta\beta_{0\nu}$)

Conclusion



- Model building- very much a work in progress ! though we have learnt several important things.
- Seesaw- a dominant paradigm with many interesting features: e.g. origin of matter, grand unification, low energy tests in LFV etc.
- Quark lepton symmetry and $B - L$ - Nature's new symmetries.
- $SO(10)$ GUT is a natural group and testable.
- Bottom up scenarios do reveal many new symmetries which can throw light on the flavor structure of quarks. Bottom-Up and Top-Down approaches must meet in the middle.
- High precision searches for θ_{13} , mass ordering, CP phase, $\beta\beta_{0\nu}$ will "weed" out a lot of models.

• Another hint of quark-lepton connection

☞ Seesaw gives m_ν at high scale; measurements are weak scale. Extrapolation

☞ Suppose at high scale quark-lepton unification gives $\theta^q = \theta^l$ - if $m_1 \simeq m_2 \simeq m_3$, radiative corrections can magnify mixing angles.

Parida, Rajasekaran and RNM (03)

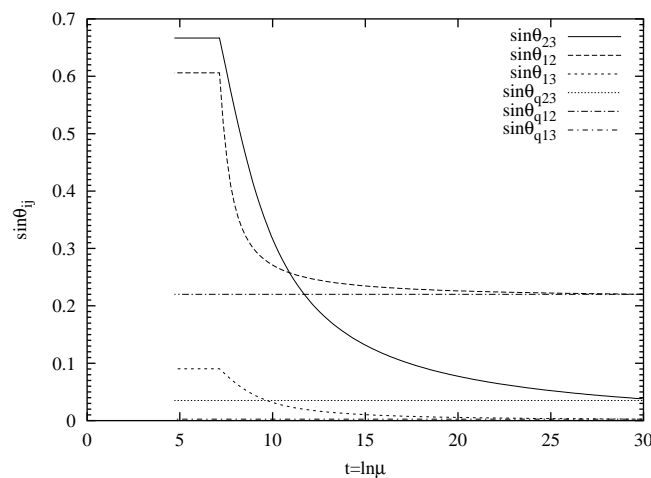


Figure 7: High scale mixing unification and Radiative magnification of mixing angles for degenerate neutrinos: from

☞ **Requires $m_1 \simeq m_2 \simeq m_3 \geq 0.1$ eV; Can be tested in the next round of $\beta\beta_{0\nu}$ experiments.**

Other model building issues

- ☞ ● If LSND is confirmed by Mini BooNe, we will need sterile neutrinos;

Other possible arguments for a sterile neutrino:

- Anomalous Pulsar Speed (up to 1000 Km/sec) have been observed: Is it hydrodynamics [Janka et al.](#) or [Neutrino Physics ?](#) (Kusenko, Segre)

;

- ☞ Possible problems with Cold Dark Matter: Predicts large number of Dwarf Galaxies and density cusp at the center of galaxies- neither well supported by observations;
- KeV sterile neutrino dark matter to the rescue
[Dodelson, Widrow; Abazajian, Fuller, Patel; ...](#)

Evidence for a sterile neutrino will revolutionize the field again.

See Kusenko's talk.

Issues with sterile neutrinos

☞ Theoretical and Cosmological

1. ν_s is singlet under standard model. Why its mass is not M_{Pl} ?
2. BBN allows at most 3.3 neutrinos; 3+1 or 3+2 scenarios for LSND \rightarrow 4 or 5. (see however Cyburt et al (2004) for a more relaxed bound of 4.5). How to reconcile if BBN allows only $\delta N_\nu \leq 4$?
3. WMAP and SDSS limit $\sum_i m_i \leq 0.3 - 1$ eV or so (2σ) for ν 's in equilibrium at BBN epoch or contributing to energy density at the recombination era.

WMAP 3 year data \rightarrow : $N_\nu \leq 3.3$.

4. For KeV WDM scenario, possible tension between structure formation and X-ray background constraints ?

Mirror models provide a consistent framework.

☞ Coexisting duplicate of the standard model-inspired by superstring theories

➤ Standard model \otimes Standard model-prime .

visible sector	mirror sector
$SU(2)_L \times U(1)_{I_{3R}}$ $\times U(1)_{B-L}$	$SU(2)_L \times U(1)_{I_{3R}}$ $\times U(1)_{B-L}$
$W, Z, \gamma, \text{ gluons}$ $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R Higgs H, Δ_R	$W, Z, \gamma, \text{ gluons}$ $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R H', Δ'_R

Requires a light scalar boson

☞ $\mathcal{L} = h\nu_s\nu\phi;$

mass $m_\phi \sim 10 - 50$ KeV.

Speculations about a scalar couplings to neutrinos:

Goldman, Stephenson and McKeller;
Chacko, Hall, Oliver, Perelstein; RNM, Nasri (2005) et al