

Electro- and neutrino production of resonances

(including the second resonance region)

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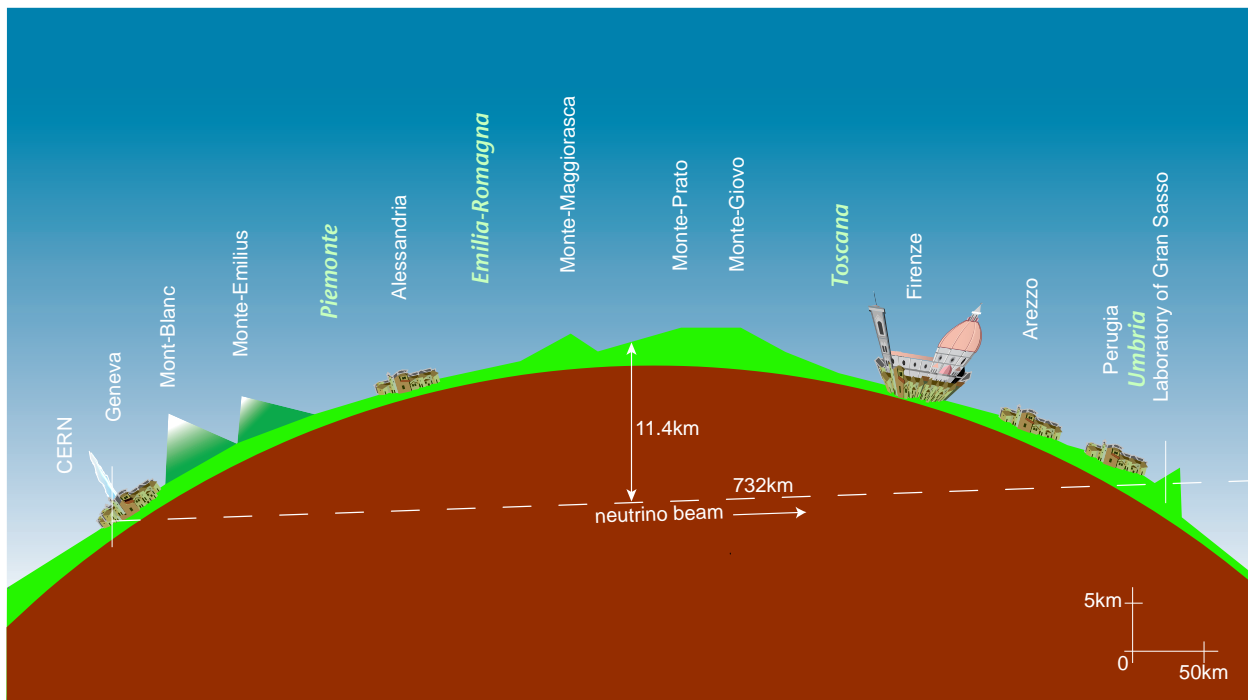
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Outline

- Why do we need to study weak resonance production and go beyond the Delta-peak
- Electromagnetic and weak vertices of resonance production: how they are related.
- General way to determine the electromagnetic form factors from JLab and Mainz accelerator electroproduction data: $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$ resonances
- Results for the cross section
- Checking quark-hadron duality and Adler sum rules
- Conclusions

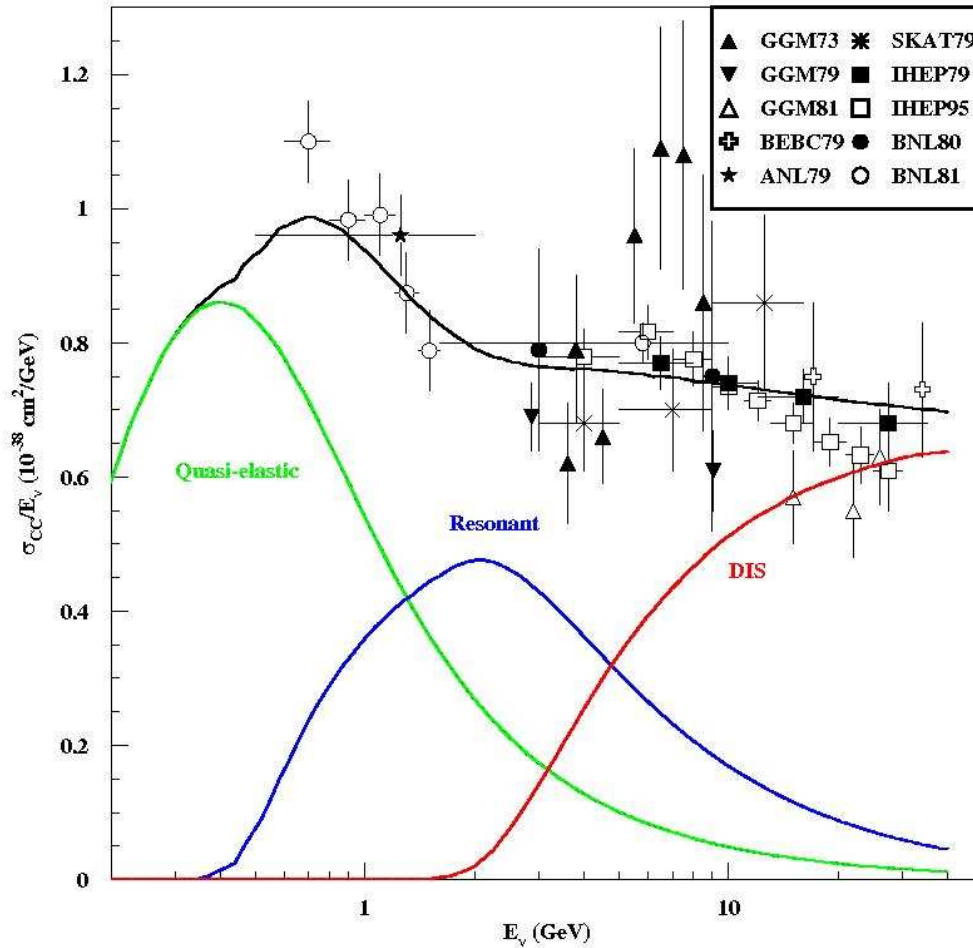
Neutrino oscillations in LBL experiments

- T2K (Tokai to Kamioka) $\langle E_\nu \rangle \sim 0.7$ GeV (planned)
- K2K (KEK to Kamioka) $\langle E_\nu \rangle \sim 1$ GeV (operating)
- MINOS (Fermilab to Soudan) $\langle E_\nu \rangle \sim 3$ GeV (operating)
- CNGS (CERN to GranSasso) $\langle E_\nu \rangle \sim 17$ GeV (under construction)



[http://proj-cngs.web.cern.ch/
proj-cngs/Download/
Download.htm](http://proj-cngs.web.cern.ch/proj-cngs/Download/Download.htm)

The total cross section



$$\sigma_{tot} = \sigma_{QE} + \sigma_{RES} + \sigma_{DIS}$$

1) quasi-elastic (QE)



2) one-pion-production \equiv
resonance production (RES)



3) deep inelastic (DIS)



Resonance production

$R_{2I 2J}$	$M_R, \text{ GeV}$	$\Gamma_{R(tot)}, \text{ GeV}$	$\Gamma_R(R \rightarrow \pi N)/\Gamma_{R(tot)}$
$P_{33}(1232)(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$	1.232	0.120	0.995
$P_{11}(1440)(P_{11}^+, P_{11}^0)$	1.440	0.350	0.65
$D_{13}(1520)(D_{13}^+, D_{13}^0)$	1.520	0.125	0.56
$S_{11}(1535)(S_{11}^+, S_{11}^0)$	1.535	0.150	0.45
...			

Accurate measurements and several theoretical approaches are available for the leading $P_{33}(1232)$ resonance for both electro- and neutrino-production

What about resonances with higher masses?

Accurate measurements and several theoretical approaches for electroproduction

Higher mass resonances in neutrino production

1) Old experiments have large errorbars

- ANL ($E_\nu \sim 0.5 - 1.5$ GeV)
- BNL ($E_\nu \sim 1 - 3$ GeV)
- SKAT ($E_\nu \sim 4 - 12$ GeV)
- BEBC ($E_\nu \sim 50$ GeV)

2) Modern experiments intended to study exclusive one-pion production

- K2K ($E_\nu \sim 0.2 - 4$ GeV)
- MiniBOONE
($E_\nu \sim 0.3 - 2.5$ GeV)
- Minerva
($E_\nu \sim 0.3 - 2.5$ GeV;
C, Fe, Pb nuclear targets)

3) Theory:

- Rein–Sehgal model (1980), based on the relativistic quark model;
update by K.Kuzmin et al (Dubna), K.Hagiwara et al (KEK)
- phenomenological model of Dortmund group (hep-ph/0604132)

Higher mass resonances in neutrino production

4) Fitting the total neutrino cross section $\sigma_{tot} = \sigma_{QE} + \sigma_{RES} + \sigma_{DIS}$: to avoid double counting one should separate RES and DIS invariant-mass regions

● V. Naumov, (Dubna) Max Born Symp, Wroclaw, Dec 2005

RES $W < 1.5$ GeV

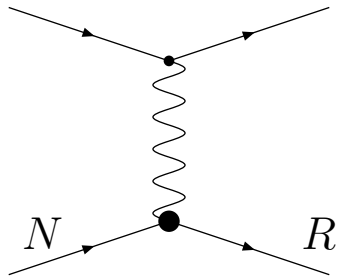
DIS $W > 1.5$ GeV

● Y. Nowak, (Wroclaw) Max Born Symp, Wroclaw, Dec 2005

△ resonance and smooth transition to DIS single pion channel

The opportunity to make such separation relies on the phenomenon of quark–hadron duality. It would be nice to study duality in a direct way

Phenomenological description



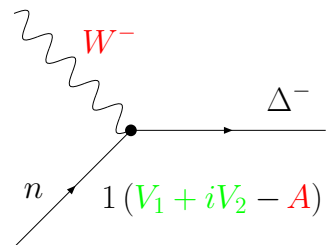
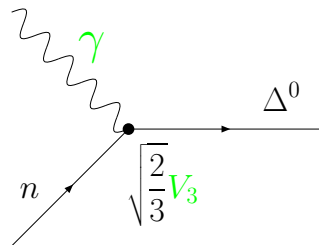
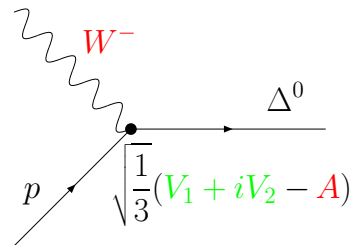
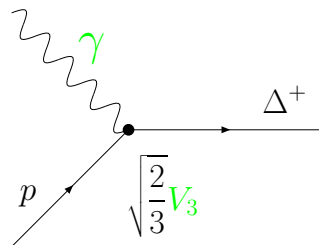
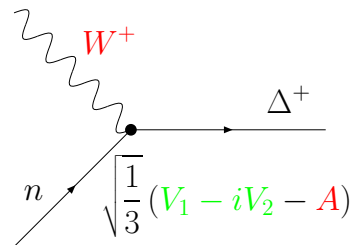
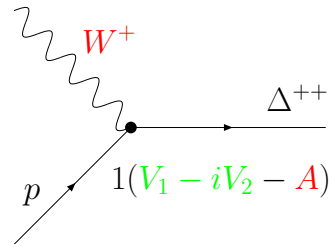
The electromagnetic hadronic vertex is parametrized in terms of the *electromagnetic nucleon-resonance form factors* $C_i^{(p)}$ and $C_i^{(n)}$, which depend on the momentum transferred squared $q^2 = -Q^2$ and in general case do not coincide for proton and neutron

The weak hadronic vertex is parametrized in terms of the *weak nucleon-resonance vector* (C_i^V) and *axial* (C_i^A) form factors

The form factors characterize the hadronic vertex and are independent of the leptonic one.

R	$M_R, \text{ GeV}$	$\Gamma_{R(tot)}, \text{ GeV}$	$\Gamma_R(R \rightarrow \pi N)/\Gamma_{R(tot)}$ elasticity
$P_{33}(1232)(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$	1.232	0.114	0.995
$P_{11}(1440)(P_{11}^+, P_{11}^0)$	1.440	0.350(250 – 450)	0.6(0.6 – 0.7)
$D_{13}(1520)(D_{13}^+, D_{13}^0)$	1.520	0.125(110 – 135)	0.5(0.5 – 0.6)
$S_{11}(1535)(S_{11}^+, S_{11}^0)$	1.535	0.150(100 – 250)	0.4(0.35 – 0.55)

Isospin relations for isospin-3/2 states



1) electromagnetic amplitudes (and as a consequence form factors) are the same for p and n

$$A(\gamma p \rightarrow R^+) = A(\gamma n \rightarrow R^0)$$

$$C^{(p)} = C^{(n)}$$

2) Isospin triplet $V_a = (V_1, V_2, V_3)$

Weak and el-m amplitudes

$$\begin{aligned} A(W^+ n \rightarrow R^+)^V &= \\ &= \sqrt{\frac{1}{3}} \sqrt{2} \frac{V_1 - iV_2}{\sqrt{2}} = A(\gamma p \rightarrow R^+) \end{aligned}$$

Form factors

$$C^{(p)} = C^V$$

Form factors for $P_{33}(1232)$ ($J^P = \frac{3}{2}^+$)

Earlier articles in this notation: Dufner, Tsai, PR 168, 1801; Lewellyn Smith, PR 3 (1972) 261; Schreiner, von Hippel, NPB58 (1983) 333; Paschos, Sakuda, Yu, PRD 69 (2004) 014013; Singh, Athar, Ahmad, hep-ph/0507016;

The resonance field is described by a Rarita-Schwinger spinor $\psi_\lambda^{(R)}$. Quite generally the weak vertex for the resonance production may be written as

$$\begin{aligned} \langle \Delta | V^\nu | N \rangle = \bar{\psi}_\lambda^{(R)} & \left[\frac{C_3^V}{m_N} (\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^V}{m_N^2} (q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) \right. \\ & \left. + \frac{C_5^V}{m_N^2} (q \cdot p' g^{\lambda\nu} - q^\lambda p'^\nu) + C_6^V g^{\lambda\nu} \right] \gamma_5 u_{(N)} \\ \langle \Delta | A^\nu | N \rangle = \bar{\psi}_\lambda & \left[\frac{C_3^A}{m_N} (\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^A}{m_N^2} (q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) + C_5^A g^{\lambda\nu} + \frac{C_6^A}{m_N^2} q^\lambda q^\nu \right] u_{(N)} \end{aligned}$$

dictated by gauge invariance

How the form factors can be determined

Axial FF: PCAC $i\overline{\Delta}_\mu^+ q^\mu \left[C_5^A + \frac{C_6^A}{m_N^2} q^2 \right] u_N, = -i\sqrt{\frac{1}{3}} \frac{m_\pi^2 f_\pi}{q^2 - m_\pi^2} \overline{\Delta}_\mu^+ g_\Delta q^\mu u_N.$

$$\implies C_5^A(Q^2 = 0) = \frac{g_\Delta f_\pi}{\sqrt{3}} = 1.2 \quad C_6^A = m_N^2 \frac{C_5^A}{m_\pi^2 + Q^2}$$

Vector FF: CVC $q^\mu J_\mu = 0 \implies C_6^V = 0,$

comparison with electroproduction cross section (1968 - 1971) and magnetic multipole

dominance leads to $C_3^V(0) = 2.05 \pm 0.04,$ $C_4^V(0) = -\frac{m_N}{W} C_3^V,$ $C_5^V = 0$

FF fall down with Q^2 faster than dipole $\frac{C_3^V(0)}{D} \frac{1}{1 + \frac{Q^2}{4M_V^2}}$

with $(1 + \frac{Q^2}{M_V^2})^2 \equiv D_V$ with $M_V = 0.84$ GeV

2001: unambiguous evidences from the JLab for the contribution of the electric

$E2 \sim -2.5\%$, of scalar multipoles $S2 \sim -5\%$. They are taken into account by extracting

the form factors from the helicity amplitudes O.L., Paschos, Piranishvili, 2006

Helicity amplitudes for $P_{33}(1232)$

Helicity amplitudes evaluated from the electroproduction data on proton at $W = M_R$ Tiator et al. (Mainz), EPJA 19 (2004); Burkert, Li (JLab), IJMP 13 (2004); Aznauryan (JLab) (private comm, 2005) The relations to C_i^V are calculated by our group at arbitrary Q^2 and W

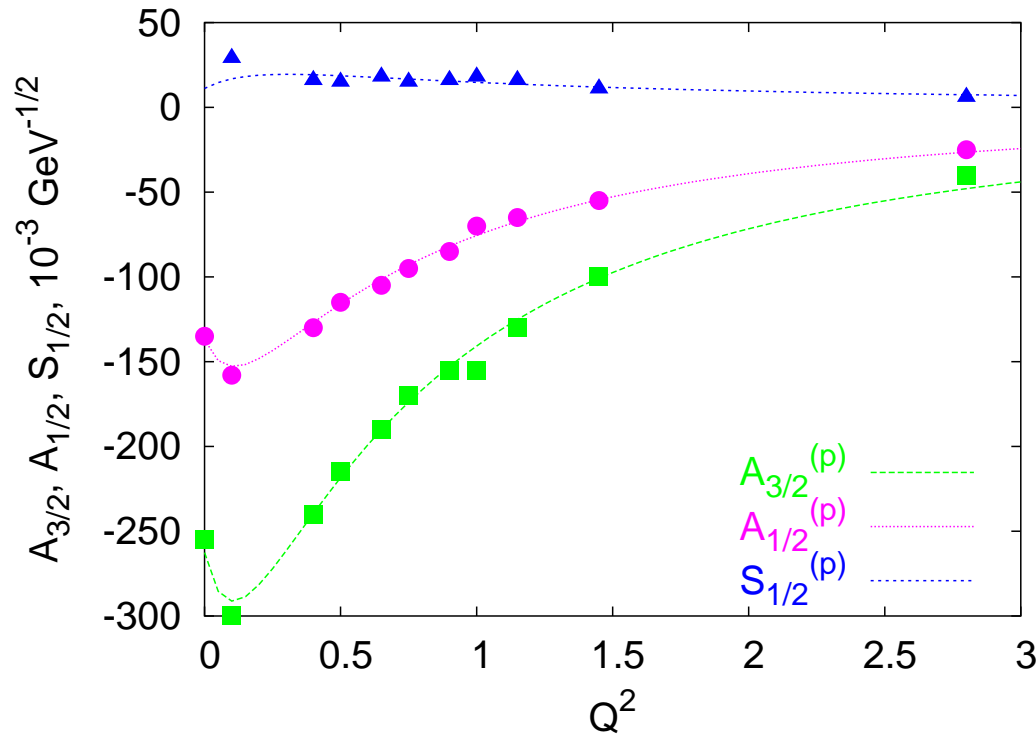
$$\begin{aligned}
 A_{3/2} &= \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2 - m_N^2)}} \langle R, +\frac{3}{2} | J_{em} \cdot \varepsilon^{(R)} | N, +\frac{1}{2} \rangle = \\
 &= -\sqrt{N} \frac{|\vec{q}|}{p'^0 + M_R} \left[\frac{C_3^{(p)}}{m_N} (m_N + M_R) + \frac{C_4^{(p)}}{m_N^2} (m_N \nu - Q^2) + \frac{C_5^{(p)}}{m_N^2} m_N \nu \right]
 \end{aligned}$$

$$\begin{aligned}
 A_{1/2} &= \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2 - m_N^2)}} \langle R, +\frac{1}{2} | J_{em} \cdot \varepsilon^{(R)} | N, -\frac{1}{2} \rangle \\
 &= \sqrt{\frac{N}{3}} \frac{|\vec{q}|}{p'^0 + M_R} \left[\frac{C_3^{(p)}}{m_N} (m_N + M_R - 2\frac{m_N}{M_R} (p'^0 + M_R)) + \frac{C_4^{(p)}}{m_N^2} (m_N \nu - Q^2) + \frac{C_5^{(p)}}{m_N^2} m_N \nu \right]
 \end{aligned}$$

$$\begin{aligned}
 S_{1/2} &= \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2 - m_N^2)}} \frac{q_z}{\sqrt{Q^2}} \langle R, +\frac{1}{2} | J_{em} \cdot \varepsilon^{(S)} | N, +\frac{1}{2} \rangle = \\
 &= \sqrt{\frac{2N}{3}} \frac{|\vec{q}|^2}{M_R(p'^0 + M_R)} \left[\frac{C_3^{(p)}}{m_N} M_R + \frac{C_4^{(p)}}{m_N^2} W^2 + \frac{C_5^{(p)}}{m_N^2} m_N (m_N + \nu) \right]
 \end{aligned}$$

$$N = \frac{\pi\alpha_{em}}{m_N(W^2 - m_N^2)} 2m_N(p'^0 + M_R)$$

Beyond the magnetic dominance



Helicity amplitudes for $P_{33}(1232)$ excitation
on proton target at $W = M_{P_{1232}}$

$$C_3^V = \frac{2.13}{D_V} \cdot \frac{1}{1+Q^2/4M_V^2}$$

$$C_4^V = \frac{-1.5}{D_V} \cdot \frac{1}{1+Q^2/4M_V^2},$$

$$C_5^V = \frac{-0.58}{D_V} \cdot \frac{1}{1+Q^2/0.76M_V^2}$$

where

$$D_V = (1 + Q^2/0.71 \text{ GeV}^2)^2$$

Coincide with the "magnetic dominance" values with 4% accuracy

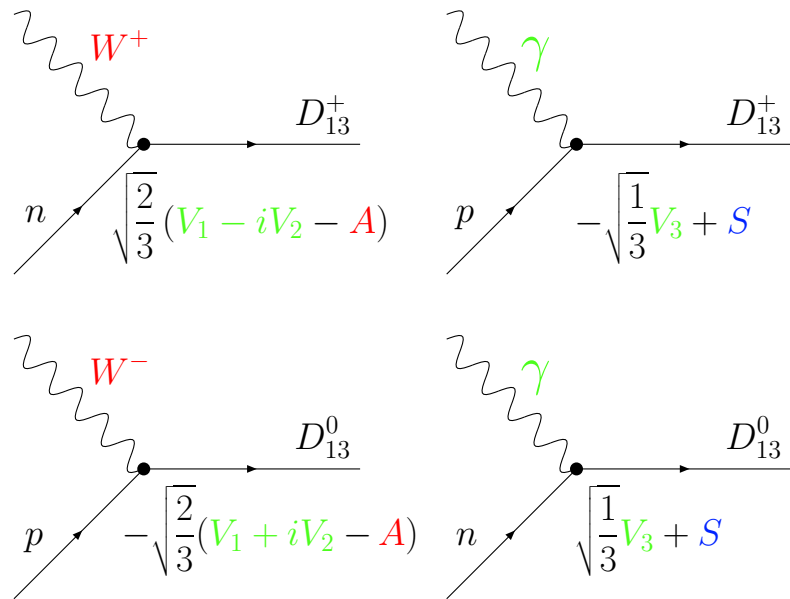
$$D_{13}(1520): J^P = 3/2^-, I = 1/2$$

The formulas for this resonance are similar to that for P_{33} with the γ_5 changing the place: there are 3 independent vector form factors and 3 independent axial form factors

$$\langle D_{13} | V^\nu | N \rangle = \bar{\psi}_\lambda^{(R)} \left[\frac{C_3^V}{m_N} (\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^V}{m_N^2} (q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) + \frac{C_5^V}{m_N^2} (q \cdot p' g^{\lambda\nu} - q^\lambda p'^\nu) \right] u_{(N)}$$

$$\langle D_{13} | A^\nu | N \rangle = \bar{\psi}_\lambda \left[\frac{C_3^A}{m_N} (\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^A}{m_N^2} (q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) + C_5^A g^{\lambda\nu} + \frac{C_6^A}{m_N^2} q^\lambda q^\nu \right] \gamma_5 u_{(N)}$$

Isospin relations for isospin-1/2 states



1) electromagnetic amplitudes (and as a consequence form factors) are different for p and n

2) Isospin triplet $V_a = (V_1, V_2, V_3)$

Weak and el-m amplitudes

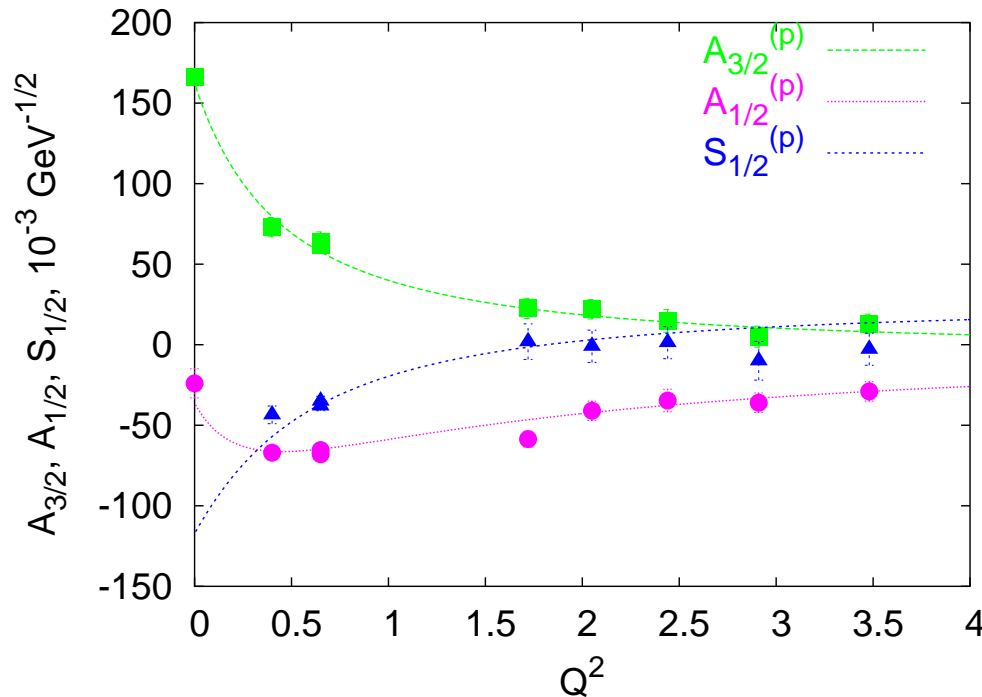
$$\begin{aligned}
 A(W^+ n \rightarrow R^+) &= \\
 &= \sqrt{\frac{2}{3}} \sqrt{2} \frac{V_1 - iV_2}{\sqrt{2}} = 2\sqrt{\frac{1}{3}} V_3 = \\
 &= A(\gamma n \rightarrow R^0) - A(\gamma p \rightarrow R^+)
 \end{aligned}$$

Form factors

$$C^V = C^{(n)} - C^{(p)}$$

$D_{13}(1520)$: Vector form factors

Helicity amplitudes (and as a consequence form factors) for electroproduction are different for the proton and neutron.



Helicity amplitudes for $D_{13}(1520)$ excitation on proton target at $W = M_{D1520}$,

latest data from Aznauryan (JLab), 2005, private communication are shown

$$C_3^{(p)} = \frac{2.95}{D_V} \cdot \frac{1}{1+Q^2/8.9M_V^2}$$

$$C_4^{(p)} = \frac{-1.05}{D_V} \cdot \frac{1}{1+Q^2/8.9M_V^2},$$

$$C_5^{(p)} = \frac{-0.48}{D_V}$$

$$C_3^{(n)} = \frac{-1.14}{D_V} \cdot \frac{1}{1+Q^2/8.9M_V^2}$$

$$C_4^{(n)} = \frac{0.46}{D_V} \cdot \frac{1}{1+Q^2/8.9M_V^2},$$

$$C_5^{(n)} = \frac{-0.17}{D_V}$$

where

$$D_V = (1 + Q^2/0.71 \text{ GeV}^2)^2$$

Axial form factors for $D_{13}(1520)$

From PCAC $C_6^A(Q^2) = m_N^2 \frac{C_5^A(Q^2)}{m_\pi^2 + Q^2}$, $C_5^A(D_{13}) = -\sqrt{\frac{2}{3}} g_{\pi NR} f_{D_{13}} = -2.1$

The Q^2 dependence for axial form factors cannot be determined from experiment because of the lack of the data.

Instead, we consider two cases: (i) “fast fall-off”, in which the Q^2 dependence is chosen the same as for the P_{33} resonance, and (ii) “slow fall-off”, in which the Q^2 dependence is flatter and is taken to be the same as for the vector form factors for each resonance.

$$D_{13}(1520) : C_5^A = \frac{-2.1/D_A}{1+Q^2/3M_A^2} \text{ (“fast fall-off”)}$$

$$C_5^A = \frac{-2.1/D_A}{1+Q^2/8.9M_A^2} \text{ (“slow fall-off”).}$$

with the axial mass common for all the resonances $M_A = 1.05$ GeV

We also do not know C_3^A , C_4^A and take $C_3^A(Q^2) = 0$, $C_4^A(Q^2) = 0$.

For more details see [O.L., E. Paschos, G. Piranishvili, hep-ph/0602210](#)

$$P_{11}(1440), J^P = \frac{1}{2}^+ \quad \text{and} \quad S_{11}(1535), J^P = \frac{1}{2}^-$$

For the spin-1/2 resonances all formulas are simpler

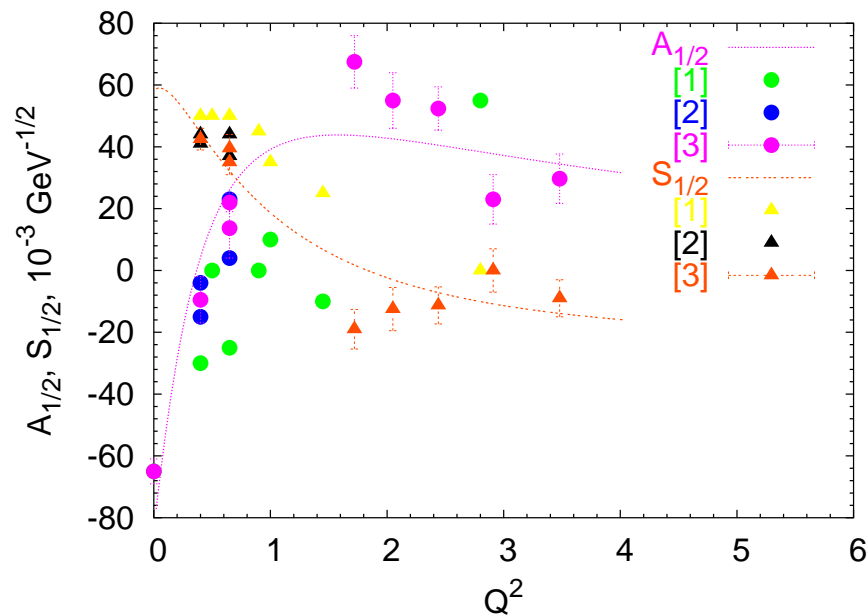
$$\langle P_{11} | J^\nu | N \rangle = \bar{u}(p') \left[\frac{g_1^V}{(m_N + M_R)^2} (Q^2 \gamma^\nu + \not{q} q^\nu) + \frac{g_2^V}{m_N + M_R} i \sigma^{\nu\rho} q_\rho \right. \\ \left. - g_1^A \gamma^\nu \gamma_5 - \frac{g_3^A}{m_N} q^\nu \gamma_5 \right] u(p),$$

where $\sigma^{\nu\rho} = \frac{i}{2} [\gamma^\nu, \gamma^\rho]$.

$$\langle S_{11} | J^\nu | N \rangle = \bar{u}(p') \left[\frac{g_1^V}{(m_N + M_R)^2} (Q^2 \gamma^\nu + \not{q} q^\nu) \gamma_5 + \frac{g_2^V}{m_N + M_R} i \sigma^{\nu\rho} q_\rho \gamma_5 \right. \\ \left. - g_1^A \gamma^\nu - \frac{g_3^A}{m_N} q^\nu \right] u(p),$$

Vector form factors for $P_{11}(1440)$

Helicity amplitudes (and as a consequence form factors) for electroproduction are different for the proton and neutron, because they include now not only isovector, but also isoscalar part.



Helicity amplitudes for $P_{11}(1440)$ excitation on proton target at $W = M_{P1440}$

[1] Tiator et al. (Mainz), EPJA 19 (2004);

[2] Burkert, Li (JLab), IJMP 13 (2004);

[3] Aznauryan (JLab) (private comm, 2005)

$$g_1^{(p)} = \frac{2.3/D_V}{1 + Q^2/4.3M_V^2},$$

$$g_2^{(p)} = \frac{-0.76}{D_V} \left[1 - 2.8 \ln \left(1 + \frac{Q^2}{1 \text{ GeV}^2} \right) \right]$$

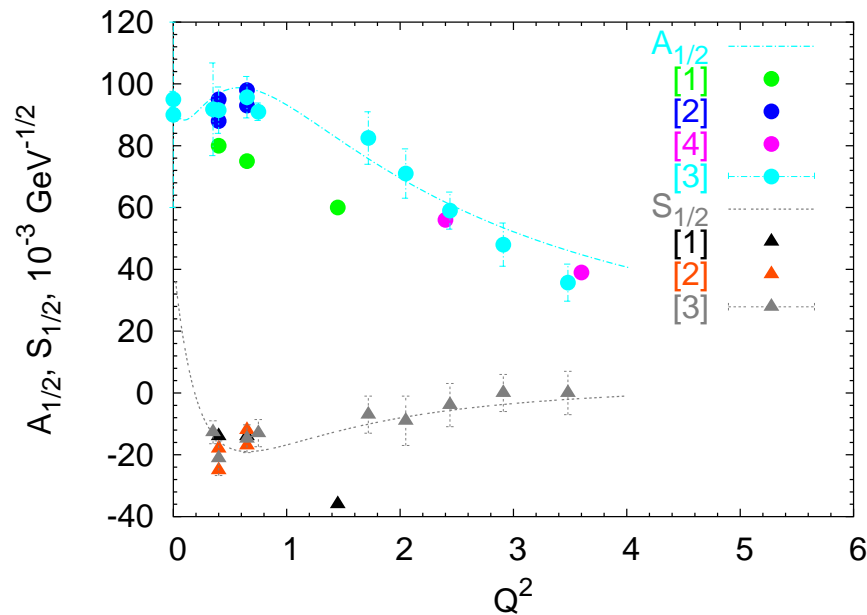
$$g_i^V = g_i^{(n)} - g_i^{(p)}$$

neutron: neglecting isoscalar contribution (which makes sense within the accuracy of the data available)

$$g_i^{(n)} = -g_i^{(p)}$$

Vector form factors for $S_{11}(1535)$

Helicity amplitudes (and as a consequence form factors) for electroproduction are different for the proton and neutron.



Helicity amplitudes for $S_{11}(1535)$ excitation on proton target at $W = M_{S_{1535}}$

[1] Tiator et al. (Mainz), EPJA 19 (2004);

[2] Burkert, Li (JLab), IJMP 13 (2004);

[3] Aznauryan (JLab)(private comm, 2005)

[4] Armstrong et al. (JLab), PRD 60 (2000)

$$g_1^{(p)} = \frac{2.0/D_V}{1 + \frac{Q^2}{1.2M_V^2}} \left[1 + 7.2 \ln \left(1 + \frac{Q^2}{1 \text{ GeV}^2} \right) \right]$$

$$g_2^{(p)} = \frac{0.84}{D_V} \left[1 + 0.11 \ln \left(1 + \frac{Q^2}{1 \text{ GeV}^2} \right) \right]$$

$$g_i^V = g_i^{(n)} - g_i^{(p)}$$

neutron: neglecting isoscalar contribution (which makes sense within the accuracy of the data available)

$$g_i^{(n)} = -g_i^{(p)}$$

Axial form factors

Axial form factors are related by PCAC to the strong πNR couplings g_{P11} and g_{S11} , which in turn are determined from the elastic resonance width

$$P_{11}(1440) : \quad g_3^A(Q^2) = \frac{m_N(M_R + m_N)}{Q^2 + m_\pi^2} g_1^A(Q^2), \quad g_1^A(0) = -\sqrt{\frac{2}{3}} \frac{g_{P11} f_\pi}{M_R + m_N} = -0.51$$

$$S_{11}(1535) : \quad g_3^A(Q^2) = \frac{m_N(M_R - m_N)}{Q^2 + m_\pi^2} g_1^A(Q^2), \quad g_1^A(0) = -\sqrt{\frac{2}{3}} \frac{g_{S11} f_\pi}{M_R - m_N} = -0.21$$

The Q^2 dependence for g_1^A is not known, so we again consider “fast fall-off” and “slow fall-off” cases:

$$P_{11}(1440) : \quad g_1^A(Q^2) = \frac{-0.51/D_A}{1 + Q^2/3M_A^2} \text{ (“fast fall-off”)}$$

$$g_1^A(Q^2) = \frac{-0.51/D_A}{1 + Q^2/4.3M_V^2} \text{ (“slow fall-off”) ,}$$

$$S_{11}(1535) : \quad g_1^A(Q^2) = \frac{-0.21/D_A}{1 + Q^2/3M_A^2} \text{ (“fast fall-off”)}$$

$$g_1^A(Q^2) = \frac{-0.21/D_A}{1 + Q^2/1.2M_A^2} \left[1 + 7.2 \ln \left(1 + \frac{Q^2}{1 \text{ GeV}^2} \right) \right] \text{ (“slow fall-off”)}$$

How to calculate the cross section

The calculations have been done [Schreiner, von Hippel, NPB 58 \(1973\) 333](#), neglecting lepton, that is a valid approximation at $Q^2 \gg m_\mu^2$. Recent formulas including muon mass [Paschos, O.L., PRD 71 \(2005\) 074003](#) we present it in a form close to DIS. The cross section in this form is the same for all the resonances

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{4\pi} \cos^2 \theta_C \frac{W}{m_N E^2} \left\{ \mathcal{W}_1 (Q^2 + m_\mu^2) + \frac{\mathcal{W}_2}{m_N^2} \left[2(pk)(pk') - \frac{1}{2} m_N^2 (Q^2 + m_\mu^2) \right] - \frac{\mathcal{W}_3}{m_N^2} \left[Q^2 (pk) - \frac{1}{2} (pq)(Q^2 + m_\mu^2) \right] + \frac{\mathcal{W}_4}{m_N^2} m_\mu^2 \frac{(Q^2 + m_\mu^2)}{2} - 2 \frac{\mathcal{W}_5}{m_N^2} m_\mu^2 (pk) \right\}$$

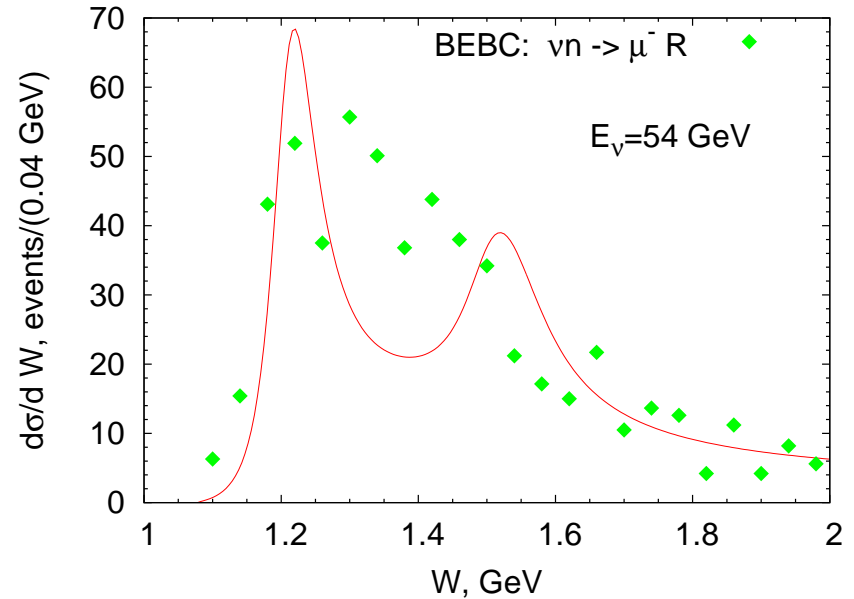
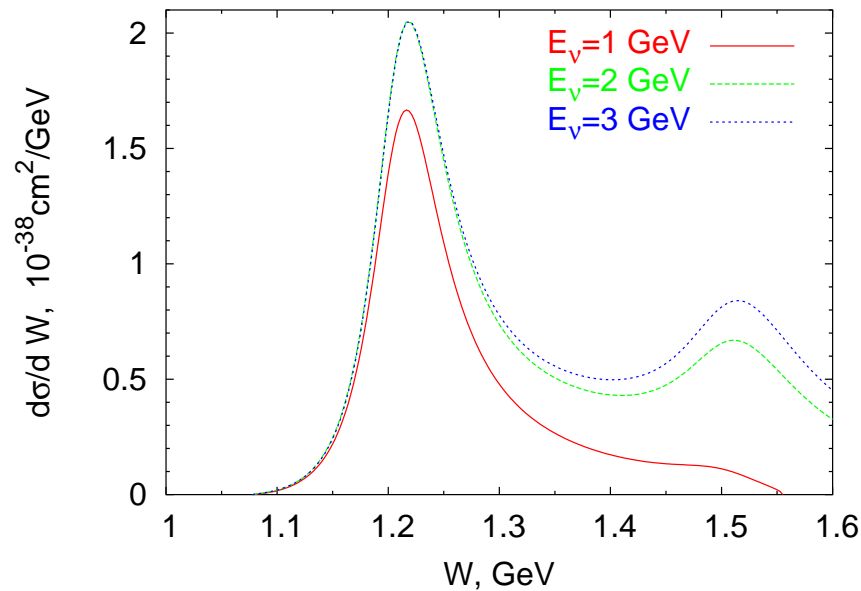
and the hadronic structure functions are defined as usual

$$\mathcal{W}^{\mu\nu} = -g^{\mu\nu} \mathcal{W}_1 + p^\mu p^\nu \frac{\mathcal{W}_2}{m_N^2} - i\varepsilon^{\mu\nu\sigma\lambda} p_\sigma q_\lambda \frac{\mathcal{W}_3}{2m_N^2} + \frac{\mathcal{W}_4}{m_N^2} q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2} (p^\mu q^\nu + p^\nu q^\mu)$$

The functional dependence of the structure functions on the form factors vary with resonance.

- $\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$ give main contribution; \mathcal{W}_i in terms of C_i are given in our paper
- \mathcal{W}_3 describe the vector-axial interference
- $\mathcal{W}_4, \mathcal{W}_5$ contribute to the Xsec proportional to the lepton mass

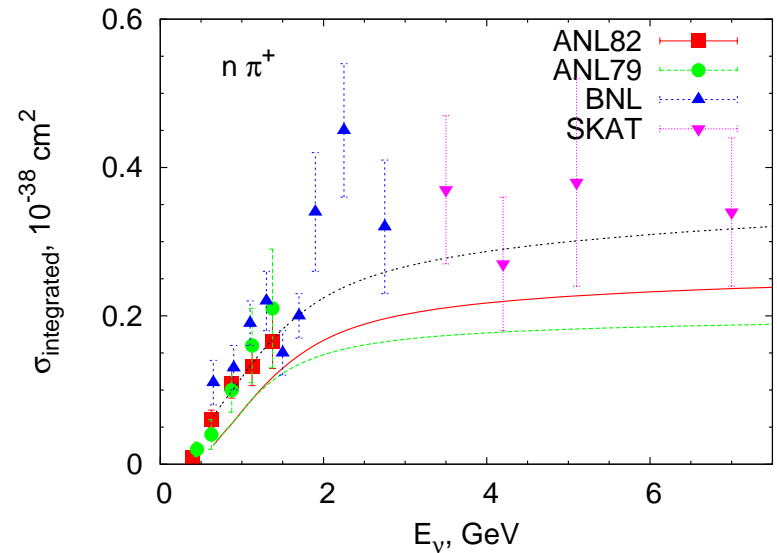
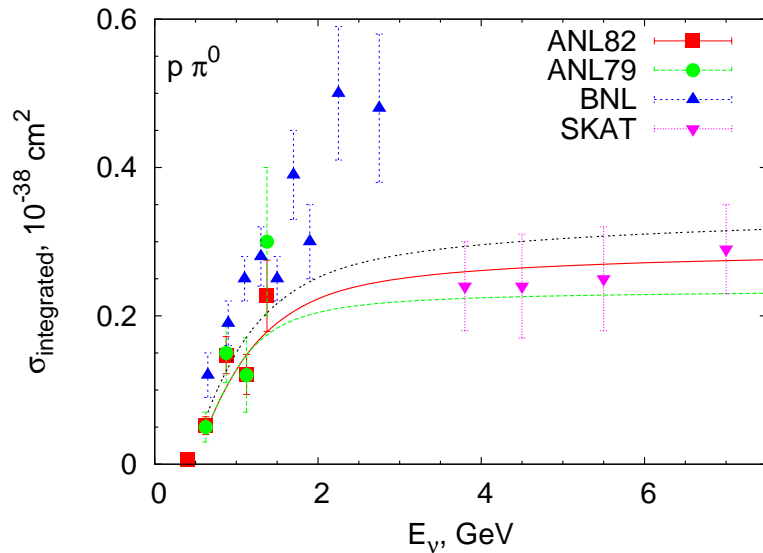
Neutrino production at different E_ν



- At $E_\nu < 1 \text{ GeV}$ the second resonance region is negligible in neutrino scattering. It will not be seen in K2K and MiniBOONE.
- At $E_\nu \sim 50 \text{ GeV}$ the two peaks are clearly seen. However, BEBC experiment [Allasia et al, NPB 343 \(1990\) 285](#) didn't resolve them.



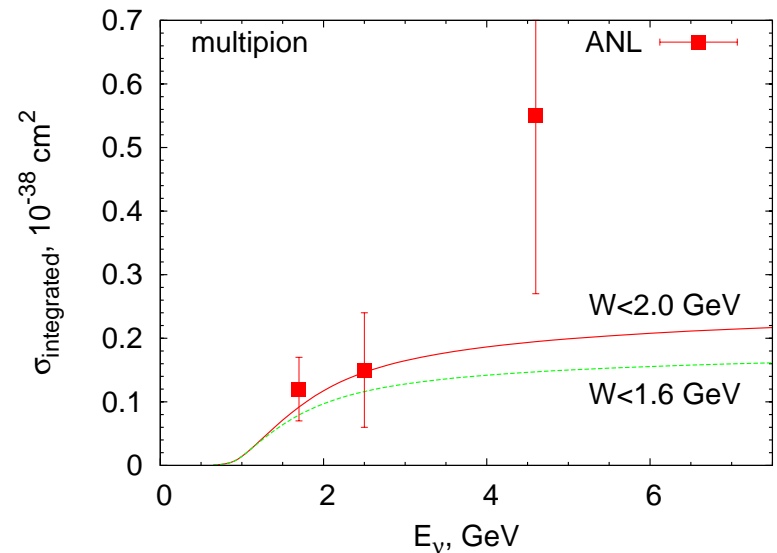
BNL data points are consistently higher than those of ANL and SKAT, errorbars are large



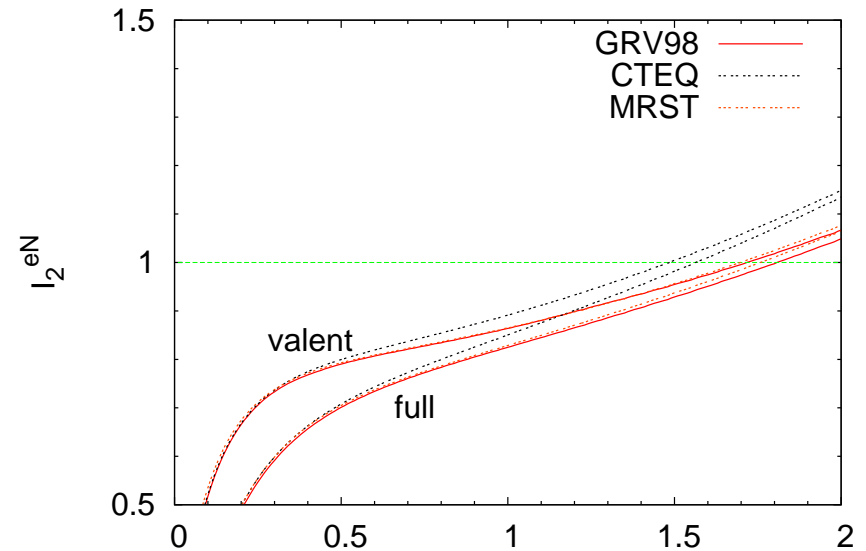
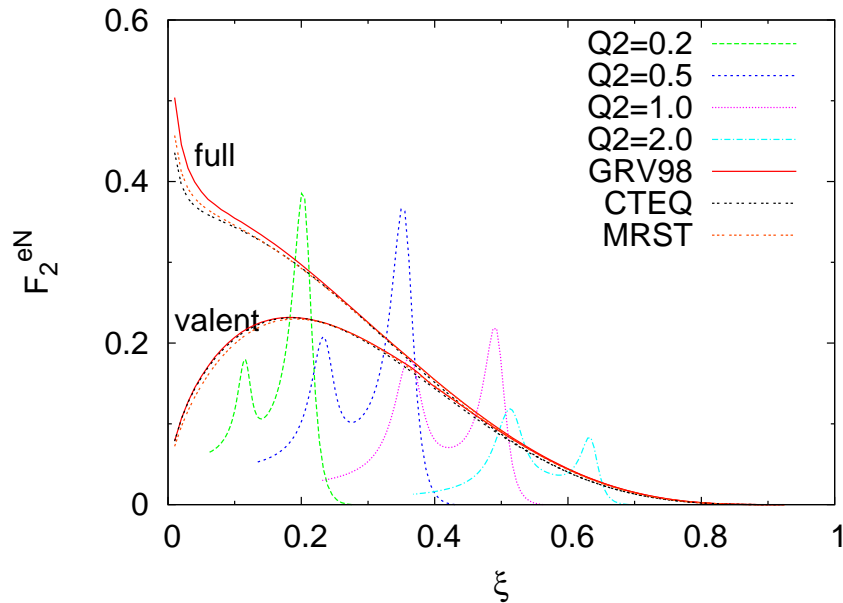
For π^+n channel our curve is a little lower than experimental points: either contributions from higher resonances or a smooth isospin-1/2 incoherent background, for example

$$\sigma_{bgr}^{\pi^+n} = 5 \cdot 10^{-40} \left(\frac{E_\nu}{1 \text{ GeV}} - 0.28 \right)^{1/4} \text{ cm}^2,$$

$$\sigma_{bgr}^{\pi^0p} = \frac{1}{2} \sigma_{bgr}^{\pi^+n}$$



Duality for electron scattering: F_2^{eN}



The use of the Nachtmann scaling variable

$\xi = \frac{2x}{1+(1+4x^2m_N^2/Q^2)^{1/2}}$ includes some of the target mass corrections

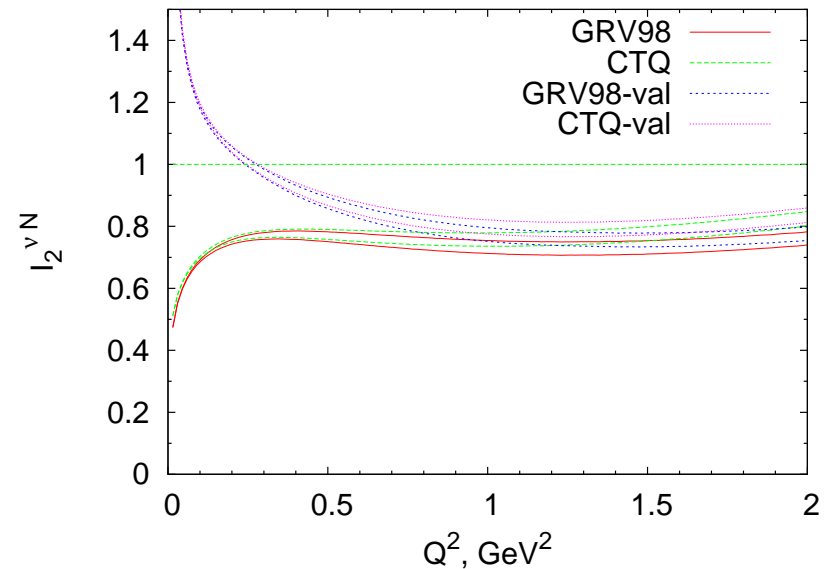
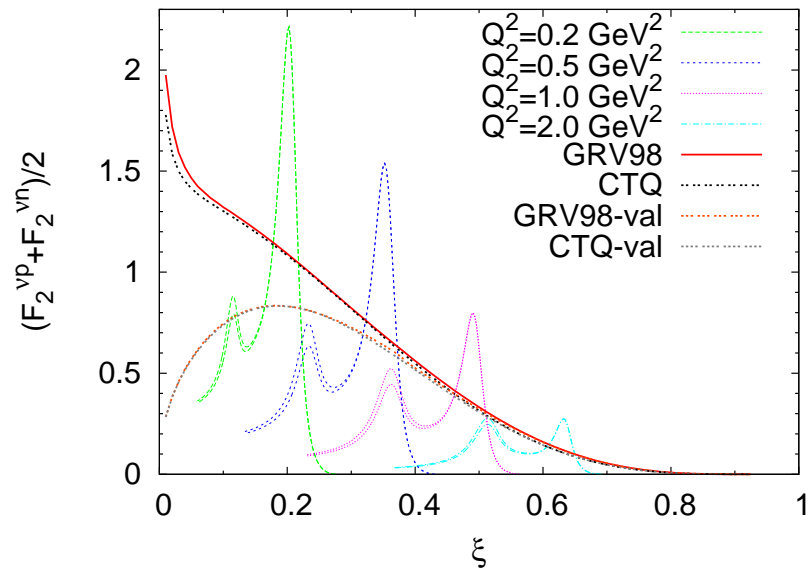
As Q^2 increases, the resonance curves should slide along the DIS curve

$$I_2^{eN}(Q^2) = \frac{\int_{\xi_i}^{\xi_f} d\xi F_2^{eN(\text{res})}(\xi, Q^2)}{\int_{\xi_i}^{\xi_f} d\xi F_2^{eN(\text{LT})}(\xi, Q^2)},$$

$$\xi_i = \xi(W = 1.6 \text{ GeV}, Q^2),$$

$$\xi_f = \xi(W = 1.1 \text{ GeV}, Q^2)$$

Duality for CC neutrino scattering: $F_2^{\nu N}$



Similar results for $P_{33}(1232)$ are in Matsui, Sato, Lee, PRC 72 and for Rein-Sehgal model in Graczyk, Juszczak, Sobczyk, hep-ph/0601077

$$I_2(\text{res/DIS}) = \frac{\int_{\xi_i}^{\xi_f} F_2^{\text{res}}(\xi, Q^2) d\xi}{\int_{\xi_i}^{\xi_f} F_2^{\text{DIS}}(\xi, Q^2) d\xi},$$

$$\xi_i = \xi_i(W = 1.6 \text{ GeV}, Q^2),$$

$$\xi_f = \xi_f(W = 1.1 \text{ GeV}, Q^2).$$

The discrepancy will be eliminated by including other resonances, or background, or modifying the FF at large Q^2 .

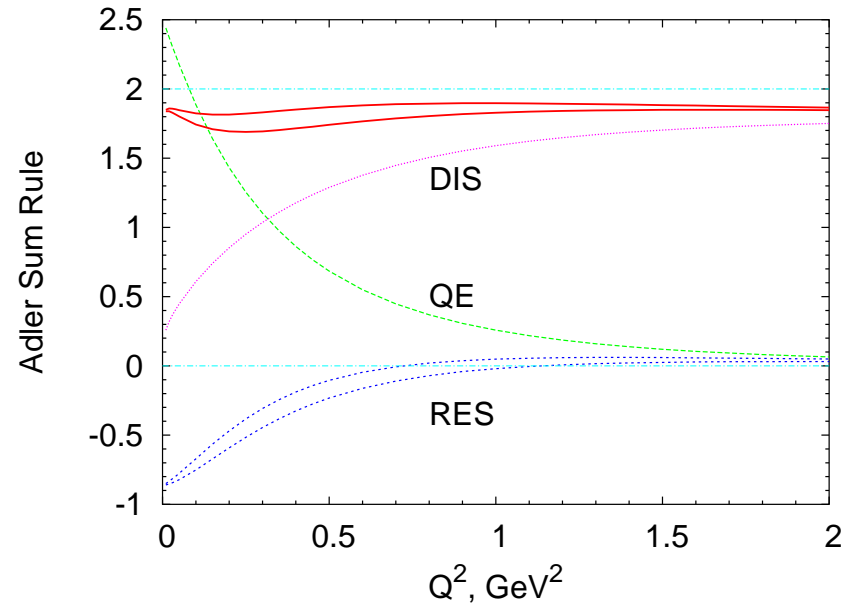
Resonance contribution to the Adler sum rule

$$\left[g_{1V}^{(QE)} \right]^2 + \left[g_{1A}^{(QE)} \right]^2 + \left[g_{2V}^{(QE)} \right]^2 \frac{Q^2}{2M^2} + \int d\nu \left[W_2^{\nu n}(Q^2, \nu) - W_2^{\nu p}(Q^2, \nu) \right] = 2$$

Using for QE

$$g_{1V}^{(QE)} = \frac{1}{D_V},$$
$$g_{2V}^{(QE)} = \frac{3.7}{D_V},$$
$$g_{1A}^{(QE)} = \frac{1.23}{D_A}.$$

Adler sum Rule is satisfied with a 10% accuracy

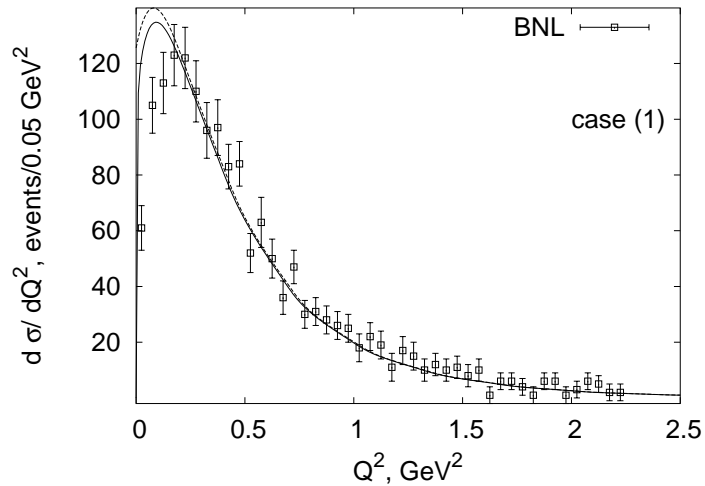


Summary

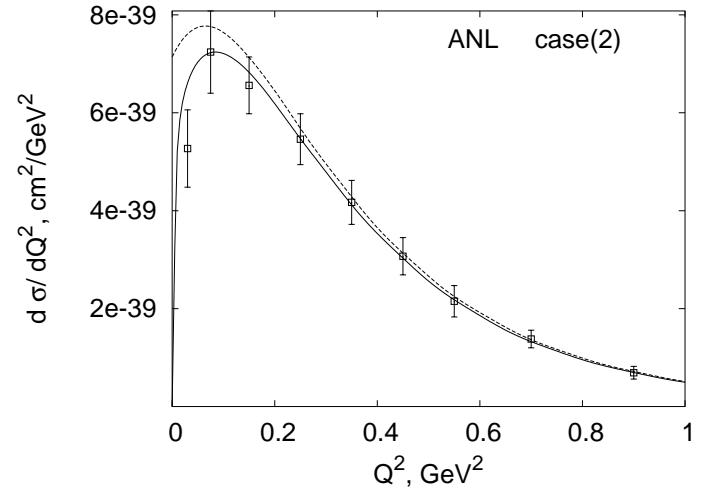
- We present a general formalism for analysing the excitation of resonances by electrons and neutrinos
- We use recent data on helicity amplitudes from JLab and Mainz accelerator to determine the electromagnetic and weak vector form factors including their Q^2 -dependences
- For the isospin-1/2 resonances the form factors fall down not so fast as for the Δ . For $P_{11}(1440)$ and $S_{11}(1535)$ fall off, at small Q^2 , is even slower than dipole.
- Δ resonance description shows good agreement with the data, second resonance region must be observable in experiments for $E_\nu > 2$ GeV
- Quark–hadron duality for CC neutrino scattering is satisfied in the region $Q^2 = 0.2 - 1.5$ GeV² for the $(F_2^{\nu p} + F_2^{\nu n})/2$ at the level of $\sim 20\%$. The Adler sum rule is satisfied with a 10% accuracy.

Other topics

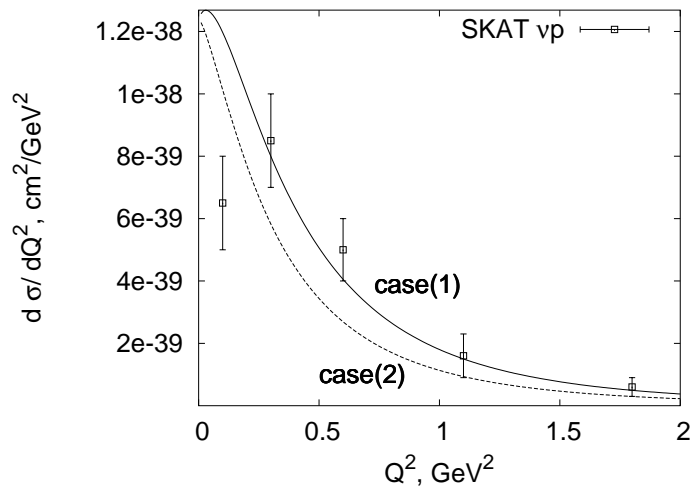
Cross section for $\nu_{\mu} p \rightarrow \mu^{-} \Delta^{+++} \rightarrow \mu^{-} p \pi^{+}$



$\langle E_{\nu} \rangle \sim 1 \text{ GeV}$



$\langle E_{\nu} \rangle \sim 1 \text{ GeV}$



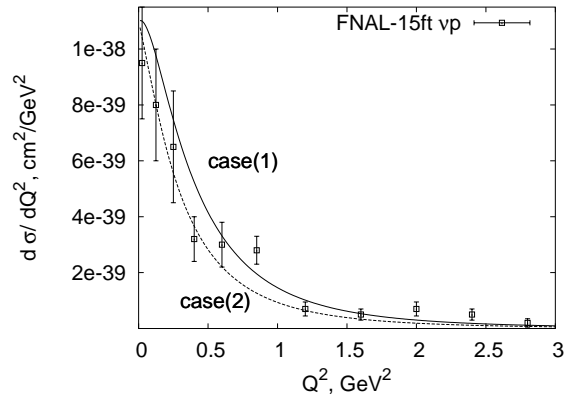
$\langle E_{\nu} \rangle \sim 7 \text{ GeV}$

$$\text{case (1): } C_5^A(Q^2) = \frac{C_5^A(0)}{D_A} \cdot \frac{1}{1 + \frac{Q^2}{3M_A^2}}$$

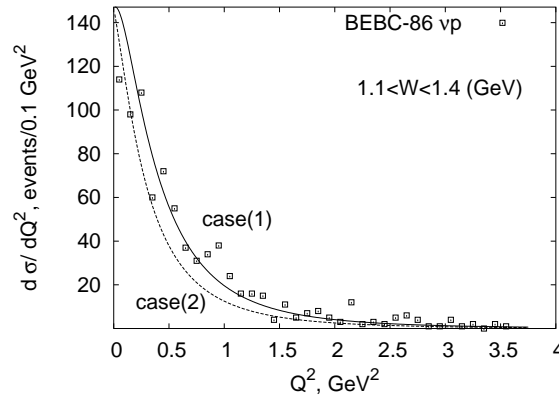
$$\text{case (2): } C_5^A(Q^2) = \frac{C_5^A(0)}{D_A} \cdot \frac{1}{1 + \frac{2Q^2}{M_A^2}}$$

Paschos, O.L. PRD 71 (2005) 074003

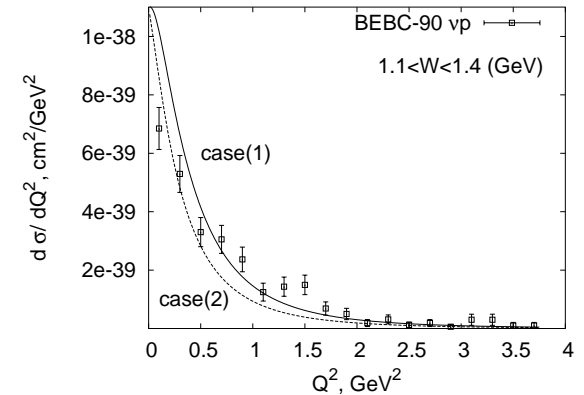
Cross section for $\nu_{\mu} p \rightarrow \mu^{-} \Delta^{+++} \rightarrow \mu^{-} p \pi^{+}$



$E_{\nu} \sim 15 - 40 \text{ GeV}$



$\langle E_{\nu} \rangle \sim 54 \text{ GeV}$



$\langle E_{\nu} \rangle \sim 54 \text{ GeV}$

The low Q^2 region still to be determined and understood precisely.

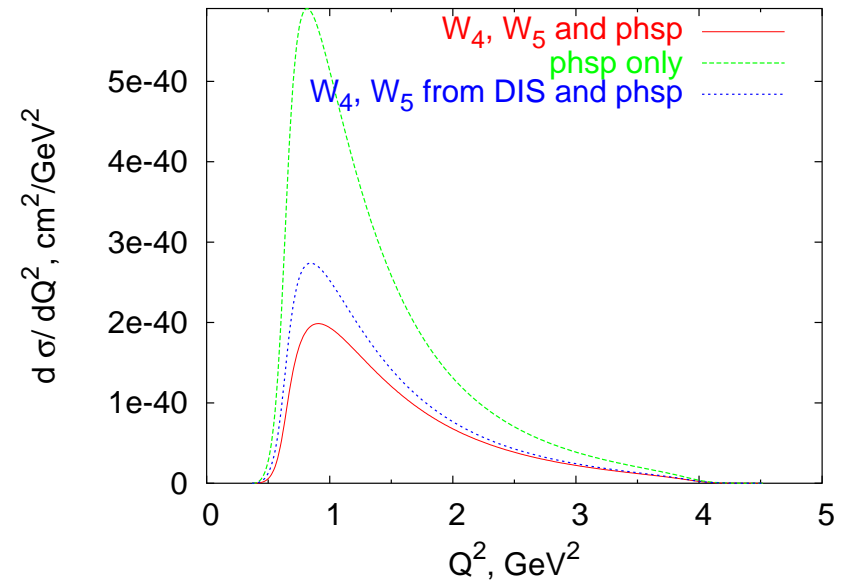
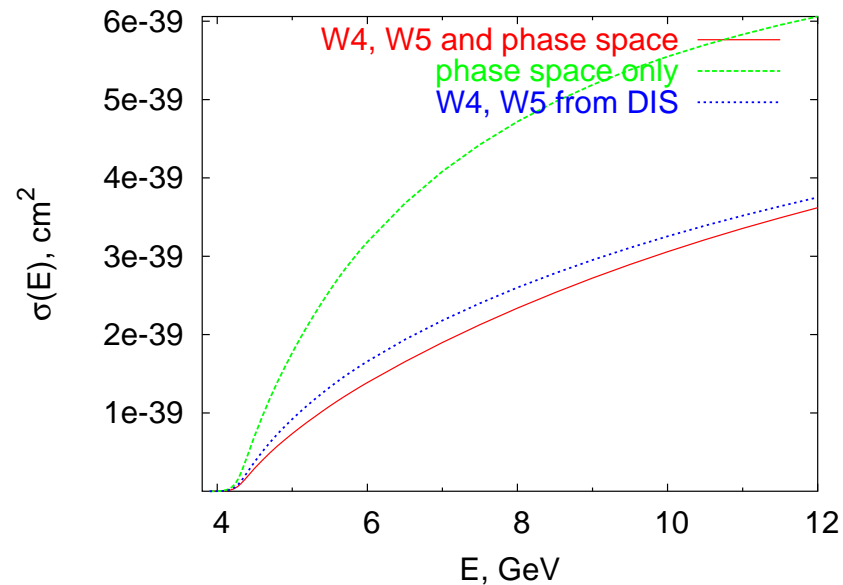
Factors, which decrease the cross section are: 1) Pauli blocking [Paschos, Sakuda, Yu PRD 69 \(2004\) 014013](#) 2) muon mass effects [Paschos, O.L., PRD 71 \(2005\) 074003](#)

For the $d\sigma/dQ^2$ lepton mass effects are noticeable at low Q^2 for small neutrino energies.

τ -production: $\nu_\tau p \rightarrow \tau^- \Delta^{++} \rightarrow \tau^- p \pi^+$

Taking into account nonzero mass if the final lepton reduces the cross section in two ways:

- 1) due to the kinematical restrictions on the phase space available
- 2) due to the "small" structure functions \mathcal{W}_4 and \mathcal{W}_5



red line: both effects are taken into account

green line: only the reduction of the phase space is taken into account; ($\mathcal{W}_4 = \mathcal{W}_5 = 0$)

blue line: structure functions in the partonic limit: $\mathcal{W}_4 = 0$, $\mathcal{W}_5 = \mathcal{W}_2/2x$

Background

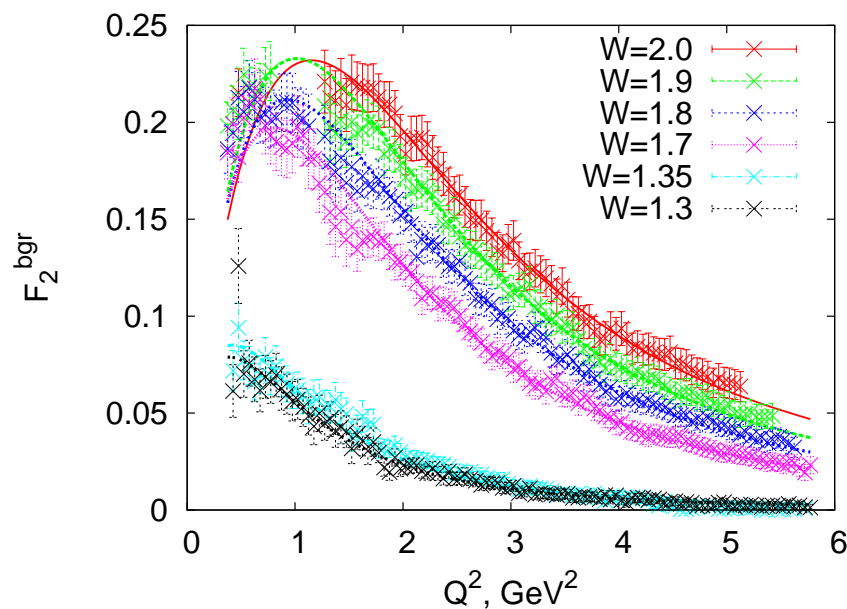
(very preliminary !)

Motivation: simplicity, only noninterfering background can be introduced in such a way

Problem: to introduce a single function $background(Q^2, W)$ in such a way that it can be added to any differential or integrated cross section

Question: for what quantity(ies) should such background be introduced?

My answer: for the structure functions!



$$F_2^{bgr} = F_2^{JLab} - F_2^{RES}$$

F_2^{JLab} is experimental data on the structure function from JLab experiment M. Osipenko et al., PR C73 (2006) 045205; hep-ex/0507098

F_2^{RES} is calculated in our model

Another way to extract F_2^{bgr} — from old virtual photoproduction data F.W. Brasse et al., NP B110 (1976) 413 — gives similar result