Global Analysis for Nuclear PDFs

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Workshop on Intersections of Nuclear Physics
with Neutrinos and Electrons

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- Parton Distribution Functions (PDFs) in the Nucleon
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    Motivation, Status, Naive expectation from data

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Introduction

Parton Distribution Functions

(PDFs)

in the Nucleon

Recent papers on unpolarized PDFs in the nucleon

CTEQ (uncertainties) D. Stump (J. Pumplin) et al., Phys. Rev. D65 (2001) 14012 & 14013.
(CTEQ6) D. Pumplin et al., JHEP, 07 (2002) 012; JHEP, 0310 (2003) 046;
Eur. Phys. J. C40 (2005) 145; Phys.Rev. D69 (2004) 114005.

GRV (GRV98) M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C5 (1998) 461.
--- no recent update

MRST A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne,

(MRST2001) Eur. Phys. J. C23 (2002) 73;
(MRST2002, MRST2003-conservative) Eur. Phys. J. C28 (2003) 455;
(theoretical errors) Eur. Phys. J. C35 (2004) 325;
(MRST2004-physical gluon) Phys. Lett. B604 (2004) 61;
(MRST2004-QED) Eur. Phys. J. C39 (2005) 155.

Alekhin S. I. Alekhin, Phys. Rev. D68 (2003) 014002.

ZEUS S. Chekanov et al., hep-ph/0503274. **H1** C. Targett-Adams, hep-ex/0507024.

It is likely that I miss some papers!

Recent activities

- uncertainties of PDFs
- NNLO analysis
- QED corrections

Parton distribution functions are determined by fitting various experimental data.

- electron/muon: $\mu + p \to \mu + X$
- neutrino: $\nu_\mu + p \to \mu + X$
- Drell-Yan: $p + p \to \mu^+ \mu^- + X$
- $\cdots$

(1) assume functional form of PDFs at fixed $Q^2 (\equiv Q_0^2)$:

e.g. $f_i(x, Q_0^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x),$

where $i = u_v,\ d_v,\ \bar{u},\ \bar{d},\ \bar{s},\ g$

(2) calculate observables at their experimental Q^2 points.

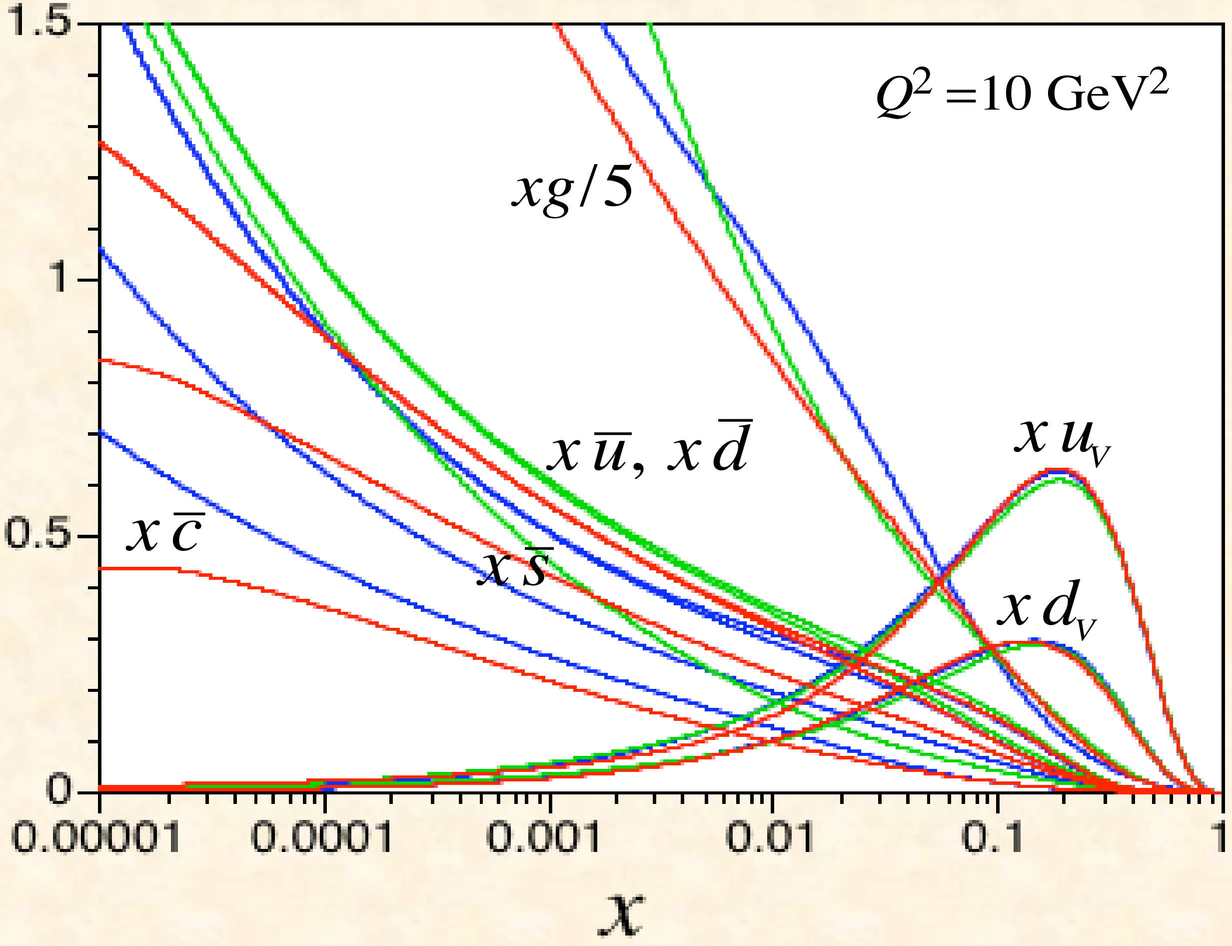
(3) then, the parameters $A_i,\ \alpha_i,\ \beta_i,\ \gamma_i$ are determined so as to minimize χ^2 in comparison with data.

Recent unpolarized distributions

see **http://durpdg.dur.ac.uk/hepdata/pdf.html**

CTEQ6, JHEP 0207 (2002) 012; **GRV98**, Eur. Phys. J. C5 (1998) 461;

MRST02, Eur. Phys. J. C28 (2003) 455.



Parton Distribution Functions

in Nuclei

Status of PDF determinations

Unpolarized PDFs in the nucleon ★ ★ ★ ★

Investigated by 3 major groups (CTEQ, GRV, MRST).

Well studied from small x to large x in the wide range of Q^2.

The details are known. **(Recent studies: NNLO, QED, error analysis, $s-\bar{s}$, …)**

"Polarized" PDFs in the nucleon ★ ★

Investigated by several groups (GS, GRSV, LSS, AAC, BB, …).

Available data are limited (DIS) at this stage. (recent: HERMES, JLab)

New data from RHIC-Spin, COMPASS, J-PARC, eRHIC, ELIC, …

PDFs in "nuclei" ★ ★

Investigated by only a few groups. Not so investigated!

Available data are limited (inclusive DIS, Drell-Yan).

New data from RHIC, LHC, JLab, J-PARC, eRHIC, ELIC…

Why nuclear parton distribution functions?

(1) Basic interest to understand nuclear structure in the high-energy region, Determination of $\sin^2\theta_W$

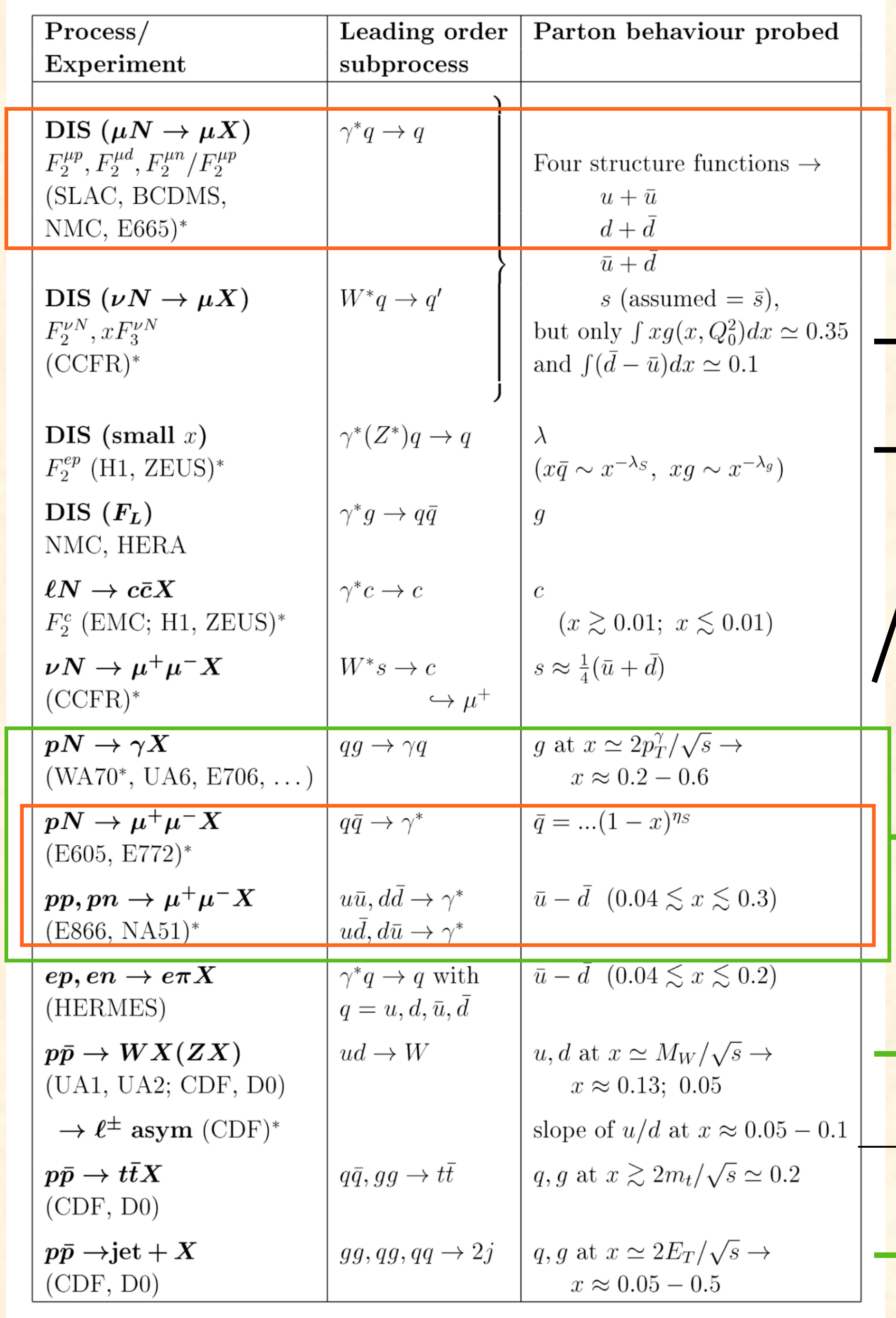
- perturbative & non-perturbative QCD
- $\sin^2\theta_W$ in neutrino scattering (NuTeV)

(2) Practical purpose to describe hadron cross sections precisely

- heavy-ion reactions: quark-gluon plasma signature
- long-baseline neutrino experiments:

  nuclear effects in $\nu + {}^{16}O$
- nuclear corrections for extracting u_v and d_v from NuTeV / CCFR

Situation of data for nuclear PDFs



Process/ Experiment	Leading order subprocess	Parton behaviour probed
DIS ($\mu N \to \mu X$) $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)*	$\gamma^* q \to q$	Four structure functions → $u + \bar{u}$ $d + \bar{d}$ $\bar{u} + \bar{d}$ s (assumed $= \bar{s}$), but only $\int xg(x, Q_0^2)dx \simeq 0.35$ and $\int(\bar{d} - \bar{u})dx \simeq 0.1$
DIS ($\nu N \to \mu X$) $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)*	$W^* q \to q'$	
DIS (small x) F_2^{ep} (H1, ZEUS)*	$\gamma^*(Z^*)q \to q$	λ ($x\bar{q} \sim x^{-\lambda_S}$, $xg \sim x^{-\lambda_g}$)
DIS (F_L) NMC, HERA	$\gamma^* g \to q\bar{q}$	g
$\ell N \to c\bar{c}X$ F_2^c (EMC; H1, ZEUS)*	$\gamma^* c \to c$	c ($x \gtrsim 0.01$; $x \lesssim 0.01$)
$\nu N \to \mu^+\mu^- X$ (CCFR)*	$W^* s \to c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{4}(\bar{u} + \bar{d})$
$pN \to \gamma X$ (WA70*, UA6, E706, …)	$qg \to \gamma q$	g at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2 - 0.6$
$pN \to \mu^+\mu^- X$ (E605, E772)*	$q\bar{q} \to \gamma^*$	$\bar{q} = ...(1-x)^{\eta_S}$
$pp, pn \to \mu^+\mu^- X$ (E866, NA51)*	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{u} - \bar{d}$ ($0.04 \lesssim x \lesssim 0.3$)
$ep, en \to e\pi X$ (HERMES)	$\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$	$\bar{u} - \bar{d}$ ($0.04 \lesssim x \lesssim 0.2$)
$p\bar{p} \to WX(ZX)$ (UA1, UA2; CDF, D0)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13$; 0.05
→ **$\ell^\pm$ asym** (CDF)*		slope of u/d at $x \approx 0.05 - 0.1$
$p\bar{p} \to t\bar{t}X$ (CDF, D0)	$q\bar{q}, gg \to t\bar{t}$	q, g at $x \gtrsim 2m_t/\sqrt{s} \simeq 0.2$
$p\bar{p} \to$jet $+ X$ (CDF, D0)	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05 - 0.5$

Available data
for nuclear PDFs

Jlab at large x

Neutrino factory: 10~15 years later ?

Small-x,
high-energy electron facility?
(ELIC, eRHIC)

RHIC, LHC, J-PARC

RHIC, LHC

Table from MRST,
hep/ph-9803445

RHIC, LHC

Current nuclear data are kinematically limited.

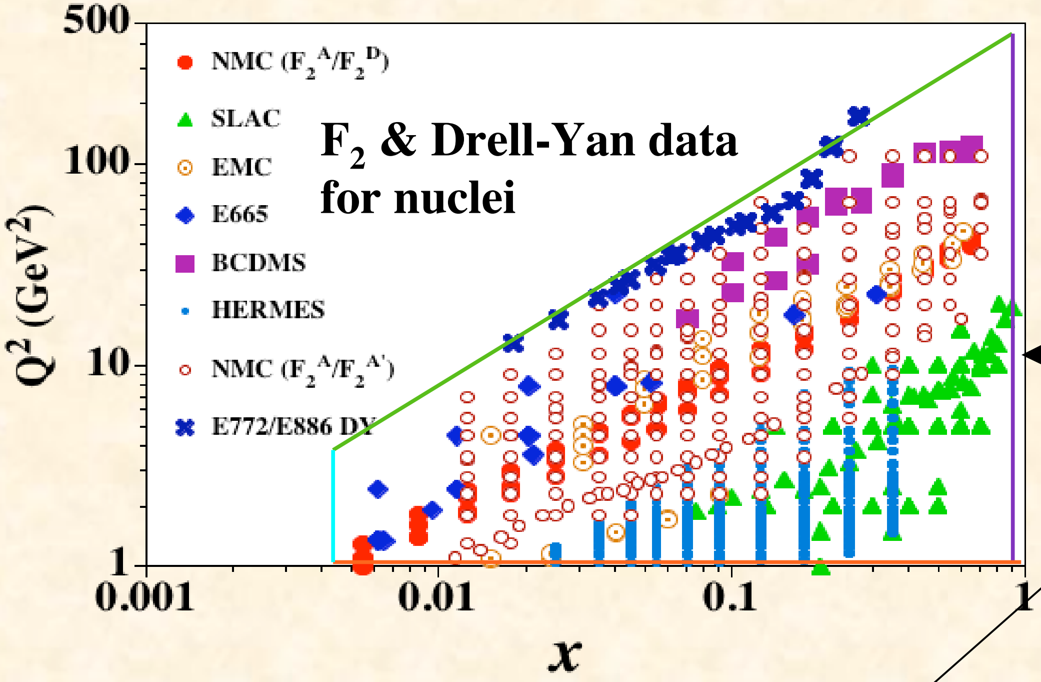
$$x = \frac{Q^2}{2p\cdot q} \simeq \frac{Q^2}{ys}$$

(from H1 and ZEUS, hep-ex/0502008)

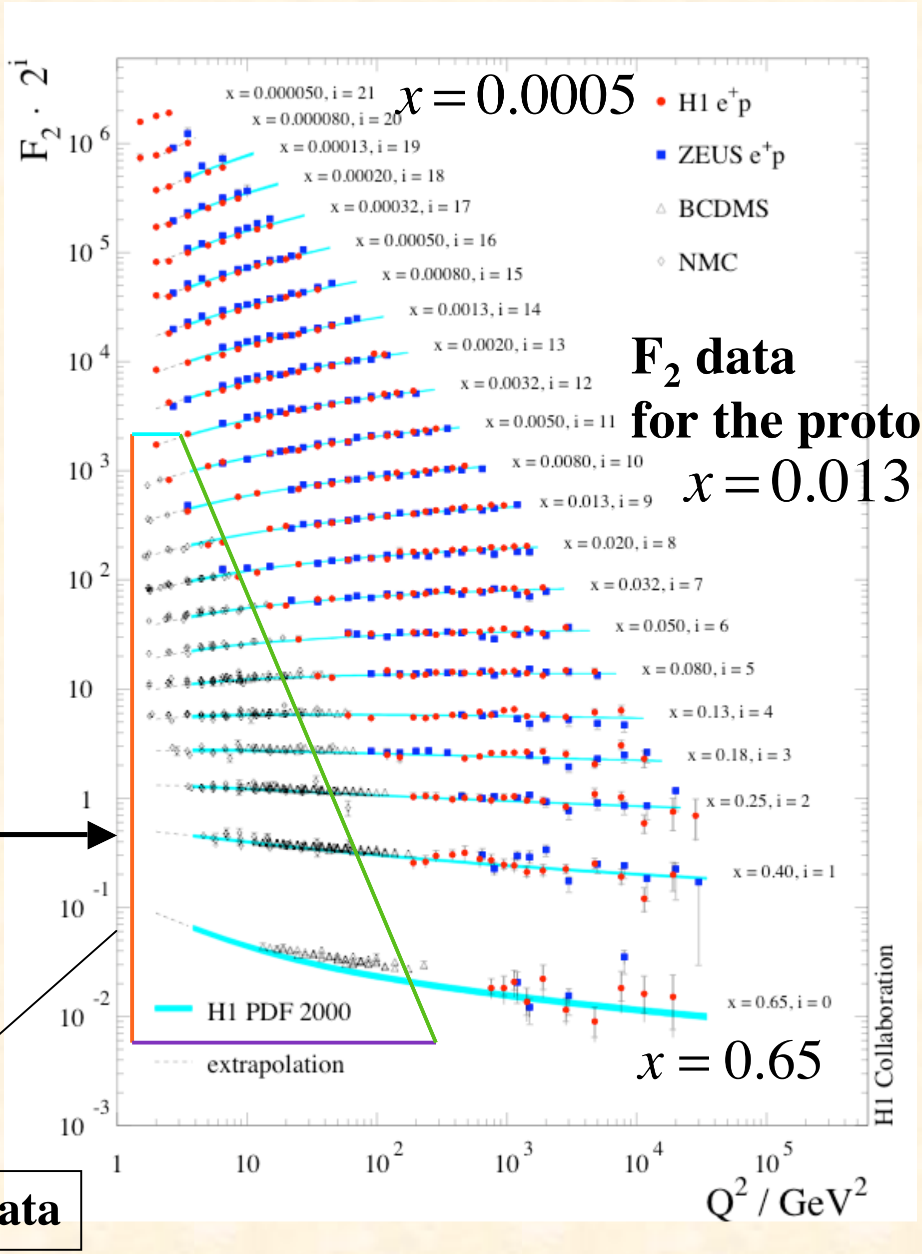
$$\text{fixed target:}\;\; \min(x) = \frac{Q^2}{2M_N E_{lepton}} \le \frac{1}{2E_{lepton}(\text{GeV})}$$

$$\text{if}\;\; Q^2 \ge 1\ \text{GeV}^2$$

$$\text{for } E_{lepton}(\text{NMC}) = 200\ \text{GeV},\;\; \min(x) = \frac{1}{2\cdot 200} = 0.003$$



F$_2$ & Drell-Yan data for nuclei



$x = 0.0005$

F$_2$ data for the proton

$x = 0.013$

$x = 0.65$

region of nuclear data

References on Nuclear PDFs

There are only a few papers on
the parametrization of nuclear PDFs!
→ Need much more works.

(EKRS) K. J. Eskola, V. J. Kolhinen, and P. V. Ruuskanen,
Nucl. Phys. B535 (1998) 351;
K. J. Eskola, V. J. Kolhinen, and C. A. Salgado,
Eur. Phys. J. C9 (1999) 61.

χ^2 analysis

(HKM, HKN) M. Hirai, SK, M. Miyama, Phys. Rev. D64 (2001) 034003;
M. Hirai, SK, T.-H. Nagai, Phys. Rev. C70 (2004) 044905.

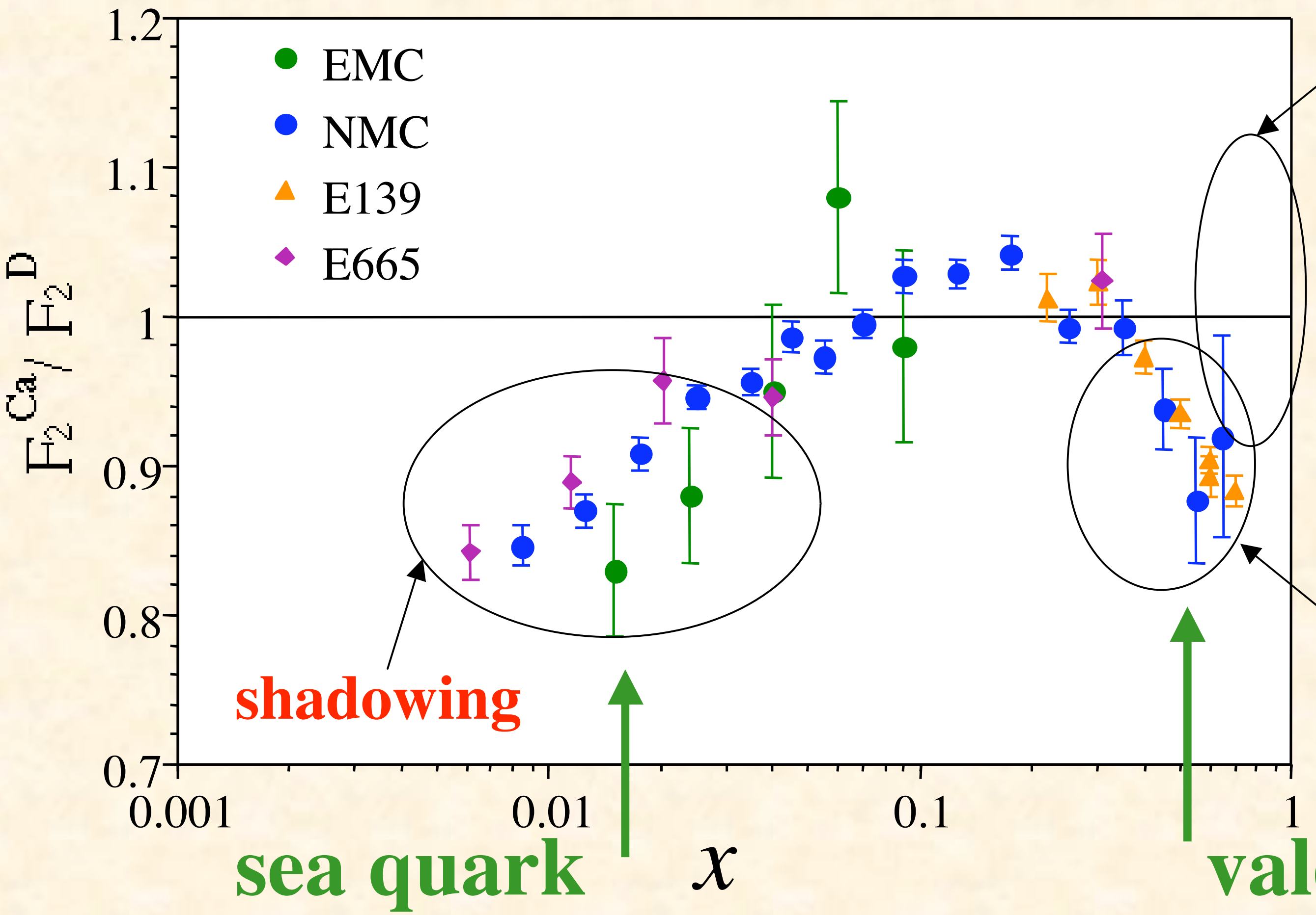
(DS) D. de Florian and R. Sassot, Phys. Rev. D69 (2004) 074028.

talks by R. Petti

Nuclear modification

For review,
D. F. Geesaman, K. Saito, A. W. Thomas,
Ann. Rev. Nucl. Part. Sci. 45 (1995) 337.

Nuclear modification of F_2^A / F_2^D is well known in electron/muon scattering.



Fermi motion

shadowing

Binding,
N modification

sea quark

valence quark

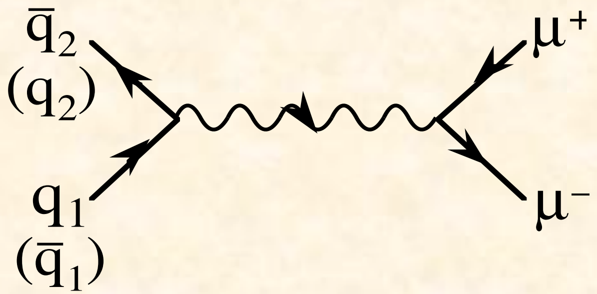
Sea quark

e/μ scattering

$$F_2^N = \frac{F_2^p + F_2^n}{2} = \frac{5}{18} x\left(u + \bar{u} + d + \bar{d}\right) + \frac{2}{18} x\left(u + \bar{u} + d + \bar{d}\right)$$

$$= \frac{5}{18} xV + \underline{\frac{4}{18} xS} \quad \text{if } \bar{q} \text{ distributions are flavor symmetric}$$

Drell-Yan (lepton-pair production)



$$p_1 + p_2 \to \mu^+ \mu^- + X$$

$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$

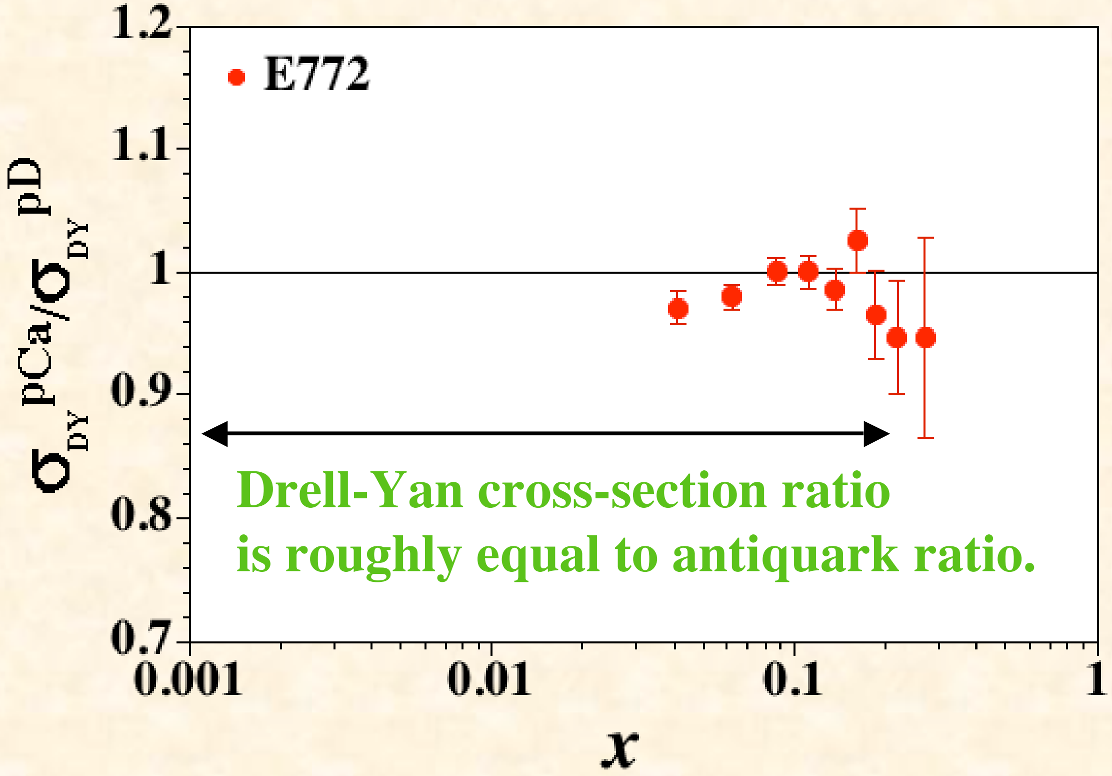
$$\text{at large } x_F = \underset{\substack{\uparrow \\ \text{projectile}}}{x_1} - \underset{\substack{\uparrow \\ \text{target}}}{x_2}$$

$$d\sigma \propto q_V(x_1)\bar{q}(x_2)$$

$\bar{q}(x_2)$ can be obtained if $q_V(x_1)$ is known.

Drell-Yan and Antiquark Distributions

$$p + A \rightarrow \mu^{+}\mu^{-} + X$$



Drell-Yan cross-section ratio
is roughly equal to antiquark ratio.

$$\frac{\sigma_{DY}^{pCa}}{\sigma_{DY}^{pD}} \approx \frac{\bar{q}^{Ca}}{\bar{q}^{D}}$$

The Fermilab E772 Drell-Yan data suggested that nuclear modification of antiquark distributions should be small in the region, x≈0.1.

Scaling Violation and Gluon Distributions

$$\frac{\partial}{\partial \log Q^2} q_i^+(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\cancel{\sum_j P_{q_iq_j}(x/y)\,q_j^+(y,Q^2)} + \underline{P_{qg}(x/y)\,g(y,Q^2)}\right]$$

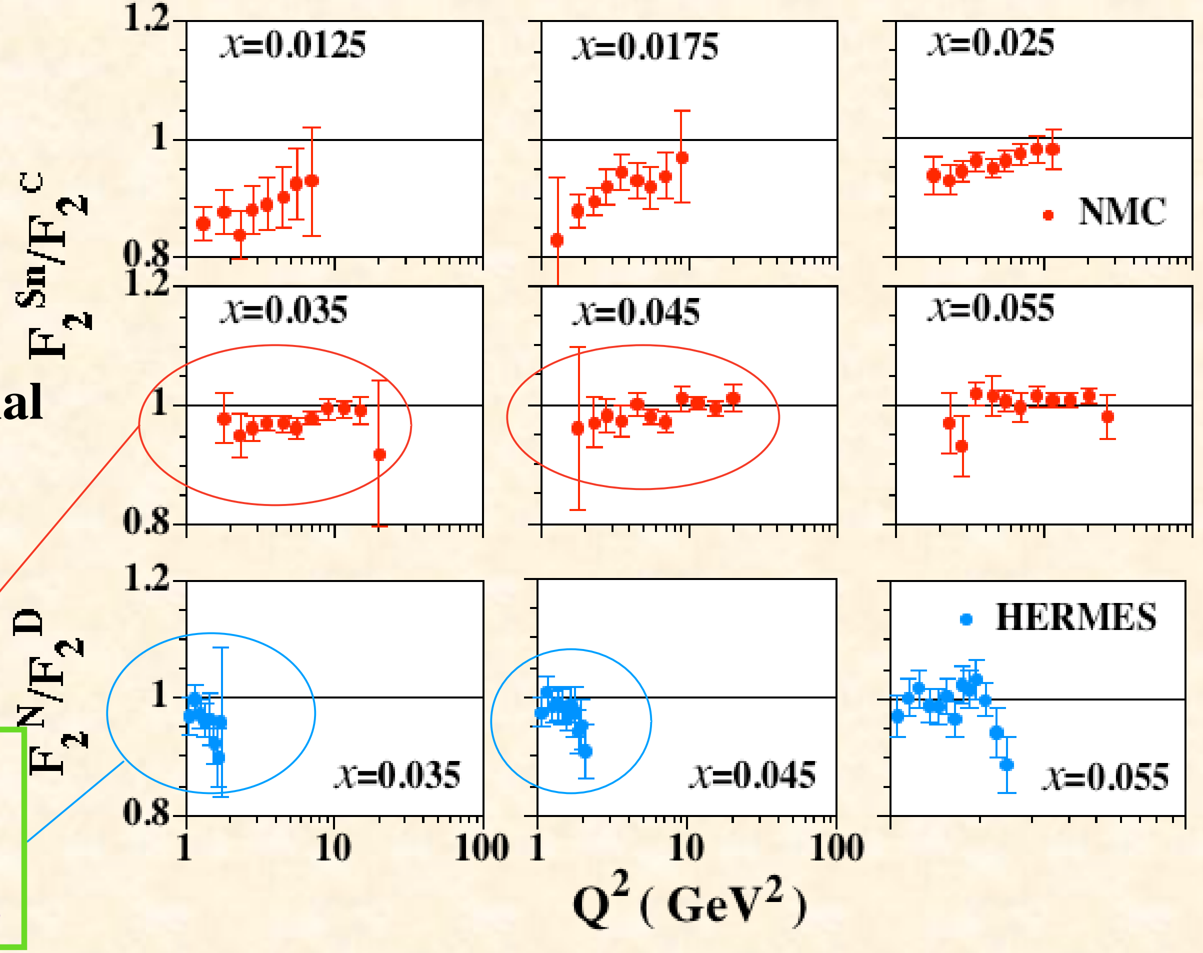
dominant term at small x

$$q_i^+ = q_i + \bar{q}_i$$

at small x

$$\frac{\partial F_2}{\partial\left(\ln Q^2\right)} \approx \frac{20\,\alpha_s}{27\pi}\,xg$$

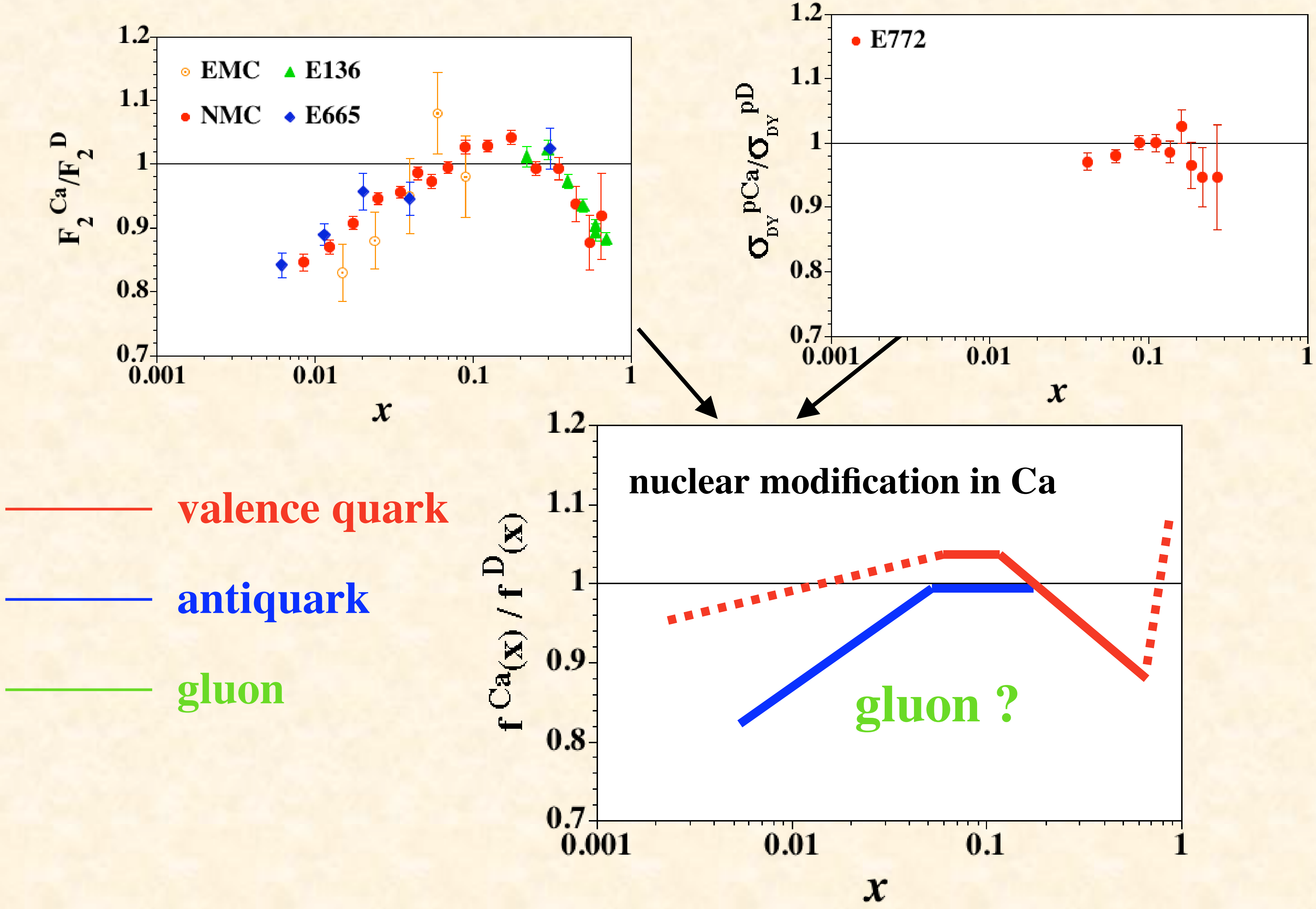
Q^2 dependence of F_2 is proportional to the gluon distribution.



No experimental consensus of Q^2 dependence!

→ $G^A(x)$ determination is difficult.

Nuclear modification of PDFs without any numerical analysis



Our Analysis

Our works

The optimum nuclear PDFs are determined by χ^2 analysis of nuclear F_2 and Drell-Yan data.

(1) M. Hirai, SK, M. Miyama, Phys. Rev. D64 (2001) 034003.

(2) M. Hirai, SK, T.-H. Nagai, Phys. Rev. C70 (2004) 044905.

(3) Research in progress (better fit, NLO effects, …)

NPDF code could be obtained from
http://research.kek.jp/people/kumanos/nuclp.html

The code can be used for calculating nuclear PDFs for a given nucleus, x, and Q^2.

Application: Nuclear PDF effects on $\sin^2\theta_W$

(1) SK, Phys. Rev. D66 (2002) 111301.

(2) M. Hirai, SK, T.-H. Nagai, Phys. Rev. D71 (2005) 113007.

Nuclear parton distributions (per nucleon)

if there *were* no modification

$$A\, u^A = Z\, u^p + N\, u^n, \quad A\, d^A = Z\, d^p + N\, d^n$$

Isospin symmetry: $u^n = d^p \equiv d\,,\ d^n = u^p \equiv u$

$$\rightarrow u^A = \frac{Z\, u + N\, d}{A}, \qquad d^A = \frac{Z\, d + N\, u}{A}$$

Take into accont the nuclear modification

by the factors $w_i(x,A)$

$$u_v^A(x) = w_{u_v}(x,A)\ \frac{Z\, u_v(x) + N\, d_v(x)}{A}$$

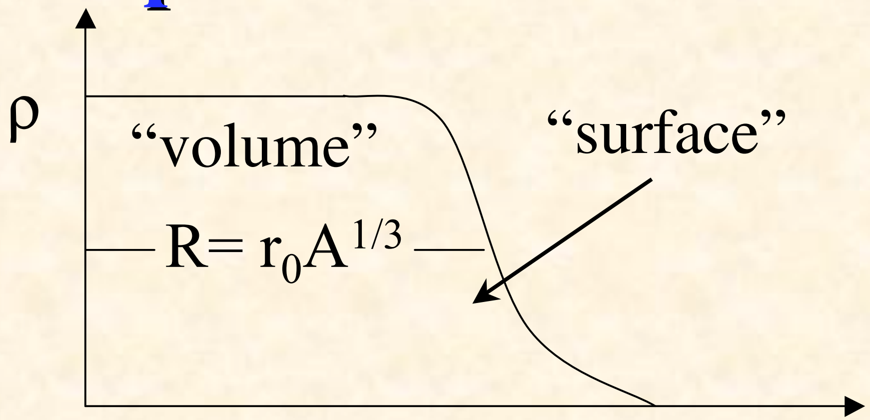
$$d_v^A(x) = w_{d_v}(x,A)\ \frac{Z\, d_v(x) + N\, u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\ \bar{q}(x)$$

$$g^A(x) = w_g(x,A)\ g(x)$$

A dependence

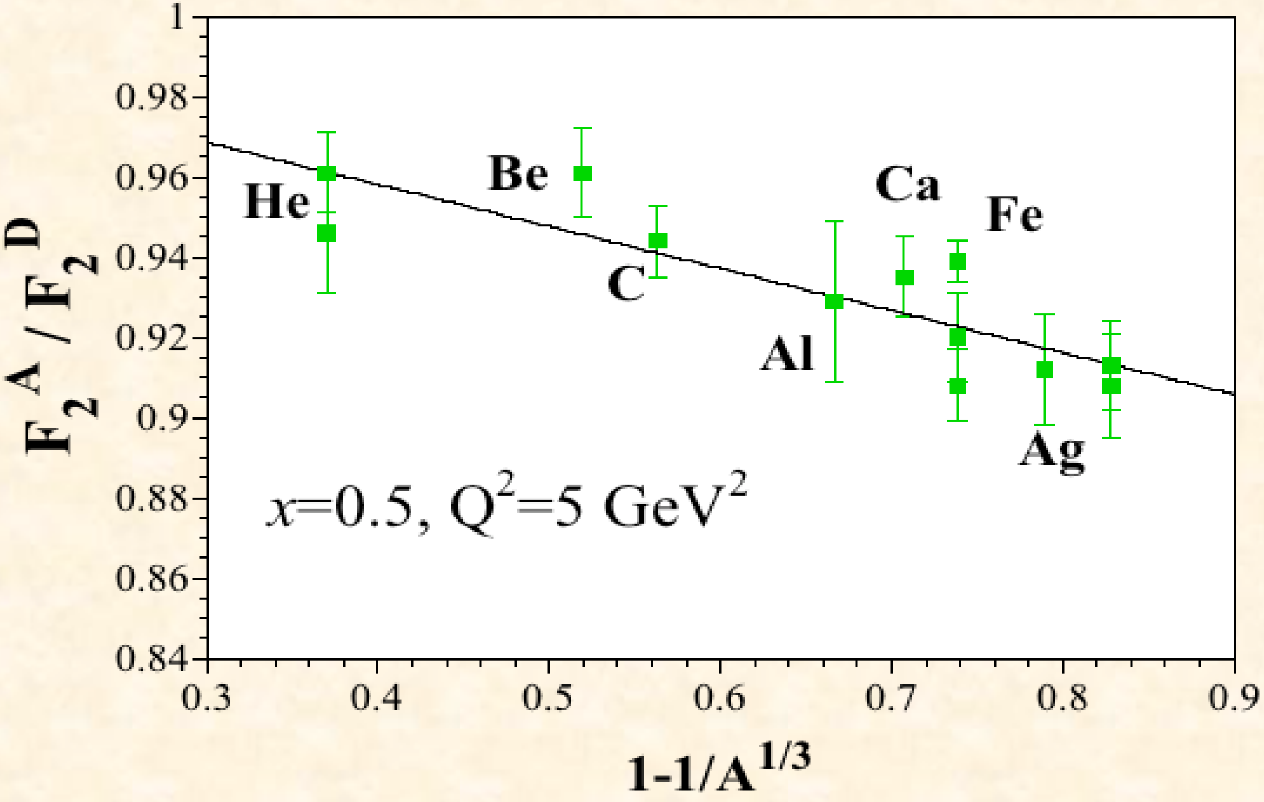
Ref. I. Sick and D. Day, Phys. Lett. B 274 (1992)



roughly speaking $\sigma_A = A\,\sigma_V + A^{2/3}\,\sigma_S$

$$\rightarrow \frac{\sigma_A}{A} = \sigma_V + \frac{1}{A^{1/3}}\,\sigma_S$$

$$\sim \frac{1}{A^{1/3}} \text{ dependence}$$



We are trying
a more complicated
A dependence.

Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A)\, f_i(x), \quad i = u_v,\, d_v,\, \bar{q},\, g$$

first, assume the A dependence as $1/A^{1/3}$

then, use $w_i(x,A) = 1 + (1-1/A^{1/3}) \dfrac{a_i+b_i x+c_i x^2+d_i x^3}{(1-x)^{\beta_i}}$

$a_i, b_i, c_i, d_i, \beta_i$: parameters to be determined by χ^2 analysis

Fermi motion: $\dfrac{1}{(1-x)^{\beta_i}} \to \infty$ as $x \to 1$ if $\beta_i > 0$

Shadowing: $w_i(x \to 0, A) = 1 + (1-1/A^{1/3})\, a_i < 1$

Fine tuning: b_i, c_i, d_i

Constraints

- **Nuclear charge**

$$Z = A \int dx \left[\frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right]$$

$$= A \int dx \left(\frac{2}{3} u_v^A - \frac{1}{3} d_v^A \right)$$

- **Baryon number:** $A = A \int dx \, \frac{1}{3} (u_v^A + d_v^A)$

- **Momentum:** $A = A \int dx \, x \, (u_v^A + d_v^A + 6 \, \bar{q}^A + g^A)$

Three parameters can be determined by these conditions.

Experimental data: total number=951

We have been waiting for JLab data, but …

(1) F_2^A / F_2^D 606 data

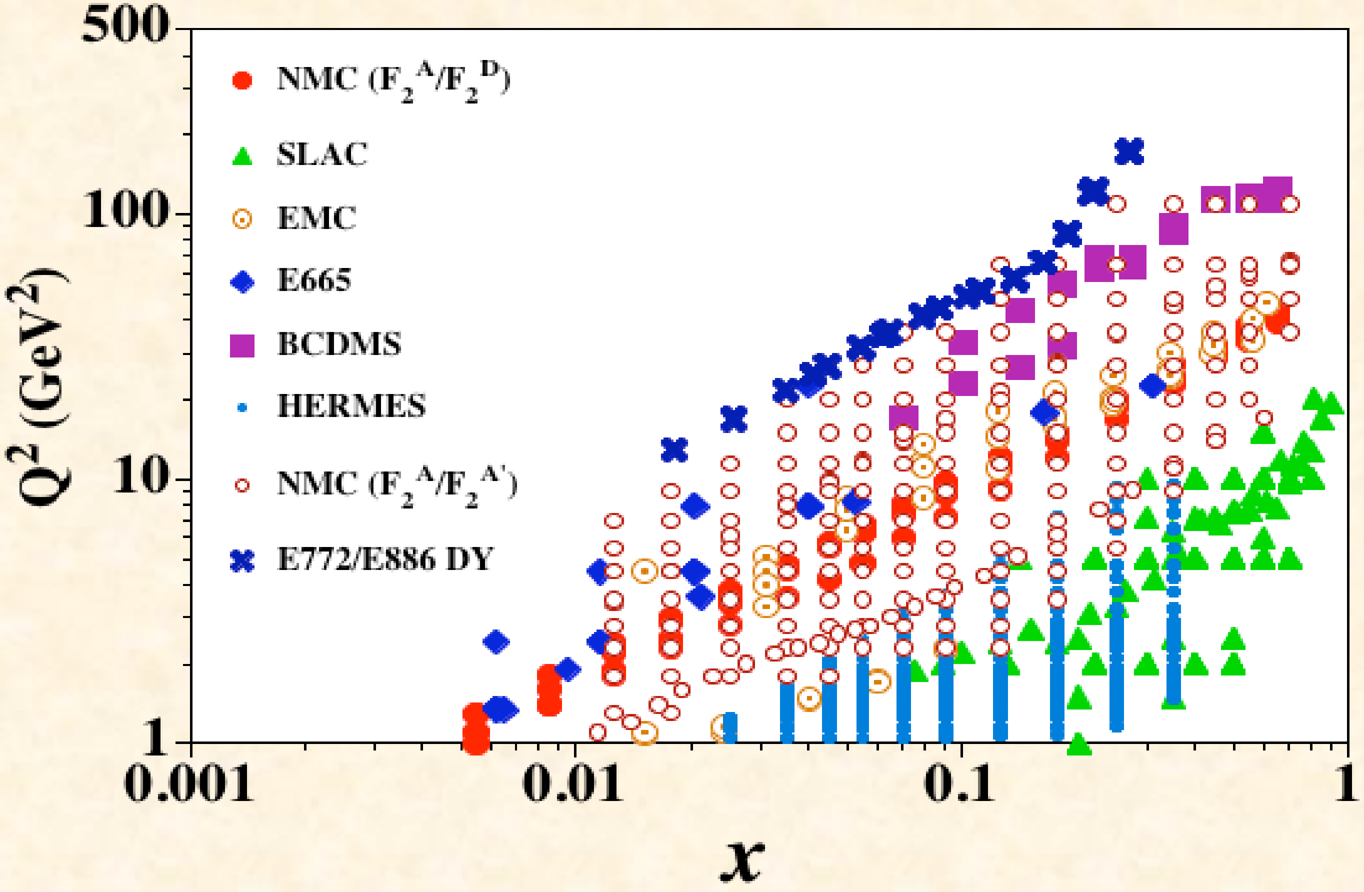
NMC: He, Li, C, Ca
SLAC: He, Be, C, Al, Ca, Fe, Ag, Au
EMC: C, Ca, Cu, Sn
E665: C, Ca, Xe, Pb
BCDMS: N, Fe
HERMES: N, Kr

(2) $F_2^A / F_2^{A'}$ 293 data

NMC: Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$ 52 data

E772: C / D, Ca / D, Fe / D, W / D
E866: Fe / Be, W / Be



Analysis conditions

- **parton distributions in the nucleon**

  MRST01 (Λ_{QCD}=220 MeV)

- **Q^2 point at which the parametrized PDFs are defined: $Q^2 = 1\ \mathrm{GeV}^2$**

- **used experimental data: $Q^2 \geq 1\ \mathrm{GeV}^2$**

- **total number of data: 951**

  606 (F_2^A/F_2^D) + 293 ($F_2^A/F_2^{A'}$) + 52 (Drell–Yan)

- **subroutine for the χ^2 analysis: CERN - Minuit**

$$\chi^2 = \sum_i \frac{(R_i^{data} - R_i^{calc})^2}{(\sigma_i^{data})^2}, \qquad R = \frac{F_2^A}{F_2^D}, \ \frac{F_2^A}{F_2^{A'}}, \ \frac{\sigma_{DY}^{pA}}{\sigma_{DY}^{pA'}}, \qquad \sigma_i^{data} = \sqrt{(\sigma_i^{sys})^2 + (\sigma_i^{stat})^2}$$

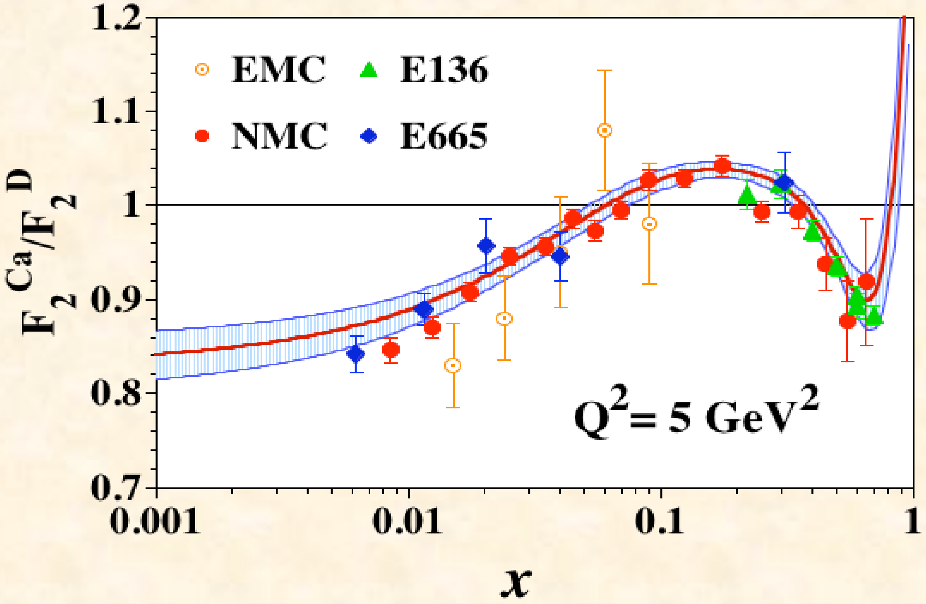
The error of a distribution $F(x)$ is given by

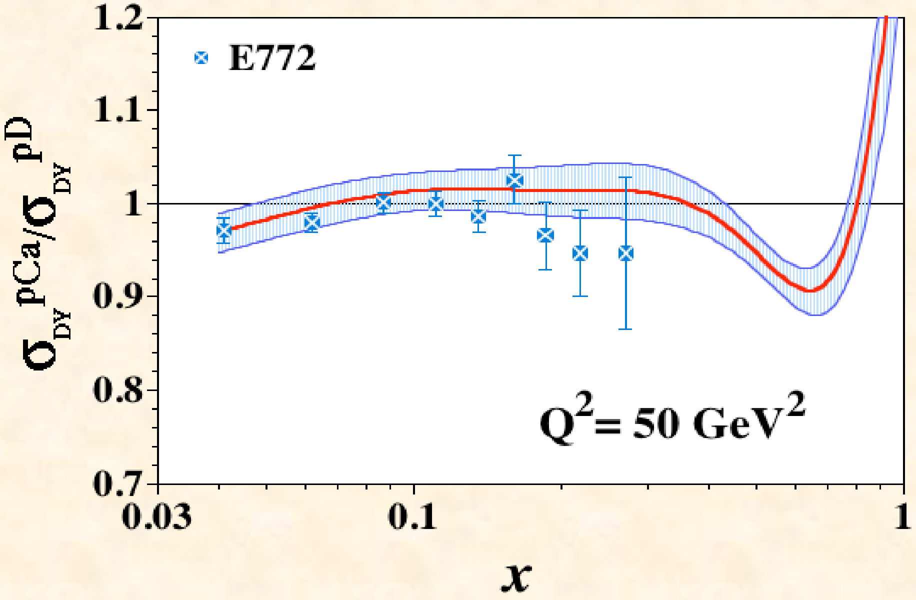
$$\left[\delta F(x)\right]^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F(x)}{\partial \xi_i} H_{ij}^{-1} \frac{\partial F(x)}{\partial \xi_j}$$

H = Hessian

ξ_i = parameter

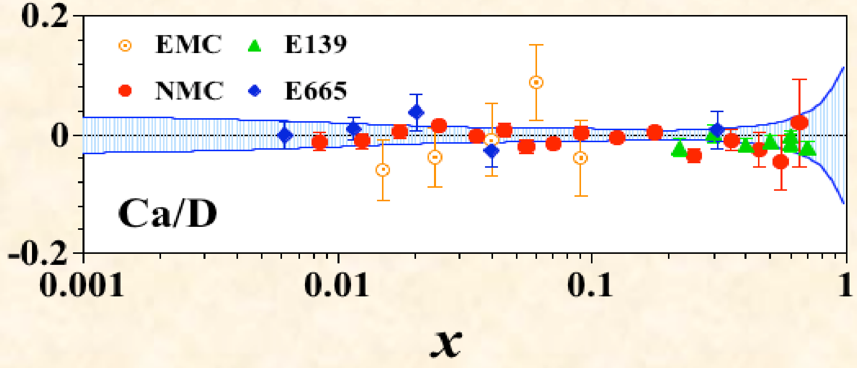
Comparison with F_2^{Ca}/F_2^{D} & $\sigma_{DY}^{pCa}/\sigma_{DY}^{pD}$ data

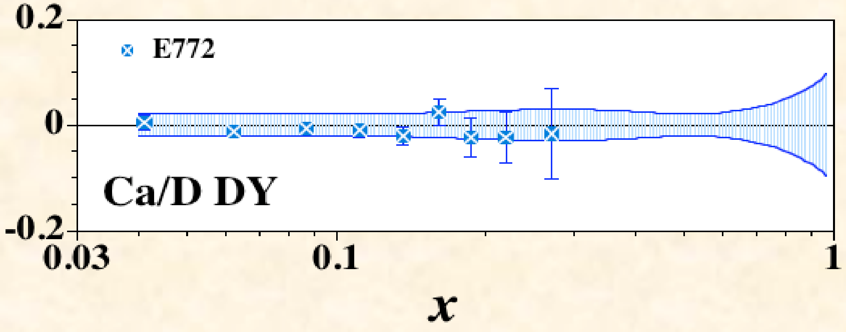




$(R^{exp}-R^{theo})/R^{theo}$ at the same Q^2 points

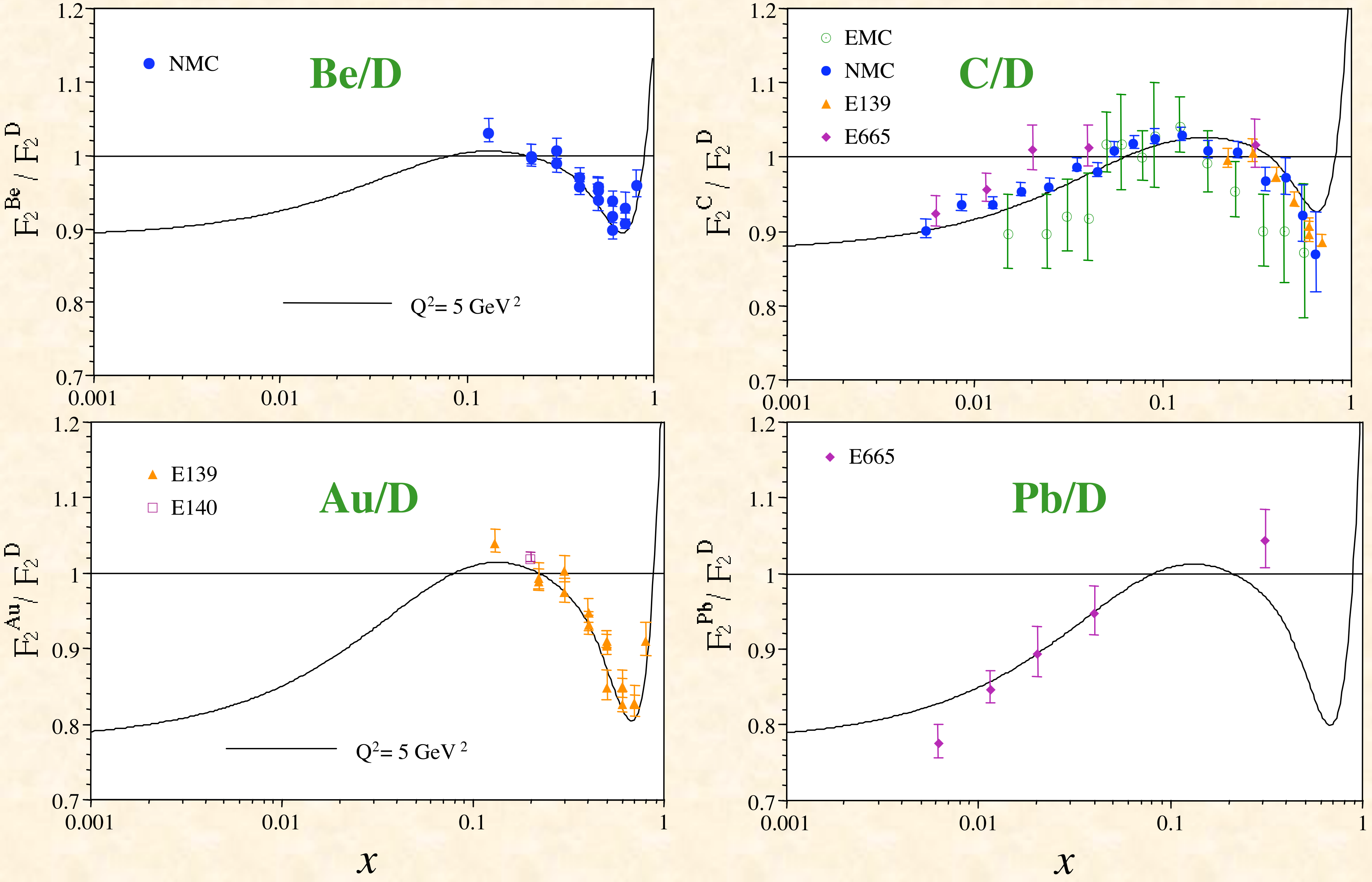
$R= F_2^{Ca}/F_2^{D}, \sigma_{DY}^{pCa}/\sigma_{DY}^{pD}$





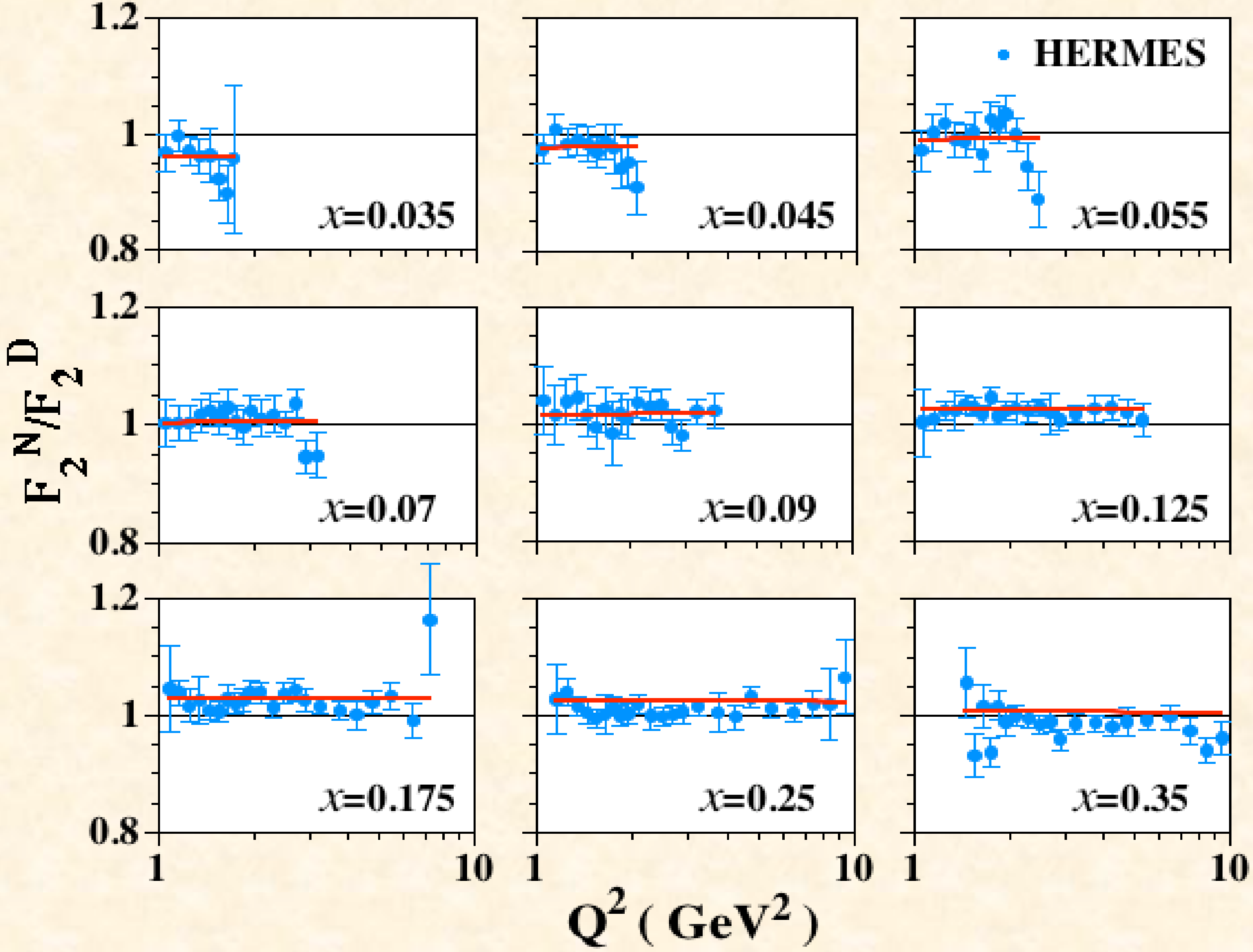
χ^2 / d.o.f. = 1.58 (HKN04) → trying to reduce (1.1~1.2)

Small and large nuclei

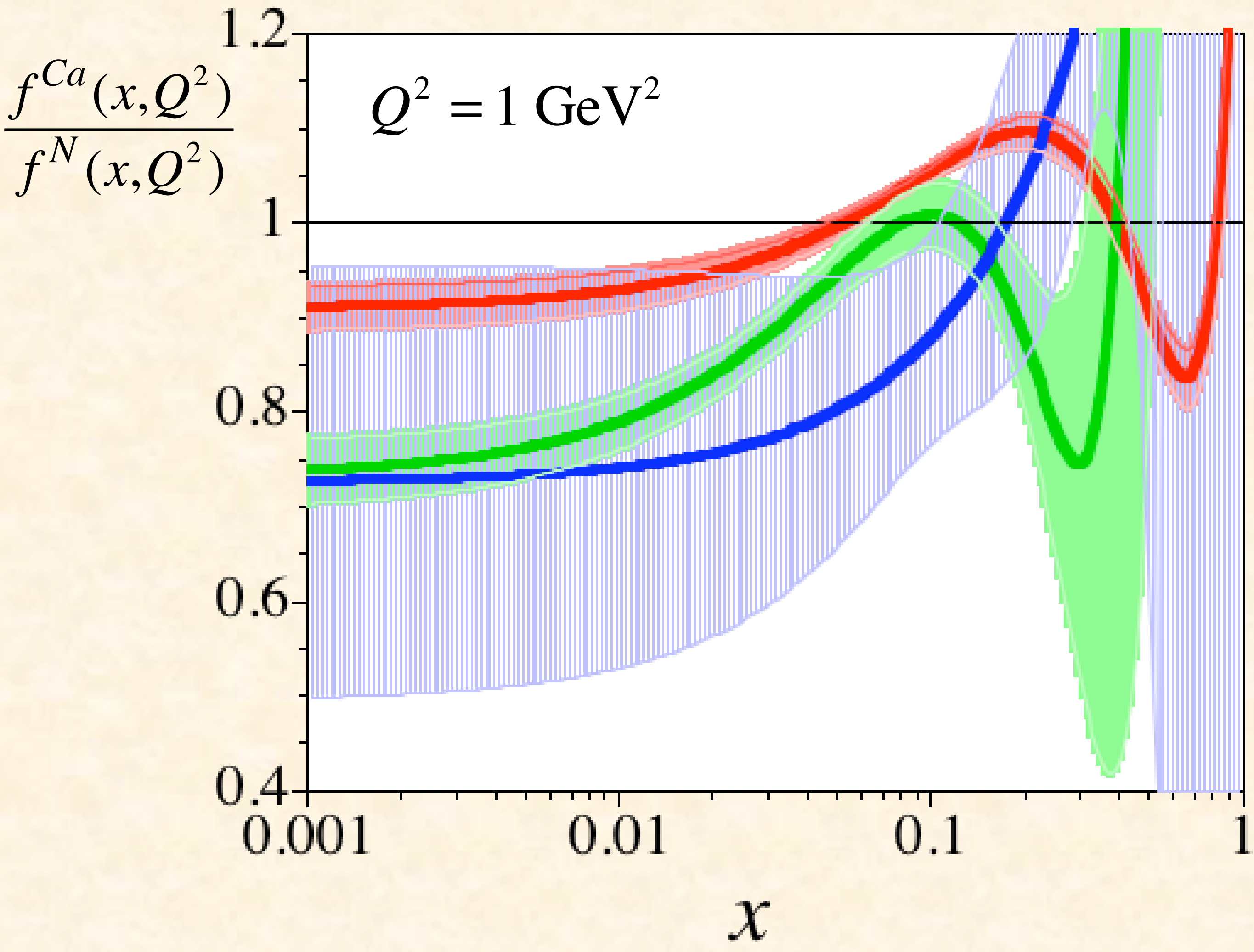


Q^2 dependence

Due to the lack of accurate Q^2 dependent data, the determination of g(x) is very difficult.



Nuclear corrections for ^{40}Ca with uncertainties



Valence quark

Antiquark

Gluon

PDF uncertainties

🟥

🟩

🟦

PDF-library code can be obtained from
http://research.kek.jp/people/kumanos/nuclp.html

Research in progress

(1) NLO analysis

(2) Improvements of the fit

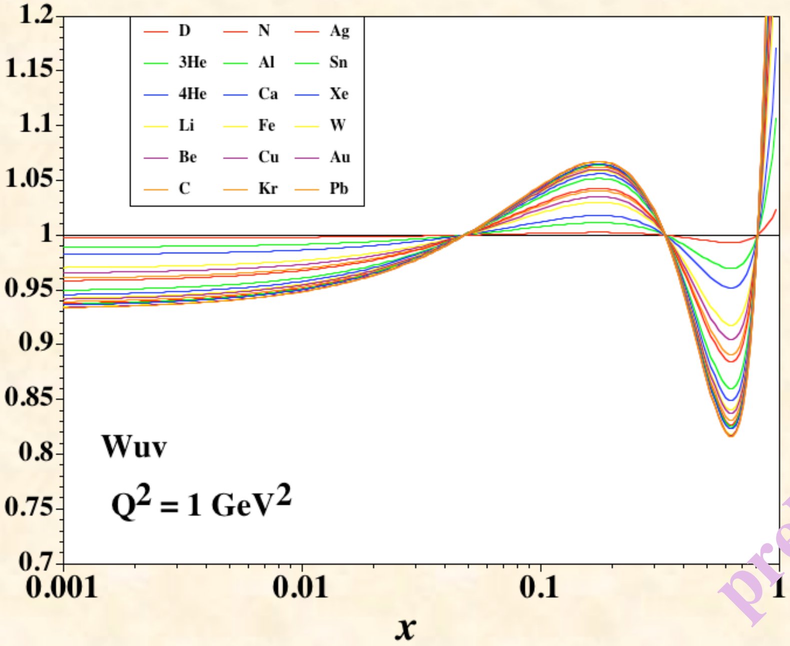
• • •

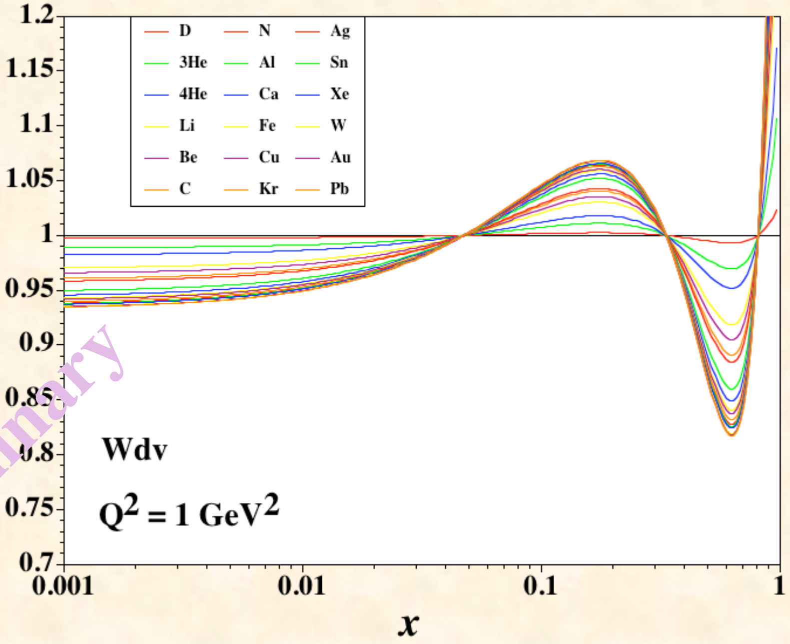
We should investigate the small Q^2 region for ν interactions.

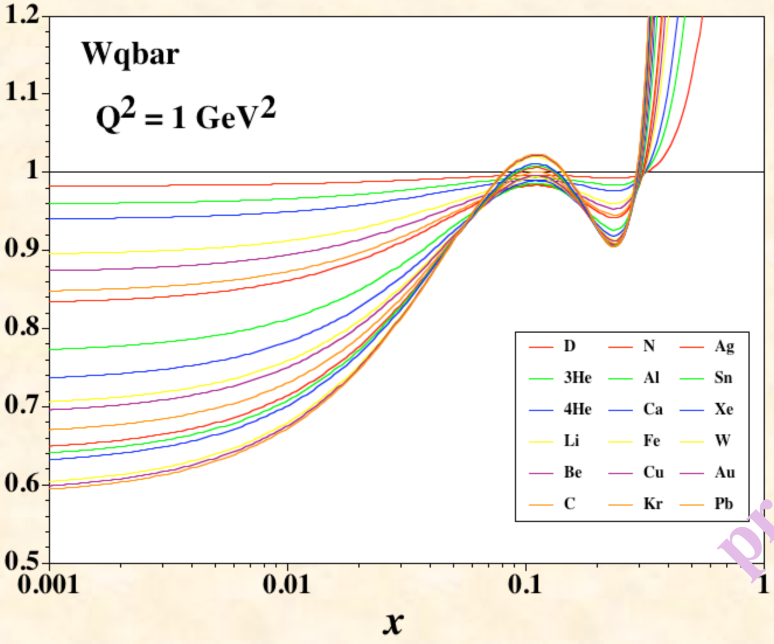
Refs. W. Melnitchouk, R. Ent, C. Keppel,
Phys. Rept. 406 (2005) 127.

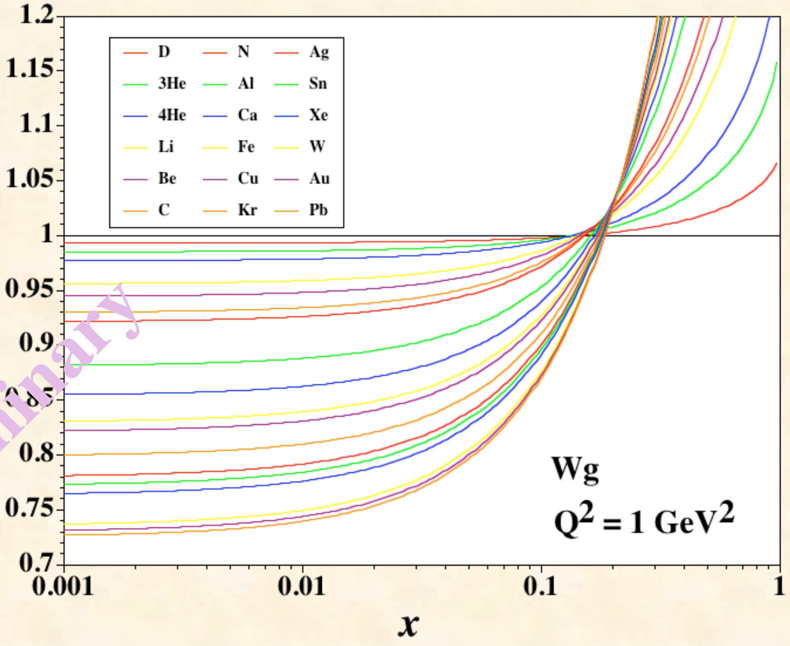
A. Bodek, U. K. Yang, Nucl. Phys. B112 (2002) 70.

Nuclear modifications





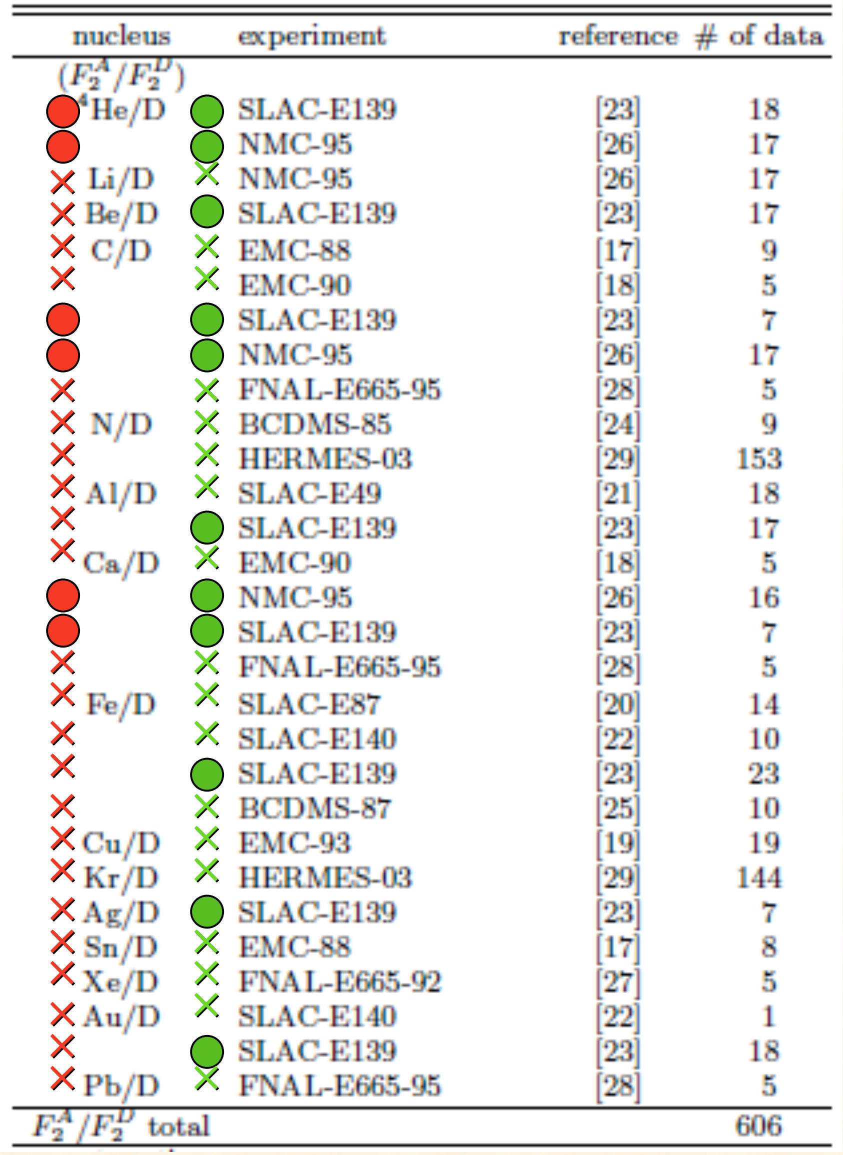




Comparison with Other PDFs

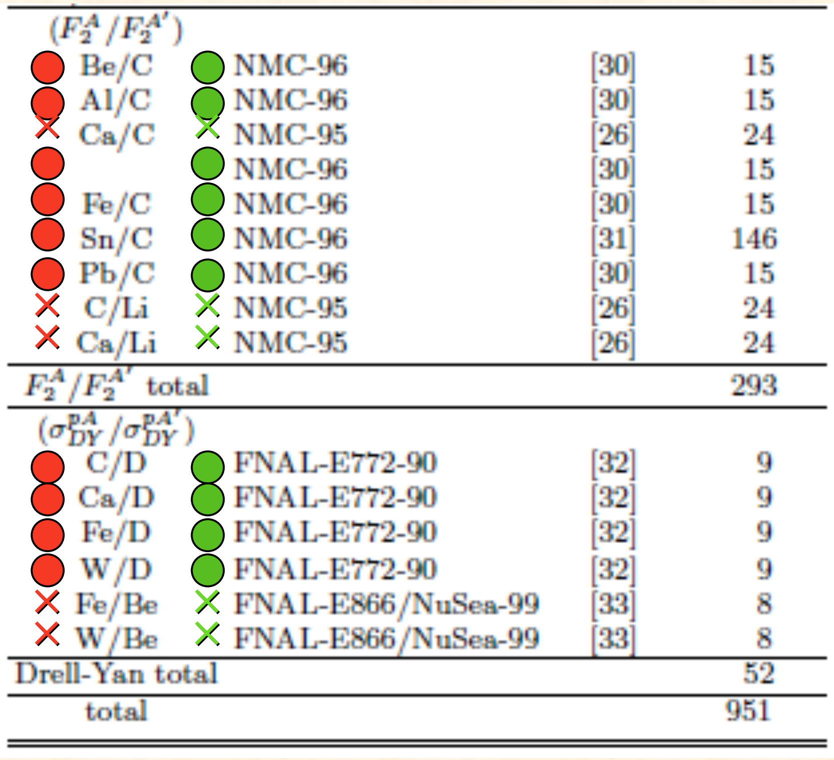
Comparison of used data set

HKN data table



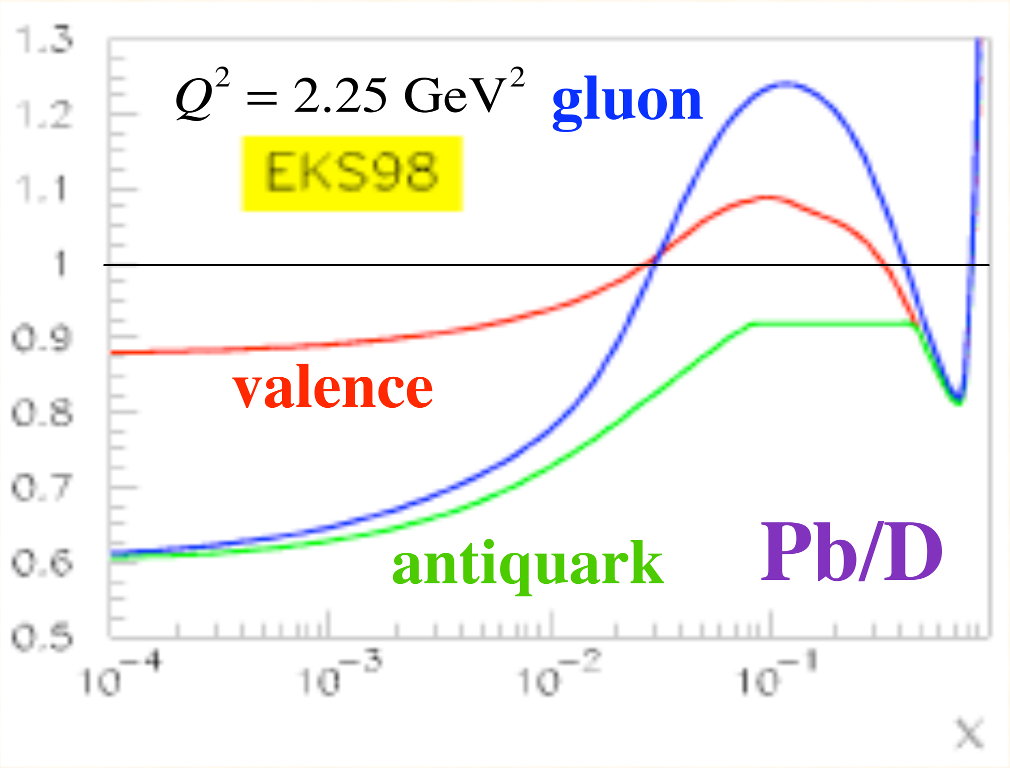
nucleus	experiment	reference	# of data
(F_2^A/F_2^D)			
🔴 ^{4}He/D	🟢 SLAC-E139	[23]	18
🔴	🟢 NMC-95	[26]	17
× Li/D	× NMC-95	[26]	17
× Be/D	🟢 SLAC-E139	[23]	17
× C/D	× EMC-88	[17]	9
×	× EMC-90	[18]	5
🔴	🟢 SLAC-E139	[23]	7
🔴	🟢 NMC-95	[26]	17
×	× FNAL-E665-95	[28]	5
× N/D	× BCDMS-85	[24]	9
×	× HERMES-03	[29]	153
× Al/D	× SLAC-E49	[21]	18
×	🟢 SLAC-E139	[23]	17
× Ca/D	× EMC-90	[18]	5
🔴	🟢 NMC-95	[26]	16
🔴	🟢 SLAC-E139	[23]	7
×	× FNAL-E665-95	[28]	5
× Fe/D	× SLAC-E87	[20]	14
×	× SLAC-E140	[22]	10
×	🟢 SLAC-E139	[23]	23
×	× BCDMS-87	[25]	10
× Cu/D	× EMC-93	[19]	19
× Kr/D	× HERMES-03	[29]	144
× Ag/D	🟢 SLAC-E139	[23]	7
× Sn/D	× EMC-88	[17]	8
× Xe/D	× FNAL-E665-92	[27]	5
× Au/D	× SLAC-E140	[22]	1
×	🟢 SLAC-E139	[23]	18
× Pb/D	× FNAL-E665-95	[28]	5
F_2^A/F_2^D total			606

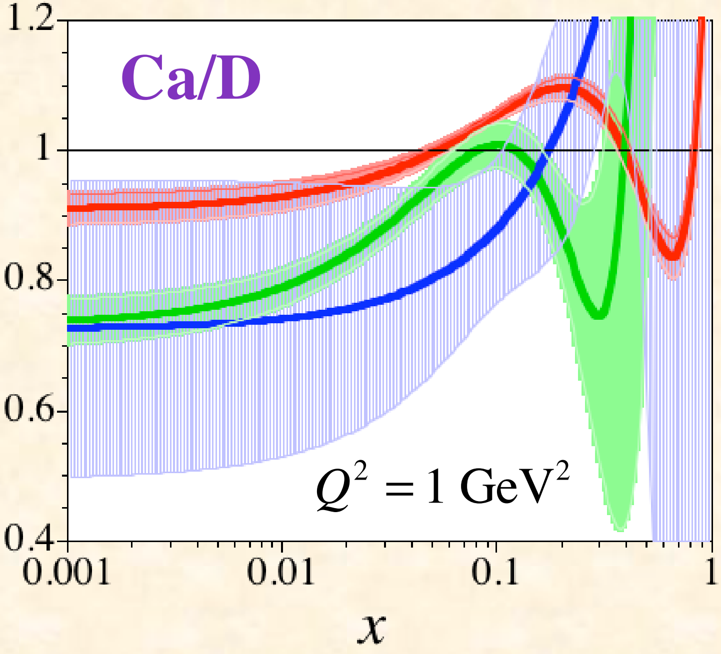
- 🔴 **data in EKRS**
- 🟢 **data in DS**
- × × **data not included**



$(F_2^A/F_2^{A'})$			
🔴 Be/C	🟢 NMC-96	[30]	15
🔴 Al/C	🟢 NMC-96	[30]	15
× Ca/C	× NMC-95	[26]	24
🔴	🟢 NMC-96	[30]	15
🔴 Fe/C	🟢 NMC-96	[30]	15
🔴 Sn/C	🟢 NMC-96	[31]	146
🔴 Pb/C	🟢 NMC-96	[30]	15
× C/Li	× NMC-95	[26]	24
× Ca/Li	× NMC-95	[26]	24
$F_2^A/F_2^{A'}$ total			293
$(\sigma_{DY}^{pA}/\sigma_{DY}^{pA'})$			
🔴 C/D	🟢 FNAL-E772-90	[32]	9
🔴 Ca/D	🟢 FNAL-E772-90	[32]	9
🔴 Fe/D	🟢 FNAL-E772-90	[32]	9
🔴 W/D	🟢 FNAL-E772-90	[32]	9
× Fe/Be	× FNAL-E866/NuSea-99	[33]	8
× W/Be	× FNAL-E866/NuSea-99	[33]	8
Drell-Yan total			52
total			951

Results for nuclear PDFs





Summary

(1) χ^2 analysis for the nuclear PDFs, and their uncertainties.

Valence quark: well determined except for the small-x region.

Antiquark: determined at small x,
large uncertainties at medium and large x.

Gluon: large uncertainties in the whole-x region.

NPDF code could be obtained from
http://research.kek.jp/people/kumanos/nuclp.html

Fit improvement, NLO, … in progress.

(2) Important applications for various high-energy processes
→ e.g. heavy-ion and neutrino reactions

(3) Need future experiments
→ LHC, J-PARC, high-energy electron facility, ν factory, …