Electromagnetic Structure Functions and Neutrino Nucleon Scattering

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May 5, 2006

Introduction

- It has been know for a long time that in the few GeV energy region, the quasi-elastic, few pion and inclusive contributions to the cross section are nearly equal. Lipari, Lusignoli and Sartogo, 1995 made the standard plot

- All components important to understand neutrino oscillation experiments, the balance of which depends on e.g., the minimum invariant mass of the final hadronic state, $W^2_{\rm min}$. Recent work by Kuzmin, Lyubushkin, Naumov, hep-ph/0511308 attempts to find the $W_{\rm min}$ so that the components best represent current neutrino measurements.

- The inelastic component is not currently well calculated in this energy regime because of the necessity of low-Q^2 structure functions.

- This talk is about extrapolations to low-Q^2 of structure functions for $W^2 > W^2_{\rm min}$.

- I'll assume local quark-hadron duality.

- Target mass corrections: work with Stefan Kretzer, Phys. Rev. D66 and Phys. Rev. D 69.

Plan

- Brief review neutrino scattering in NLO QCD with target mass corrections (TMC) and the importance of the low-Q^2 contribution to the cross section.

- Comparison of NLO+TMC with a parameterization of F_2^{ep}. (NLO+TMC overestimates F_2 at low Q^2.)

- The Capella, Kaidalov, Merino and Thanh Van (CKMT) parameterization of F_2^{ep} and the Bodek-Yang-Park parameterization.

- The translation to νN scattering.

- Reevaluated cross sections with these two extrapolations at low-Q^2.

- Summary.

Mass Corrections

Differential cross section (CC) m=muon mass, M=nucleon mass:

$$\begin{aligned}\frac{d\sigma}{dx\,dy} &= \frac{G_F^2 ME}{\pi(1+Q^2/M_W^2)^2}\left[\left(xy^2+\frac{m^2 y}{2ME}\right)F_1^{TMC}\right.\\ &+ \left(1-\frac{m^2}{4E^2}-y-\frac{Mxy}{2E}\right)F_2^{TMC}+\left(xy-\frac{xy^2}{2}-\frac{m^2 y}{4ME}\right)F_3^{TMC}\\ &+ \left.\left(\frac{m^2 y}{2ME}+\frac{m^4}{4M^2E^2x}\right)F_4^{TMC}-\frac{m^2}{ME}F_5^{TMC}\right]\end{aligned}$$

TMC

TMC corrections come from:

- $x \to \xi$ with

$$\frac{1}{\xi} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2} + \frac{M^2}{Q^2}} \iff \xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}}$$

- A "mismatch" between quark momentum p and nucleon momentum P: proton momentum $P^2 = M^2$ and incident parton momentum $p^2 = 0$, then $p^+ = \xi P^+$, but $p^- \neq \xi P^-$.

- Including non-collinear partons in the nucleon, with $k_T < M$. R.K. Ellis et al.

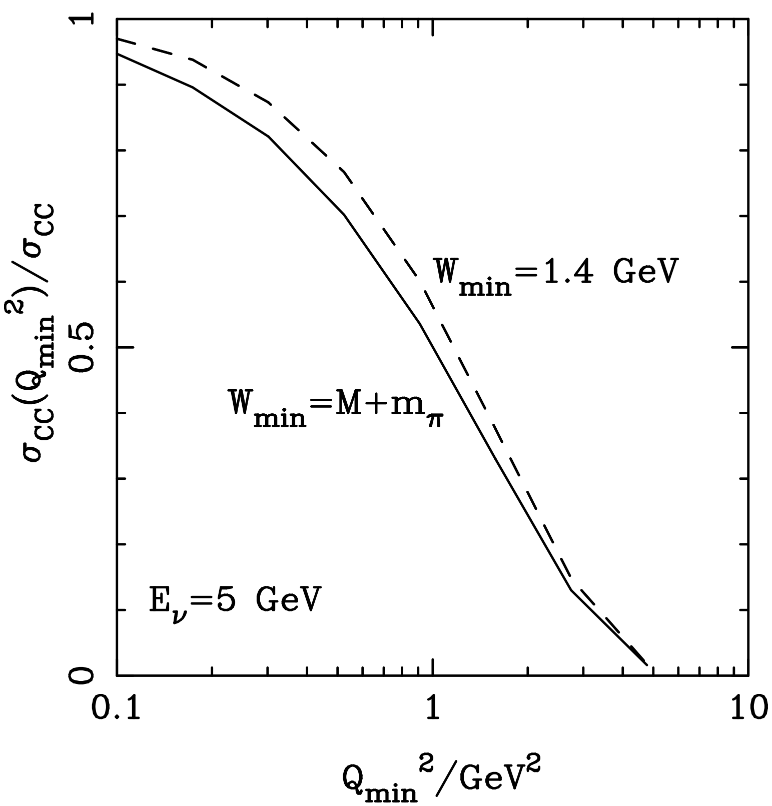
With the identifications:

$$\begin{aligned} \rho^2 &= 1 + \frac{4M^2x^2}{Q^2} \\ \mathcal{F}_2 &= q(\xi, Q^2) + \bar{q}(\xi, Q^2) \end{aligned}$$

Georgi and Politzer [PRD 14 (1976)], Barbieri et al., and Georgi, Politzer and deRujula [Ann. Phys. 103 (1977), where $2xF_1 = F_2$ is not assumed]. The results, for example, for F_2:

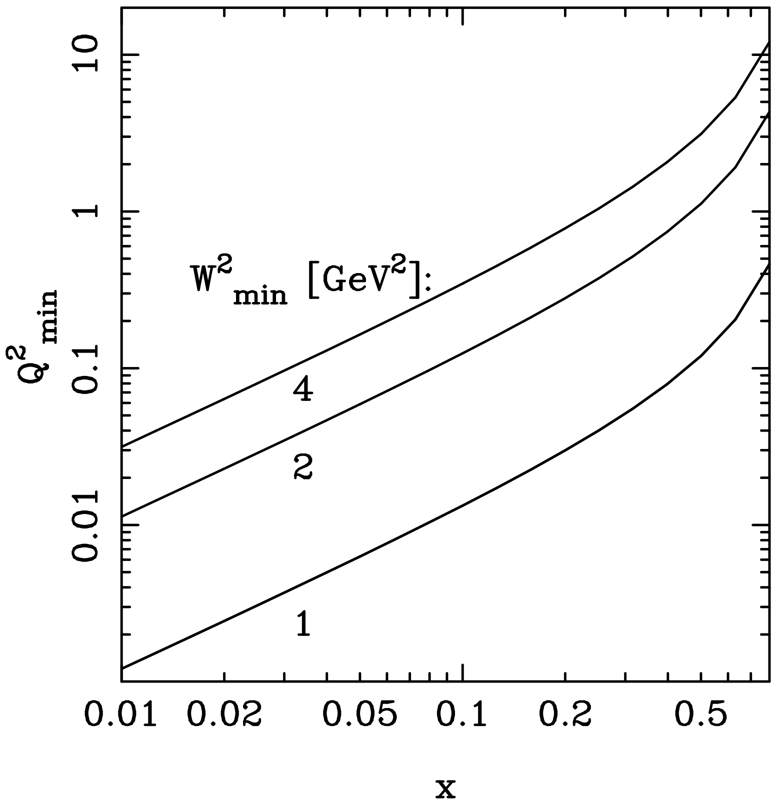
$$\begin{aligned} F_2^{TMC}(x, Q^2) &= 2\,\frac{x^2}{\rho^3}\frac{\mathcal{F}_2(\xi, Q^2)}{\xi} + 12\,\frac{M^2}{Q^2}\,\frac{x^3}{\rho^4}\int_\xi^1 d\xi' \frac{\mathcal{F}_2(\xi', Q^2)}{\xi'} \\ &\quad + 24\,\frac{M^4}{Q^4}\,\frac{x^4}{\rho^5}\int_\xi^1 d\xi' \int_{\xi'}^1 \frac{\mathcal{F}_2(\xi'', Q^2)}{\xi''} \end{aligned}$$

DIS CC cross sections



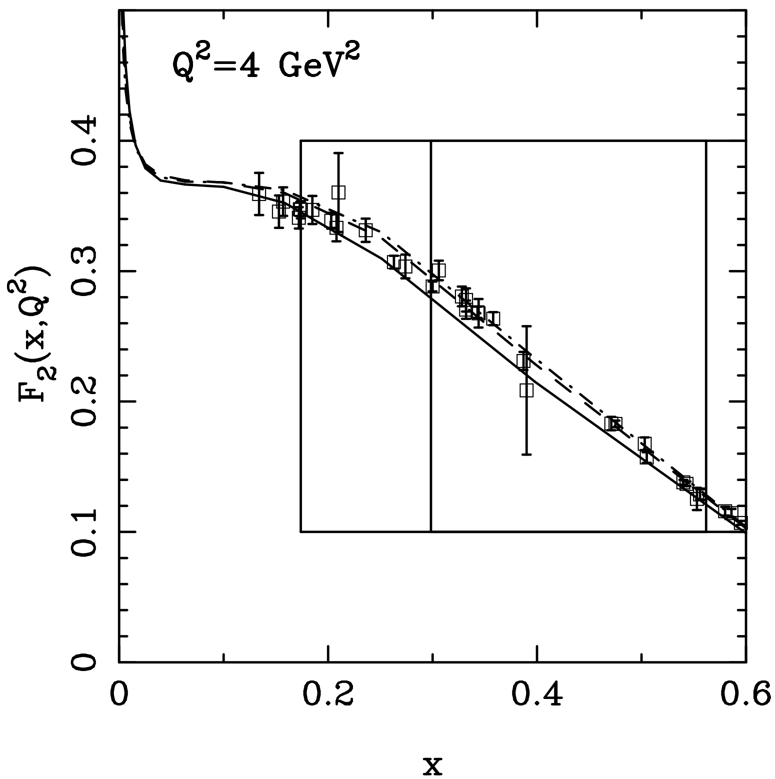
- Neutrino-nucleon CC cross section for $Q^2 > Q^2_{\rm min}$ normalized to the νN cross section.
- Calculated using NLO+TMC.
- Half the cross section comes from $Q^2 < 1$ GeV^2.

What values of x?



- $W^2 = Q^2(\frac{1}{x} - 1) + M^2$
- $x > Q^2/(2ME_\nu)$

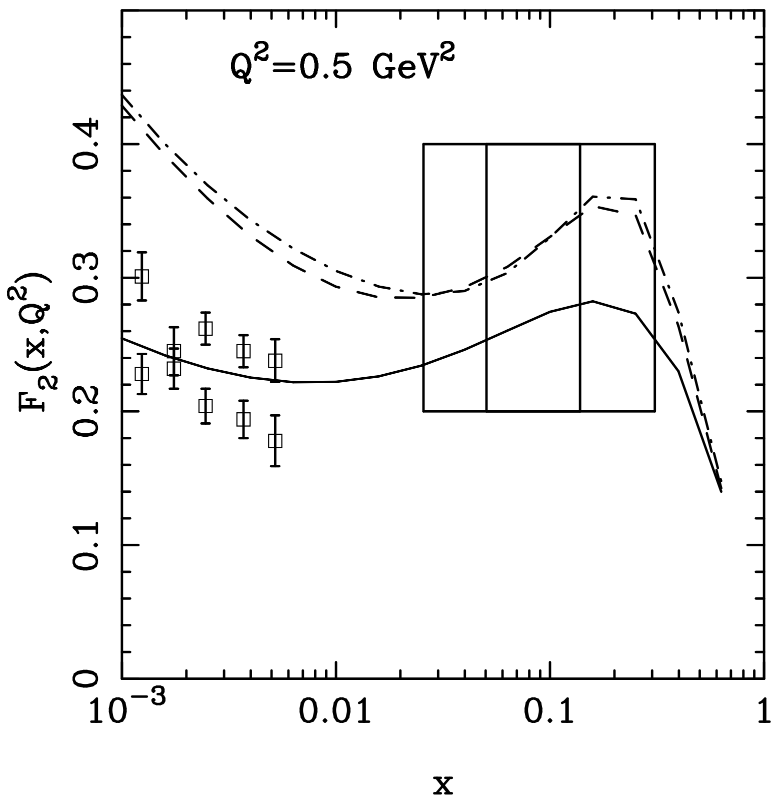
$F_2^{ep},\ Q^2 = 4$ **GeV^2**



Use the Abramowicz, Levin, Levy and Maor (ALLM) parameterization (solid) of F_2 represent ep data. ALLM, Phys. Lett. 1991, AL hep-ph/9712415. This has 23 parameters.

Also shown, NLO+TMC and NNLO+TMC and SLAC data for $Q^2 = 3.7 - 4.3$ GeV^2. L. Whitlow et al., Phys. Lett. B (1990).

$F_2^{ep},\ Q^2 = 0.5$ GeV^2



ALLM (solid), data from E665 M. Adams et al., Phys. Rev. D 54 (1996) with $Q^2 = 0.43,\ 0.59$ GeV^2 NLO+TMC, NNLO+TMC.

Capella, Kaidalov, Merino and Thanh Van

CKMT, Phys. Lett. B 337, 358 (1994), Moriond 1994, 7 parameters in

$$
\begin{aligned}
F_2(x,Q^2) &= F_2^{sea}(x,Q^2) + F_2^{val}(x,Q^2) \\
&= Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} \\
&+ Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R} \\
&\times \Big(1+f(1-x)\Big)
\end{aligned}
$$

CKMT Valence in ep scattering

CKMT fit $\alpha_R = 0.4250$ and $b = 0.6452$ GeV^2.

$$F_2^{val}(x,Q^2) = Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}\Big(1+f(1-x)\Big)$$

$B = B_u$ is calculated to be 1.2064, $f = B_d/B_u = 0.15$ is also calculated. They are calculated invoking valence counting rules at $Q^2 = 2$ GeV^2. Also fit is $c = 3.5489$ GeV^2 in

$$n(Q^2) = \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right)$$

CKMT "Sea" in ep scattering

CKMT fit $A = 0.1502$ and $a = 0.2631$ GeV^2.

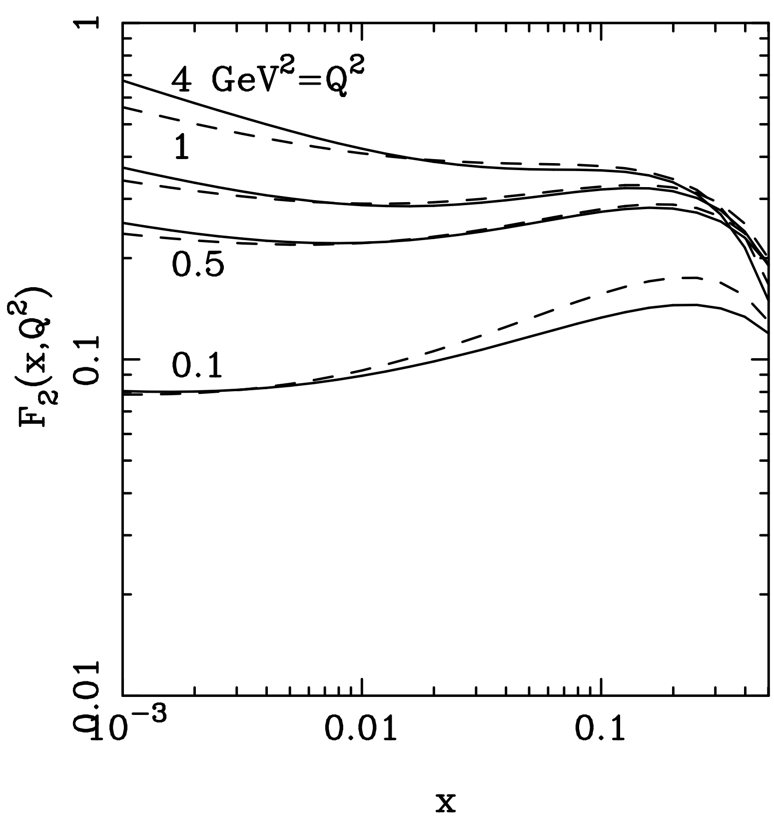
$$F_2^{sea}(x,Q^2) = Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}$$

Also fit is $\Delta_0 = 0.07684$ and $d = 1.1170$ GeV^2 in

$$\Delta(Q^2) = \Delta_0\left(1+\frac{2Q^2}{Q^2+d}\right)$$

Δ_0 is similar to power law in generalized vector meson dominance at low Q^2, where it is pomeron dominated.

Comparison: ALLM and CKMT in ep scattering



ALLM (solid), and CKMT (dashed).

CKMT in νN scattering

See CKMT Moriond Proceedings.

- F_2^{sea} changes only overall normalization: $A \to A_\nu = 0.60$, which I fixed at $Q^2 = 10$ GeV^2 to match reasonably well with the NLO+TMC evaluation.

- Note: $A_\nu / A \simeq 4$ in sea part. For electromagnetic case

$$\frac{1}{9}x(d+\bar{d}+s+\bar{s})+\frac{4}{9}x(u+\bar{u}) \simeq \left(\frac{3}{9}+\frac{8}{9}\right)xq_{sea} = \frac{11}{9}xq_{sea}$$

For CC case, with $\bar{u} = \bar{d} \simeq 2s = q_{sea}$,

$$2x(d+s+\bar{u}) \simeq 5xq_{sea}$$

CKMT in νN scattering

- Expect that the underlying non-perturbative process is governed by the same $\Delta(Q^2)$ and form factor $\left(Q^2/(Q^2+a)\right)^{1+\Delta}$.

- For the valence part, recalculate B and f at $Q^2 = 2$ GeV^2. I get

$$B_\nu = 2.695 \qquad f_\nu = 0.595$$

- Valence x and Q^2 dependence shouldn't change between electromagnetic and charged current scattering.

CKMT for F_1

For F_1, use

$$R = \frac{F_2}{2xF_1}\left(1 + \frac{4M^2x^2}{Q^2}\right) - 1$$

with a parameterization of R from Whitlow et al., Phys. Lett. 1990. Below $Q^2 = 0.3$ GeV^2, rescale the value at 0.3 GeV^2 by $Q^2/(0.3$ GeV$^2)$.

CKMT for F_3

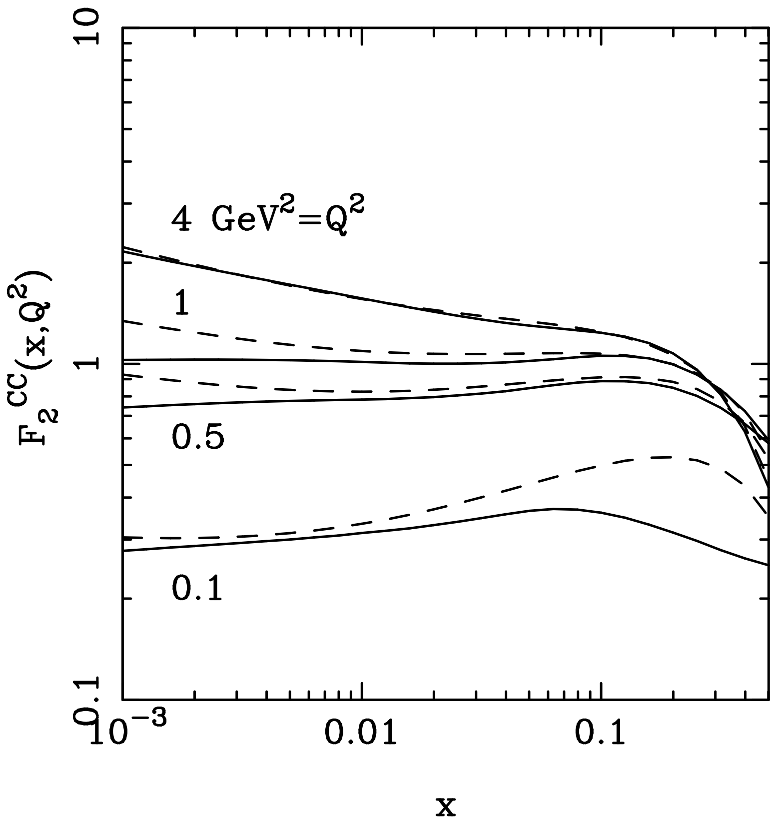
For F_3, use

$$
\begin{aligned}
F_3(x,Q^2) &= \Bigg[\frac{A_\nu}{15}x^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} \\
&+ B_\nu x^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R} \\
&\times \Big(1+f_\nu(1-x)\Big)\Bigg]/(1.1x)\ .
\end{aligned}
$$

- The denominator of 1.1 adjusts the integral of the valence part to give a Gross-Llewellyn-Smith sum rule result of 3×0.9 (QCD corrected).

- The normalization of the sea term is a little ad-hoc. It should be the s quark contribution. The choice above is not bad in comparison to NLO+TMC at $Q^2 = 4$ GeV^2. (I did not try to fine tune this parameter.)

Comparison: BYP and CKMT in νN scattering



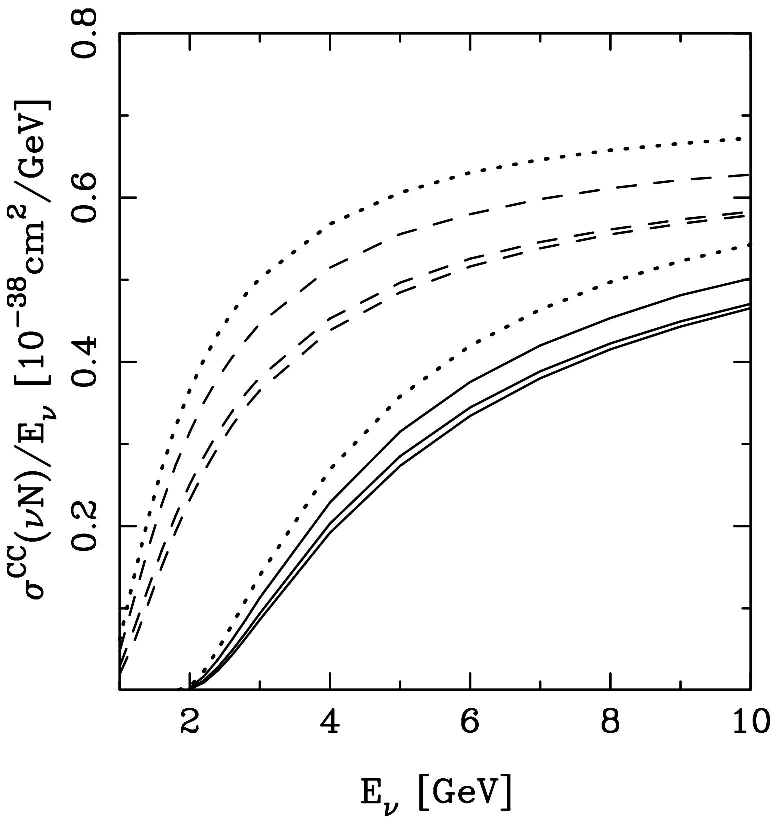
Bodek-Yang-Park (BYP) (solid), and CKMT (dashed).

Strategy for cross sections

- Use NLO+TMC in for $Q^2 > Q_0^2$. Attach a parameterization for $Q^2 < Q_0^2$. Should be insensitive to Q_0^2.

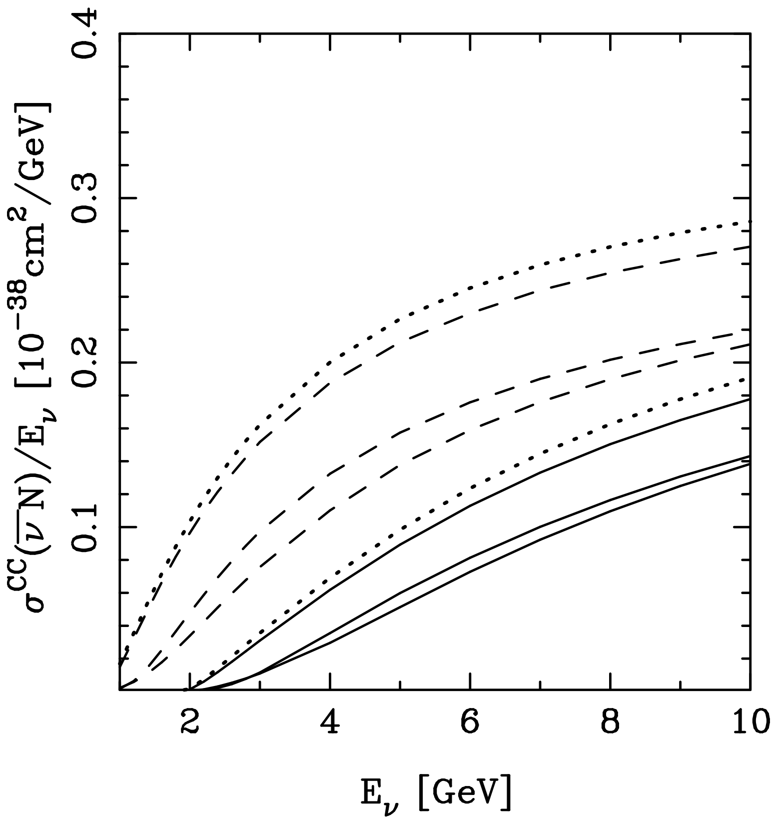
- Results shown for $Q_0^2 = 4$ GeV^2.

νN CC cross section



- Solid lines, $W^2_{\rm min} = 4$ GeV^2, dashed lines for $W^2_{\rm min} = 2$ GeV^2.
- Upper solid and dashed are NLO+TMC, lower two are CKMT and BYP extrapolations below Q_0^2.
- Dotted lines show LO+TMC.

$\bar{\nu}N$ CC cross section



- Solid lines, $W^2_{\min} = 4$ GeV^2, dashed lines for $W^2_{\min} = 2$ GeV^2.
- Upper solid and dashed are NLO+TMC, lower two are CKMT and BYP extrapolations below Q^2_0.
- Dotted lines show LO+TMC.

Summary

- The CKMT and BYP extrapolations yield similar results on the cross sections. CKMT is slightly larger.
- The neutrino cross section is reduced by 7-8% for $W^2_{\rm min} = 2$ GeV^2 at 10 GeV, 11-13% at 5 GeV, relative to the NLO+TMC result.
- Antineutrino scattering is impacted more, with changes of order 20% at 10 GeV.
- CKMT parameterization has a simple interpretation. One can rescale the standard sea and valence PDFs by the same Q^2 dependent factors in the CKMT parameterization and get essentially the same results.

- Calculate $F_i(x, Q^2)$ using NLO+TMC above Q_0^2.
- For $Q^2 < Q_0^2$, rescale the separate sea and valence components of $F_i(x, Q_0^2)$ by e.g. $F_2^{sea}(x, Q^2)/F_2^{sea}(x, Q_0^2)$.

- I look forward to more measurements of neutrino structure functions and cross sections!