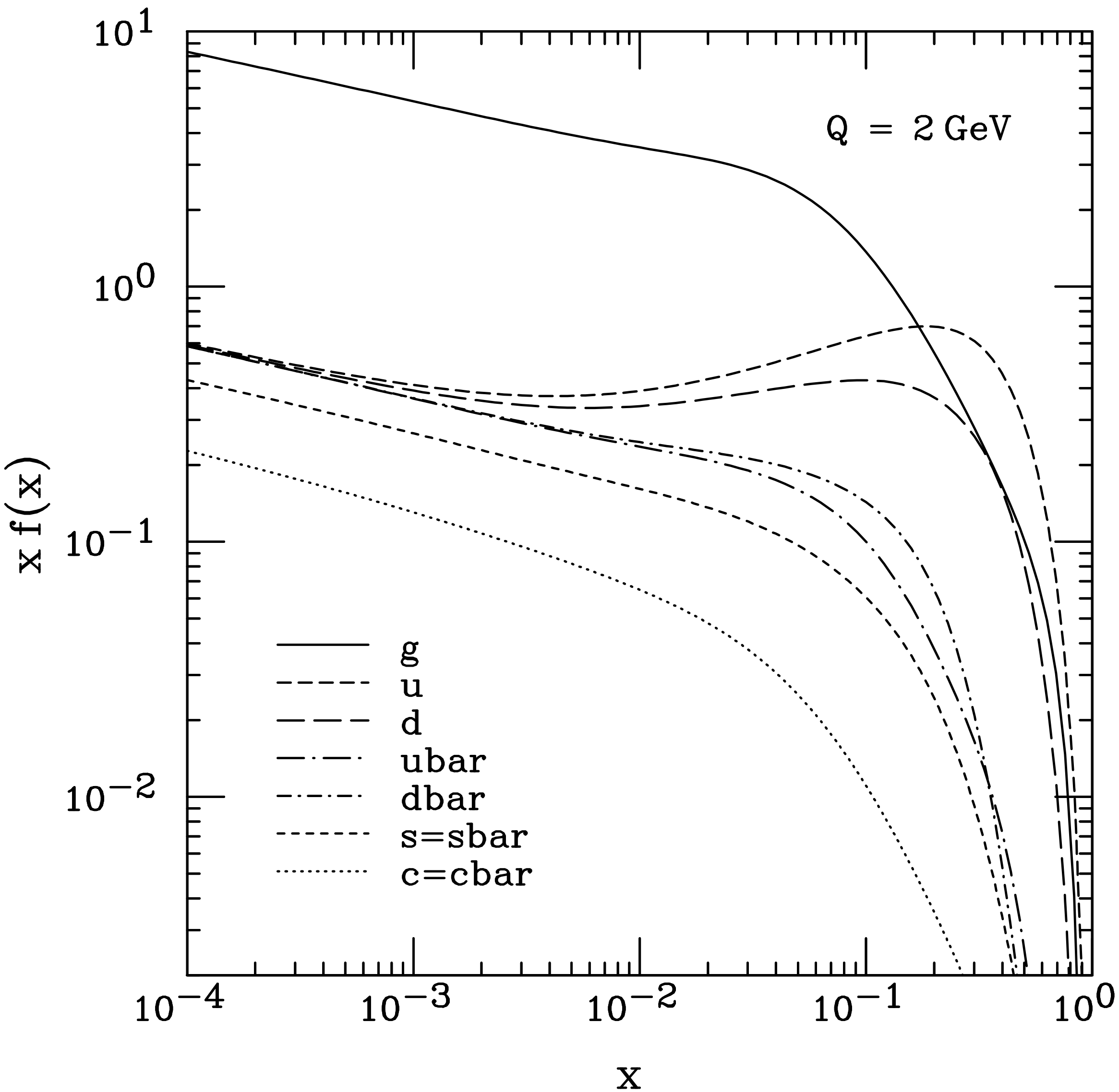
Workshop on Intersections of Nuclear Physics
with Neutrinos and Electrons
JLab, May 4-5, 2006

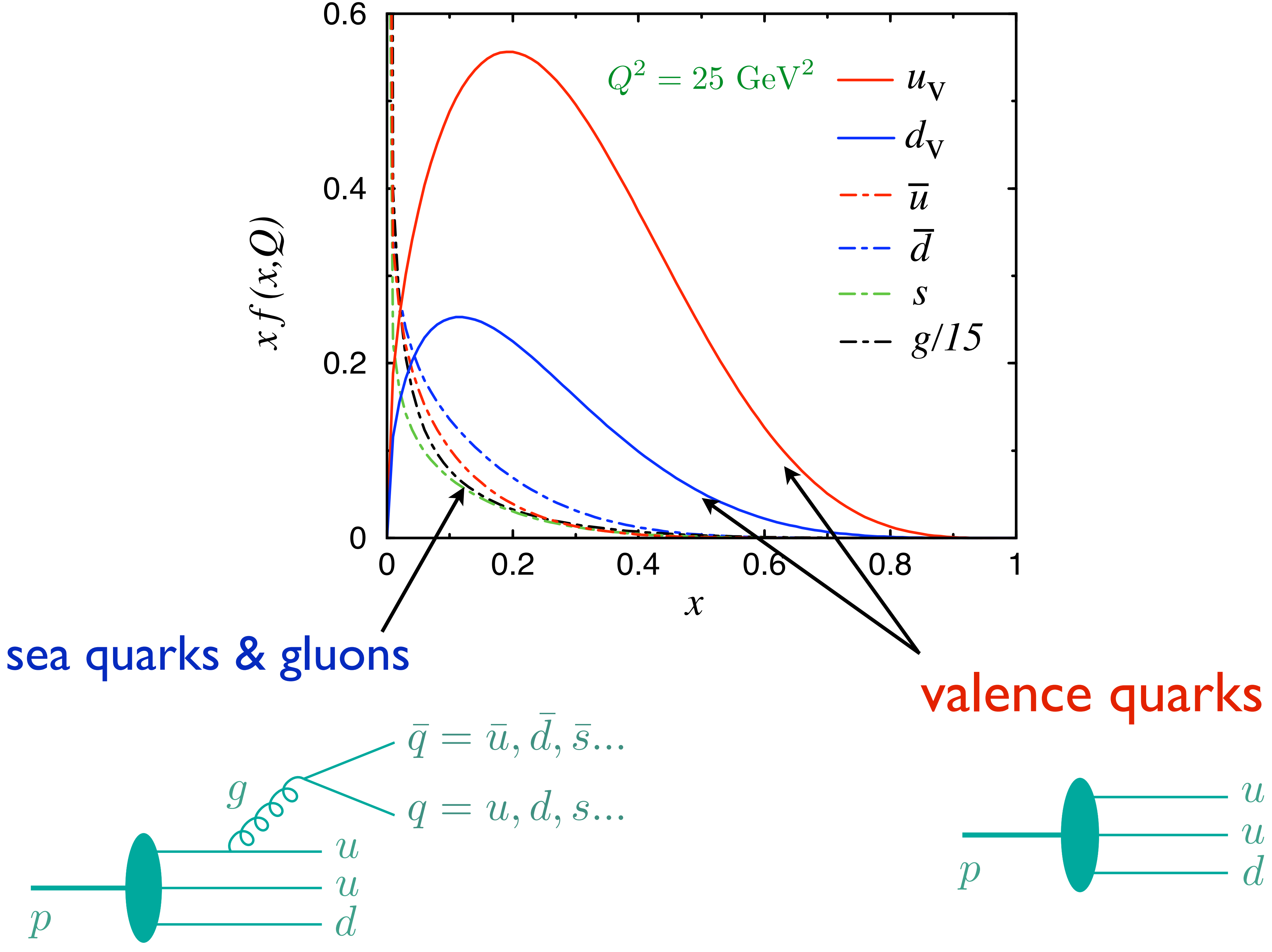
Quark distributions at large x

Wally Melnitchouk
Jefferson Lab



Lai et al., Eur. Phys. J. C12 (2000) 375

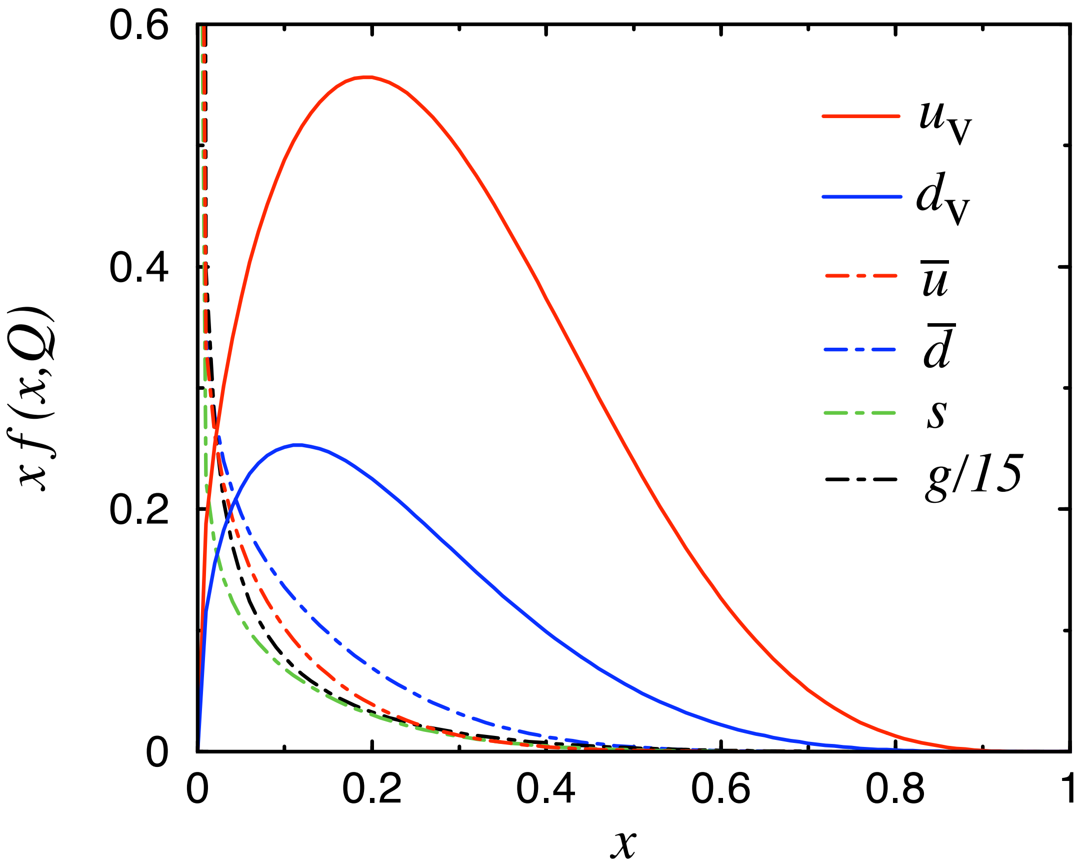
Significant advances in determination of quark and gluon distributions at small x in recent years



0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
$x\,f(x,Q)$
x
$Q^2 = 25\ \mathrm{GeV}^2$
u_V
d_V
$\bar{u}$
$\bar{d}$
s
$g/15$
sea quarks & gluons
valence quarks
$\bar{q} = \bar{u}, \bar{d}, \bar{s}...$
$q = u, d, s...$
g
u
u
d
p
u
u
d
p

Valence quarks

- Nucleon structure at intermediate & large x dominated by *valence* quarks
- Most direct connection between quark distributions and models of the nucleon is through valence quarks



Valence quarks

- At large x, valence u and d distributions extracted from p and n structure functions

$$F_2^p \approx \frac{4}{9} u_v + \frac{1}{9} d_v$$

$$F_2^n \approx \frac{4}{9} d_v + \frac{1}{9} u_v$$

- u quark distribution well determined from p

- d quark distribution requires n structure function

$$\Longrightarrow \qquad \frac{d}{u} \approx \frac{4 - F_2^n / F_2^p}{4 F_2^n / F_2^p - 1}$$

Valence quarks

- Ratio of d to u quark distributions particularly sensitive to quark dynamics in nucleon

- SU(6) spin-flavour symmetry

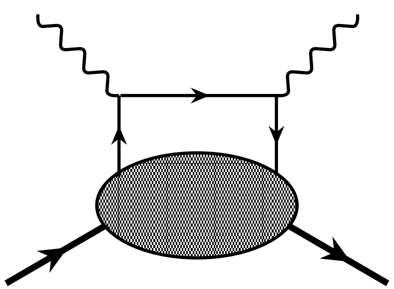
proton wave function

$$p^\uparrow = -\frac{1}{3}d^\uparrow(uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow(uu)_1 + \frac{\sqrt{2}}{6}u^\uparrow(ud)_1 - \frac{1}{3}u^\downarrow(ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow(ud)_0$$

interacting quark

spectator diquark

diquark spin



Valence quarks

- Ratio of d to u quark distributions particularly sensitive to quark dynamics in nucleon

- SU(6) spin-flavour symmetry

proton wave function

$$p^{\uparrow} = -\frac{1}{3}d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3}d^{\downarrow}(uu)_1$$
$$+ \frac{\sqrt{2}}{6}u^{\uparrow}(ud)_1 - \frac{1}{3}u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}}u^{\uparrow}(ud)_0$$

$\longrightarrow$ $u(x) = 2\ d(x)$ for all x

$\longrightarrow$ $\frac{F_2^n}{F_2^p} = \frac{2}{3}$

Valence quarks

- scalar diquark dominance

$M_\Delta > M_N \implies (qq)_1$ has larger energy than $(qq)_0$

$\implies$ scalar diquark dominant in $x \to 1$ limit

since only u quarks couple to scalar diquarks

$\longrightarrow \quad \dfrac{d}{u} \to 0$

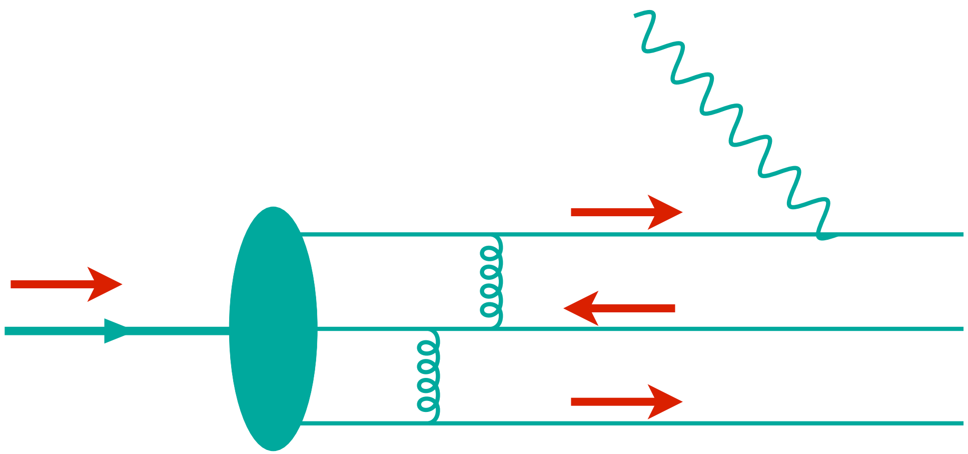
$\longrightarrow \quad \dfrac{F_2^n}{F_2^p} \to \dfrac{1}{4}$

Feynman 1972, Close 1973, Close/Thomas 1988

Valence quarks

- <u>hard gluon exchange</u>

at large x, helicity of struck quark = helicity of hadron



$$q^{\uparrow} \gg q^{\downarrow}$$

$\Longrightarrow$ <u>helicity-zero diquark dominant in $x \to 1$ limit</u>

$\longrightarrow$ $$\frac{d}{u} \to \frac{1}{5}$$

$\longrightarrow$ $$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$$

Farrar, Jackson 1975

Valence quarks

- BUT no free neutron targets!
  (neutron half-life ~ 12 mins)

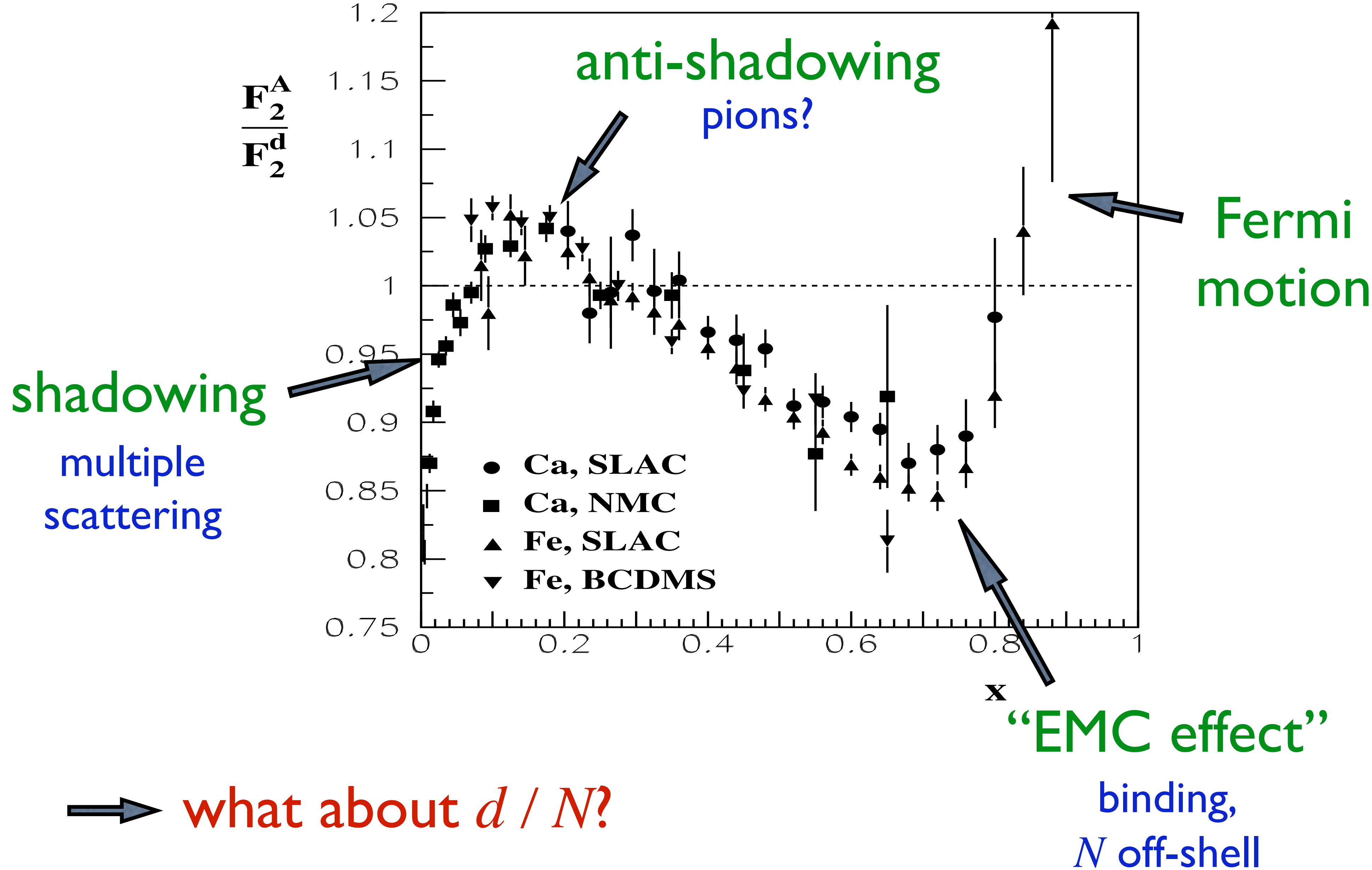
  ⟹ use deuteron as "effective neutron target"

- However: deuteron is a nucleus, and $F_2^d \neq F_2^p + F_2^n$

  ⟹ nuclear effects (nuclear binding, Fermi motion, shadowing) *obscure neutron structure* information

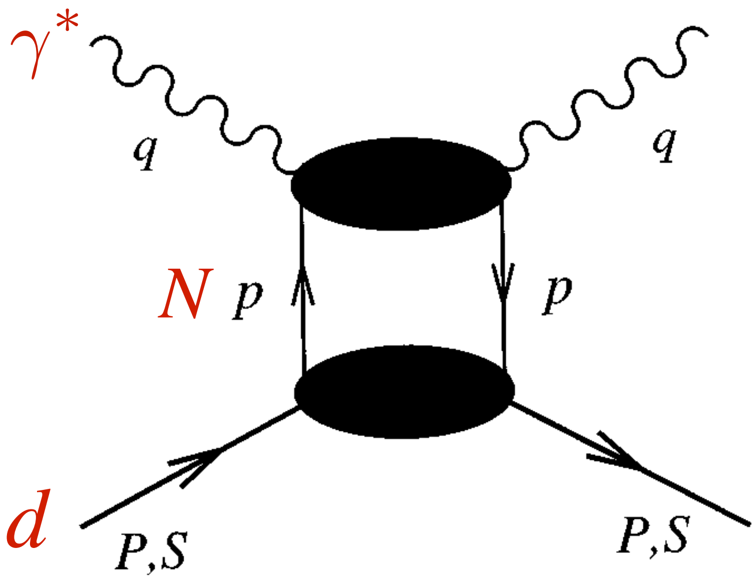
  ⟹ "nuclear EMC effect"

Nuclear “EMC effect”



→ what about *d* / *N*?

EMC effect in deuteron



Nuclear "impulse approximation"

→ incoherent scattering from individual nucleons in deuteron

$$F_2^d(x) = \int dy\ f_{N/d}(y)\ F_2^N(x/y)\ + \delta^{(\text{off})} F_2^d(x)$$

nucleon momentum distribution

off-shell correction

EMC effect in deuteron

Nucleon momentum distribution in deuteron

→ relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\rm max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

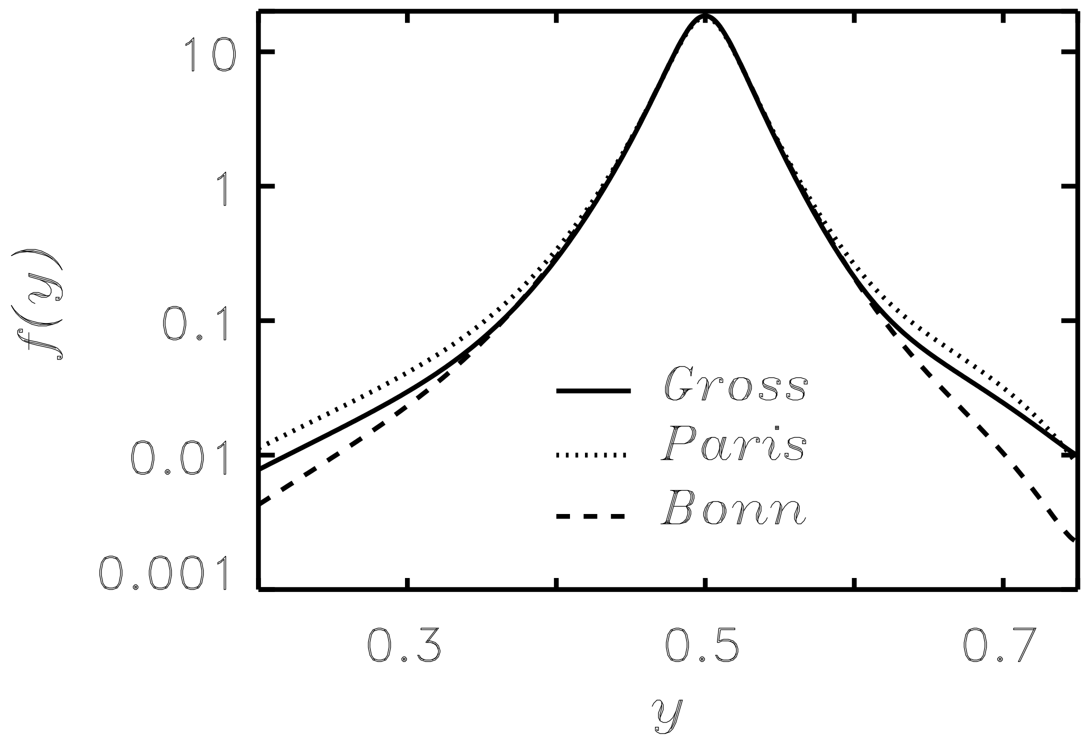
momentum fraction of deuteron carried by nucleon

EMC effect in deuteron

Nucleon momentum distribution in deuteron

→ relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$



EMC effect in deuteron

Nucleon momentum distribution in deuteron

➡ relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

Wave function dependence only at large $|y\text{-}1/2|$

➡ sensitive to large p components of wave function

➡ not very well known

EMC effect in deuteron

Nucleon momentum distribution in deuteron

→ relativistic dNN vertex function

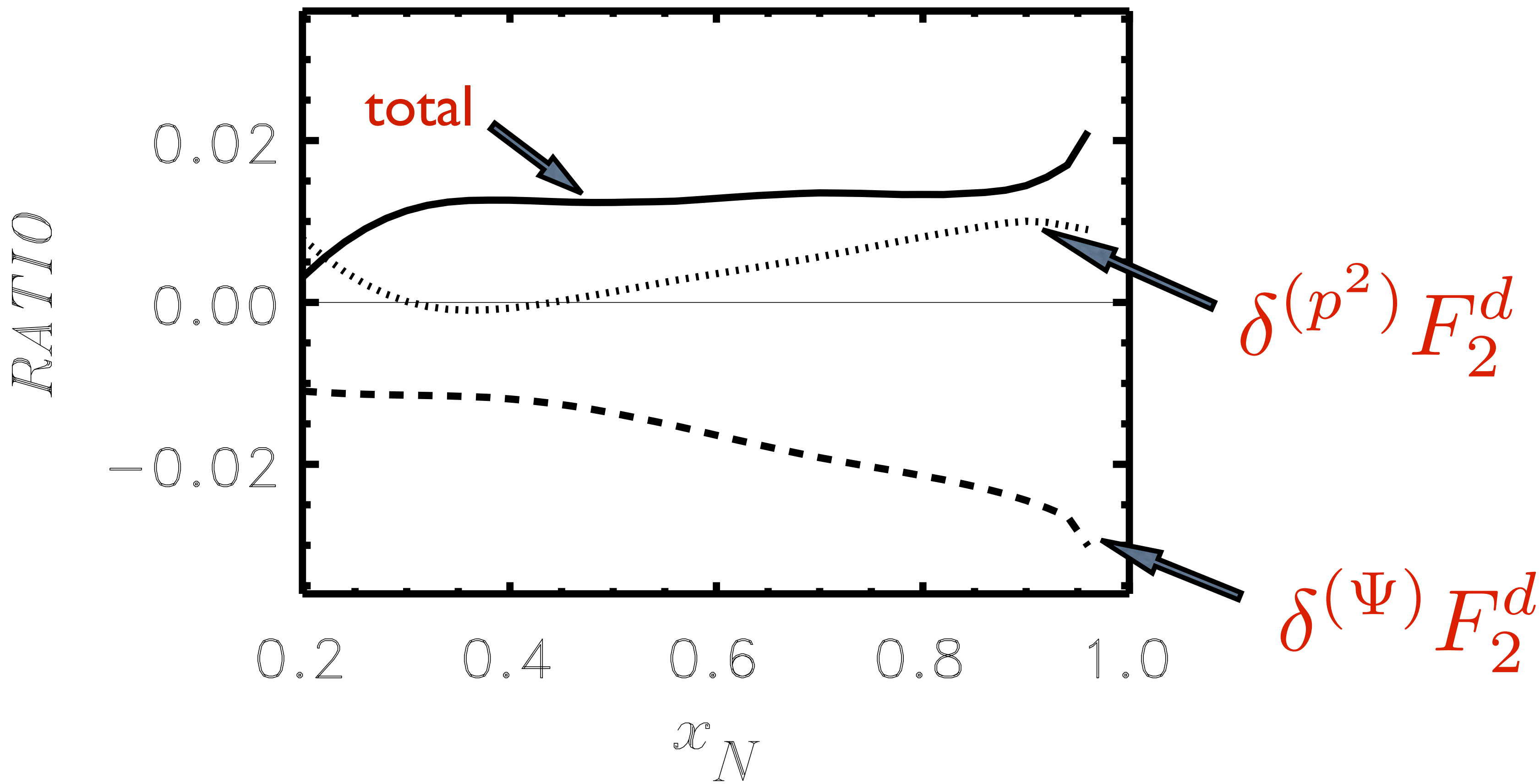
$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\rm max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

Nucleon off-shell correction

$\delta^{(\rm off)} F_2^d$ → $\delta^{(\Psi)} F_2^d$ negative energy components of d wave function

→ $\delta^{(p^2)} F_2^d$ off-shell N structure function

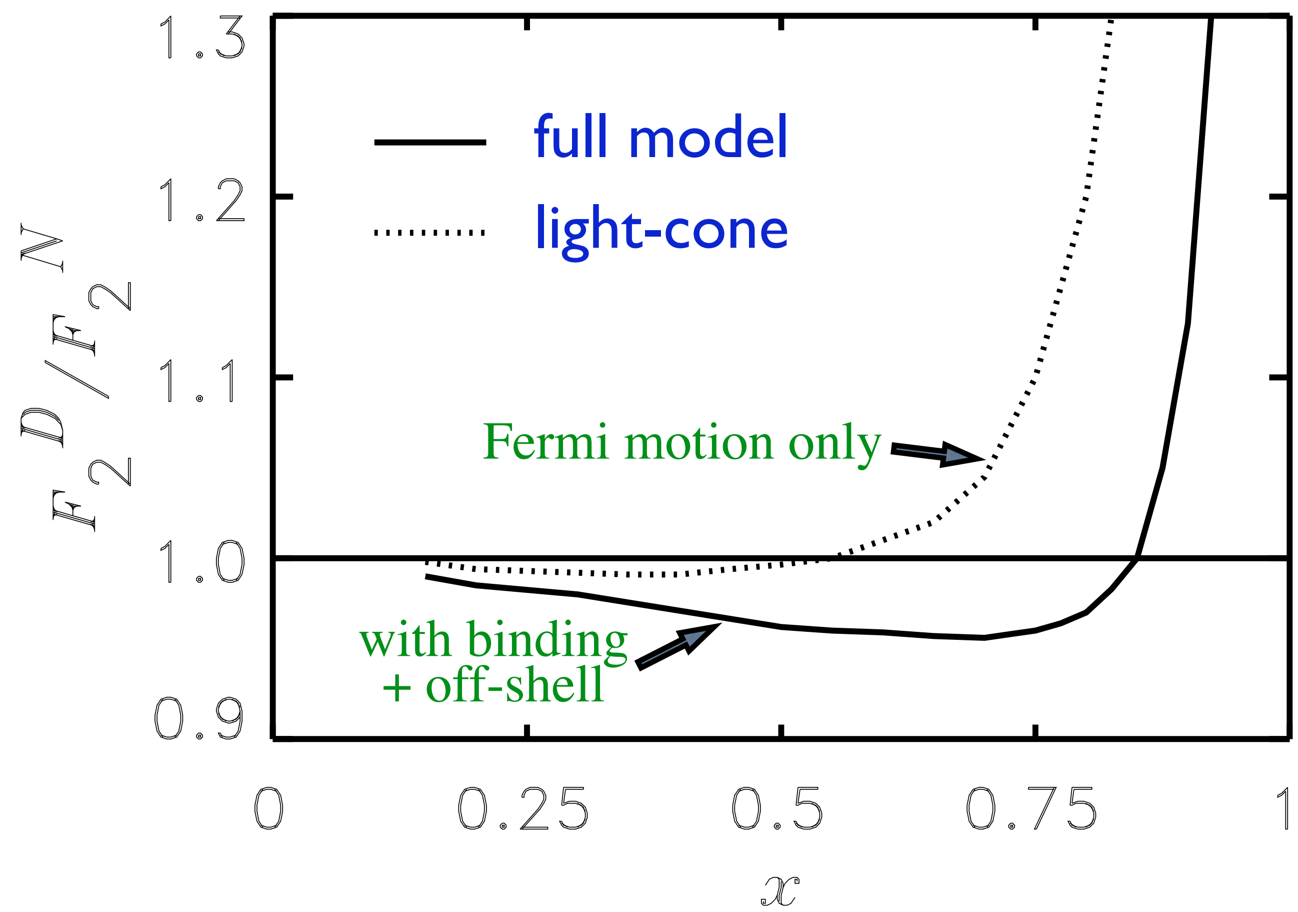
Off-shell correction



→ $\leq 1 - 2\ \%$ **effect**

WM, Schreiber, Thomas, Phys. Lett. B 335 (1994) 11

EMC effect in deuteron



Larger EMC effect (smaller d/N ratio)

→ F_2^n underestimated at large x

Unsmearing

Note: calculated d/N ratio depends on input F_2^n

⟹ extracted n depends on input n ... cyclic argument

Solution: iteration procedure

0. subtract $\delta^{(\mathrm{off})} F_2^d$ from d data: $F_2^d \rightarrow F_2^d - \delta^{(\mathrm{off})} F_2^d$

1. smear F_2^p with $f_{N/d}$: $f_{N/d} \otimes F_2^p \equiv S_p\, F_2^p$

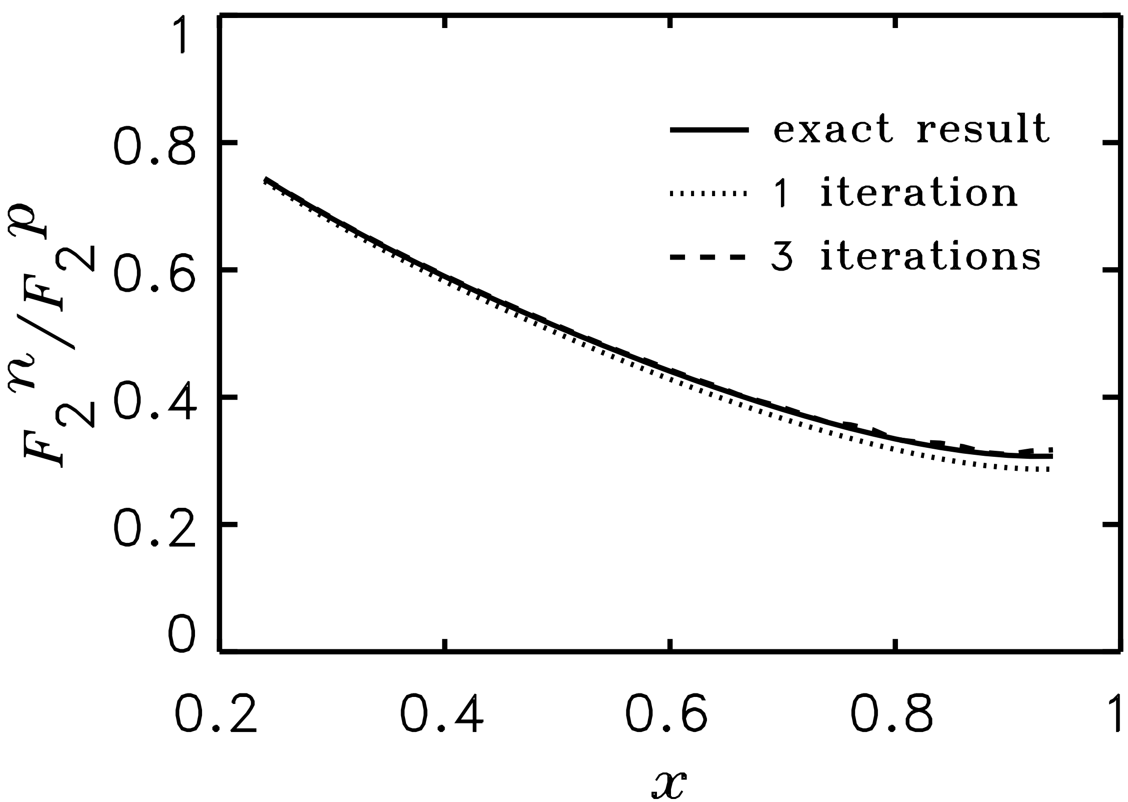
2. extract neutron via $F_2^n = S_n (F_2^d - F_2^p / S_p)$

   starting with $e.g.$ $S_n = S_p$

3. smear F_2^n with $f_{N/d}$ to get new S_n

4. repeat 2-3 until convergence

Unsmearing

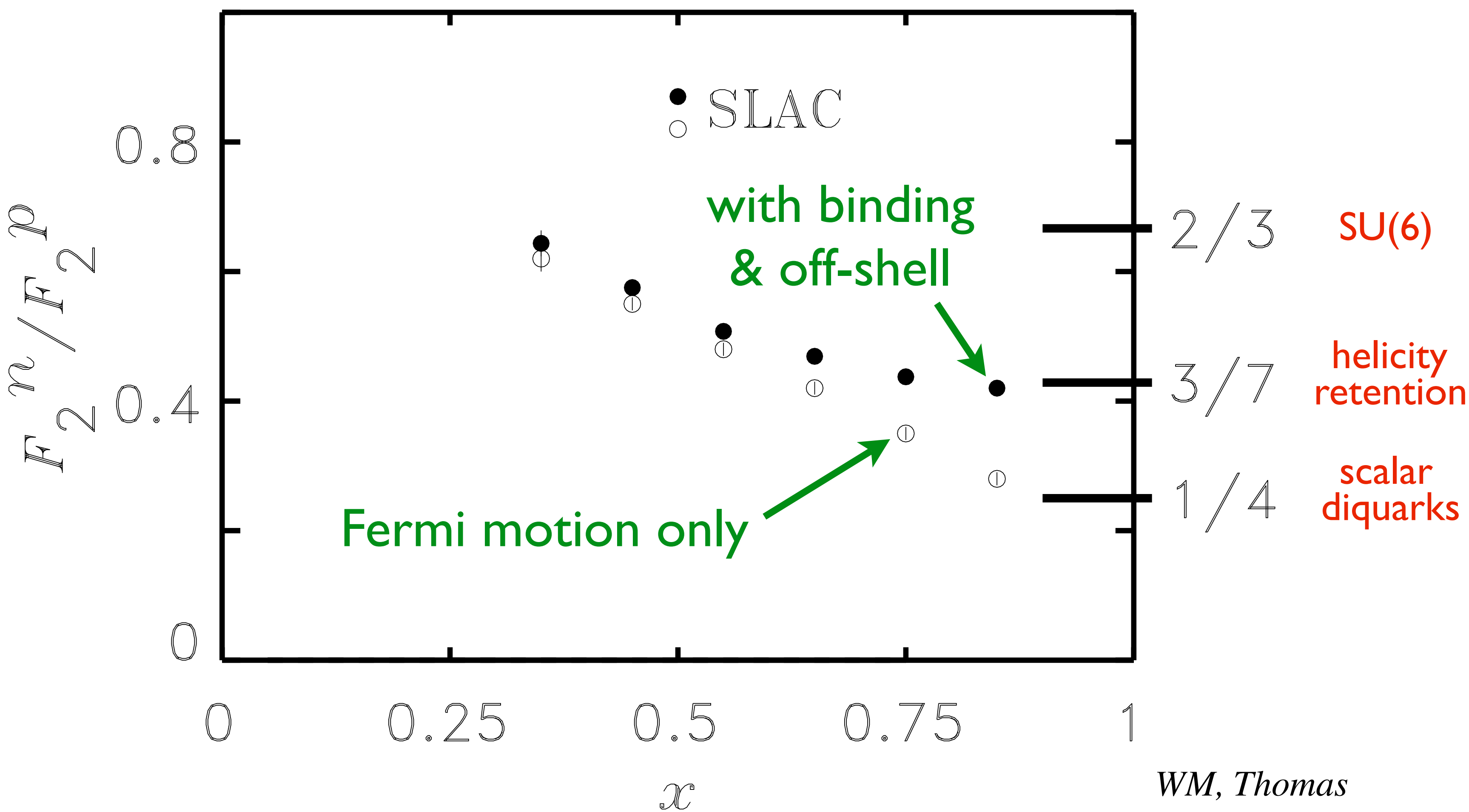


Afnan, Bissey, Gomez, Liuti, WM, Thomas et al., Phys. Rev. C68 (2003) 035201

- good convergence after several iterations
- resulting F_2^n <u>independent</u> of starting assumptions

depends only on smearing function $f_{N/d}$

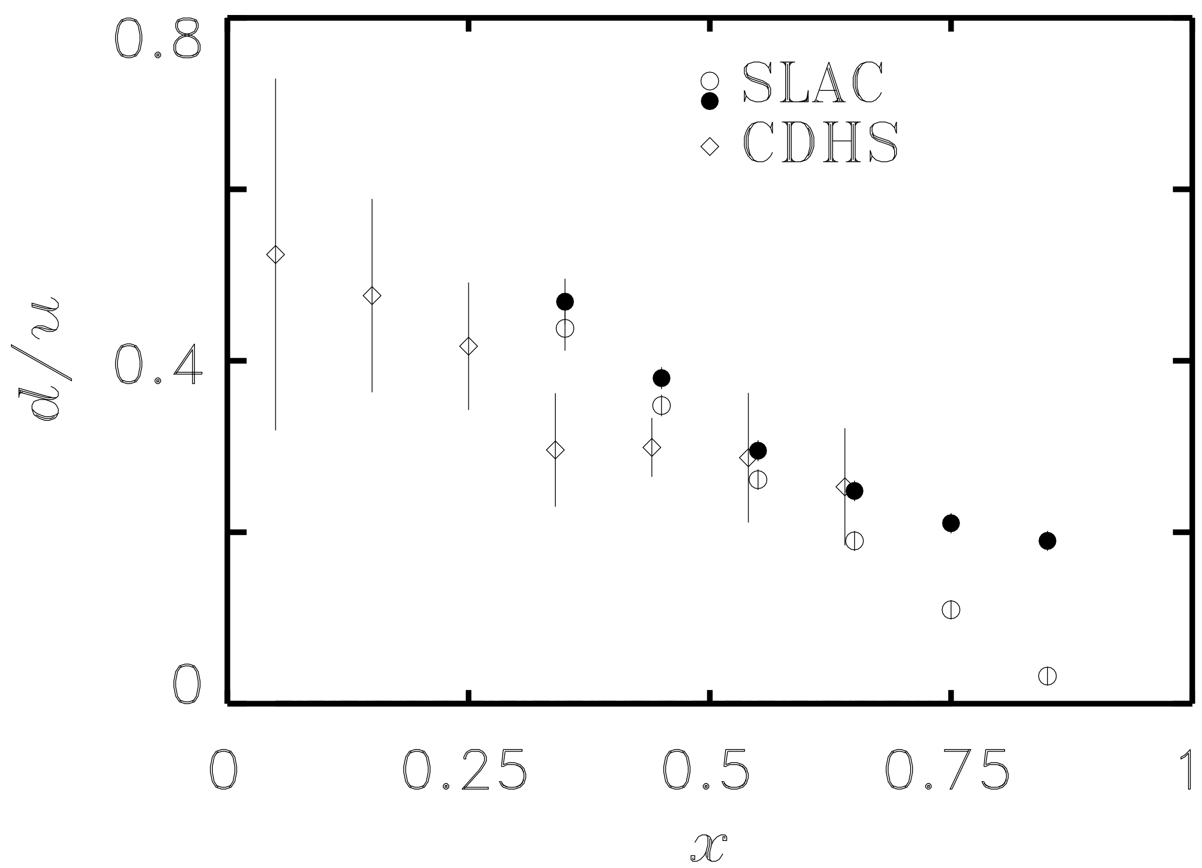
Effect on n/p ratio



WM, Thomas
Phys. Lett. B 377 (1996) 11

→ without EMC effect in d, F_2^n underestimated at large x

Effect on n/p ratio



$$F_2^{\nu p} = 2x\ (d + \bar{u}) \qquad xF_3^{\nu p} = x\ (d - \bar{u})$$

$$F_2^{\bar{\nu} p} = 2x\ (u + \bar{d}) \qquad xF_3^{\bar{\nu} p} = x\ (u - \bar{d})$$

Diquarks as Inspiration and as Objects

Frank Wilczek*

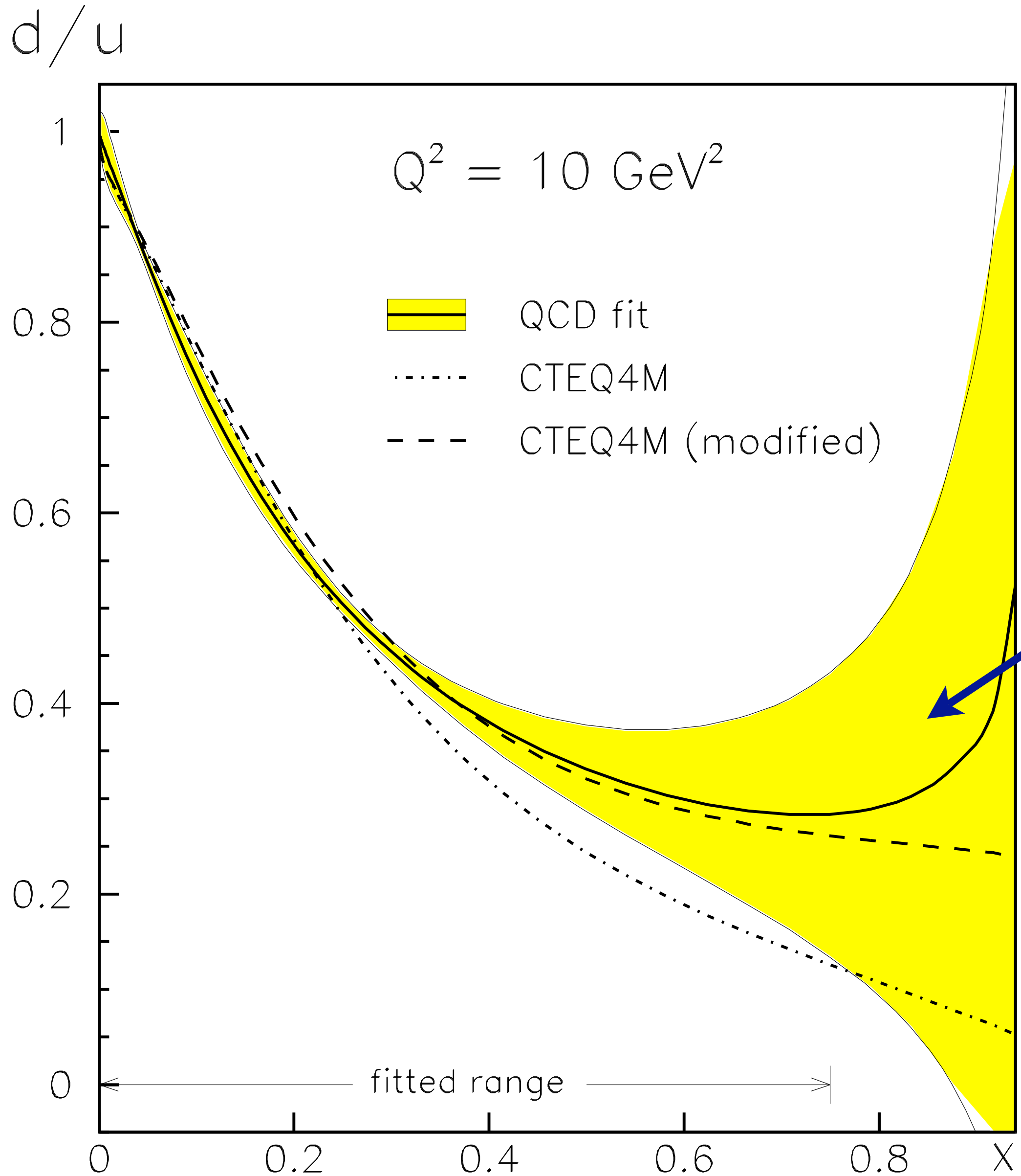
September 17, 2004

hep-ph/0409168

One of the oldest observations in deep inelastic scattering is that the ratio of neutron to proton structure functions approaches $\frac{1}{4}$ in the limit $x \to 1$

$$\lim_{x \to 1} \frac{F_2^n(x)}{F_2^p(x)} \to \frac{1}{4} \tag{1.1}$$

Folklore that experiment gives 1/4 limiting ratio...



uncertainty due to nuclear effects in neutron (full range of nuclear models)

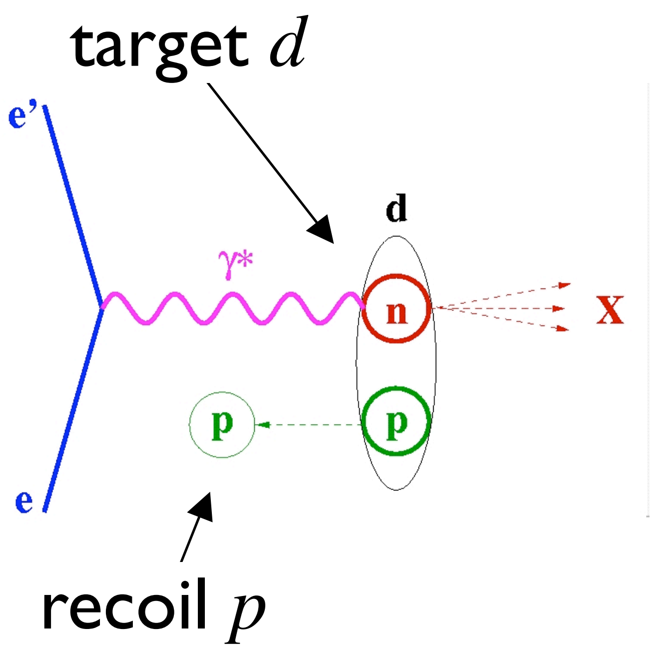
$\Rightarrow$ d distribution poorly known beyond $x \sim 0.5$

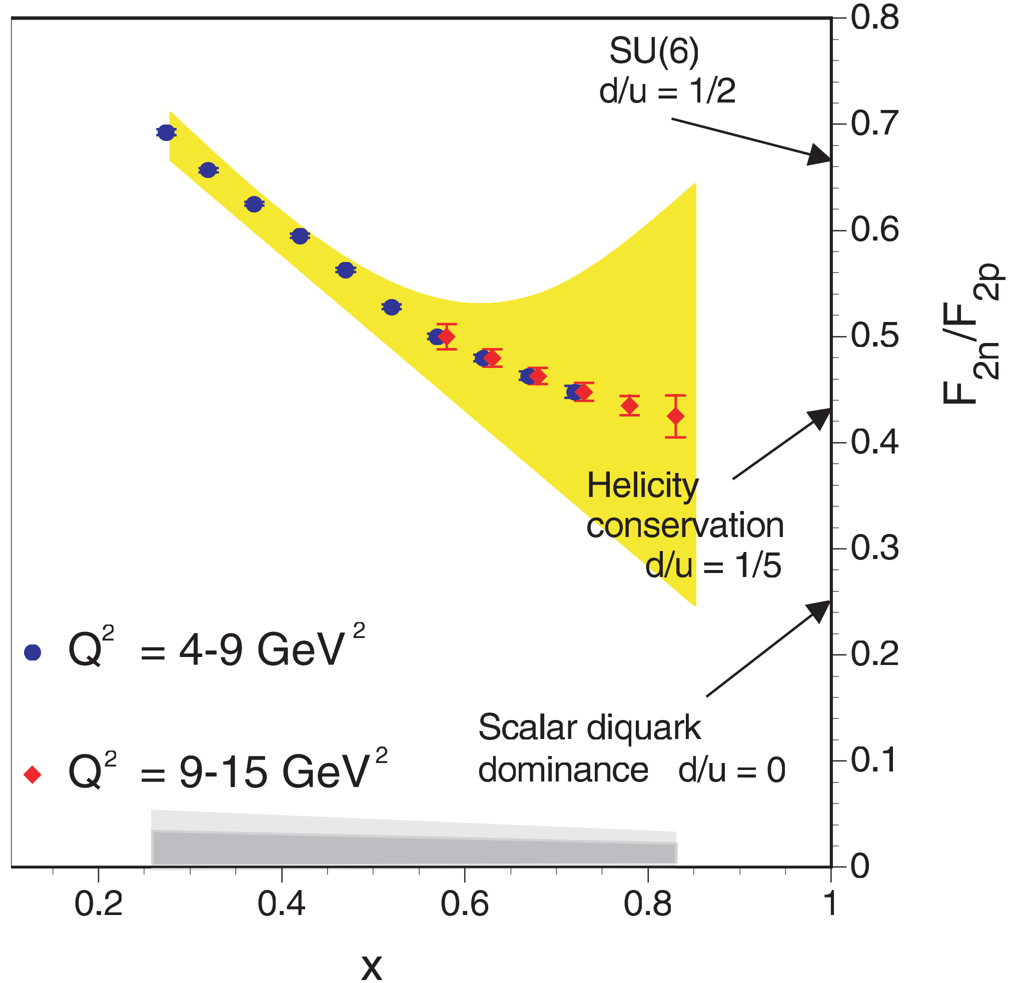
"Cleaner" methods of determining d/u

$e^{\mp}\ p \to \nu(\bar{\nu})X$	need high luminosity
$\nu(\bar{\nu})\ p \to l^{\mp}\ X$	low statistics
$p\ p(\bar{p}) \to W^{\pm}X$	need large lepton rapidity
$\vec{e}_L(\vec{e}_R)\ p \to e\ X$	low count rate
$e\ p \to e\ \pi^{\pm}\ X$	need $z \sim 1$, factorization
$e\ {}^3\mathrm{He}({}^3\mathrm{H}) \to e\ X$	tritium target

"Cleaner" methods of determining d/u

$$e\ d \rightarrow e\ p\ X$$





slow backward p

→ neutron nearly on-shell

→ minimize rescattering

JLab Hall B experiment ("BoNuS")
completed run Dec. 2005

Issues at large x

- **Target mass corrections**
  - ⇒ finite M^2/Q^2 effects (but leading twist!)

    Georgi, Politzer, PRD14 (1976) 1829
    Kretzer, Reno, PRD69 (2004) 034002

- **Higher twists**
  - ⇒ dynamical quark-gluon correlations, $1/Q^2$ suppressed

- **Quark-hadron duality**
  - ⇒ low-W resonances conspire to produce scaling function

    WM, Ent, Keppel, Phys. Rept. 406 (2005) 127

- **Large-x resummation**
  - ⇒ extend validity of pQCD by resumming large-x logs arising from soft & collinear gluons

    Sterman, NPB281 (1987) 310; Catani, Trentadue, NPB327 (1989) 323
    Corcella, Magnea, hep-ph/050742; Vogelsang, AIP Conf. Proc. 747 (2005) 9

Issues at large x

- Target mass corrections

→ Georgi-Politzer (GP) prescription

$$F_2^{\rm GP}(x,Q^2) = \frac{x^2}{r^3} F(\xi) \; + \; 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_\xi^1 d\xi' \; F(\xi') \; + 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

$$\xi = \frac{2x}{1+r}$$

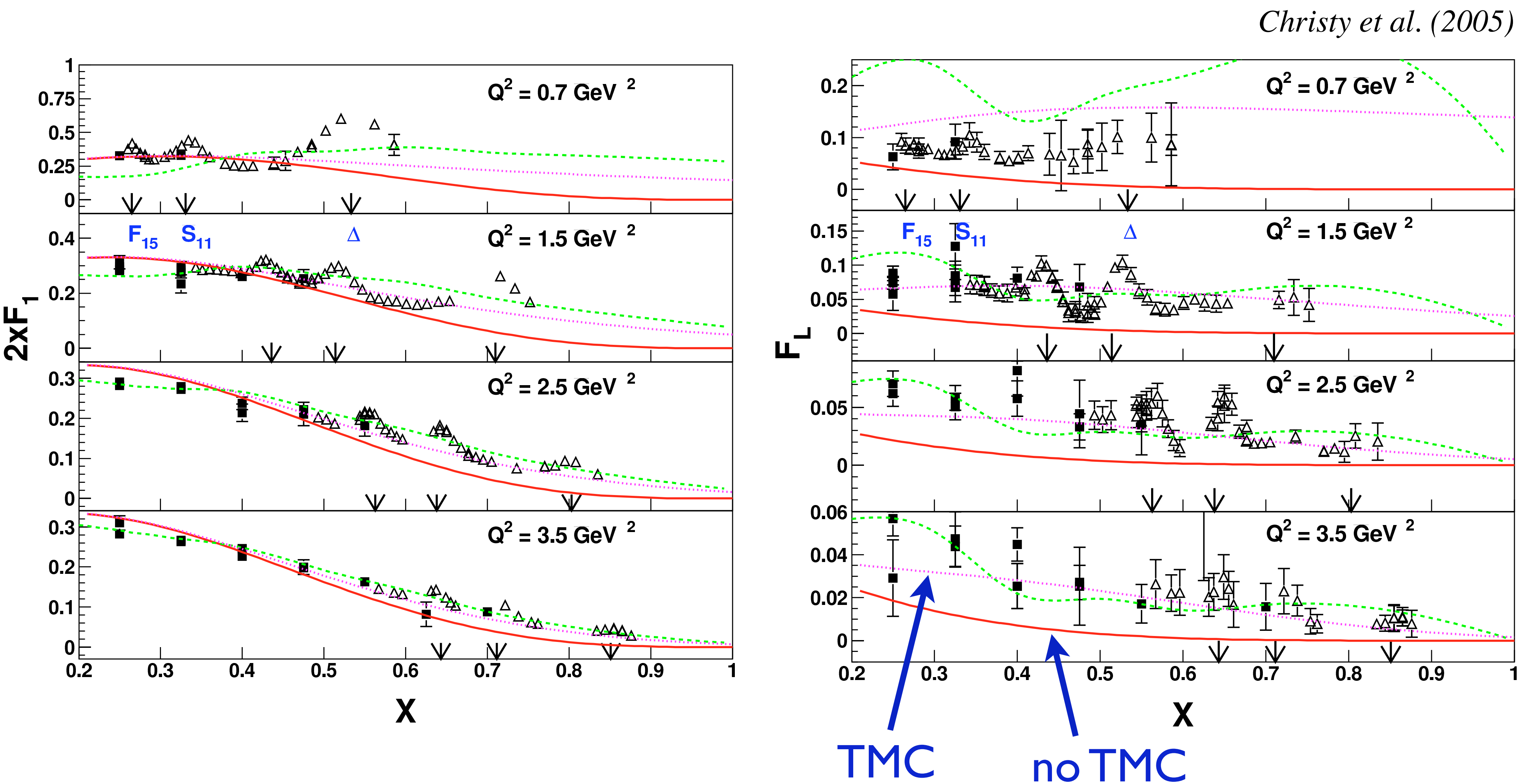
$$r = \sqrt{1 + 4x^2 M^2/Q^2}$$

"quark distribution function"

$$F(y) = \frac{F_2(y)}{y^2}$$

... and similar for other structure functions

numerically...



➡ TMCs significant at large x^2/Q^2, especially for F_L

Threshold problem

- if $F(y) \sim (1-y)^{\beta}$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

  ⇨ $F(\xi_0) > 0$

  ⇨ $F_i^{\mathrm{TMC}}(x=1, Q^2) > 0$

  is this physical?

  ⇨ problem with GP formulation?

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ n fixed, $Q^2 \to \infty$

$$\mu_2^n(Q^2) \to (\ln Q^2/\Lambda^2)^{-\lambda_n} \ A_n$$

$$A_n = \int_0^1 d\xi \ \xi^n \ F(\xi)$$

→ $n \to \infty, \ Q^2$ fixed

$$\mu_2^n(Q^2) \to \xi_0^n(Q^2) \ \widetilde{\mu}_2^n(Q^2)$$

"regularized" amplitudes (threshold-independent)

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

ansatz $\mu_2^n(Q^2) = \xi_0^n(Q^2)\ (\ln Q^2/\Lambda^2)^{-\lambda_n}\ A_n$

→ consistent with asymptotic pQCD behavior

→ not unique!

Bitar, Johnson, Tung
PLB 83B (1979) 114

Possible solutions

- Johnson/Tung - modified threshold factor

moreover, if identify A_n with $M_2^n = \int_0^1 dx\ x^{n-2}\ F_2(x)$

$$\mu_2^n(Q^2) = \xi_0^n(Q^2)\ M_2^n(Q^2)$$

$$\Longrightarrow \quad M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \cdots$$

cf. exact expression

$$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n+2}\frac{M^2}{Q^2} M_2^{n+2} + \cdots$$

$\Longrightarrow$ inconsistency at low Q^2 ?

Possible solutions

- Kulagin/Petti - expand expressions in $1/Q^2$

$$F_2^{\rm TMC}(x,Q^2) \;=\; \left(1-\frac{4x^2M^2}{Q^2}\right)F_2^{\rm LT}(x,Q^2)$$

$$+\quad \frac{x^3M^2}{Q^2}\left(6\int_x^1\frac{{\rm d}z}{z^2}F_2^{\rm LT}(z,Q^2)-\frac{\partial}{\partial x}F_2^{\rm LT}(x,Q^2)\right)$$

Kulagin, Petti, NPA765 (2006) 126

→ has correct threshold behavior

Alternative solution

- work with ξ_0 dependent PDFs

→ n-th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\max}} d\xi\ \xi^n\ F(\xi)$$

→ what is $\xi_{\max}$?

- GP use $\xi_{\max} = 1, \quad \xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\max} = \xi_0$

Steffens, WM
nucl-th/060314, PRC (2006)

Alternative solution

- what is effect on phenomenology?

  $\longrightarrow$ try several "toy distributions"

standard TMC ("sTMC")

$$q(\xi) = \mathcal{N}\, \xi^{-1/2}\, (1-\xi)^3\,, \qquad \xi_{\max} = 1$$

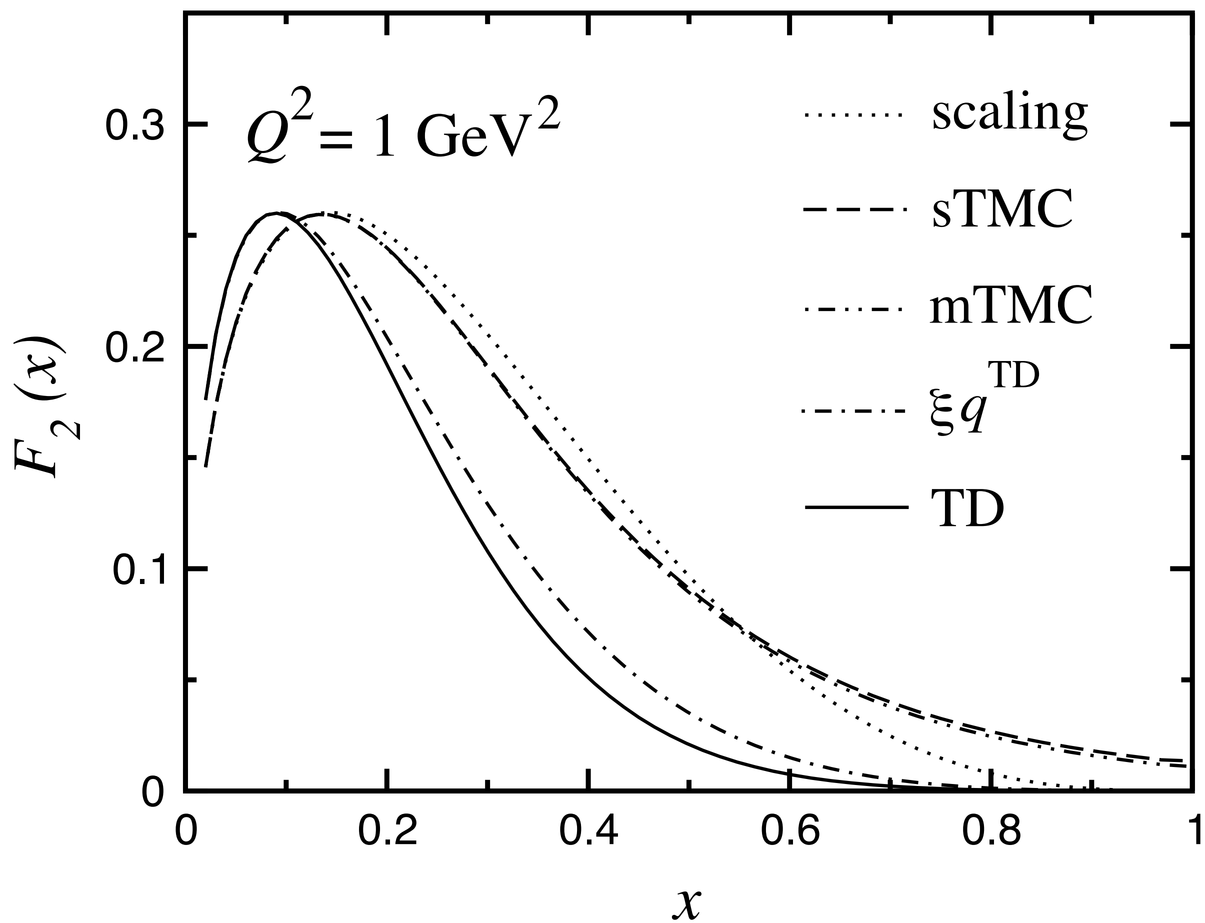
modified TMC ("mTMC")

$$q(\xi) = \mathcal{N}\, \xi^{-1/2}\, (1-\xi)^3\, \Theta(\xi - \xi_0), \qquad \xi_{\max} = \xi_0$$

threshold dependent ("TD")

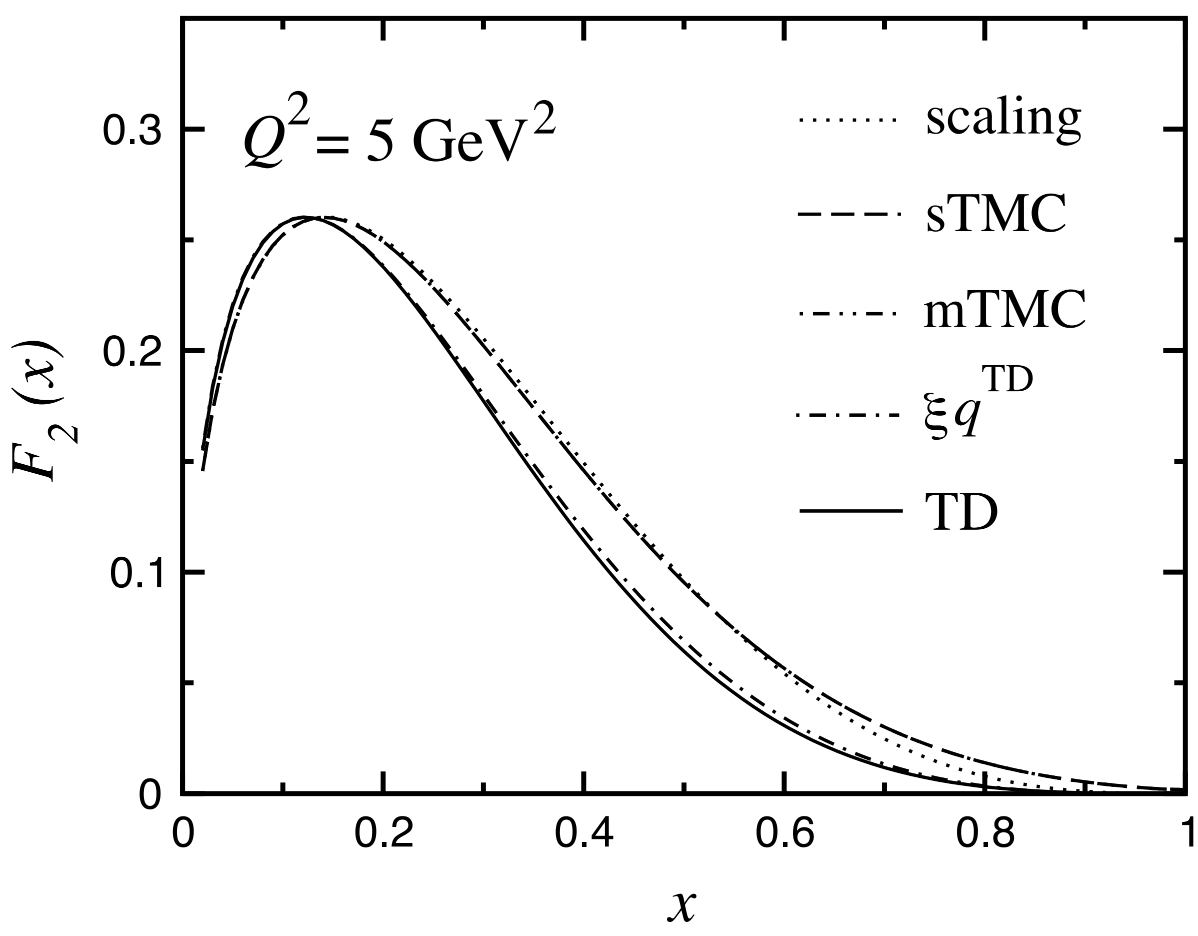
$$q^{\rm TD}(\xi) = \mathcal{N}\, \xi^{-1/2}\, (\xi_0 - \xi)^3\,, \qquad \xi_{\max} = \xi_0$$

TMCs in F_2



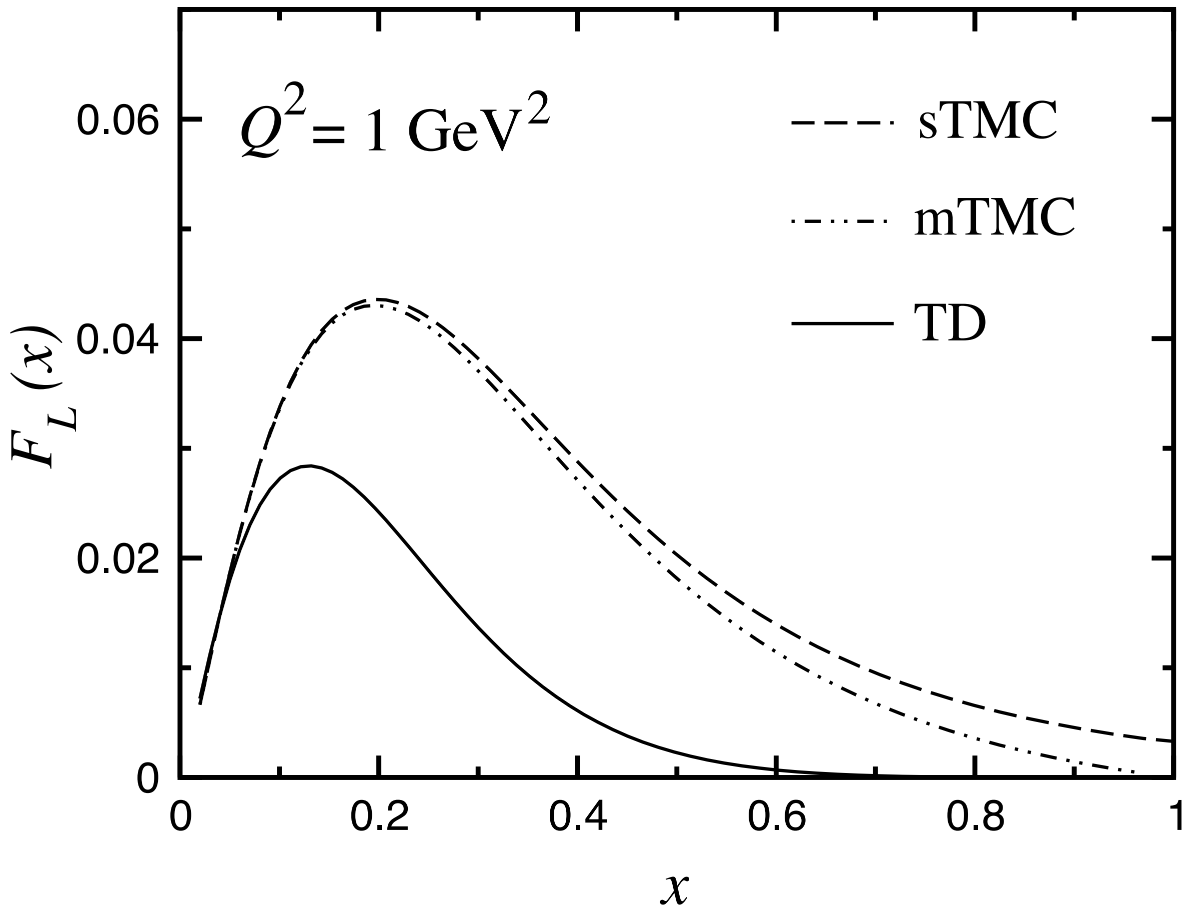
→ correct threshold behavior for “TD” correction

TMCs in F_2



→ effect small at higher Q^2

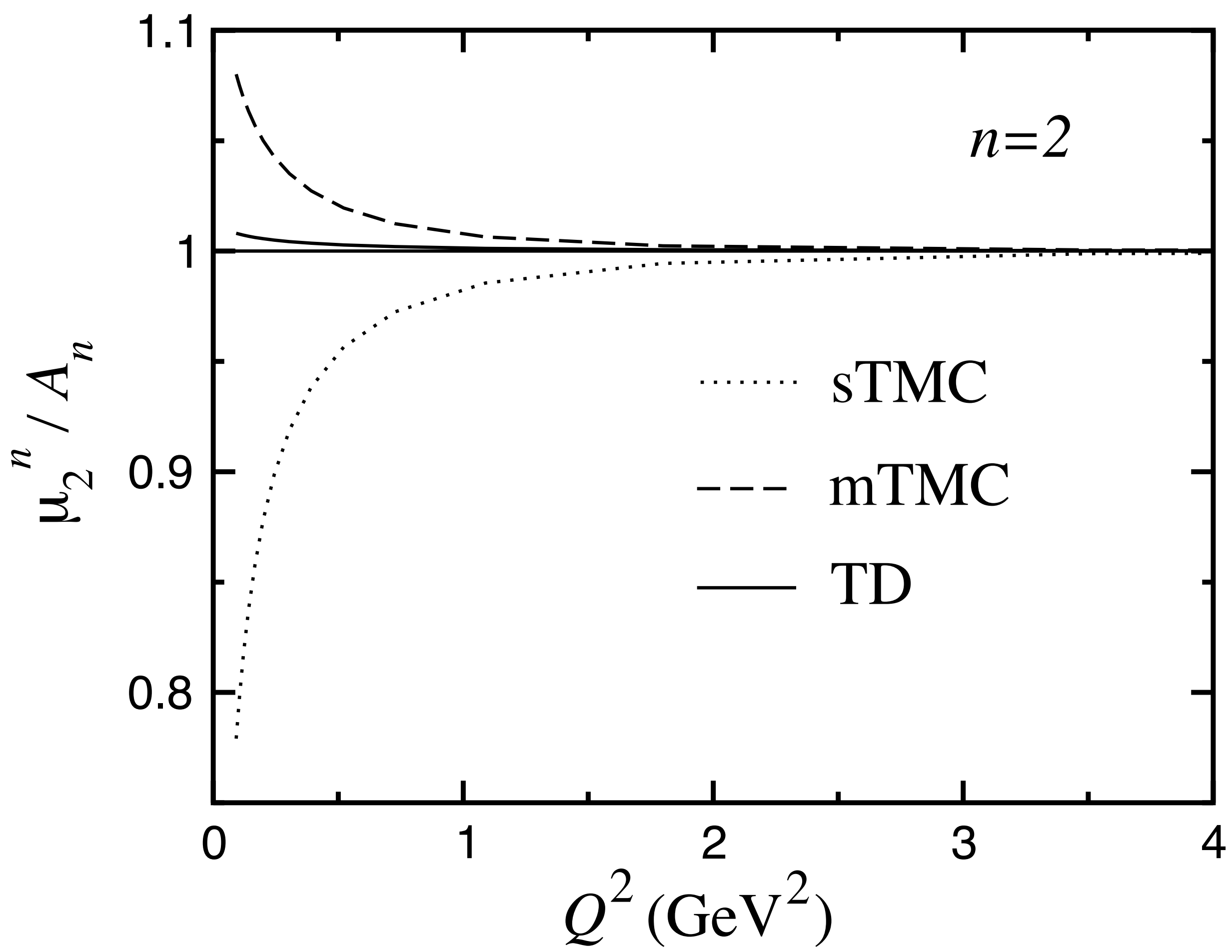
TMCs in F_L



→ correct threshold behavior for "TD" correction

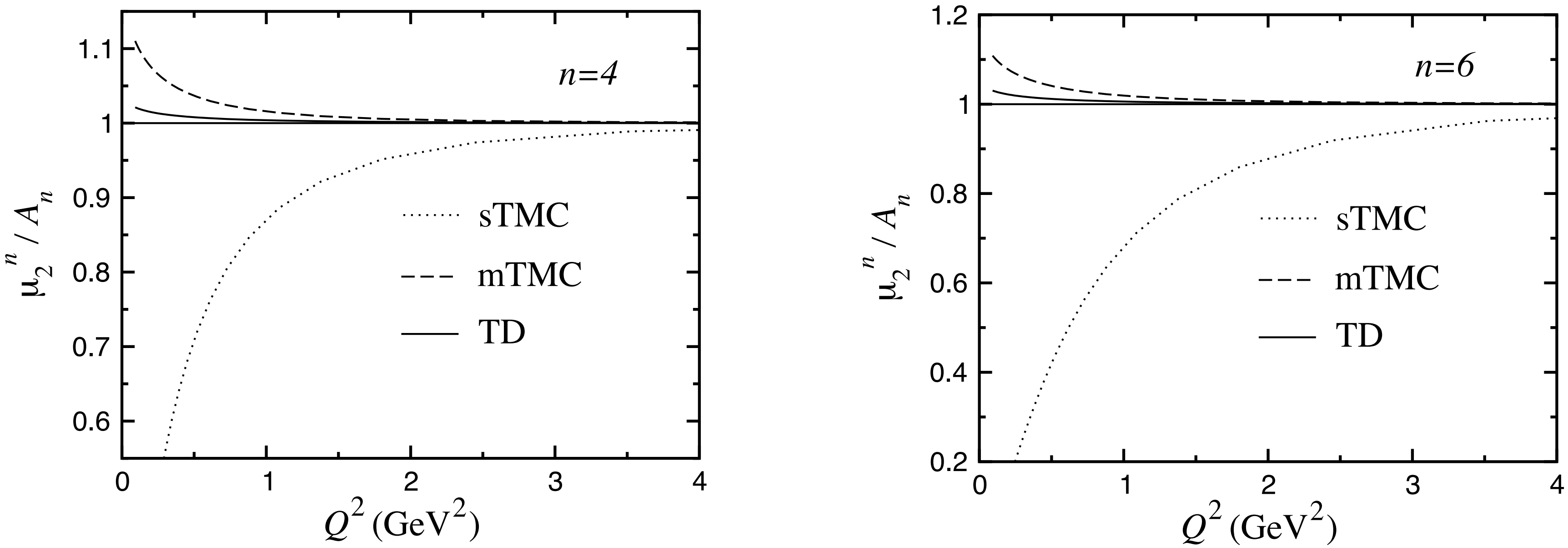
→ reduced TMC effect *cf.* sTMC and mTMC

Nachtmann F_2 moments



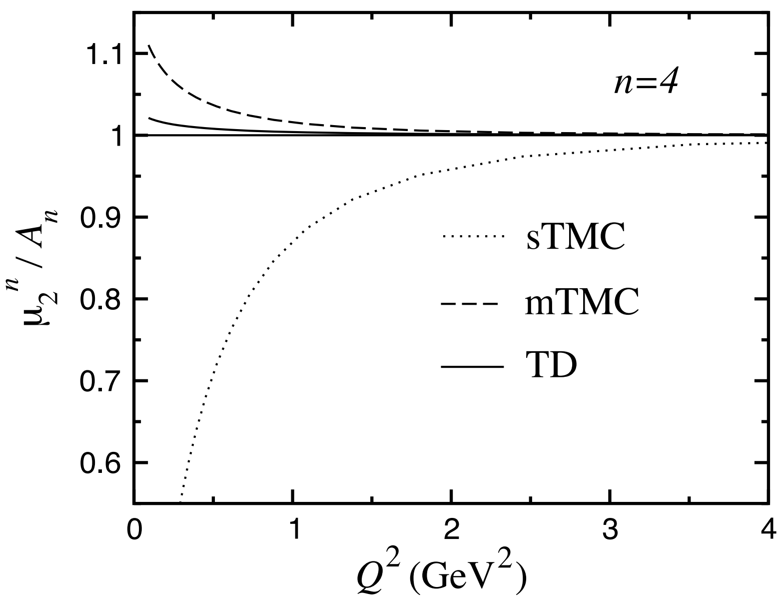
→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

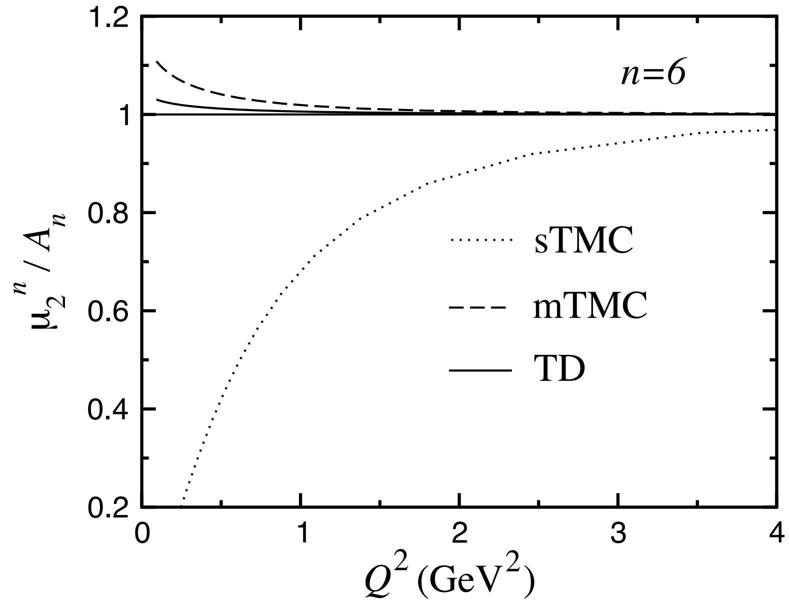
Nachtmann F_2 moments



→ higher moments show much weaker Q^2 dependence than sTMC & mTMC prescriptions

Nachtmann F_2 moments

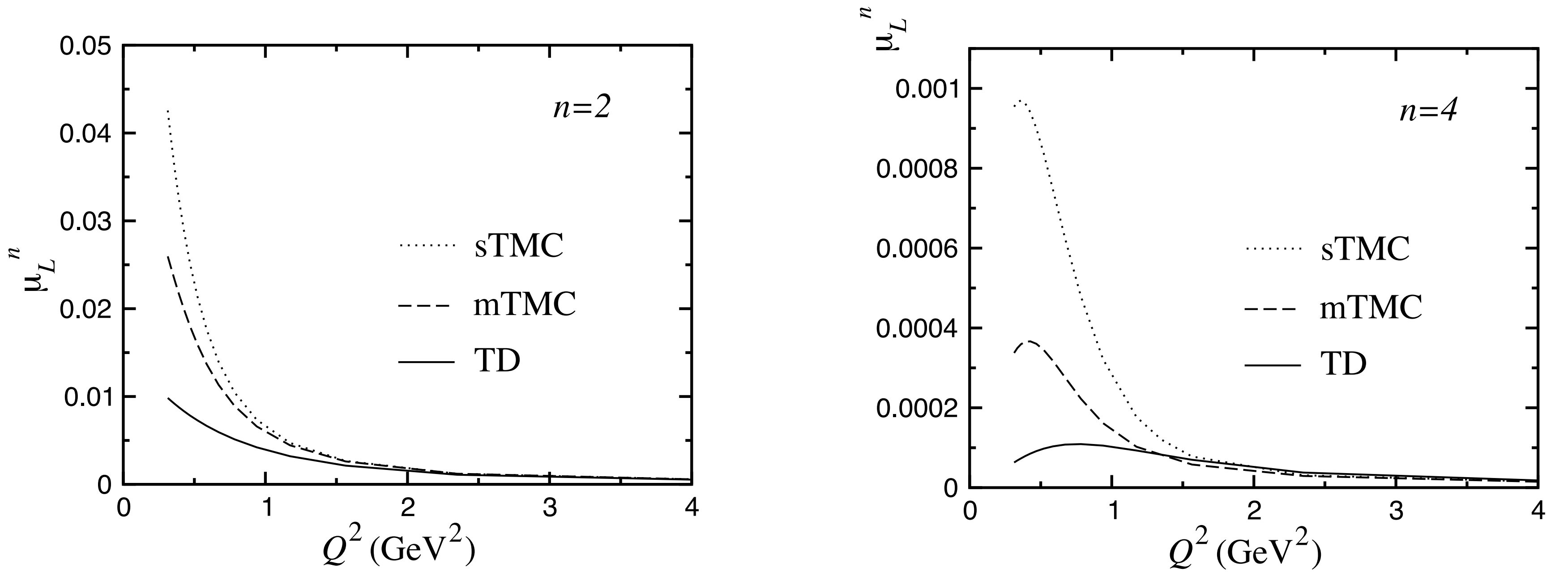




→ $$\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}$$

→ extract PDFs from structure function data at lower Q^2

Nachtmann F_L moments



→ weaker Q^2 dependence for TD prescription

Summary

- d quark distribution poorly known at large x

- (anti)neutrino data can help determine d/u ratio at large x

  $\longrightarrow$ complement e scattering data ($e.g.$ BONUS)

- alternative formulation of TMC in GP approach *without* threshold problem

  $\longrightarrow$ much faster approach to scaling for ξ_0 dependent PDF