

Vector and Axial Form Factors Applied to Neutrino Quasi-Elastic Scattering

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<http://www.pas.rochester.edu/~bodek/axial-2006.ppt>

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Outline

- Review of BBA2003 and BBBA2005 vector-form factors from electron scattering
- Reanalyze the previous neutrino-deuterium quasi-elastic data by calculating M_A with their assumptions and with BBBA2006-form factor to extract a new value of M_A
- Compare to M_A from pion electro-production
- Use the previous deuterium quasi-elastic data to extract F_A and compare axial form factor to models
- **Future: Look at what MINERvA can do**
- **See what information anti-neutrinos can give**

The hadronic current for QE neutrino scattering is given by [2]

Vector and axial form factors

$$\langle p(p_2) | J_\lambda^+ | n(p_1) \rangle =$$

$$\bar{u}(p_2) \left[\gamma_\lambda F_V^1(q^2) + \frac{i\sigma_{\lambda\nu} q^\nu \xi F_V^2(q^2)}{2M} + \gamma_\lambda \gamma_5 F_A(q^2) + \frac{q_\lambda \gamma_5 F_P(q^2)}{M} \right] u(p_1),$$

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dq^2} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \times \left[A(q^2) \mp \frac{(s-u)B(q^2)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right],$$

where

$$A(q^2) = \frac{m^2 - q^2}{4M^2} \left[\left(4 - \frac{q^2}{M^2} \right) |F_A|^2 - \left(4 + \frac{q^2}{M^2} \right) |F_V^1|^2 - \frac{q^2}{M^2} |\xi F_V^2|^2 \left(1 + \frac{q^2}{4M^2} \right) - \frac{4q^2 \text{Re} F_V^{1*} \xi F_V^2}{M^2} \right],$$

$$B(q^2) = -\frac{q^2}{M^2} \text{Re} F_A^* (F_V^1 + \xi F_V^2), \quad C(q^2) = \frac{1}{4} \left(|F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2} \left| \frac{\xi F_V^2}{2} \right|^2 \right).$$

Vector

$$F_V^1(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad \xi F_V^2(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - \frac{q^2}{4M^2}}.$$

We use the CVC to determine $G_E^V(q^2)$ and $G_M^V(q^2)$ from the electron scattering form factors $G_E^P(q^2)$, $G_E^n(q^2)$, $G_M^P(q^2)$, and $G_M^n(q^2)$:

$$G_E^V(q^2) = G_E^P(q^2) - G_E^n(q^2), \quad G_M^V(q^2) = G_M^P(q^2) - G_M^n(q^2).$$

The axial form factor F_A and the pseudoscalar form factor F_P (related to F_A by PCAC) are given by

$$F_A(q^2) = \frac{g_A}{\left(1 - \frac{q^2}{M_A^2}\right)^2}, \quad F_P(q^2) = \frac{2M^2 F_A(q^2)}{M_\pi^2 - q^2}.$$

Axial dipole approx

dipole approximation.

$$G_D(q^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71 \text{ GeV}^2$$

Vector dipole approx

$$G_E^P = G_D(q^2), \quad G_E^n = 0, \quad G_M^P = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2).$$

We refer to the above combination of form factors as ‘Dipole Form Factors’.

BBA2003-Form Factors and our constants

Our Constants

g_A	-1.267
G_F	$1.1803 \times 10^{-5} \text{ GeV}^{-2}$
$\cos \theta_c$	0.9740
μ_p	$2.793 \mu_N$
μ_n	$-1.913 \mu_N$
ξ	$3.706 \mu_N$
M_V^2	0.71 GeV^2

Table 1

The most recent values of the parameters used in our calculations (Unless stated otherwise)

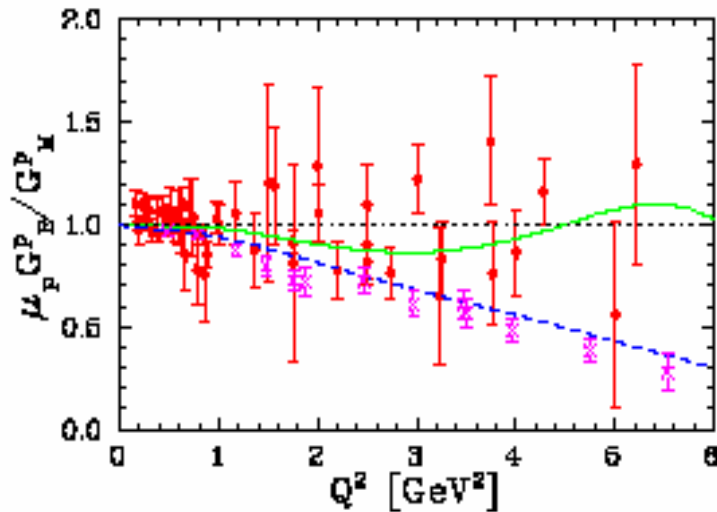


Figure 4. Ratio of G_E^p to G_M^p as extracted by Rosenbluth measurements and from polarization measurements. The lines and symbols have the same meaning as Figure 1.

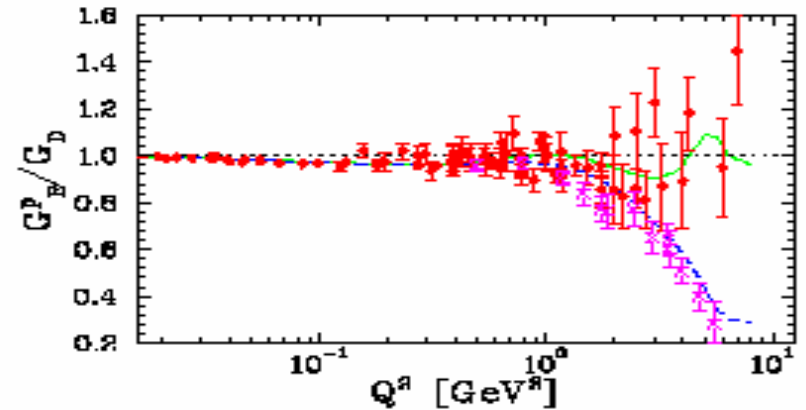


Figure 1. Our fits to G_E^p/G_D , using cross section data only (solid), and with both the cross section and polarization transfer data (dashed). The diamonds are the from Rosenbluth extractions and the crosses are the Hall A polarization transfer data. Note that we fit to cross sections, rather than fitting directly to the extracted values of G_E^p shown here.

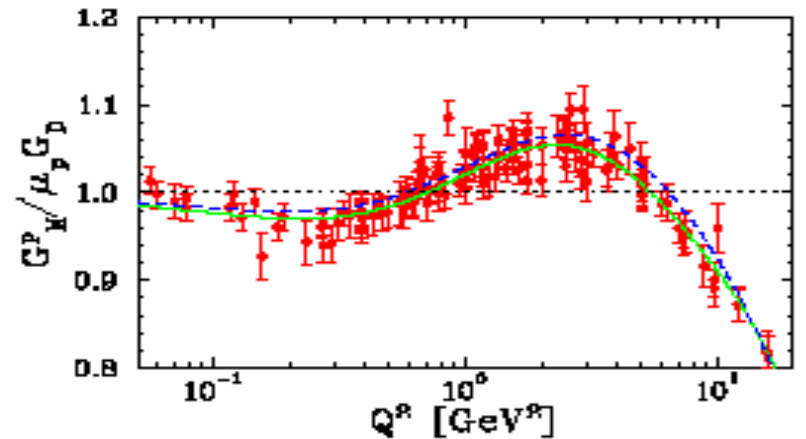
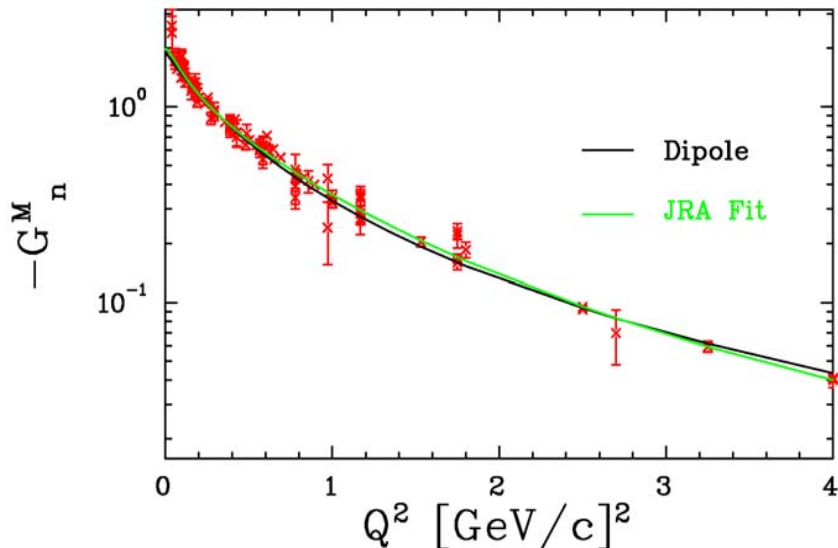
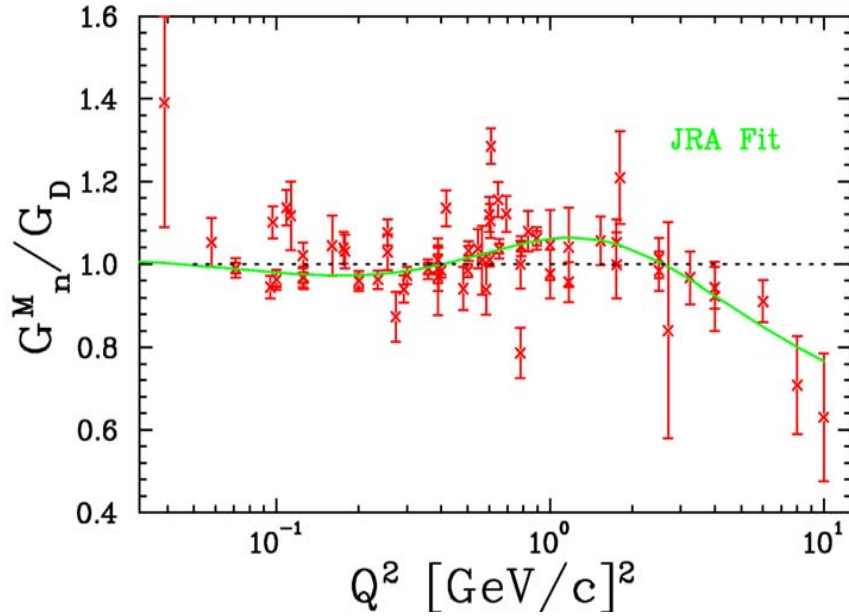


Figure 2. Our fits to $G_M^p/\mu_p G_D$. The lines and symbols have the same meaning as Figure 1.

Neutron G_M^N is negative

Neutron ($G_M^N / G_M^N \text{ dipole}$)



Neutron ($G_M^N / G_M^N \text{ dipole}$)

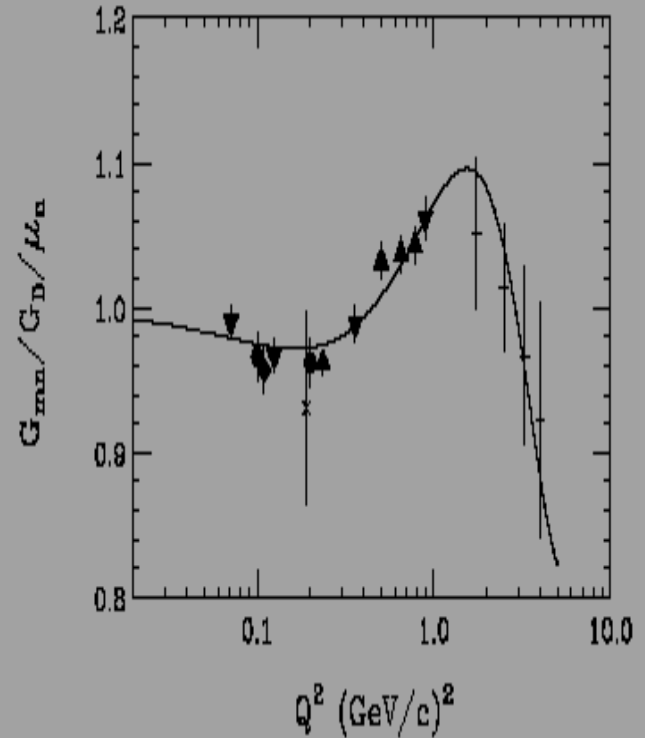


Fig. 2. The figure shows the continued fraction fit to the data. Symbols for the data as in figure 1) plus the data by Lung *et al.* (+) [23].

At low Q^2 Ratio to Dipole similar to that
 nucl-ex/0107016 G. Kubon, et al
 Phys.Lett. B524 (2002) 26-32

$$G_{mn}(Q^2) = \frac{\mu_n}{1 + \frac{Q^2 b_1}{1 + \frac{Q^2 b_2}{1 + \dots}}} \quad (2)$$

Neutron, G_E^N is positive -

Imagine N=P+pion cloud

Neutron G_E^N is positive New Polarization data gives Precise non zero G_E^N hep-ph/0202183(2002)

show_gen_new.pict

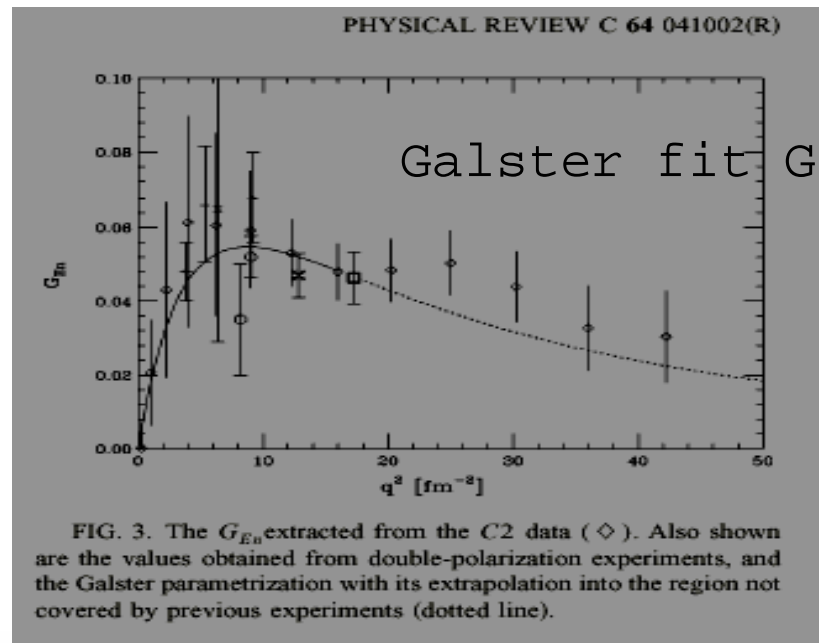
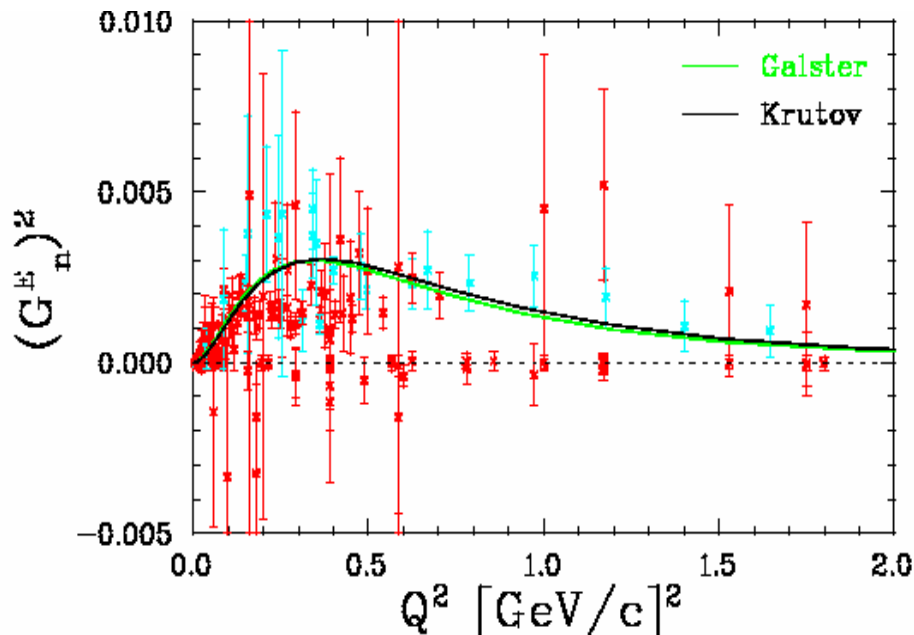
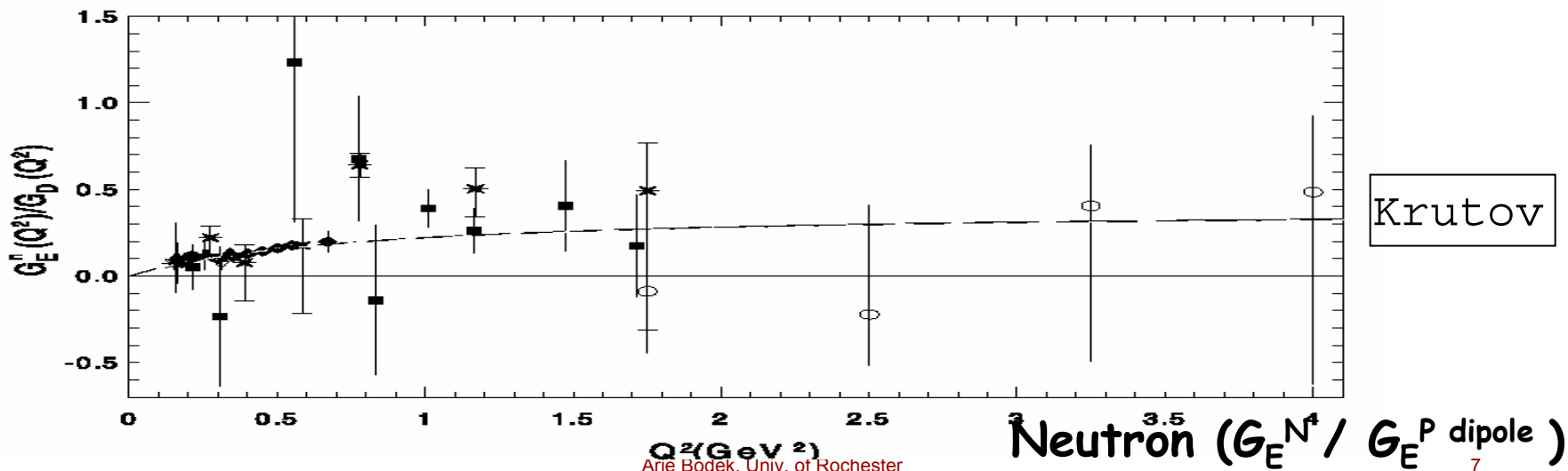


FIG. 3. The G_{E_n} extracted from the C2 data (\diamond). Also shown are the values obtained from double-polarization experiments, and the Galster parametrization with its extrapolation into the region not covered by previous experiments (dotted line).



Functional form and Values of BBA2003 Form Factors

- $G_E^{P,N}(Q^2) = \int \{e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}) d^3r\} =$ Electric form factor is the Fourier transform of the charge distribution for Proton And Neutron (therefore, odd powers of Q should not be there at low Q)

$$G_{E,M}^N(Q^2) = \frac{G_{E,M}^N(Q^2 = 0)}{1 + a_2 Q^2 + a_4 Q^4 + a_6 Q^6 + \dots} \quad G_E^P(Q^2) = G_M^P(Q^2) \frac{G_E^P(6 \text{ GeV}^2)}{G_M^P(6 \text{ GeV}^2)}$$

	data	a_2	a_4	a_6	a_8	a_{10}	a_{12}
G_E^p	CS + Pol	3.253	1.422	0.08582	0.3318	-0.09371	0.01076
G_M^p	CS + Pol	3.104	1.428	0.1112	-0.006981	0.0003705	-0.7063E-05
G_M^n		3.043	0.8548	0.6806	-0.1287	0.008912	
G_E^p	CS	3.226	1.508	-0.3773	0.6109	-0.1853	0.01596
G_M^p	CS	3.188	1.354	0.1511	-0.01135	0.0005330	-0.9005E-05

Table 2

The coefficients of the inverse polynomial fits for the G_E^p , G_M^p , and G_M^n . Fits using cross section data only, and using both cross section data and the Hall A polarization transfer data are shown separately. Note that these different polynomials replace G_D in the expression for G_E^p , G_M^p , and G_M^n . The first three rows of the table along with the fit of G_M^p Krutov *et. al.* [7] (see text) will be referred to as 'BBA-2003 Form Factors'.

$$G_E^n(Q^2) = -\mu_n \frac{a\tau}{1 + b\tau} G_D(Q^2), \quad \tau = \frac{Q^2}{4M^2},$$

with $a = 0.942$ and $b = 4.61$. This parameterization is very similar to that of Galster *et al.* [8], as shown in Figure 5.

Poor data cannot constrain
Gen very well

New innovation - Kelly Parameterization – J. Kelly, PRC 70 068202 (2004)

- Fit to sanitized dataset favoring polarization data.
- Employs the following form (Satisfies power behavior of form factors at high Q^2): --> introduce some theory constraints

$$G(q^2) = \frac{\sum_{k=0}^n a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}$$

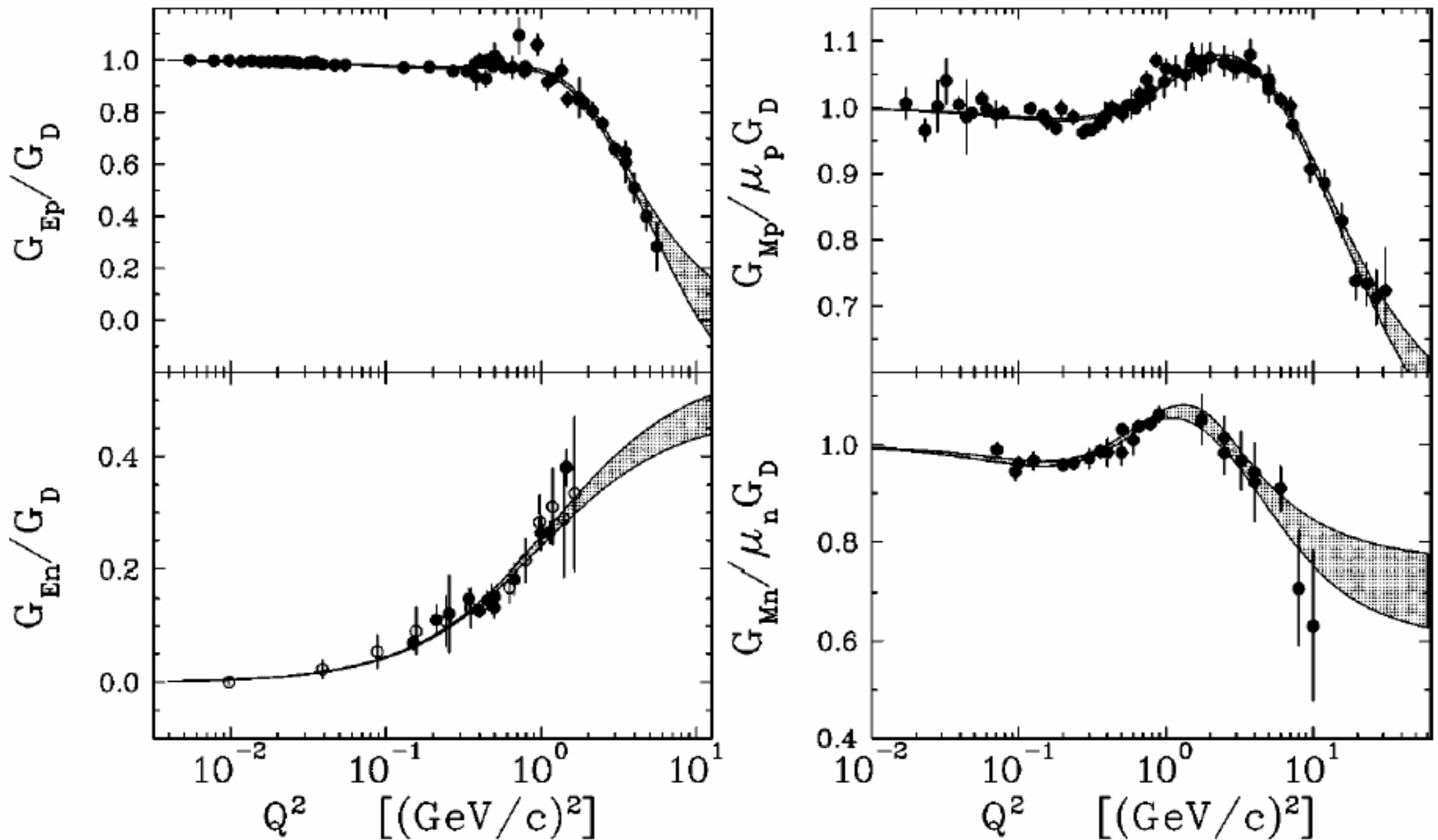
← G_{ep} , G_{mp} , and G_{mn}

$$G_{en}(Q^2) = \frac{a\tau}{1+b\tau} G_D(Q^2)$$

But uses old form for G_{en}

Kelly Parameterization

- Still not very well constrained at high Q^2 .



Source: J.J. Kelly, PRC 70 068202 (2004).
Arie Bodek, Univ. of Rochester

BBBA2005 (Bodek, Budd, Bradford, Arrington 2005)

- Fit based largely on polarization transfer data.
 - Dataset similar to that used by J. Kelly.
- **Functional form similar to that used by J. Kelly (satisfies correct power behavior at high Q^2):**

$$G(q^2) = \frac{\sum_{k=0}^n a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}$$

4 parameters for G_{ep} ,
 G_{mp} , and G_{mn} . 6
parameters for G_{en} .

use $a_0=1$ for G_{ep} , G_{mp} , G_{mn} , and $a_0=0$ for G_{en} .

- **Employs 2 additional constraints from duality to have a more constrained description at high Q^2 .**

Constraint 1: $R_p=R_n$ (from QCD)

- From local duality R for inelastic, and R for elastic should be the same at high Q^2 :

$$\left(\frac{G_{ep}}{G_{mp}}\right)^2 = \left(\frac{G_{en}}{G_{mn}}\right)^2 \quad \text{at high } Q^2.$$

- We assume that $G_{en} > 0$ continues on to high Q^2 .
- ***This constraint assumes that the QCD $R_p=R_n$ for inelastic scattering, carries over to the elastic scattering case. This constraint is may be approximate. Extended local duality would imply that this applies only to the sum of the elastic form factor and the form factor of the first resonance. (First resonance is investigated by the JUPITER Hall C program)***

Constraint 2:

From local duality:

F_{2n}/F_{2p} for Inelastic and Elastic scattering should be the same at high Q^2

- In the limit of $\nu \rightarrow \infty$, $Q^2 \rightarrow \infty$, and fixed x :

$$F_2 = x \sum_i e_i^2 f_i(x)$$

- In the elastic limit: $(F_{2n}/F_{2p})^2 \rightarrow (G_{mn}/G_{mp})^2$

$$\Rightarrow \left(\frac{G_{mn}}{G_{mp}} \right)^2 \approx \left(\frac{F_{2n}}{F_{2p}} \right)^2 \approx \frac{1 + 4 \frac{d}{u}}{4 + \frac{d}{u}}$$

We ran with $d/u=0, .2, \text{ and } .5$.

Constraint 2

- In the elastic limit: $(F_{2n}/F_{2p})^2 \rightarrow (G_{mn}/G_{mp})^2$

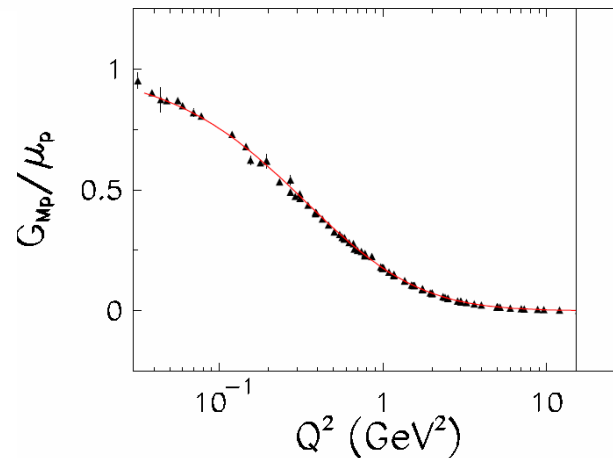
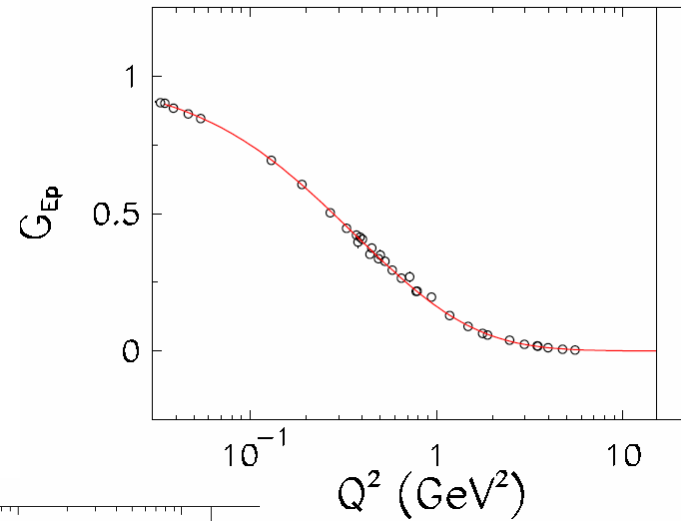
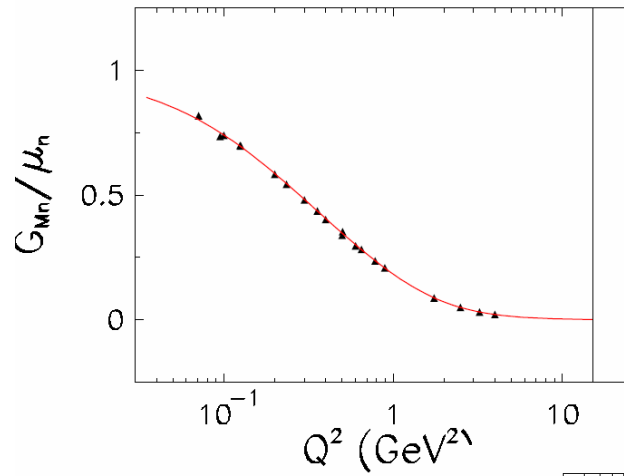
$$\Rightarrow \left(\frac{G_{mn}}{G_{mp}} \right)^2 \approx \left(\frac{F_{2n}}{F_{2p}} \right)^2 \approx \frac{1 + 4 \frac{d}{u}}{4 + \frac{d}{u}}$$

We use $d/u=0$, *This constraint assumes that the F_{2n}/F_{2p} for inelastic scattering, carries over to the elastic scattering case. **This constraint is may be approximate. Extended local duality would imply that *this* applies only to the sum of the elastic form factor and the form factor of the first resonance. (First resonance is investigated by the JUPITER Hall C program)***

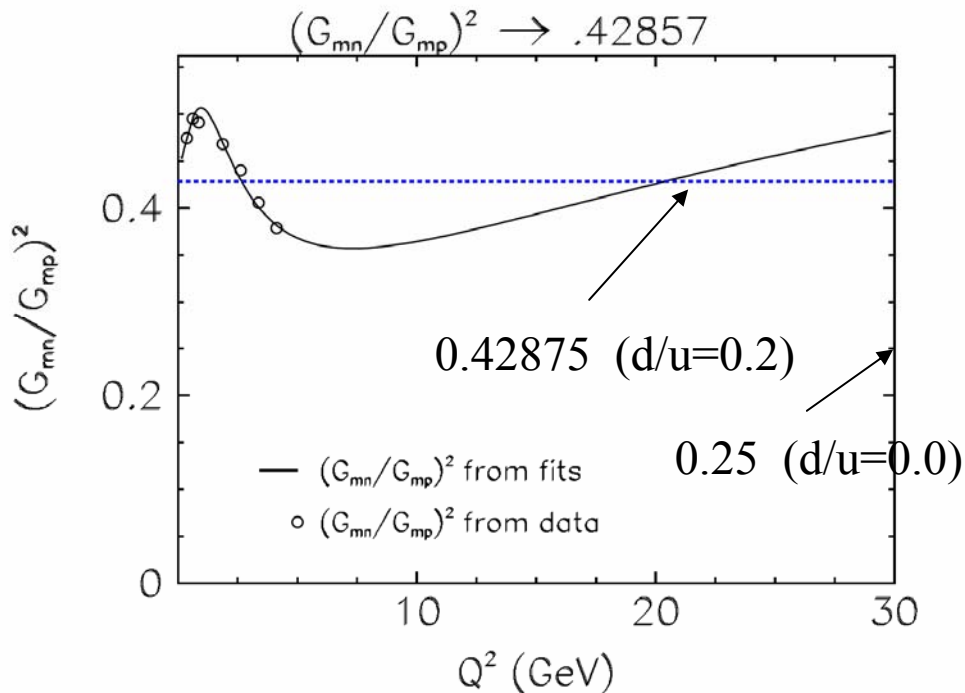
BBBA2005... NuInt05 ep-ex/0602017

- We have developed 6 parameterizations:
 - One for each value of $(d/u)=0, 0.2, 0.5$ (at high x)
 - One each for $G_{en} > 0$ and $G_{en} < 0$ at high Q^2 .
- Our preferred parameterization is for
 - $G_{en} > 0$ at high Q^2
 - $d/u=.2$, so $(G_{mn}/G_{mp})=.42857$ (if $d/u=.2$ as expected from QCD)
- Following figures based on preferred parameterization.

Sample Results BBBA2005: G_{mn} , G_{ep} , G_{mp}



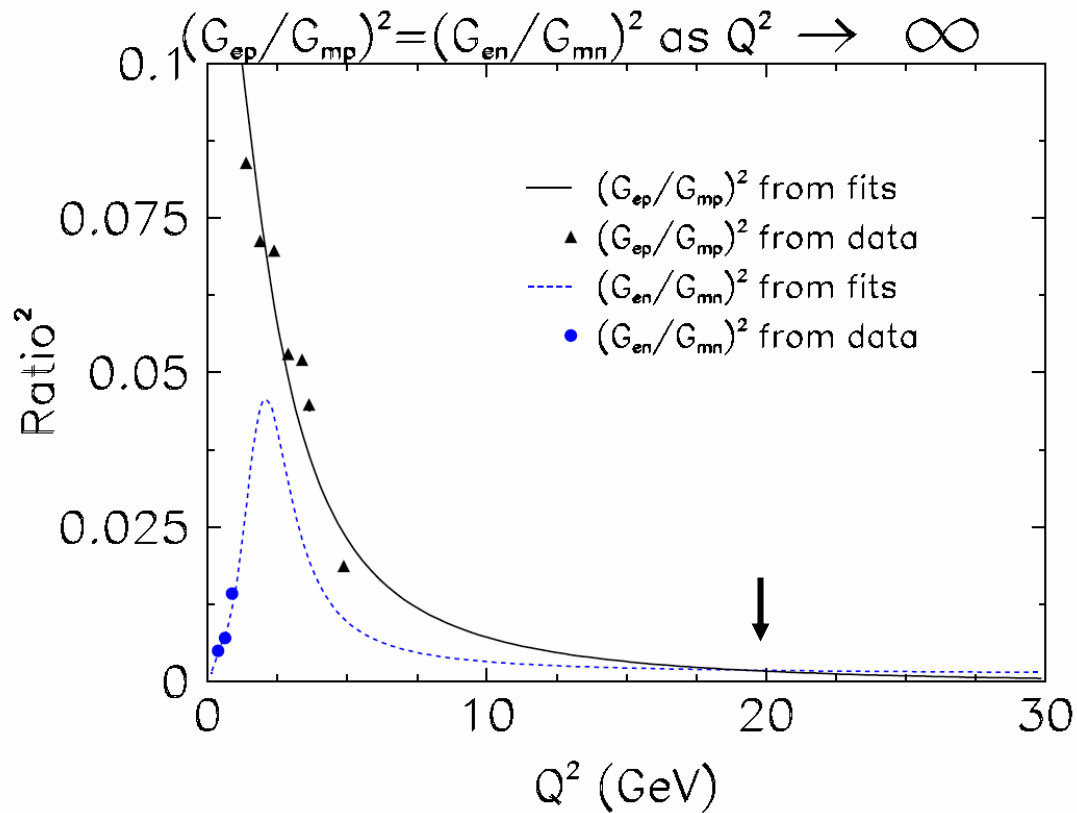
Constraints: G_{mn}/G_{mp}



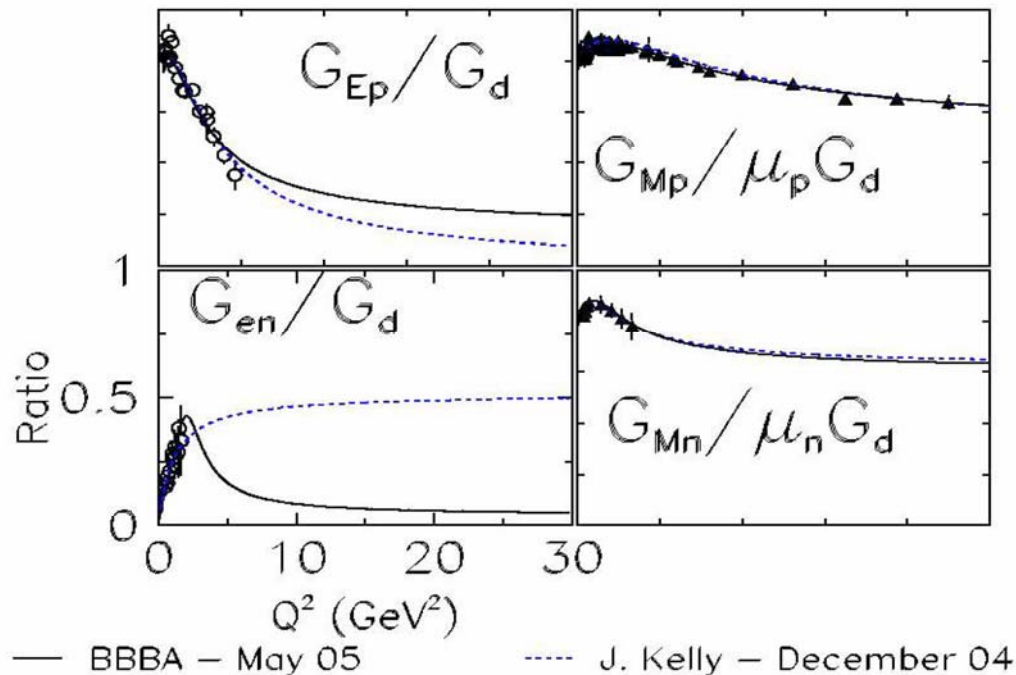
Questions: would including the first resonance make local duality work at lower Q^2 ?

Or is $d/u \rightarrow 0$ (instead of 0.2) which implies $F_2n/F_2p = 0.25$ instead of 0.43?

Constraints: $(G_{ep}/G_{mp})^2 = (G_{en}/G_{mn})^2$



Comparison with Kelly Parameterization



—— BBBA

—— Kelly

Figure 2. The solid black line shows the ratio of the BBBA05 form factors to G_d , and the dashed blue line is the ratio of the Kelly form factors to G_d . The differences in the two parameterizations for $\frac{G_{ep}}{G_d}$ and $\frac{G_{en}}{G_d}$ are due to the constraints applied to the BBBA05 form factors. All figures have a y-axis ranging from $Q^2 = 0\text{GeV}^2$ to $Q^2 = 30\text{GeV}^2$. In the lower limit ($Q^2 = 0\text{GeV}^2$), all ratios approach unity, except for G_{en} , which approaches zero.

Summary - Vector Form Factors

- We have developed new parameterizations of the nucleon form factors BBA2005.
 - Improved fitting function
 - Additional constraints extend validity to higher ranges in Q^2 (assuming local duality)
- Ready for use in simulations....
- Further tests to be done by including new F_2^n/F_2^p and R_p and R_n data from the first resonance (from new JUPITER Data)

BBBA2005 Fit Parameters (Gen>0, d/u=0.2)

Observable	a_1	a_2	b_1	b_2	b_3	b_4
G_{ep}	-0.0578 ± 0.0165		11.1 ± 0.217	13.6 ± 1.39	33.0 ± 8.95	
G_{mp}	0.0150 ± 0.0312		11.1 ± 0.103	19.6 ± 0.281	7.54 ± 0.967	
G_{en}	1.25 ± 0.368	1.30 ± 1.99	9.86 ± 6.46	305 ± 28.6	-758 ± 77.5	802 ± 156
G_{mp}	1.81 ± 0.402		14.1 ± 0.597	20.7 ± 2.54	68.7 ± 14.1	

hep-ex/0602017

STUDY OF THE REACTION $\nu_\mu d \rightarrow \mu^- pp_s$

 TABLE I. Maximum-likelihood values of M_A (GeV/c^2) for each model.

	Monopole	Dipole	Tripole	QM-AVMD
Rate	0.45 ± 0.11	0.74 ± 0.12	0.95 ± 0.16	0.69 ± 0.26
Shape	0.57 ± 0.05	1.05 ± 0.05	1.38 ± 0.06	1.25 ± 0.17
Total	0.55 ± 0.05	1.03 ± 0.05	1.35 ± 0.07	1.20 ± 0.17
Flux independent	0.54 ± 0.05	1.00 ± 0.05	1.31 ± 0.07	1.11 ± 0.16

Miller 1982: We type in their $d\sigma/dQ^2$ histogram. Fit with our
 Knowledge of their parameters : Get $M_A = 1.116 \pm 0.05$
 (A different central value, but they do event likelihood fit)
 And we do not have their the events, just the histogram.

If we put in BBBA2005 form factors and modern g_a ,
 then we get $M_A = 1.086 \pm 0.05$ or $\Delta M_A = -0.030$. So all the
 Values for M_A from this expt. should be reduced by 0.03

Do a reanalysis of old neutrino data to get ΔM_A to update using latest ga+BBBA2005 form factors.

(note different experiments have different neutrino energy Spectra, different fit region, different targets, so each experiment requires its own study).

If Miller had used Pure Dipole analysis, with $g_a=1.23$ (Shape analysis)

- the difference with BBA2003 form factors would

Have been --> $\Delta M_A = -0.050$

(I.e. results would have had to be reduced by 0.050)

But Miller 1982 did not use pure dipole (but did use $G_{\pi}=0$)

so their result only needs to be reduced by $\Delta M_A = -0.030$

Reanalysis of FOUR different neutrino experiments

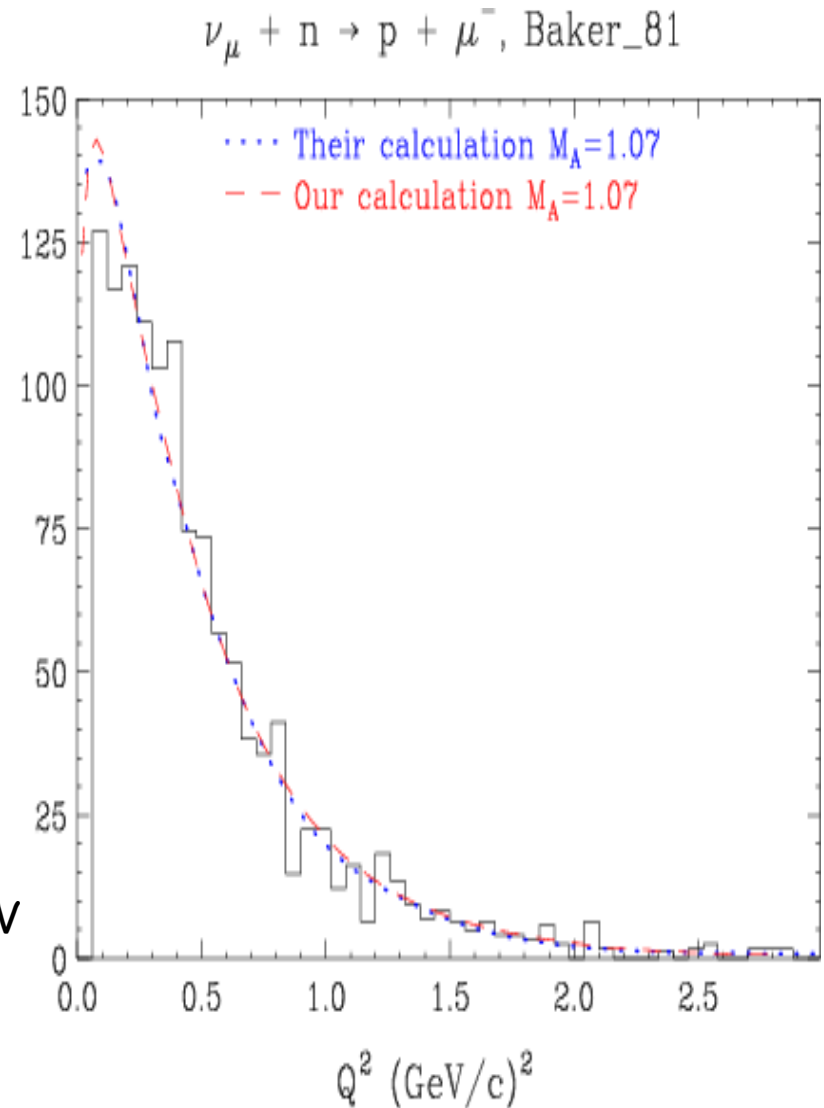
(they mostly used D2 data with Olsson vector form factors and

and older value of G_{π}) yields ΔM_A VARYING From -0.022 (FNAL energy)

to -0.030 (BNL energy)

Determining m_A , Baker et al. - 1981 BNL deuterium

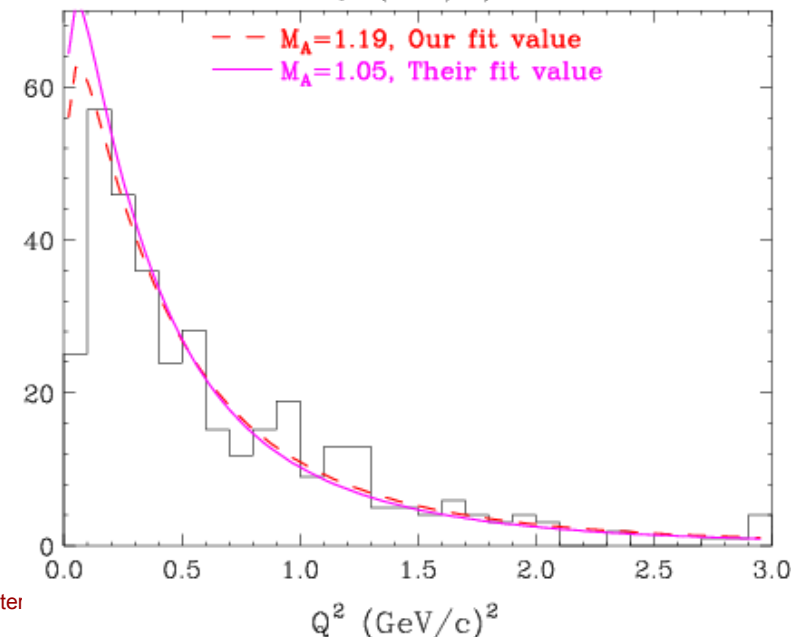
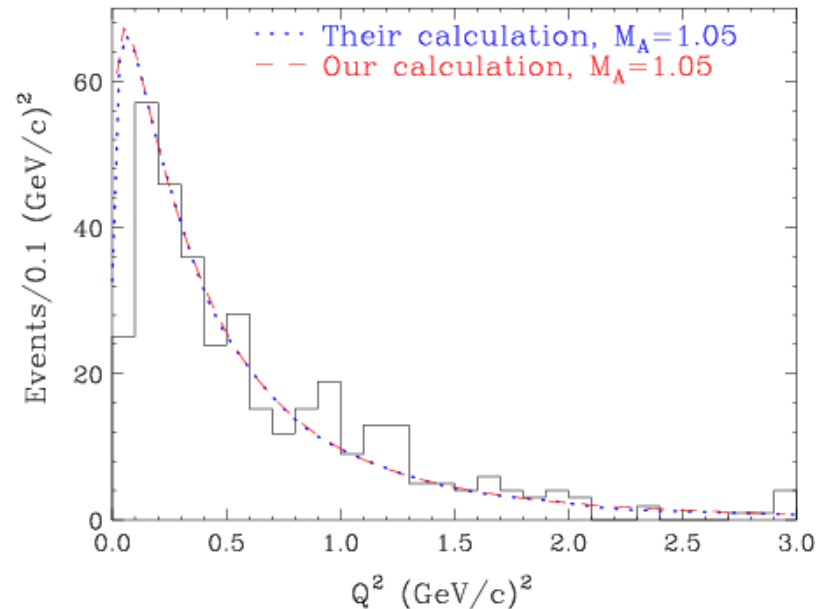
- The dotted curve shows their calculation using their fit value of 1.07 GeV
- They do unbinned likelihood to get M_A
No shape fit
- Their data and their curve is taken from the paper of Baker et al.
- The dashed curve shows our calculation using $M_A = 1.07$ GeV using their assumptions
- The 2 calculations agree.
- If we do shape fit to get M_A
- With their assumptions -- $M_A=1.079$ GeV
- We agree with their value of M_A
- If we fit with BBA Form Factors and our constants - $M_A=1.050$ GeV.
- Therefore, we must shift their value of M_A down by -0.029 GeV.
- Baker does not use a pure dipole
- The difference between BBBA2005-form factors and dipole form factors is -0.055 GeV



Kitagaki et al. 1983 FNAL deuterium

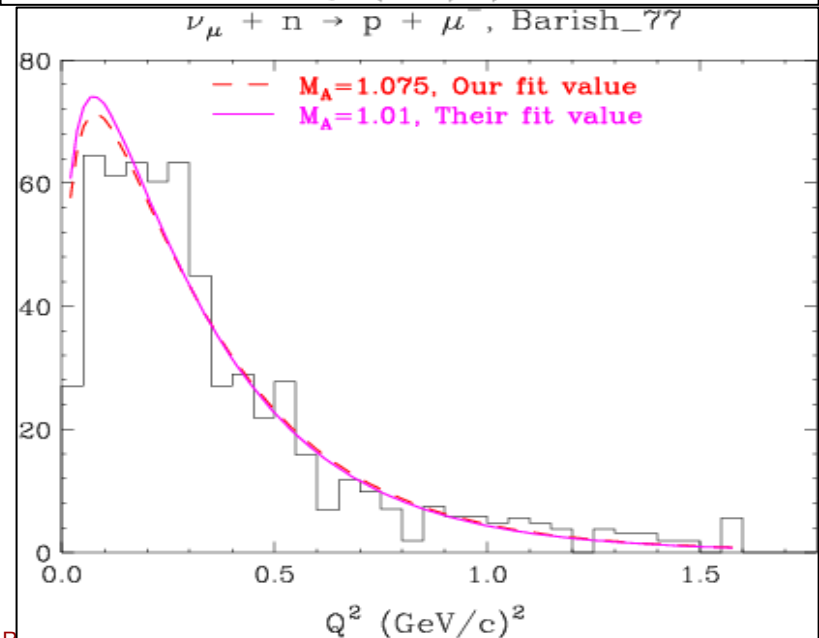
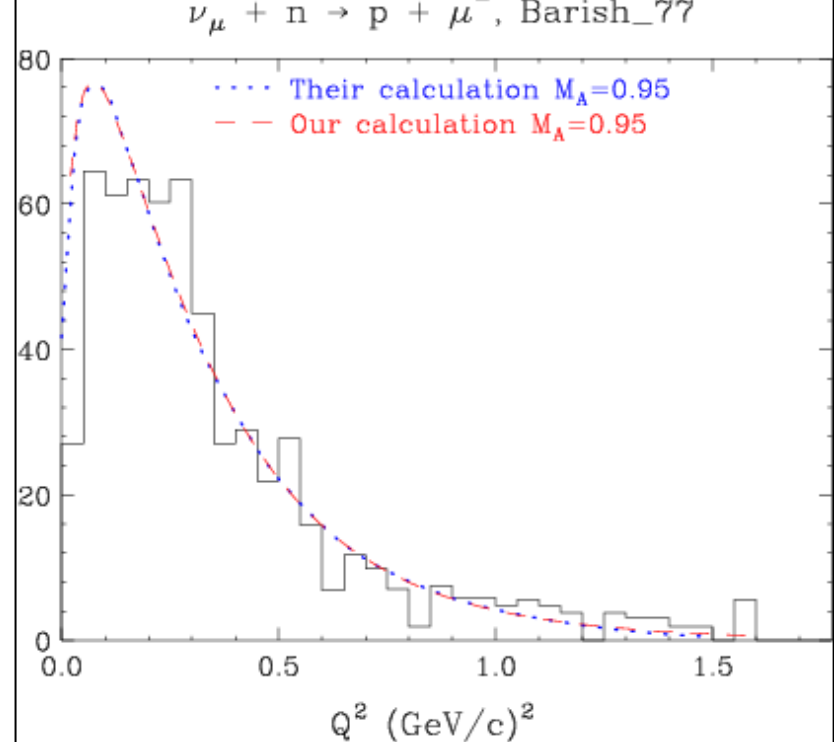
- The dotted curve shows their calculation using their fit value of $M_A=1.05$ GeV
- They do unbinned likelihood, no shape fit.
- The dashed curve shows our calculation using $M_A=1.05$ GeV and their assumptions
- The solid curve is our calculation using their fit value $M_A=1.05$ GeV
- The dash curve is our calculation using our fit value of $M_A=1.19$ GeV with their assumption
- However, we disagree with their fit value.
- Our fit value seem to be in better agreement with the data than their fit value.
- We get $M_A=1.172$ GeV when we fit with our assumptions
- Hence, -0.022 GeV should be subtracted from their M_A .

$\nu_\mu + n \rightarrow p + \mu^-$, Kitagaki 1983



Barish 1977 et al. ANL deuterium

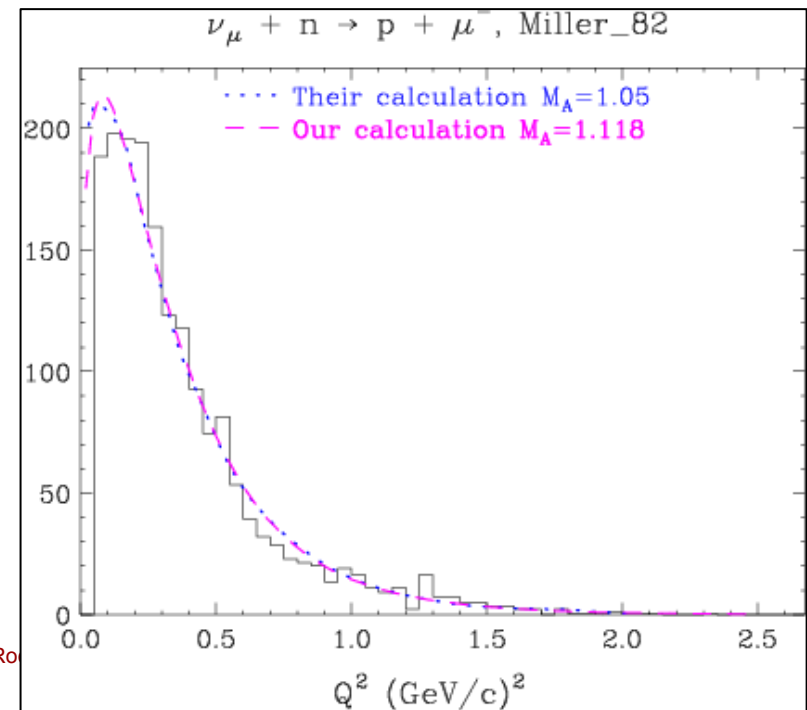
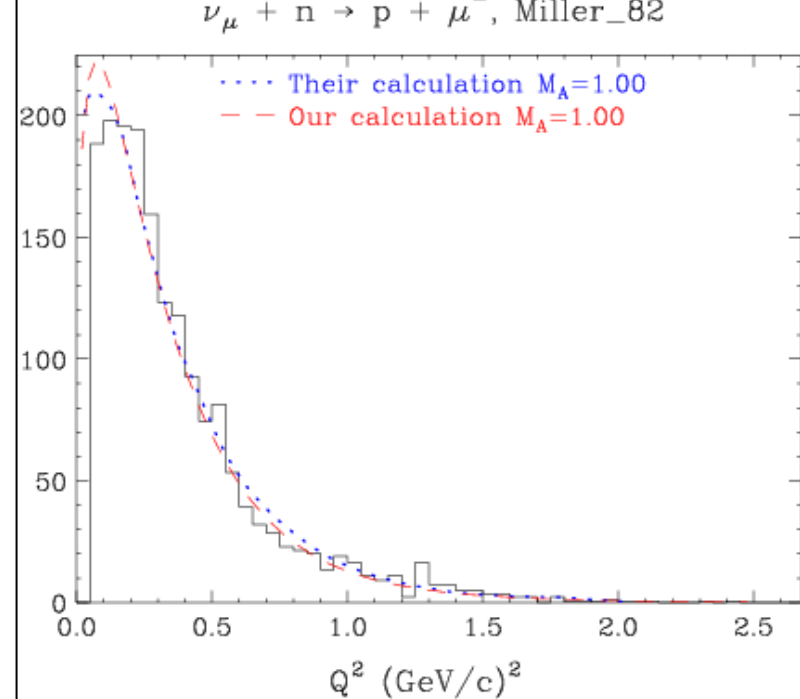
- Dotted curve - their calculation $M_A=0.95$ GeV is their unbinned likelihood fit
- The dashed curve - our calculation using their assumption
- We agree with their calculation.
- The solid curve - our calculation using their shape fit value of 1.01 GeV.
- We are getting the best fit value from their shape fit.
- The dashed curve is our calculation using our fit value $M_A=1.075$ GeV.
- We slightly disagree with their fit value.
- We get $M_A=1.046$ GeV when we fit with BBA2005 - Form Factors and our constants.
- Hence, -0.029 GeV must be subtracted from their value of M_A



Miller 1982- ANL deuterium

DESCRIBED EARLIER

- Miller is an updated version of Barish with 3 times the data
- The dotted curve - their calculation taken from their Q^2 distribution figure, $M_A=1$ GeV is their unbinned likelihood fit.
- Dashed curve is our calculation using their assumptions
- We don't quite agree with their calculation.
- Their best shape fit for M_A is 1.05
- Dotted is their calculation using their best shape M_A
- Our M_A fit of using their assumptions is 1.116 GeV
- Our best shapes agree.
- Our fit value using our assumptions is 1.086 GeV
- Hence, -0.030 GeV must be subtracted from their fit value.



Summary of Results for 4 neutrino experiments

$$\Delta M_A = -0.027 \text{ on average}$$

9

	M_A (published)	updated M_A old params.	updated M_A new params.	ΔM_A new-old	ΔM_A BBBA-2005-Dipole
Baker 1981 [9]	1.07 ± 0.06	1.079 ± 0.055	1.050 ± 0.055	-0.029	-0.055
Barish 1977 [10]	1.01 ± 0.09	1.075 ± 0.095	1.046 ± 0.099	-0.029	-0.046
Miller 1982 [11]	1.05 ± 0.05	1.116 ± 0.055	1.086 ± 0.055	-0.030	-0.050
Kitagaki 1983 [12]	$1.05^{+0.12}_{-0.16}$	$1.195^{+0.10}_{-0.11}$	$1.172^{+0.10}_{-0.11}$	-0.022	-0.053

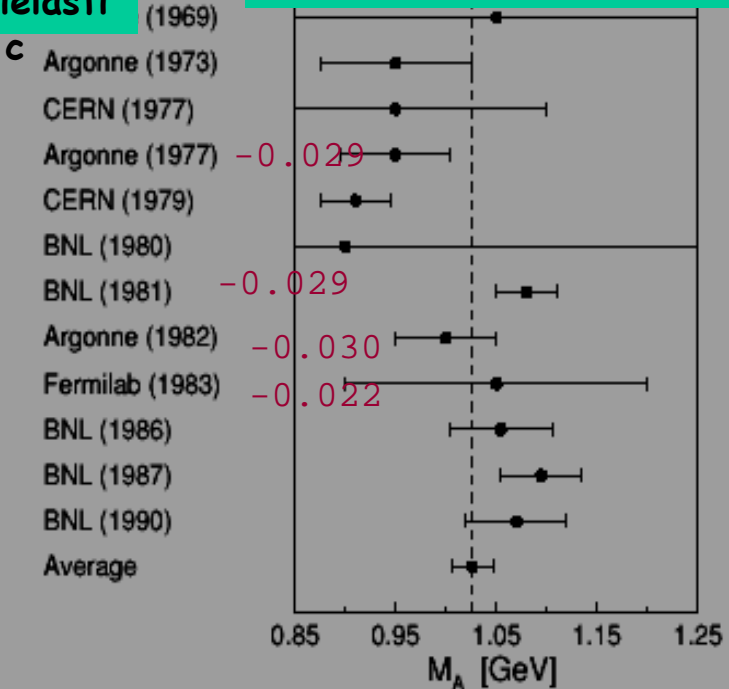
Table 3

Published and updated extractions of M_A (GeV) from deuterium experiments. The first value of M_A is the values extracted in the original publications. For Barish and Miller, we give their ‘shape fit’ value, since this value most closely reflects how we can calculate their M_A . The second value of M_A is from the analysis presented here, using the same input parameters (form factors and g_A) as in the publications, while the third uses the updated parameters from tables 2 and 1. The last two columns show the change in M_A between the new and old input parameters, and the change when comparing the BBBA-2005 and Dipole Form Factors (with g_A fixed).

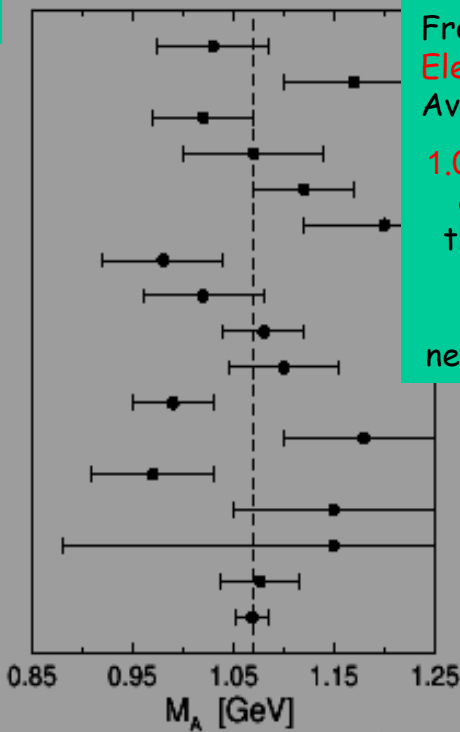
Véronique Bernard†, Latifa Elouadrhiri‡, Ulf-G Meißner§

From Neutrino quasielasti

Neutrinos $1.026 \pm 0.021 = M_A$ average



- Frascati (1970)
- Frascati (1970) GEN=0
- Frascati (1972)
- DESY (1973)
- Daresbury (1975) SP
- Daresbury (1975) DR
- Daresbury (1975) FPV
- Daresbury (1975) BNR
- Daresbury (1976) SP
- Daresbury (1976) DR
- Daresbury (1976) BNR
- DESY (1976)
- Kharkov (1978)
- Olsson (1978)
- Saclay (1993)
- MAMI (1999)
- Average



From charged Pion Electroproduction Average value of 1.069 \rightarrow 1.014 when corrected for theory hadronic effects to compare to neutrino reactions

For updated M_A we reanalyzed neutrino expt with new g_A , and BBBA2005 form factors

Got $M_A = 1.026 \pm 0.021$ (world average) minus 0.027 = $0.99 \pm 0.021 = M_A$

Difference in M_A between Electroproduction And neutrinos is understood

and antineutrino scattering experiments. The weighted average is $M_A = (1.026 \pm 0.021)$ GeV. Right panel: From charged pion electroproduction experiments. The weighted average is $M_A = (1.069 \pm 0.016)$ GeV. Note that value for the MAMI = 1.014 when corrected for hadronic effect to compare to neutrino reactions

For M_A from QE neutrino expt. On free nucleons No theory corrections needed: 0.99 ± 0.021 and 1.014 ± 0.016 are consistent \rightarrow ave = 1.007

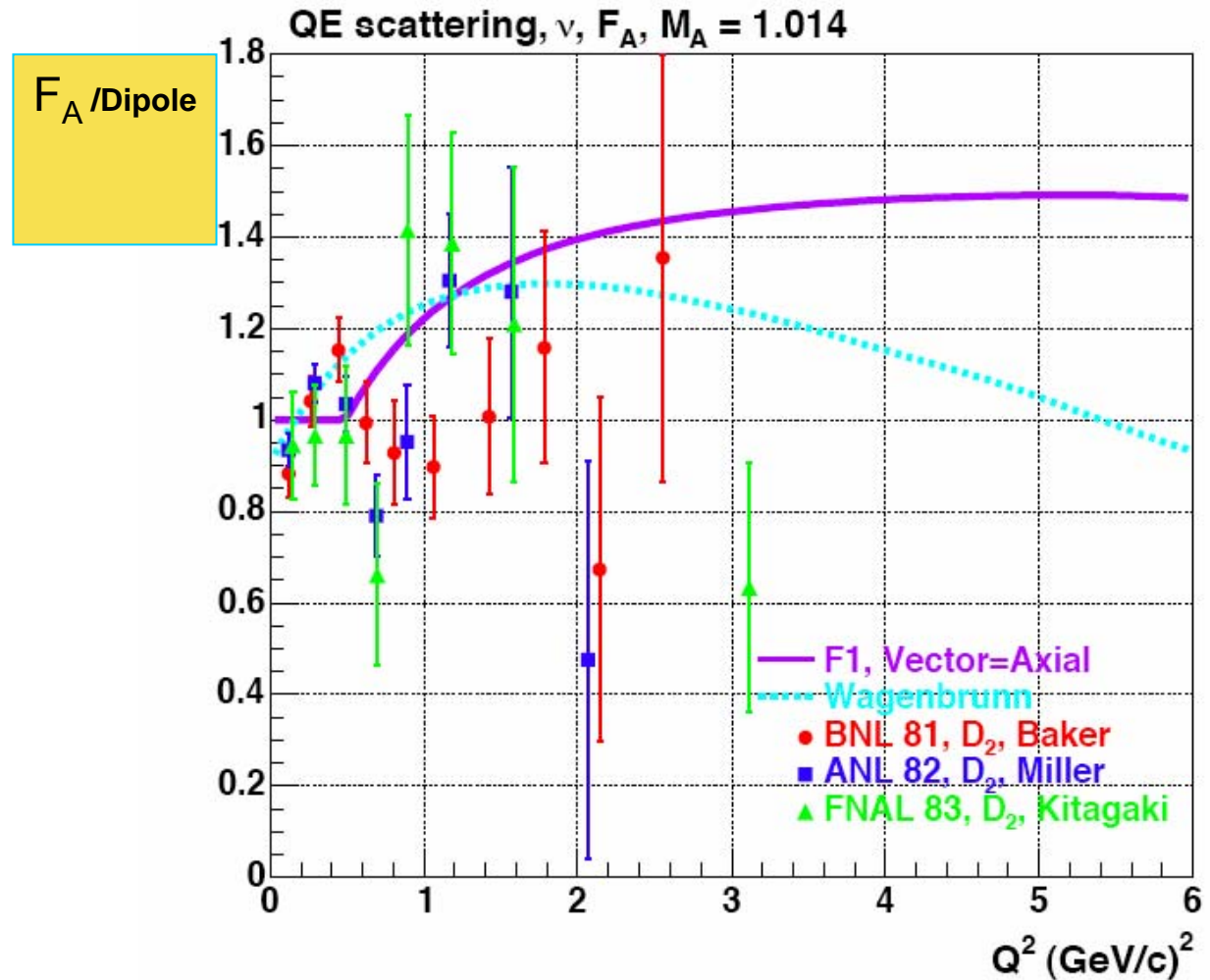
Conclusion of Reanalysis of neutrino data

- Using BBBA2005-form factors we derive a new value of $m_A = 0.99 \text{ GeV} \pm 0.021$ From the world average of Neutrino expt.
- agrees with the results from pion electroproduction:
 $m_A = 1.014 \pm 0.016 \text{ GeV}$
- We now understand the Low Q^2 behavior of F_A
- ~7-8% effect on the neutrino cross section from the new value m_A and with the updated vector form factors
- MINERvA can measure F_A and determine deviations from the dipole form at high Q^2 . Can extract F_A from neutrino data on $d\sigma/dq^2$
- The anti-neutrinos at high Q^2 serves as a check on F_A

Theory predictions for F_A

some calculations predict that F_A is may be larger than the Dipole predictions at high Q^2

1. Wagenbrum - constituent quark model (valid at intermediate Q^2)
2. Bodek - Local duality between elastic and inelastic implies that vector=axial at high Q^2
3. However, local duality may fail. We need to measure **both elastic and first resonance** vector and axial form factors. We can then test for Adler sum rule for vector and axial form scattering separately- MINERvA and JUPITER

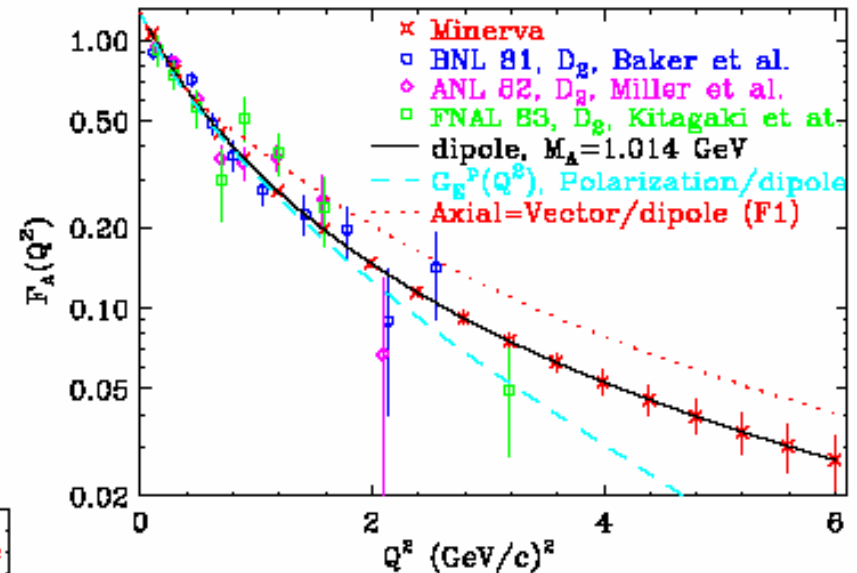


Current Neutrino data on F_A vs MINERvA

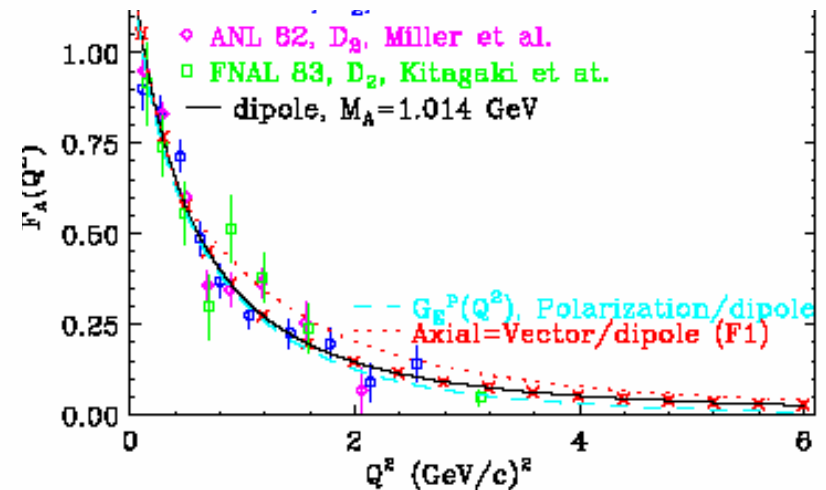
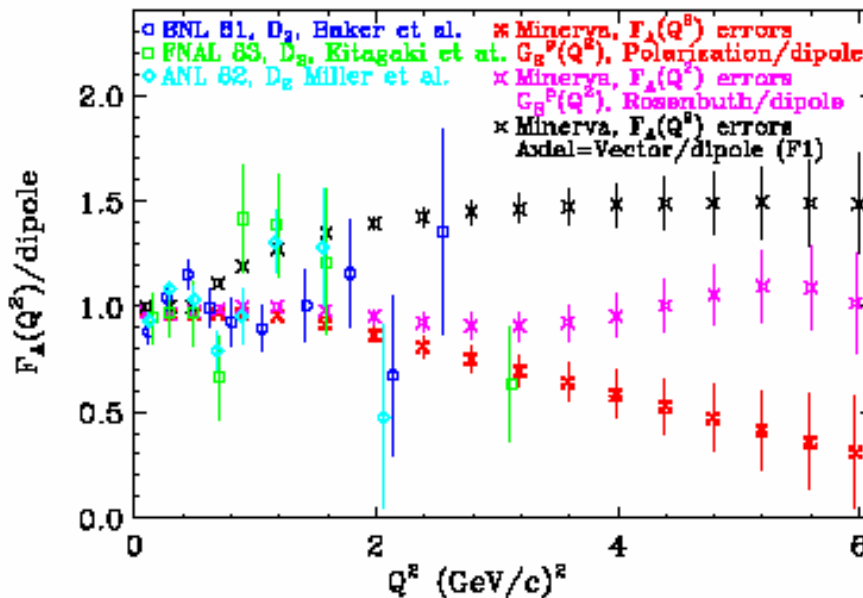
For inelastic (quarks) axial=vector
 Therefore, local duality implies that
 At high Q^2 , $2xF_1$ - elastic Axial and
 Vector are the same.

Note both F_2 and $2xF_1$ - elastic Axial and
 Vector are the same at high Q^2 - when $R \rightarrow 0$.

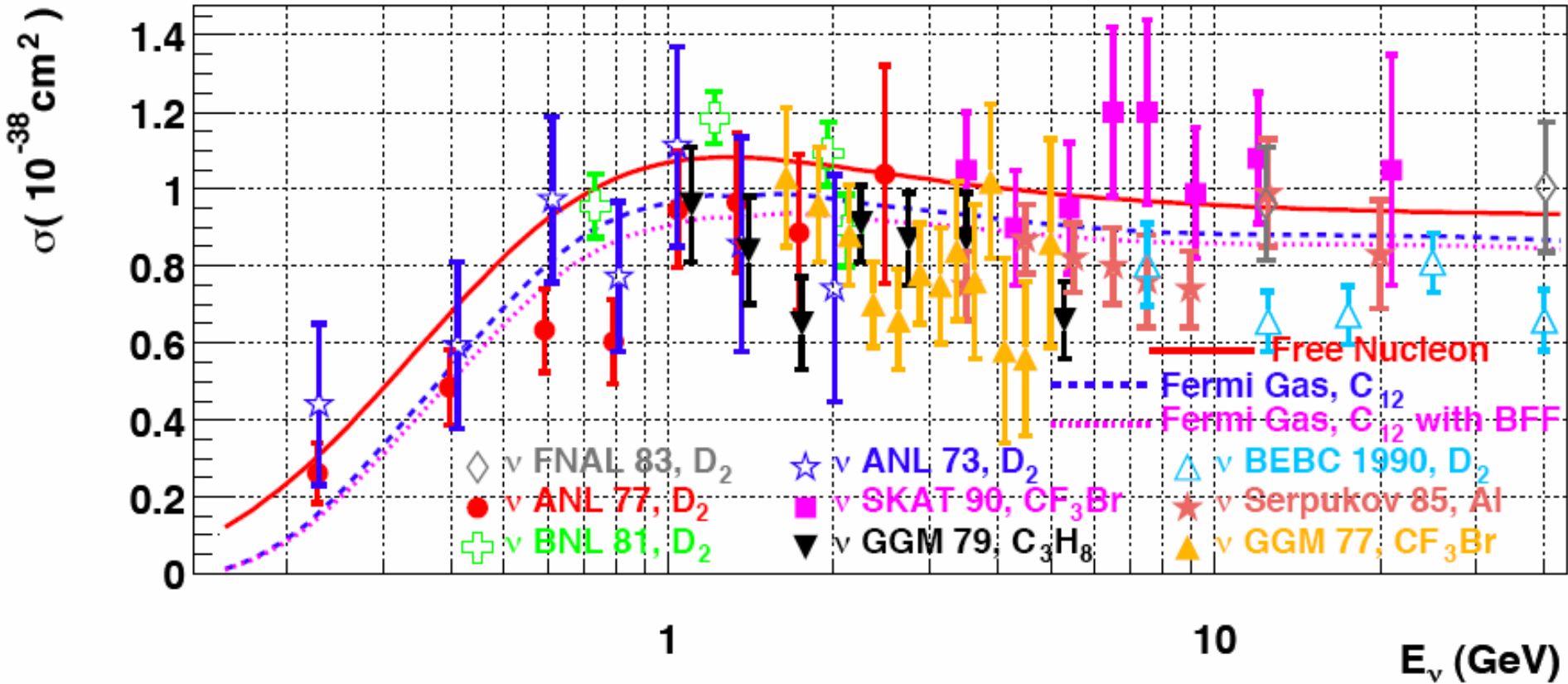
QE scattering, ν_μ , $F_A(Q^2)$



QE scattering, ν_μ , $F_A(Q^2)/\text{dipole}$, $M_A=1.014$ GeV

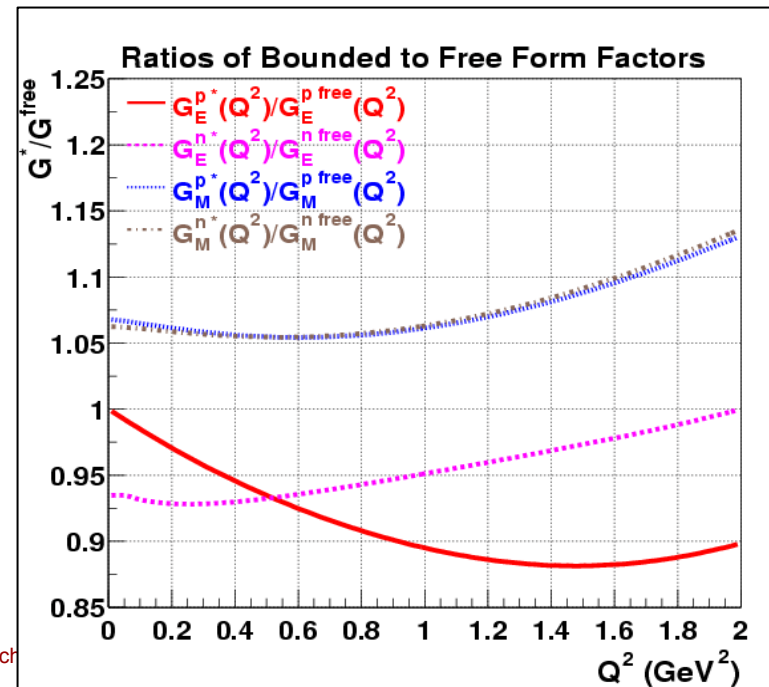
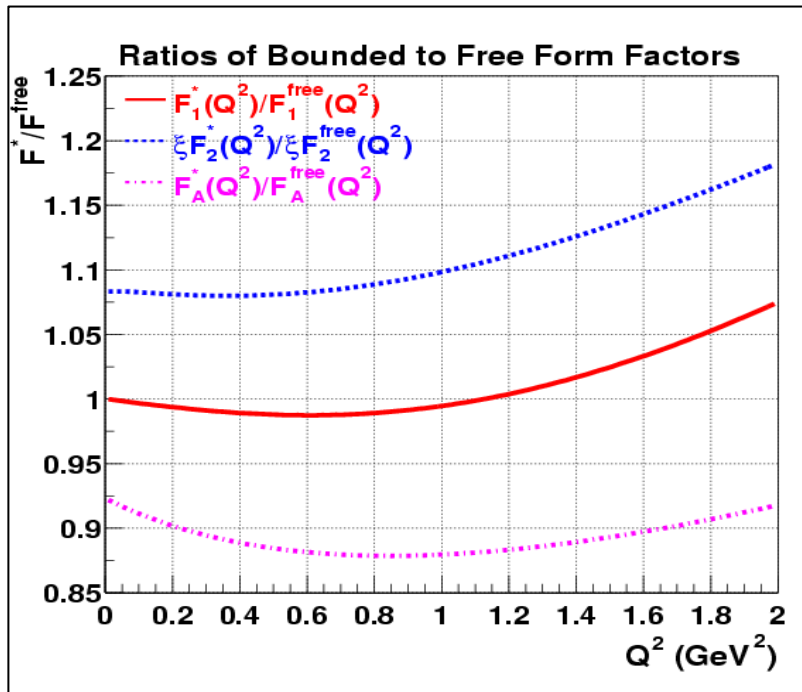
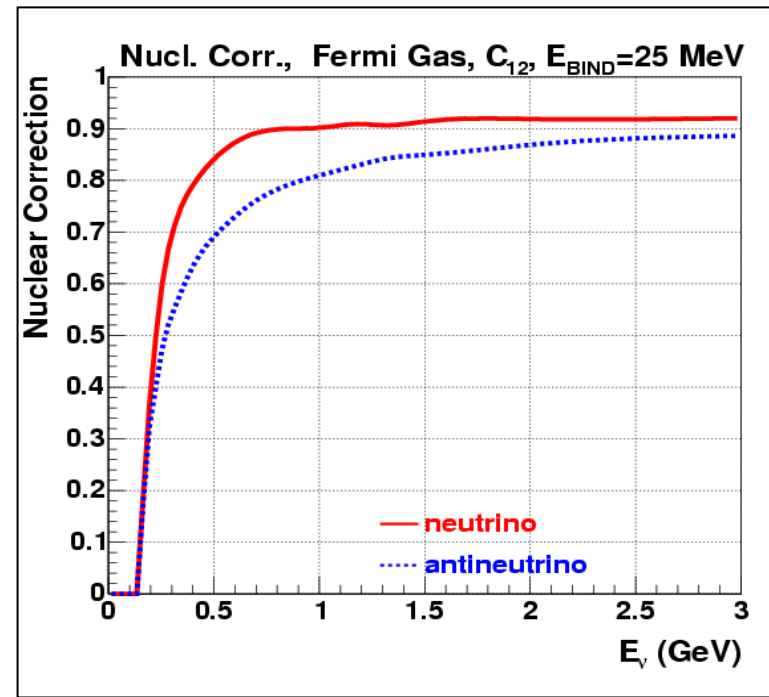


$\nu + n \rightarrow p + \mu^-$, BBA-2003 Form Factors, $m_A=1.00$



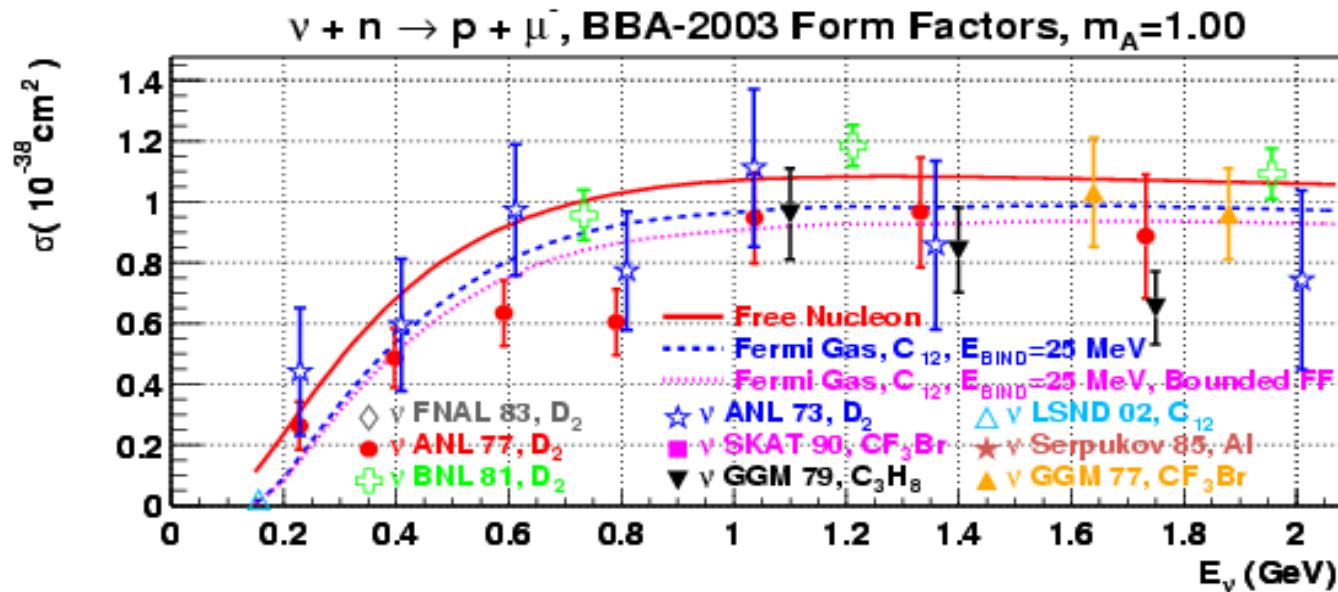
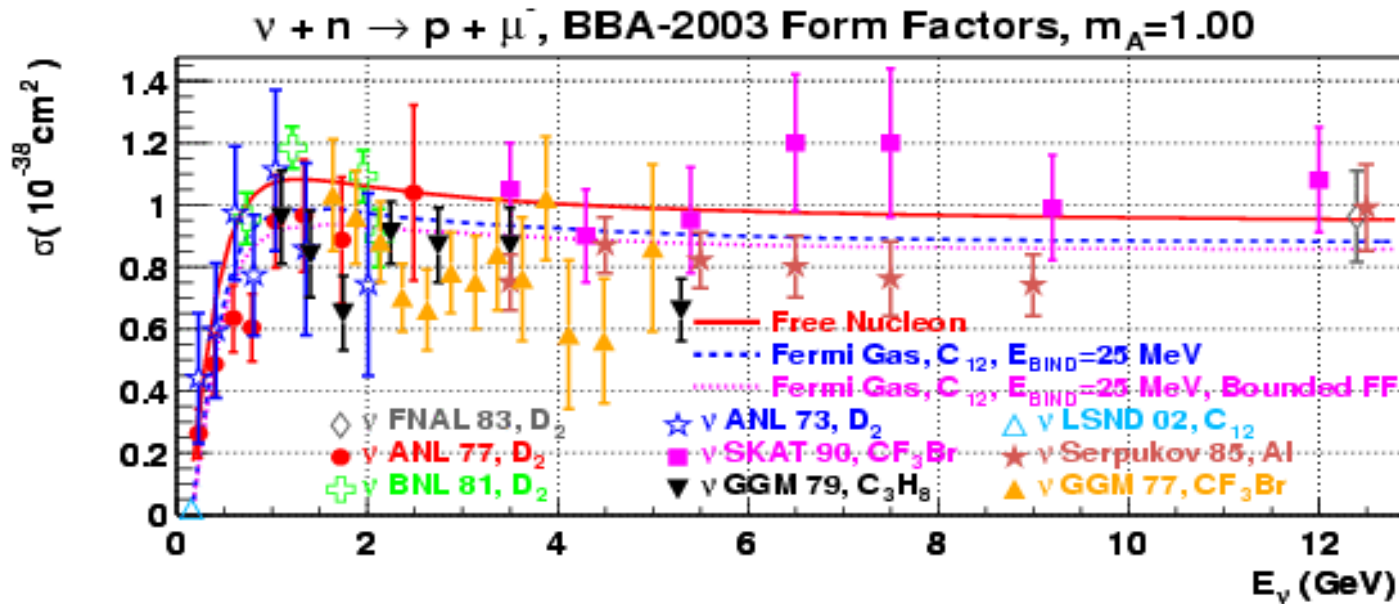
Supplemental Slides

- Nuclear correction uses NUANCE calculation
- Fermi gas model for carbon. Include Pauli Blocking, Fermi motion and 25 MeV binding energy
- Nuclear binding on nucleon form factors as modeled by Tsushima et al.
- Model valid for $Q^2 < 1$
- Binding effects on form factors expected to be small at high Q^2 .



Neutrino quasi-elastic cross section

Most of the cross section for nuclear targets low

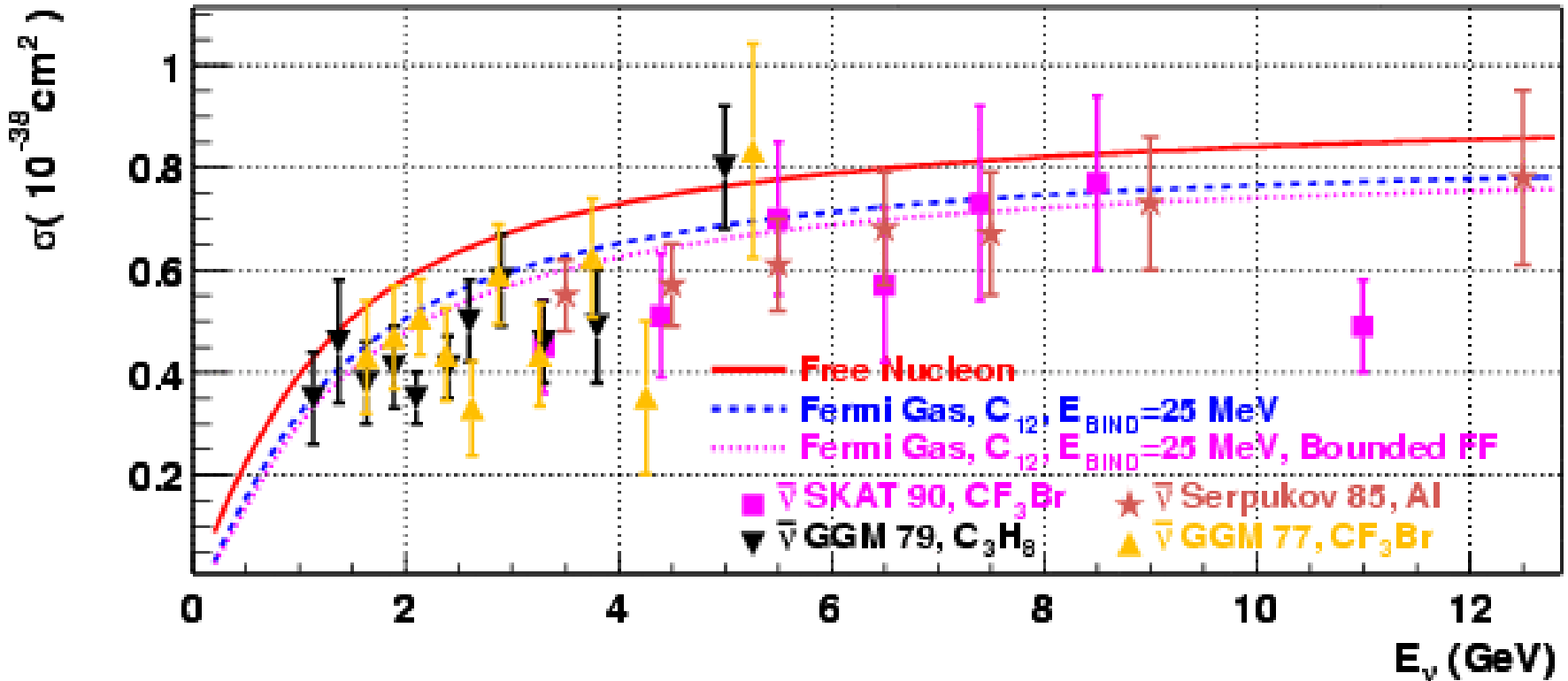


Anti-neutrino quasi-elastic cross section

Mostly on nuclear targets

Even with the most update form factors and nuclear correction, the data is low

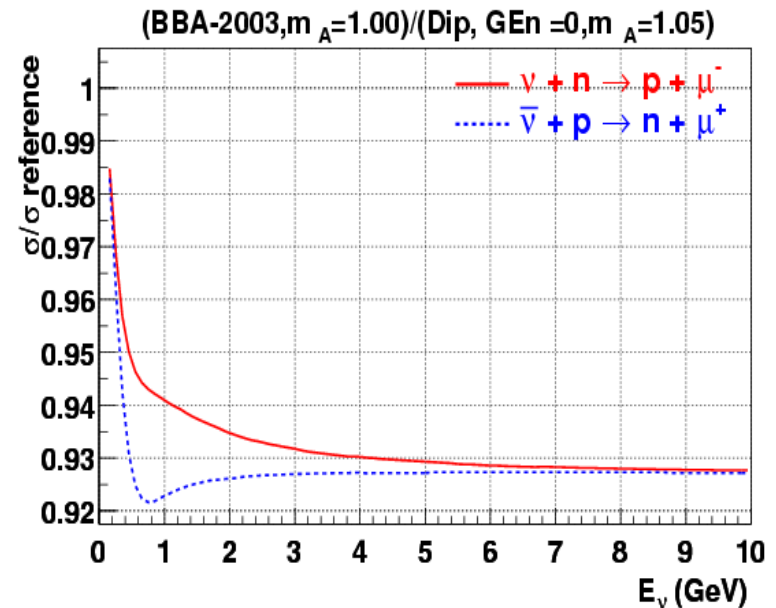
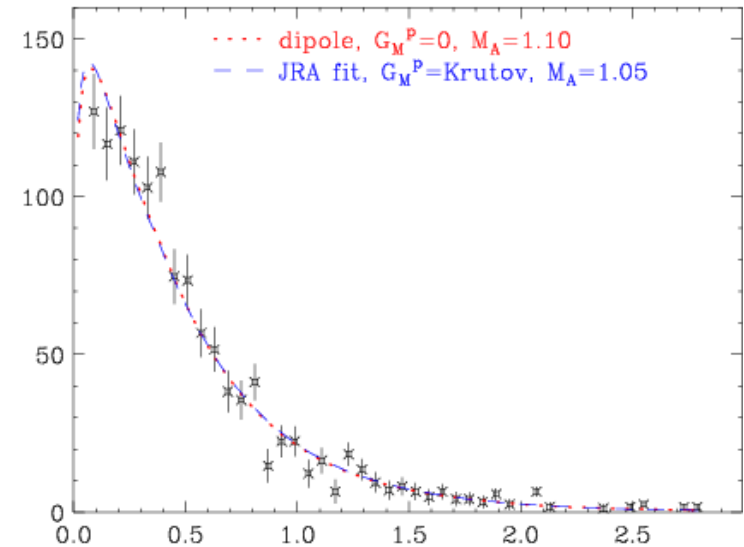
$\bar{\nu} + p \rightarrow n + \mu^+$, BBA-2003 Form Factors, $m_A=1.00$



Effects of form factors on Cross Section

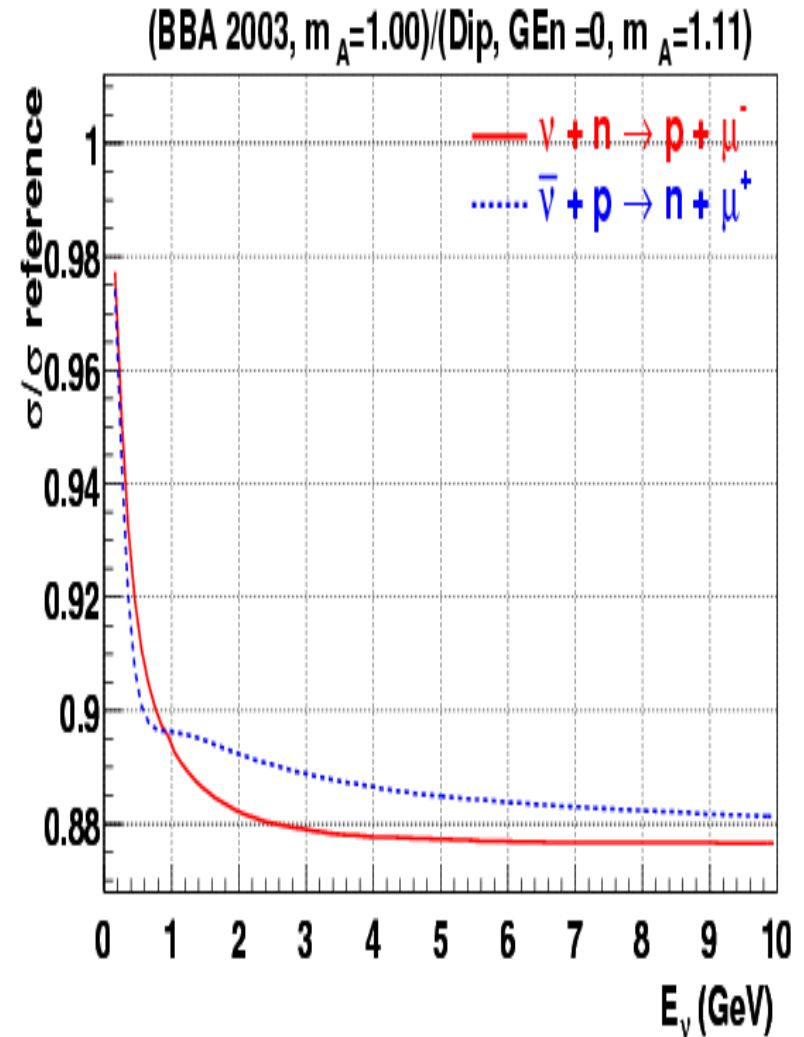
- A comparison of the Q^2 distribution using 2 different sets of form factors.
- The data are from Baker
- The dotted curve uses Dipole Form Factors with $m_A=1.10$ GeV.
- The dashed curve uses BBA-2003 Form Factors with $m_A=1.05$ GeV.
- The Q^2 shapes are the same
- However the cross sections differ by 7-8%
- Shift in m_A - roughly 4%
- Nonzero GEN - roughly 3% due
- Other vector form factor - roughly 2% at low Q^2

$\nu_\mu + n \rightarrow p + \mu^-$, Baker_81



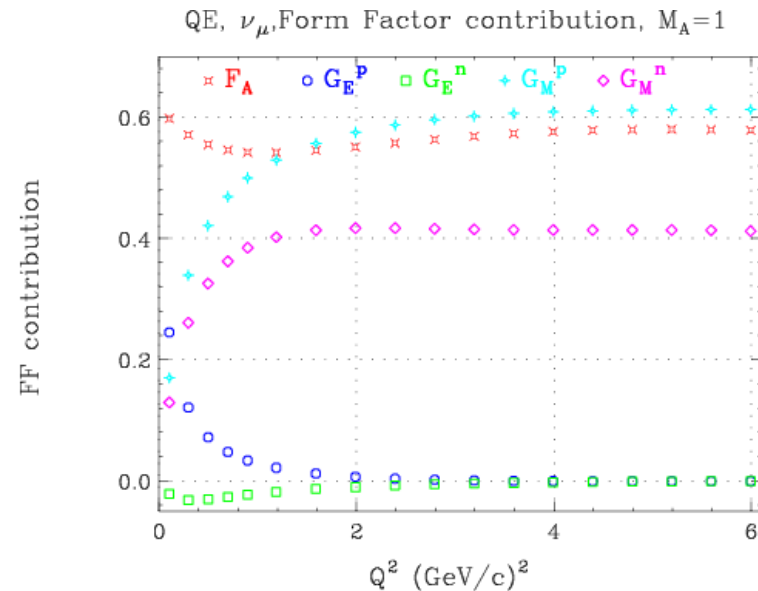
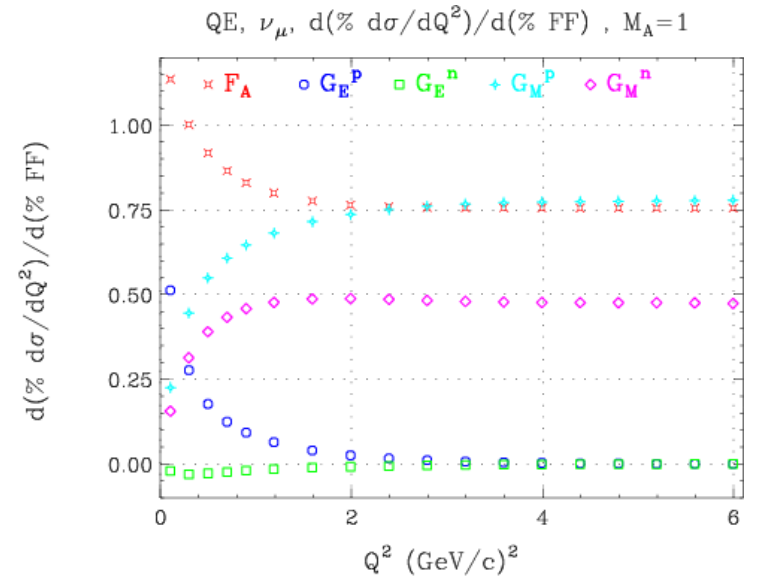
Effect of Form Factors on Cross Section

- Previously K2K used dipole form factor and set $m_A=1.11$ instead of nominal value of 1.026
- This plot is the ratio of BBA with $m_A=1$ vs dipole with $m_A=1.11$ GeV
- This gets the cross section wrong by 12%
- Need to use the best set of form factors and constants



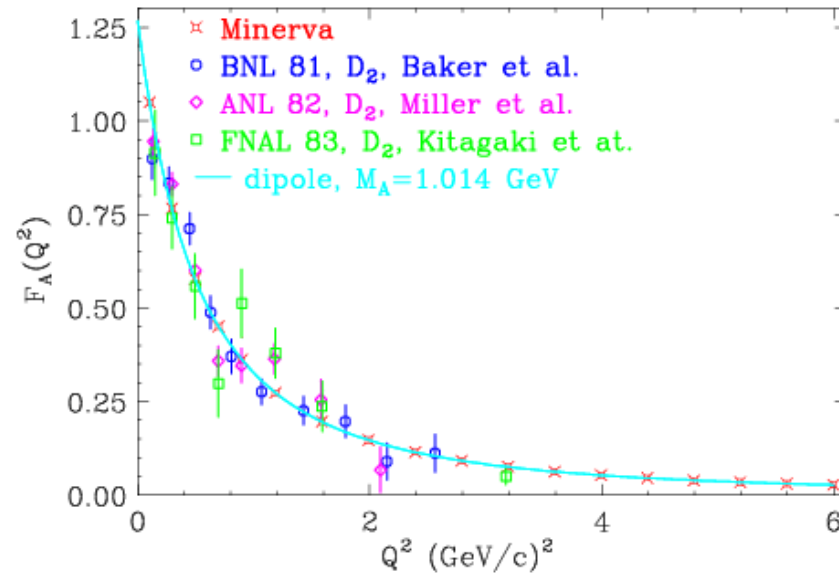
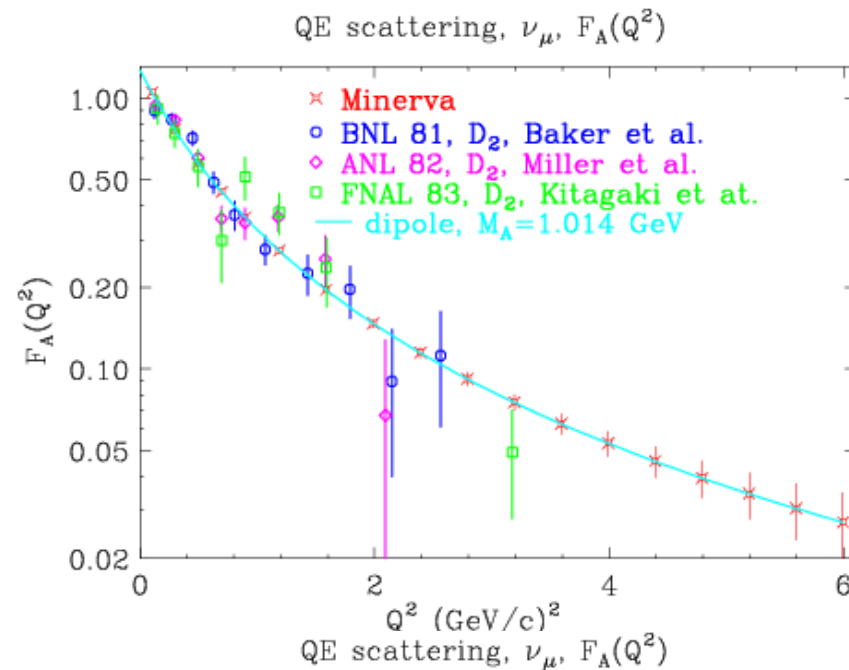
Extracting the axial form factor

- These plots show the contributions of the form factors to the cross section.
- This is $d(d\sigma/dq)/dff$ % change in the cross section vs % change in the form factors
- The form factor contribution neutrino is determined by setting the form factors = 0
- **The plots show that F_A is a major component of the cross section.**
- Also shows that the difference in G_E^P between the cross section data and polarization data will have no effect on the cross section.



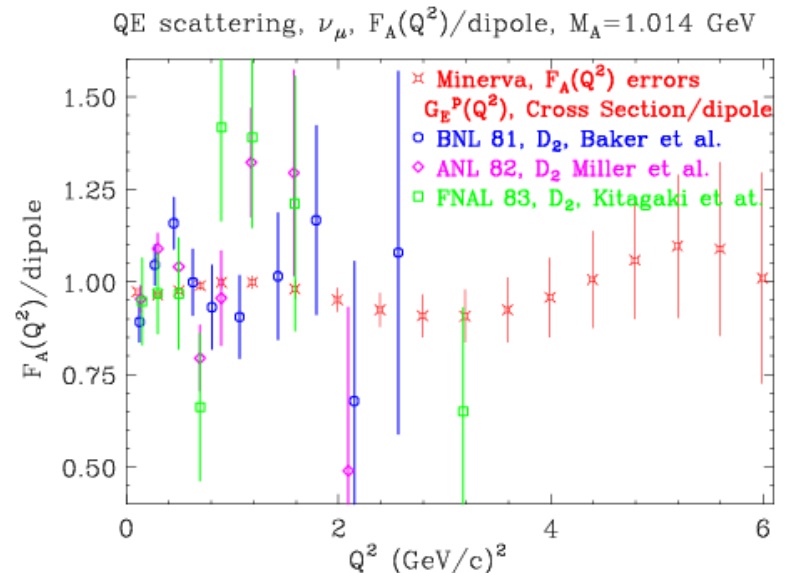
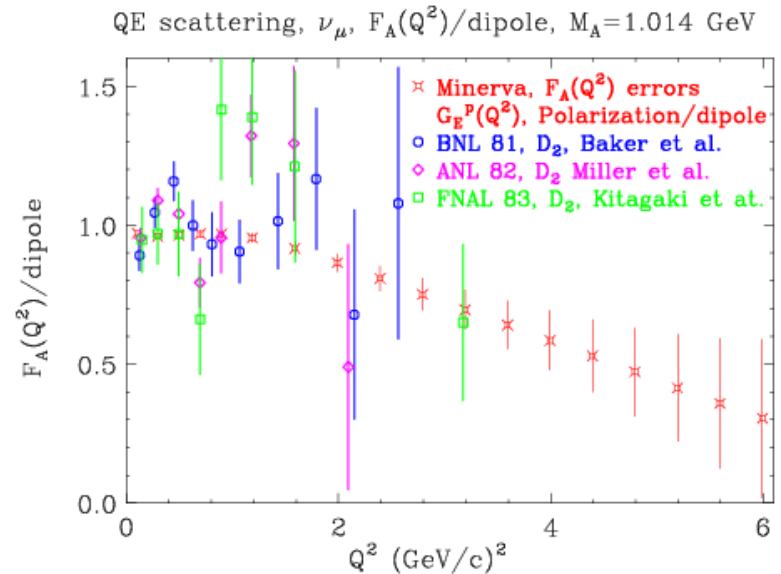
Measure $F_A(q^2)$

- We solve for F_A by writing the cross section as
- $a(q^2, E) F_A(q^2)^2 + b(q^2, E) F_A(q^2) + c(q^2, E)$
- if $(d\sigma/dq^2)(q^2)$ is the measured cross section we have:
- $a(q^2, E) F_A(q^2)^2 + b(q^2, E) F_A(q^2) + c(q^2, E) - (d\sigma/dq^2)(q^2) = 0$
- For a bin q_1^2 to q_2^2 we integrate this equation over the q^2 bin and the flux
- We bin center the quadratic term and linear term separately and we can pull $F_A(q^2)^2$ and $F_A(q^2)$ out of the integral. We can then solve for $F_A(q^2)$
- Shows calculated value of F_A for the previous experiments.
- **Show result of 4 year Minerva run**
- Efficiencies and Purity of sample is included.



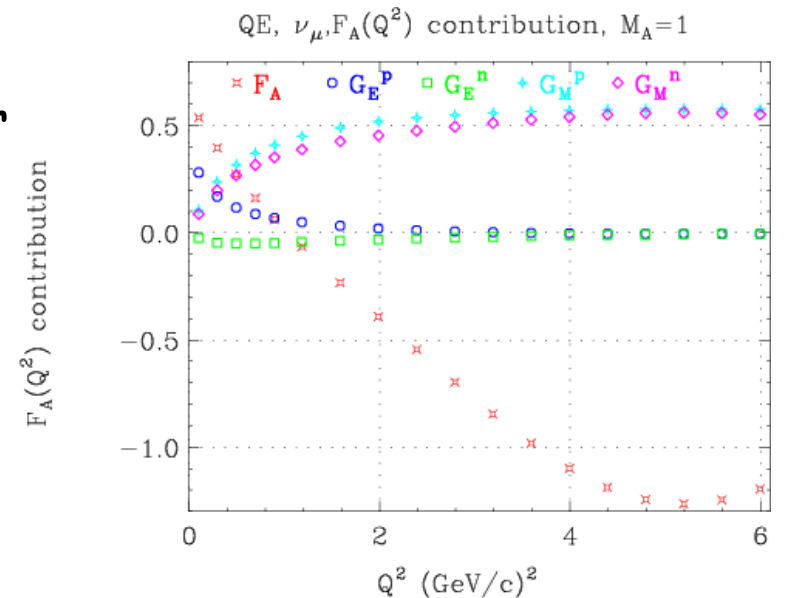
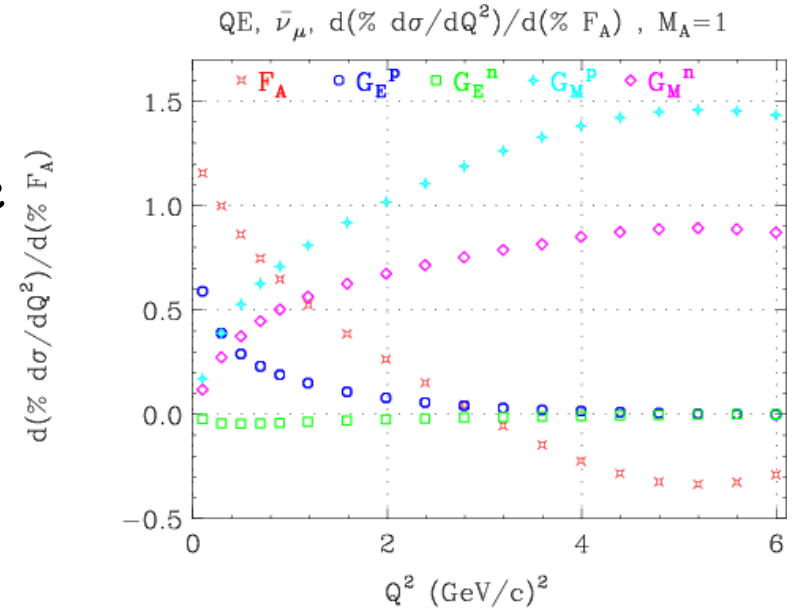
F_A /dipole

- For Minerva - show G_E^P for polarization/dipole, F_A errors, F_A data from other experiments.
- For Minerva - show G_E^P cross section/dipole, F_A errors.
- Including efficiencies and purities.
- Showing our extraction of F_A from the deuterium experiments.
- Shows that we can determine if F_A deviates from a dipole as much as G_E^P deviates from a dipole.
- However, our errors, nuclear corrections, flux etc., will get put into F_A .
- Is there a check on this?



Do we get new information from anti-neutrinos?

- $d(d\sigma/dq^2)/dff$ is the % change in the cross section vs % change in the form factors
- Shows the form factor contributions by setting $ff=0$
- At Q^2 above 2 GeV^2 the cross section become insensitive to F_A
- Therefore at high Q^2 , the cross section is determined by the electron scattering data and nuclear corrections.
- Anti-neutrino data serve as a check on F_A .



- Errors on F_A for antineutrinos
- The overall errors scale is arbitrary
- The errors on F_A become large at Q^2 around 3 GeV^2 when the derivative of the cross section wrt to F_A goes to 0
- Bottom plot shows the % reduction in the cross section if F_A is reduced by 10%
- At $Q^2 = 3 \text{ GeV}^2$ the cross section is independent of F_A

