EMC effect in neutrino DIS

Ian Cloët

Collaborators

Wolfgang Bentz and Tony Thomas

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Outline

- Nuclear structure functions
- Convolution formalism
- Nambu−Jona-Lasinio (NJL) model
  - Quark distributions
- Nucleon distributions
- Results
  - Quark distributions in nuclei
  - EMC effect for neutrino DIS
- Conclusion

Parton Model Structure Functions

- The Isoscalar Parton model expressions

$$F_2^{(\nu)JH}(x) = \sum_q x\left[q^{JH}(x) + \overline{q}^{JH}(x)\right],$$

$$F_3^{(\nu)JH}(x) = \sum_q \left[q^{JH}(x) - \overline{q}^{JH}(x)\right],$$

$$F_i^{(\nu)}(x) \equiv \frac{1}{2J+1}\sum_{H=-J}^{J} F_i^{(\nu)JH}(x).$$

- $2J+1$ quark distributions
- J integer $\Longrightarrow 2J+2$ structure functions
  J half-integer $\Longrightarrow 2J+1$ structure functions

The Calculation

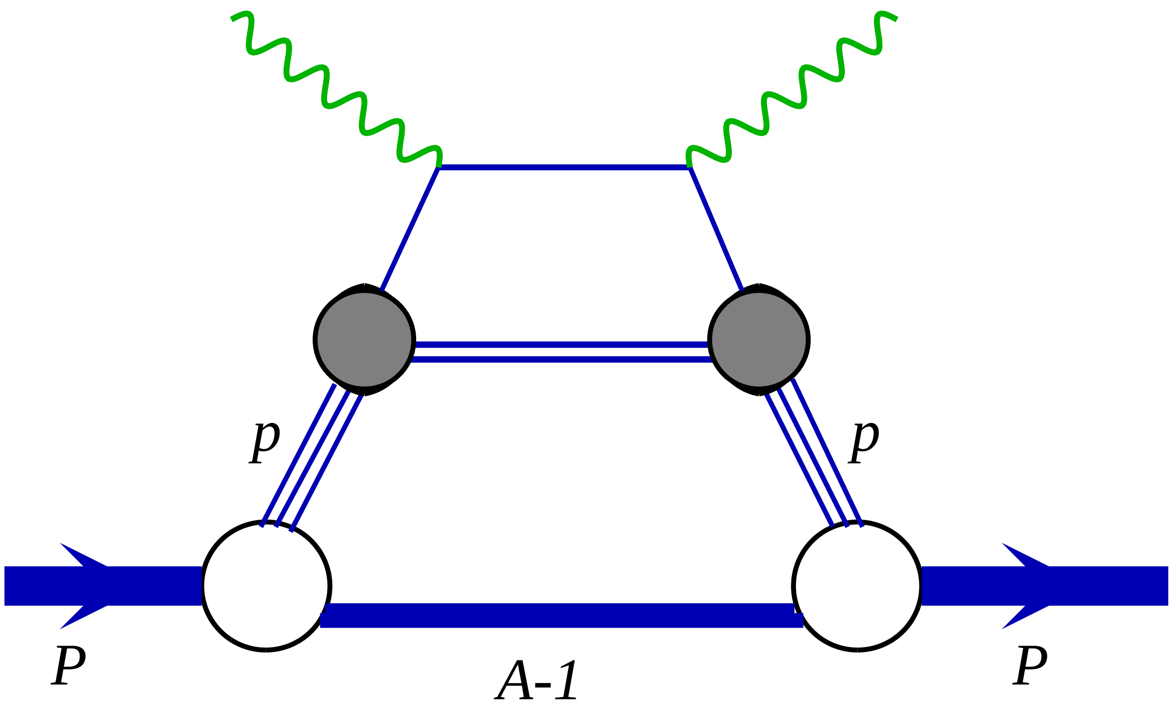
- Definition: Nuclear quark distribution functions

$$q_A^{JH}(x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^- / A} \langle A, P, H | \overline{\psi}(0) \gamma^+ \psi(\xi^-) | A, P, H \rangle .$$

- Using Convolution formalism

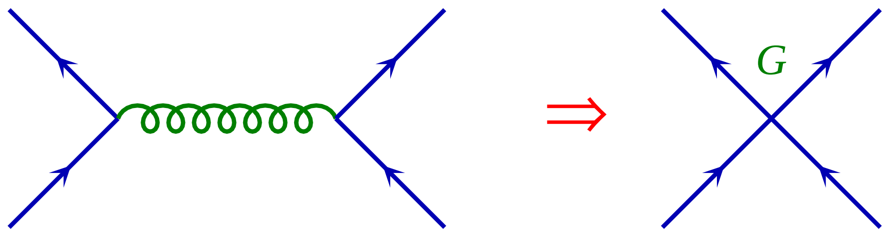
$$q_A^{JH}(x_A) = \sum_{\kappa,m} \int dy_A \int dx \; \delta(x_A - y_A x) \, f_{\kappa,m}^{(JH)}(y_A) \; q_\kappa(x) \, ,$$

Diagrammatically



The NJL Model

- Investigate the role of quark degrees of freedom.
- Low energy effective theory



- Lagrangian has same flavour symmetries as QCD:
  - Importantly chiral symmetry and CSB,
    $\longrightarrow$ Dynamically generated quark masses,
    $\longrightarrow$ Non-zero chiral condensate.

The NJL Model

- Lagrangian

$$\mathcal{L}_{NJL} = \overline{\psi}\,(i\,\partial\!\!\!/ - m)\,\psi + G\,\left(\overline{\psi}\Gamma\psi\right)^2,$$

$\Gamma =$ Dirac, colour, isospin matrices

- Using Fierz transformation can decompose $\mathcal{L}_I$ into sum of qq interaction channels

$$\mathcal{L}_{I,s} = G_s\left(\overline{\psi}\,\gamma_5\,C\,\tau_2\,\beta^A\overline{\psi}^T\right)\left(\psi^T\,C^{-1}\,\gamma_5\,\tau_2\,\beta^A\psi\right),$$

$$\mathcal{L}_{I,a} = G_a\left(\overline{\psi}\,\gamma_\mu\,C\,\vec{\tau}\tau_2\,\beta^A\overline{\psi}^T\right)\left(\psi^T C^{-1}\gamma_\mu\,\vec{\tau}\tau_2\,\beta^A\psi\right).$$

Regularization

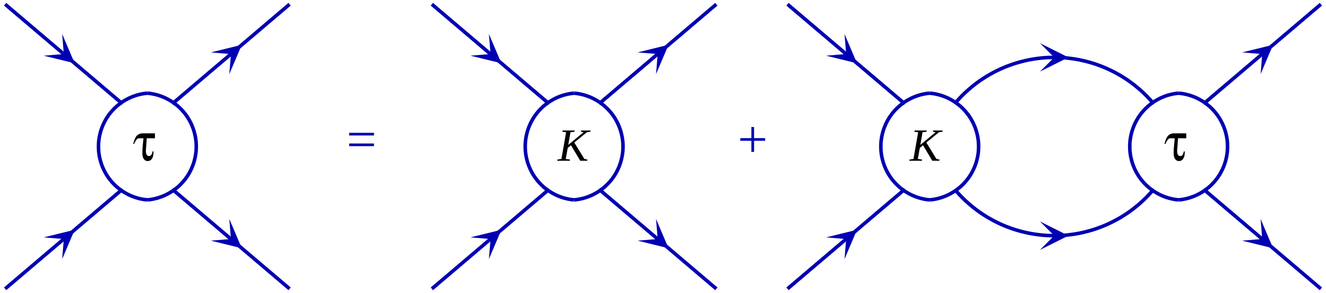
- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$
$$\longrightarrow \quad \frac{1}{(n-1)!}\int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X}.$$

- IR-cutoff eliminates unphysical thresholds for hadrons decaying into quarks and mesons.
  $\rightarrow$ simulates confinement.
- Need this to obtain nuclear matter saturation.
  W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, *138 (2001)*

The Nucleon in the NJL model

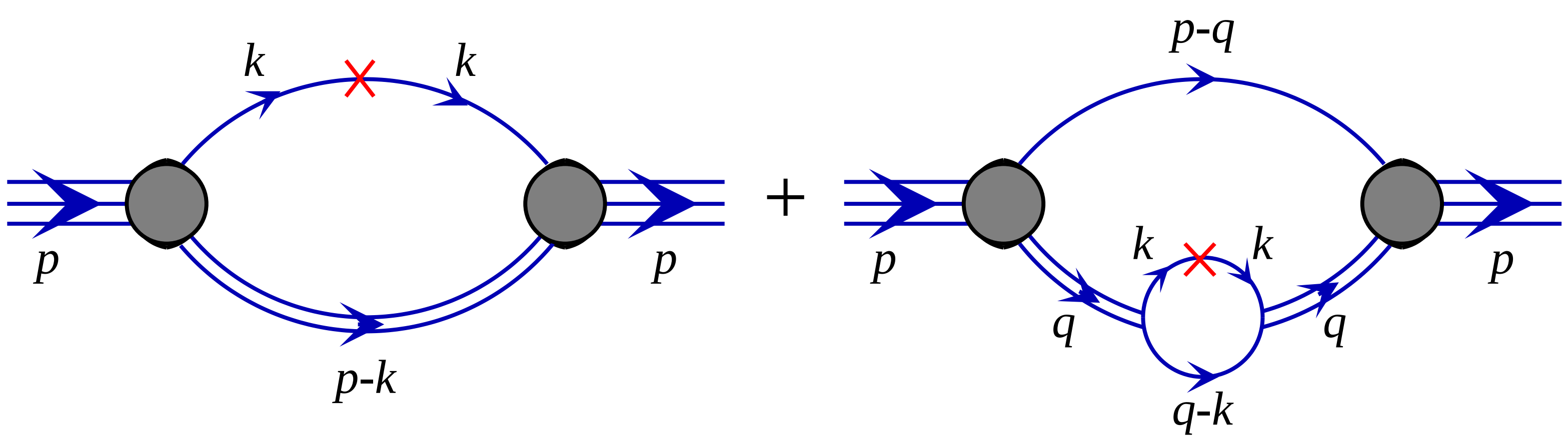
- Nucleon is approximated as a quark-diquark bound state.
- We use a relativistic Faddeev approach to describe this bound state.
- First diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.



- Here we include scalar and axial-vector diquarks.

Nucleon quark distributions

- $q(x)$ associated with a Feynman diagram calculation.



- $q(x) \rightarrow$ X $= \gamma^+ \; \delta(x - \frac{k^+}{p^+})$
- $\Delta q(x) \rightarrow$ X $= \gamma^+ \gamma_5 \; \delta(x - \frac{k^+}{p^+})$

Quark distributions in the Proton

- Spin-independent

$$u_v(x) = f^s_{q/P}(x) + \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{1}{3}\, f^a_{q/P}(x) + \frac{5}{6}\, f^a_{q(D)/P}(x),$$

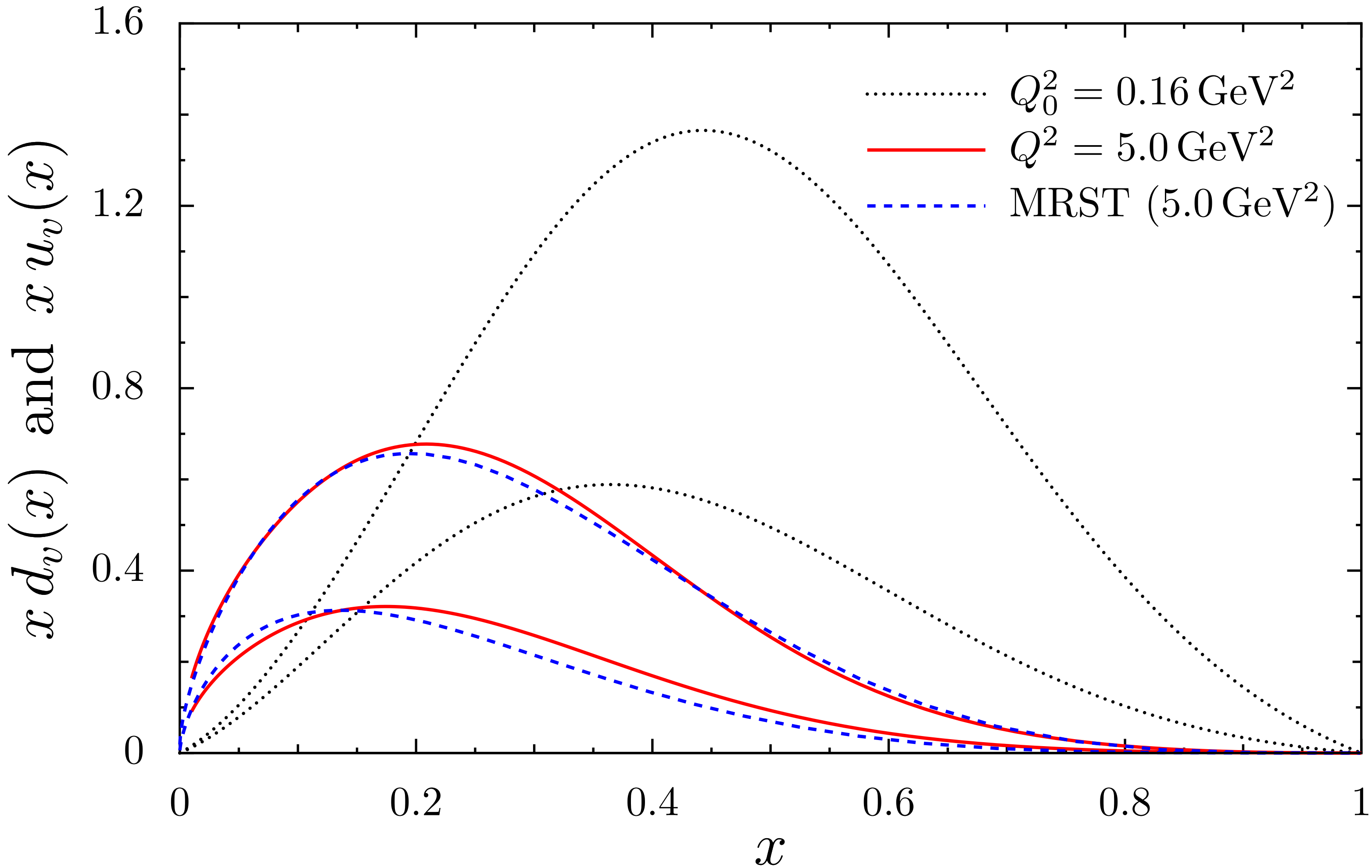
$$d_v(x) = \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{2}{3}\, f^a_{q/P}(x) + \frac{1}{6}\, f^a_{q(D)/P}(x),$$

- Spin-dependent

$$\Delta\, u_v(x) = f^s_{q/P}(x) + \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{1}{3}\, f^a_{q/P}(x) + \frac{5}{6}\, f^a_{q(D)/P}(x) + \frac{1}{2\sqrt{3}}\, f^m_{q(D)/P}(x),$$

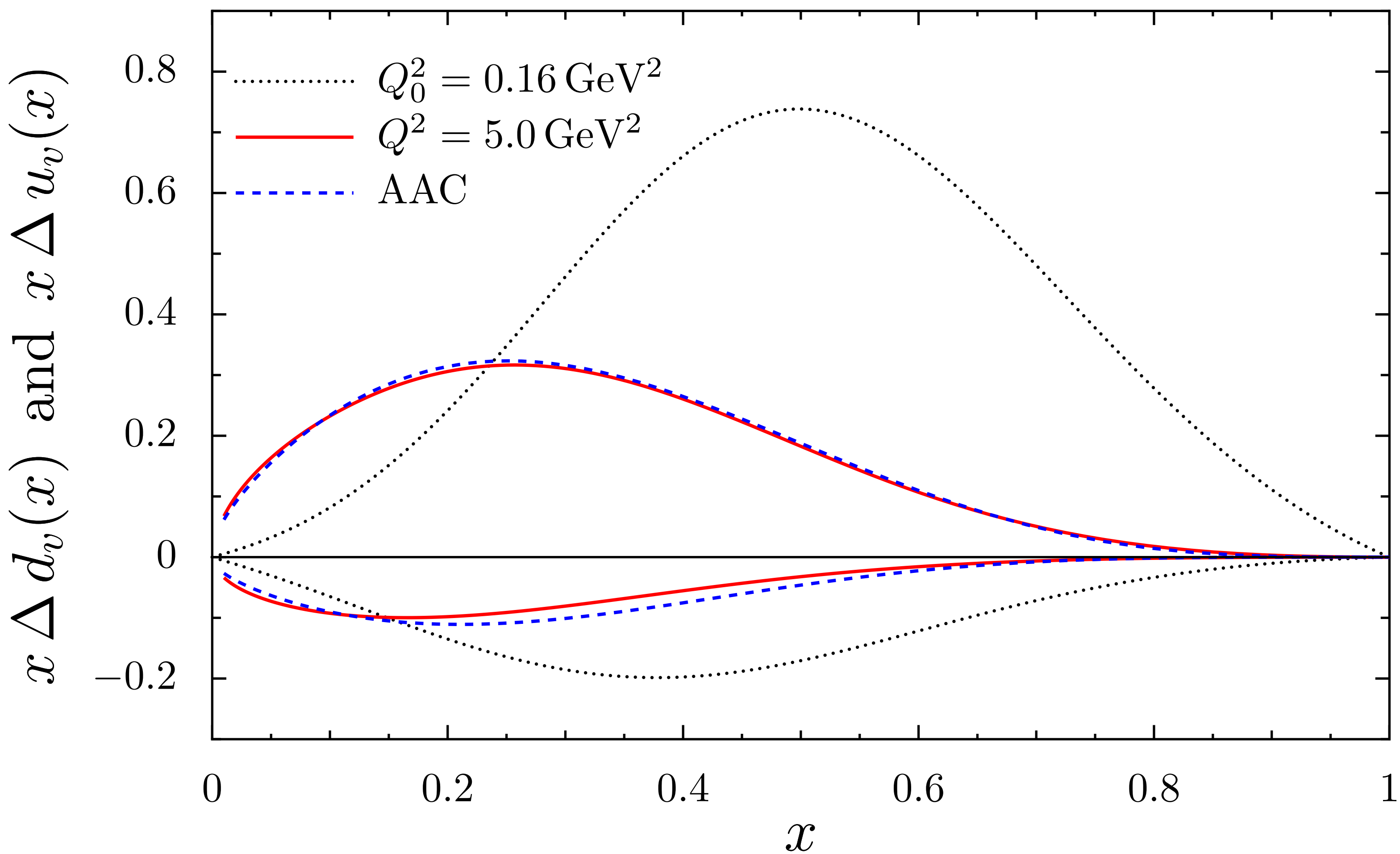
$$\Delta\, d_v(x) = \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{2}{3}\, f^a_{q/P}(x) + \frac{1}{6}\, f^a_{q(D)/P}(x) - \frac{1}{2\sqrt{3}}\, f^m_{q(D)/P}(x),$$

$u_v(x)$ and $d_v(x)$ distributions



- MRST, Phys. Lett. B **531**, 216 (2002).

$\Delta u_v(x)$ and $\Delta d_v(x)$ distributions



- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

NJL Model at Finite Density

- Re-calculate diagrams

$$\mathcal{L} = \overline{\psi}\,(i\,\partial\!\!\!/ - M^* - V\!\!\!/\,)\,\psi - \frac{(M^* - m)^2}{4G_\pi} + \frac{V_\mu V^\mu}{4G_\omega} + \mathcal{L}_I$$

- Equivalent to:
  - Scalar field: via effective masses
  - Vector field: via scale transformation
- Nuclear Matter ($\varepsilon_F = E_F + 3V_0$)

$$q_A\,(x_A) = \frac{\varepsilon_F}{E_F}\,q_{A0}\left(\frac{\varepsilon_F}{E_F}x_A - \frac{V_0}{E_F}\right),$$

- Finite Nuclei ($\hat{M}_{N\kappa} = \overline{M}_N - 3V_\kappa$)

$$q_{A,\kappa}(x_A) = \frac{\overline{M}_N}{\hat{M}_N}\,q_{A0,\kappa}\left(\frac{\overline{M}_{N\kappa}}{\hat{M}_{N\kappa}}x_A - \frac{V_\kappa}{\hat{M}_{N\kappa}}\right).$$

Nucleon distribution functions

- Definition

$$f_{\kappa m}(y_A) = \frac{\sqrt{2}\,\overline{M}_N}{A} \int \frac{d^3p}{(2\pi)^3}$$

$$\delta\left(p^3 + \varepsilon_\kappa - \overline{M}_N\, y_A\right) \overline{\Psi}_{\kappa m}(\vec{p})\ \gamma^+\, \Psi_{\kappa\, m}(\vec{p})\,,$$

- Central Potential Dirac eigenfunctions

$$\Psi_{\kappa m}(\vec{p}) = i^\ell \begin{pmatrix} F_\kappa(p)\, \Omega_{\kappa m}(\theta,\phi) \\ -G_\kappa(p)\, \Omega_{-\kappa m}(\theta,\phi) \end{pmatrix},$$

  - spherical two-spinor has the form

$$\Omega_{\kappa m}(\theta,\phi) = \sum_{m_\ell, m_s} \langle \ell\, m_\ell\, s\, m_s | j\, m\rangle\, Y_{\ell m_\ell}(\theta,\phi)\ \chi_{s m_s}\,,$$

Nucleon distributions: Results

- Spin-independent nucleon distribution

$$f_{\kappa,m}(y_A)=\sum_{k=0,2,\ldots,2j}(-1)^{j-m}\sqrt{2k+1}\begin{pmatrix} j & j & k \\ m & -m & 0 \end{pmatrix}$$

$$(2j+1)\sqrt{2k+1}\,\frac{\overline{M}_N}{16\pi^3}\int_\Lambda^\infty dp\; p\left\{2\sqrt{6}(-1)^\ell F_\kappa(p)G_\kappa(p)\right.$$

$$\sum_{L=k\pm1}(2L+1)P_L\left(\frac{\overline{M}_N\, y_A-\varepsilon_\kappa}{p}\right)\sqrt{(2\ell+1)(2\tilde{\ell}+1)}\begin{pmatrix} \ell & L & \tilde{\ell} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} L & 1 & k \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \tilde{\ell} & s & j \\ L & 1 & k \\ \ell & s & j \end{Bmatrix}$$

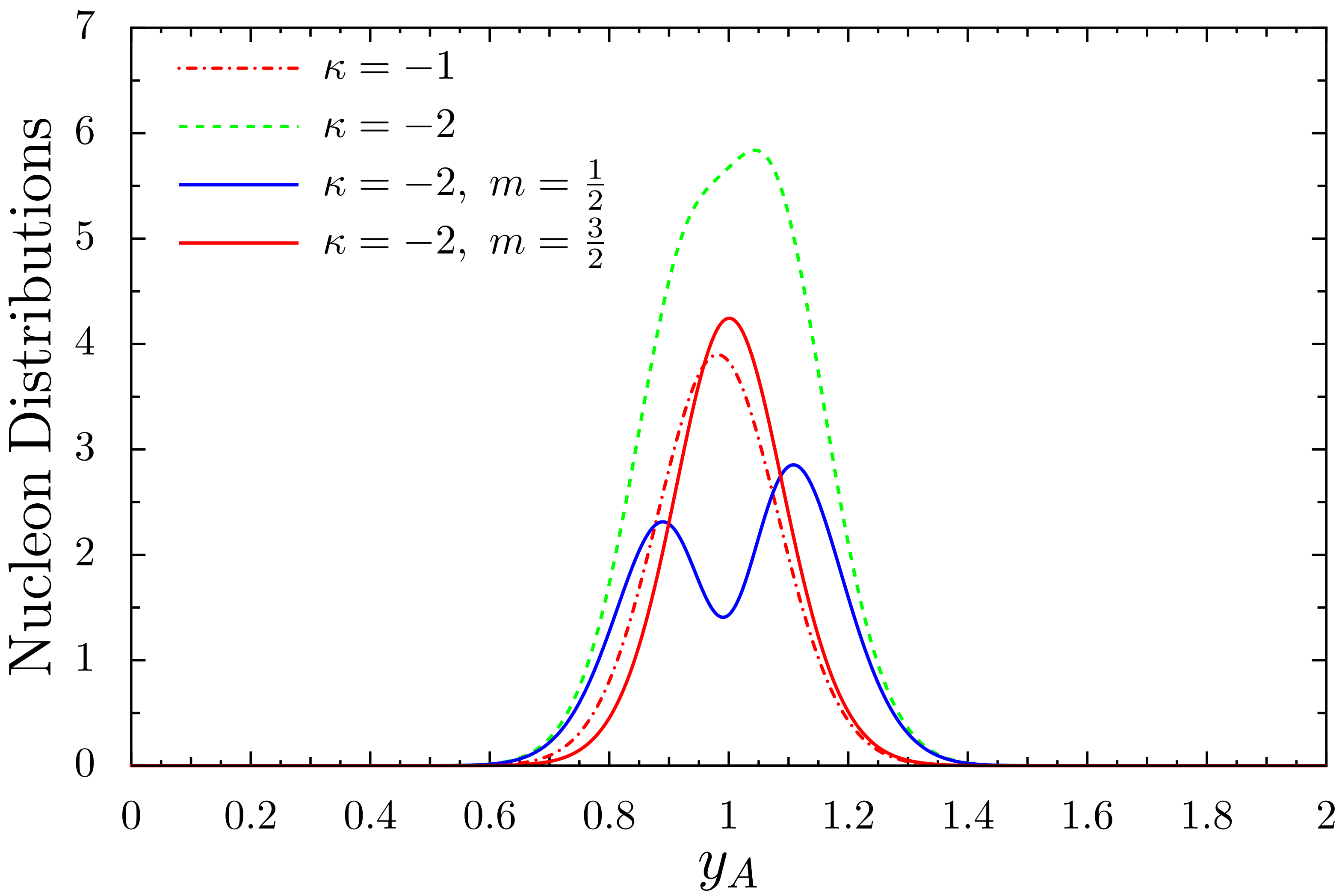
$$+(-1)^{j+\frac{1}{2}}P_k\left(\frac{\overline{M}_N\, y_A-\varepsilon_\kappa}{p}\right)\left[F_\kappa(p)^2(2\ell+1)\begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \ell & k & \ell \\ j & s & j \end{Bmatrix}\right.$$

$$\left.\left.+G_\kappa(p)^2(2\tilde{\ell}+1)\begin{pmatrix} \tilde{\ell} & k & \tilde{\ell} \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \tilde{\ell} & k & \tilde{\ell} \\ j & s & j \end{Bmatrix}\right]\right\}.$$

- Infinite nuclear matter

$$f(y_A)=\frac{3}{4}\left(\frac{\varepsilon_F}{p_F}\right)^3\left[\left(\frac{p_F}{\varepsilon_F}\right)^2-(1-y_A)^2\right].$$

Nucleon distributions: ^{12}C

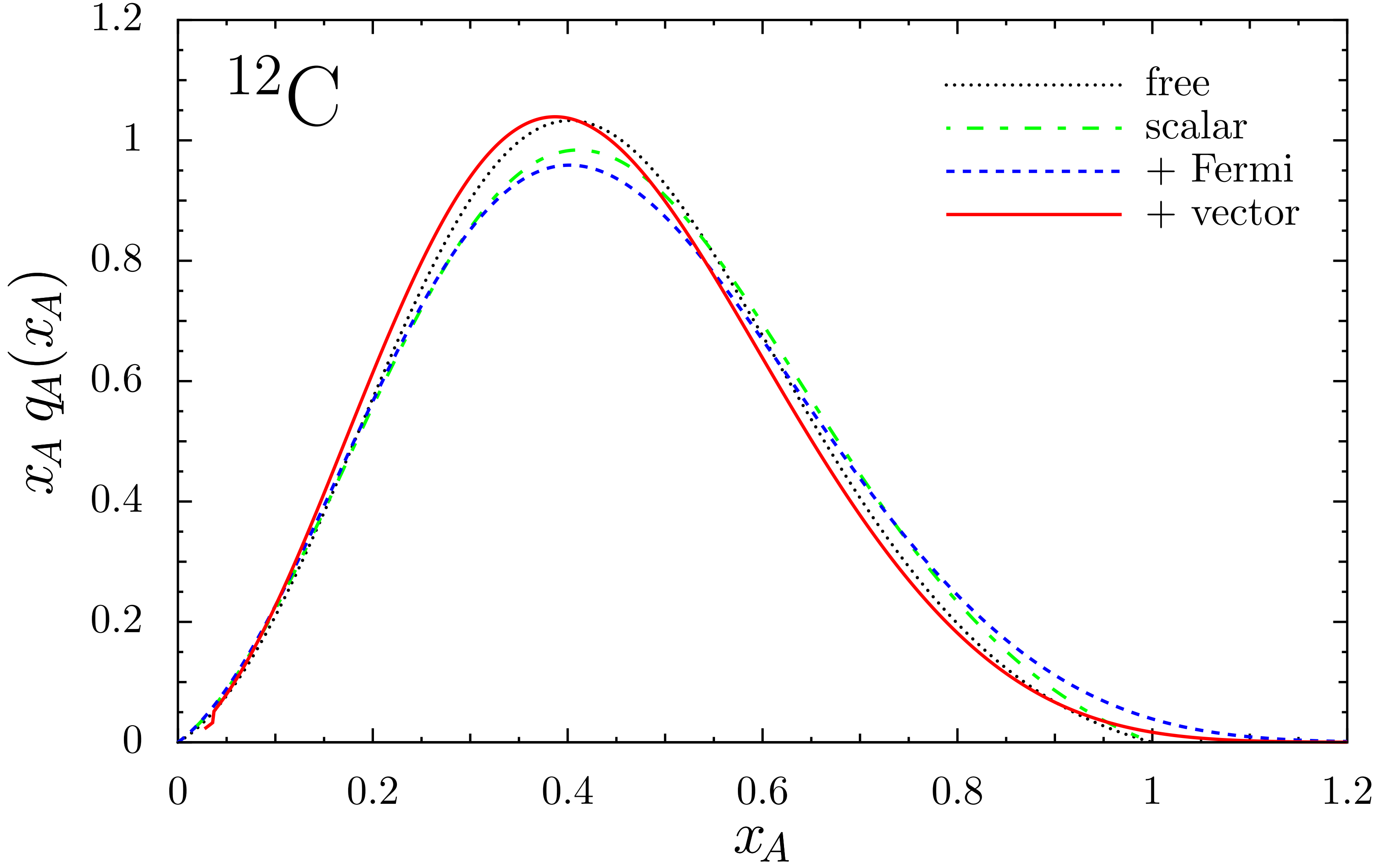


Results: Quark distributions

- Putting it all together, an example

$$u_A^{JH}(x_A) = \sum_{\kappa,m} \left[u_{p,\kappa}(x) \otimes f_{\kappa m}(y_A) \right] + \sum_{\kappa,m} \left[u_{n,\kappa} \otimes f_{\kappa m}(y_A) \right]$$

Quark distribution in ^{12}C



The EMC effect

- F_2 EMC ratio

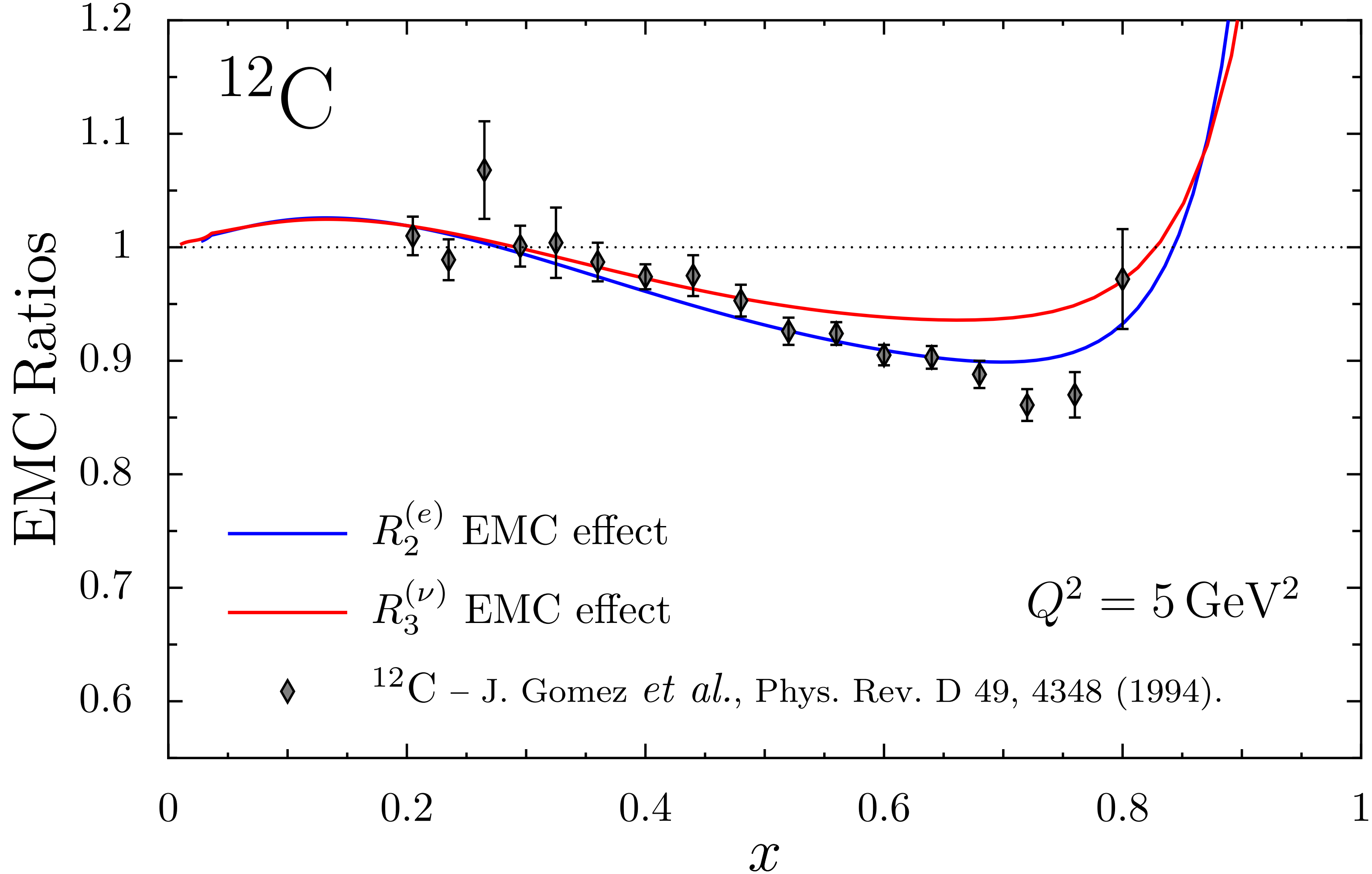
$$R_2(x) = \frac{F_{2A}(x)/A}{F_{2N}(x)}$$

- F_3 EMC ratio

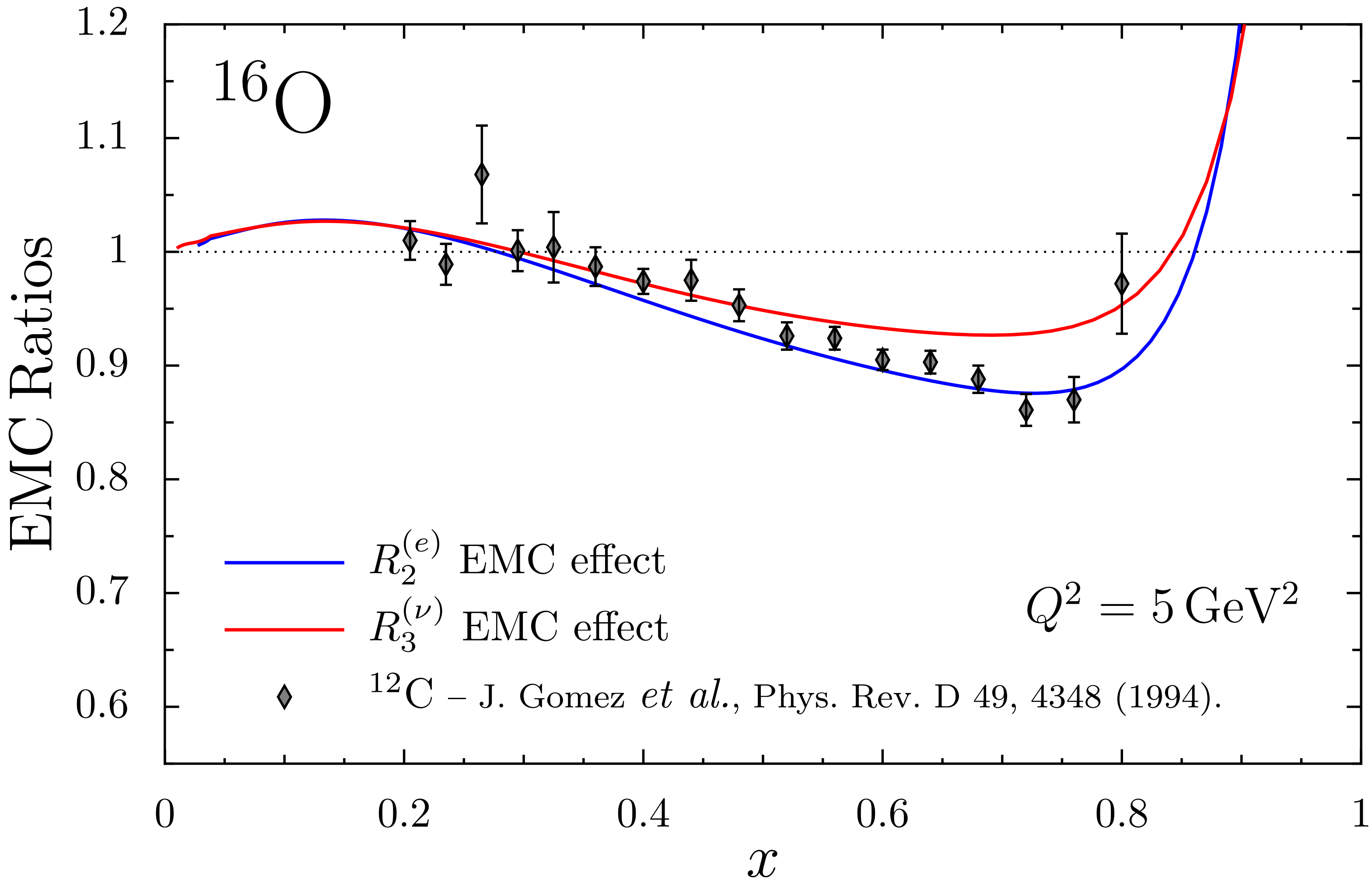
$$R_3(x) = \frac{F_{3A}(x)/A}{F_{3N}(x)}$$

- Ratios equal 1 in non-relativistic and no-medium modification limit.

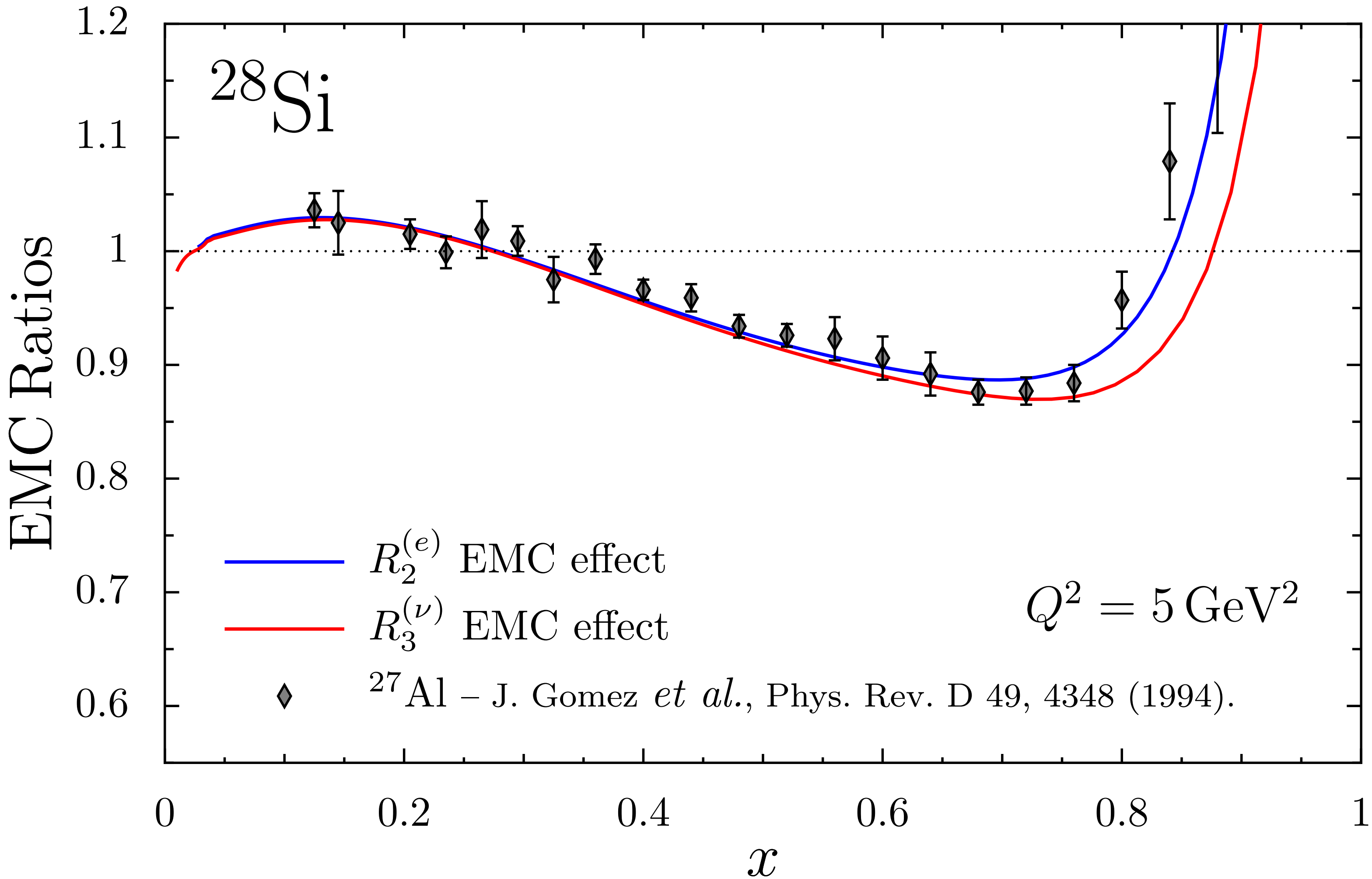
EMC ratios ^{12}C



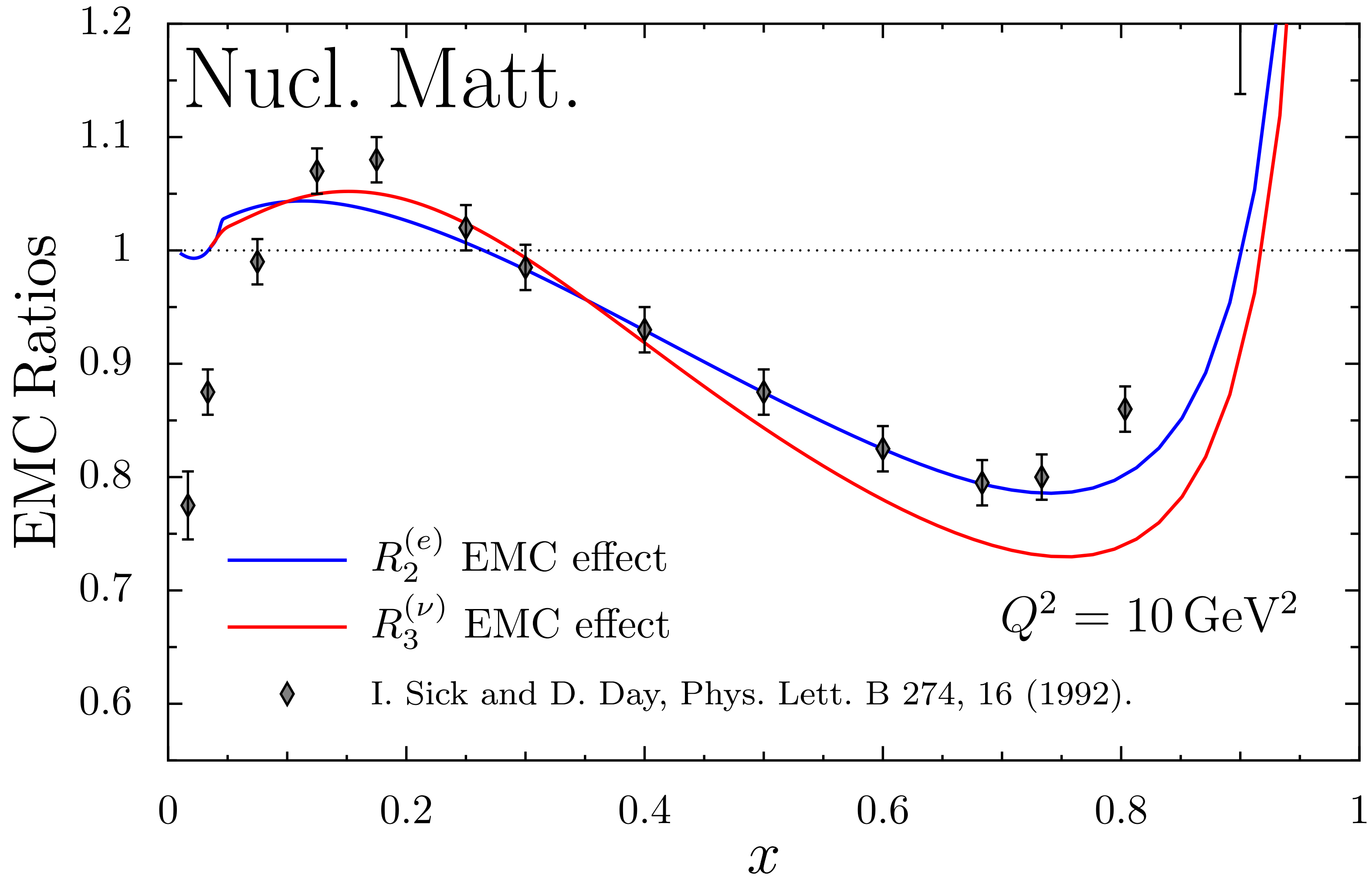
EMC ratios ^{16}O



EMC ratios ^{28}Si



Nuclear Matter



Conclusions

- Effective chiral quark theories can be used to incorporate quarks into many-body physics.
- Calculated nuclear quark distributions where the quarks bind to mean scalar and vector fields.
  - Reproduced EMC effect.
- Determined medium modifications to F_3.
  - Comparable with F_2, although increased A dependence.
- Future Work
  - include ρ mean field to calculate $N \neq Z$.
  - Solve self-consistently for nuclear potentials.
  - incorporate pions.