Munich, ν 2002

NEUTRINO OSCILLATIONS
- GLOBAL ANALYSES -

Eligio LISI
INFN, Bari, Italy

OUTLINE:

- General features
- Recent progress
- A powerful χ^2 approach
  → application to solar ν
- Conclusions

Work done with: Gianluigi Fogli
Antonio Marrone
Daniele Montanino
Antonio Palazzo

GENERAL FEATURES

- Global analyses of ν oscillation searches aim to:
  - Assess the status of a specific hypothesis → χ^2 statistics
  - Select between two (or more) hypotheses → $\Delta\chi^2$ statistics

---

- Ingredients:
  - Experimental data
  - Theoretical calculations
  - Construction and analysis of a χ^2 function

1st ingredient : **Expt. ν data**

Impressive and decisive progress in the last few years – largely covered by experimental talks at this conference

In connection with global **analyses**, let me just add a general request to experimentalists :

> Please always give enough public information to make your data expectations **calculable** outside your collaboration

Sometimes, lack of published info may prevent full understanding and proper use of data $\pm$ errors in global analyses, outside the collaborations

2nd Ingredient: **Theo. calculations**

Significant progress in so-called **unified** calculations, namely, analyses where previously disjoined hypotheses (H_1 & H_2) become sub-cases of a more general model (H) under variations of some parameters

$$H_i = H(p_i)$$

H_1	H_2	UNIFICATION
$\vartheta_{ij} \leq \pi/4$	$\vartheta_{ij} \geq \pi/4$	use both octants, $\tan^2\theta_{ij}$
SOLAR ν VACUUM	SOLAR ν MSW	VAC → Quasivacuum → MSW
ATMOSPH. $\nu_\mu \to \nu_e$	ATMOSPH. $\nu_\mu \to \nu_\tau$	ATMOSPH. 3ν oscillations
ATM. ν OSC $\Delta m^2_{solar} = 0$	SOLAR ν OSC. $\Delta m^2_{atm} = \infty$	TWO-SCALE ANALYSES
PURELY ACTIVE ν OSCILL.	PURELY STERILE ν OSCILL.	ACTIVE ⊕ STERILE OSCILLATIONS
4ν (2+2)	4ν (3+1)	Generic 4ν spectra
STANDARD INTERACTIONS ...	NONSTANDARD INTERACTIONS ...	STD + NONSTD INTERACTIONS ...

Advantages of unified analyses :

- Deeper understanding of oscillation physics
- Homogeneous comparisou of hypotheses $H_{1,2}$: $\Delta\chi^2 = \chi^2(p_2) - \chi^2(p_1)$

Obvious disadvantages :

- Number of theory parameters and of analyzed data generally increases → Results of statistical analysis less easy to decipher

---

A few examples out of a large literature :

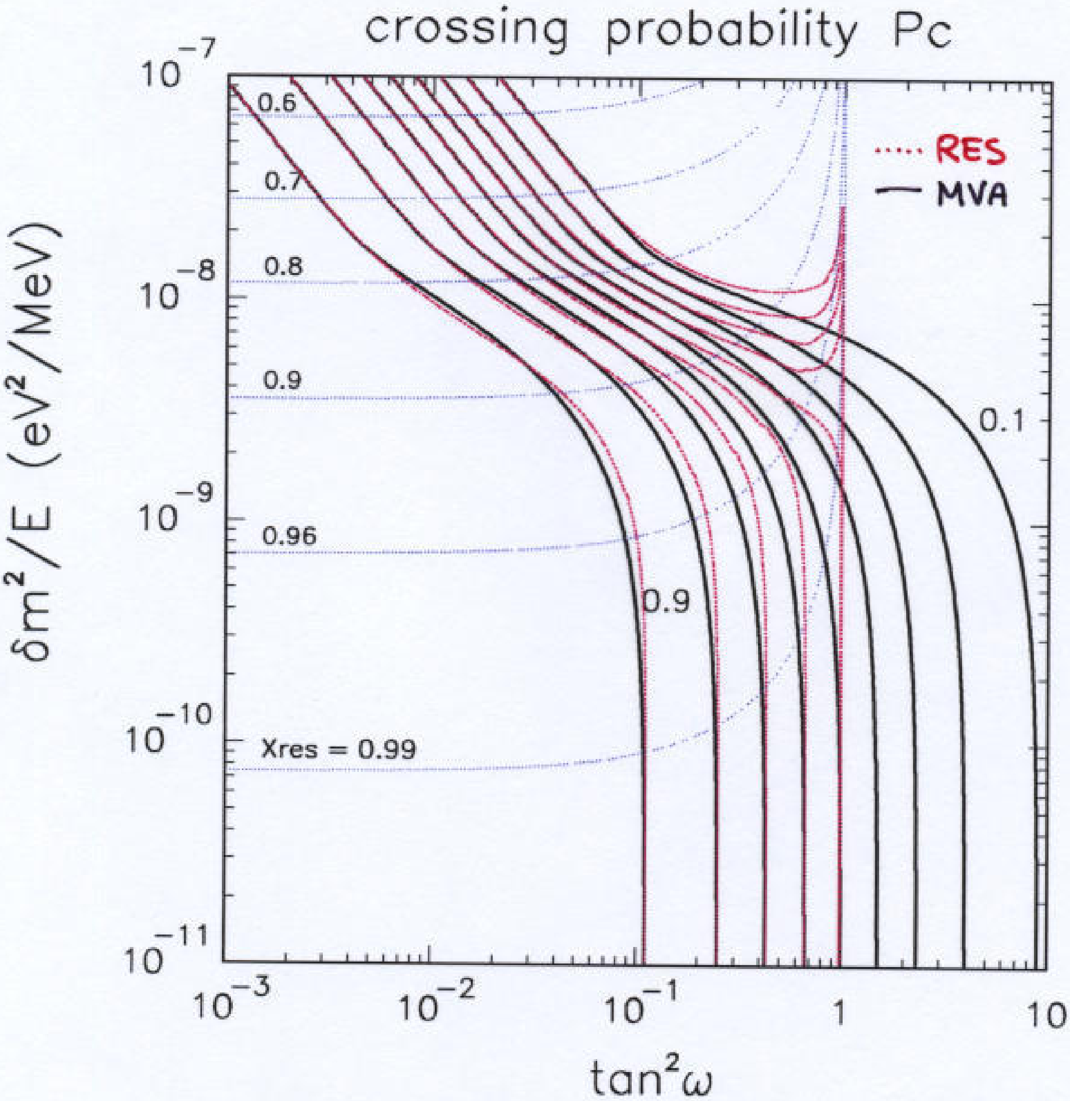
2ν : Quasivacuum solar ν osc.

3ν : Two mass scale analyses

4ν : Active + sterile ν mixing

2ν : Quasivacuum solar ν osc.

Better understanding of oscillation physics for $\delta m^2/E \sim 10^{-10} \div 10^{-8}\ eV^2/MeV$



→ Unified description of ν level crossing through the concept of "maximum violation of adiabaticity", replacing "resonance"

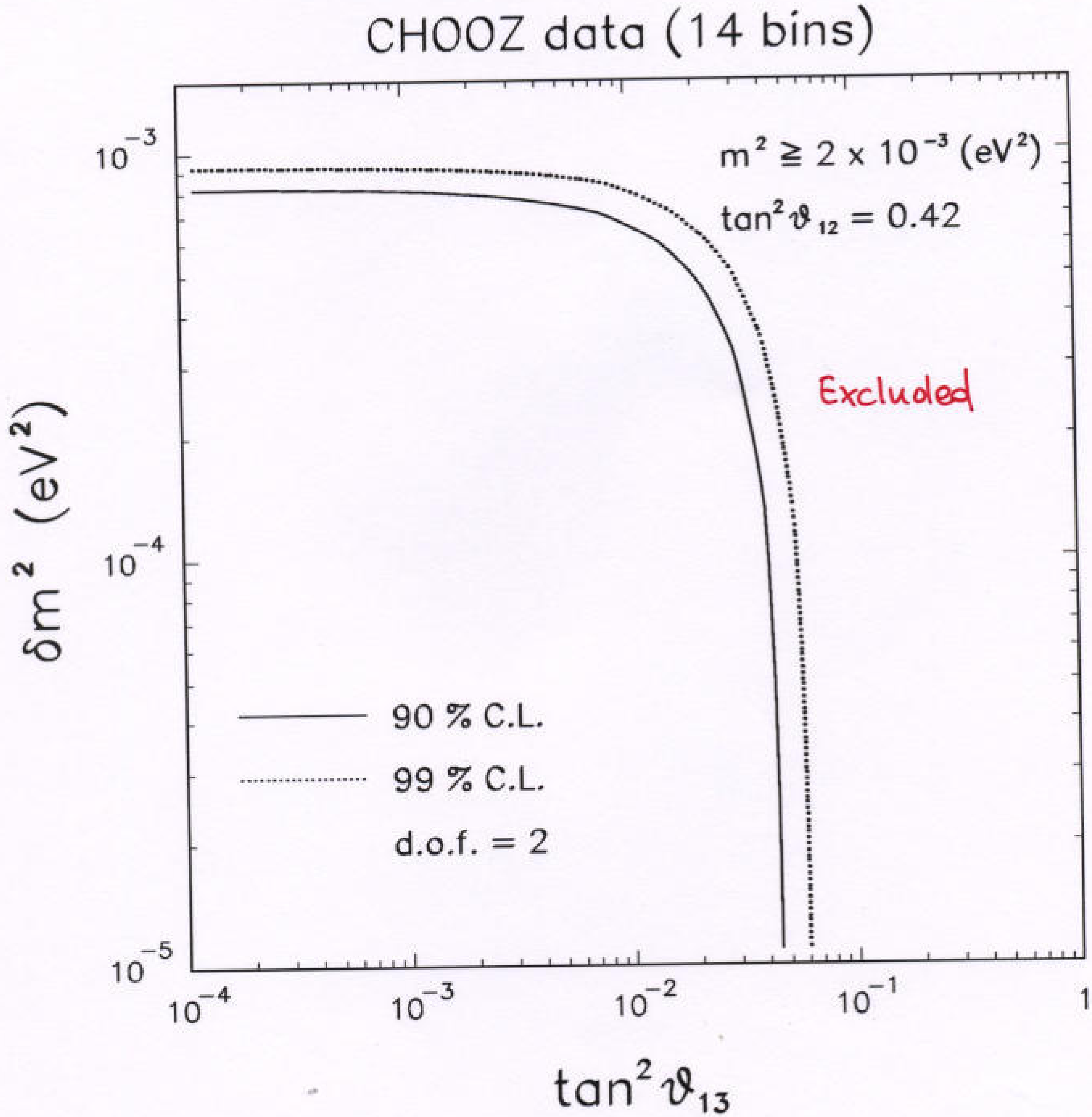
Friedland

E.L., Marrone, Montanino, Palazzo, Petcov

....

3ν: TWO MASS SCALE ANALYSIS OF CHOOZ

CHOOZ bound can be saturated either by increasing θ_{13} or δm^2_{solar} or both

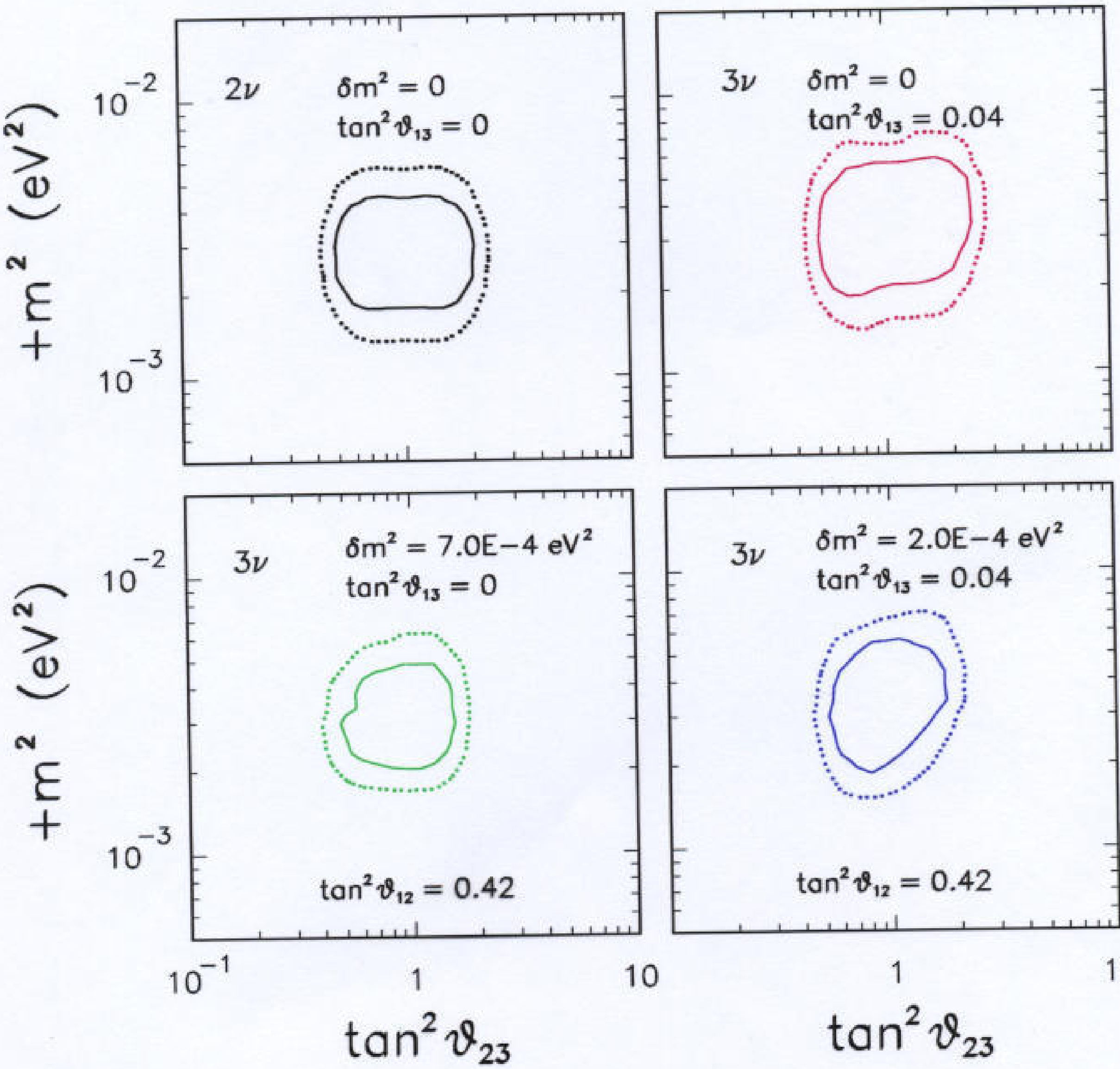


→ Relevant to assess proper 3ν upper bounds on δm^2_{solar} and θ_{13}

Bilenky, Nicolo', Petcov
Gonzalez-Garcia et al.
A. Marrone @ NOON 2001
....

3ν : TWO MASS SCALE ANALYSIS OF ATM. ν

Perturbations of atmospheric ν allowed regions for $\delta m^2_{sol} \neq 0$ and/or $\theta_{13} \neq 0$:



→ minimum value of Δm^2_{atm} stable (also for $-m^2$ case)
→ relevant for LBL physics potential

Gonzalez-Garcia & Maltoni ; Strumia; Peres & Smirnov; A. Marrone @ NOON 2001 ; ...

4ν : active + sterile ν mixing

Several results* clearly show the need to break up fits into separate data set contributions

E.g., a recent unified (pre-SNO_{NC}) 4ν global analysis shows that (Valle et al.)

GLOBAL RESULTS GIVE ...

3+1 (4ν) : BEST FIT
2+2 (4ν) : $\Delta\chi^2 = 3.7$ → acceptable
3+0 (3ν) : $\Delta\chi^2 = 19.8$ → formally rejected!

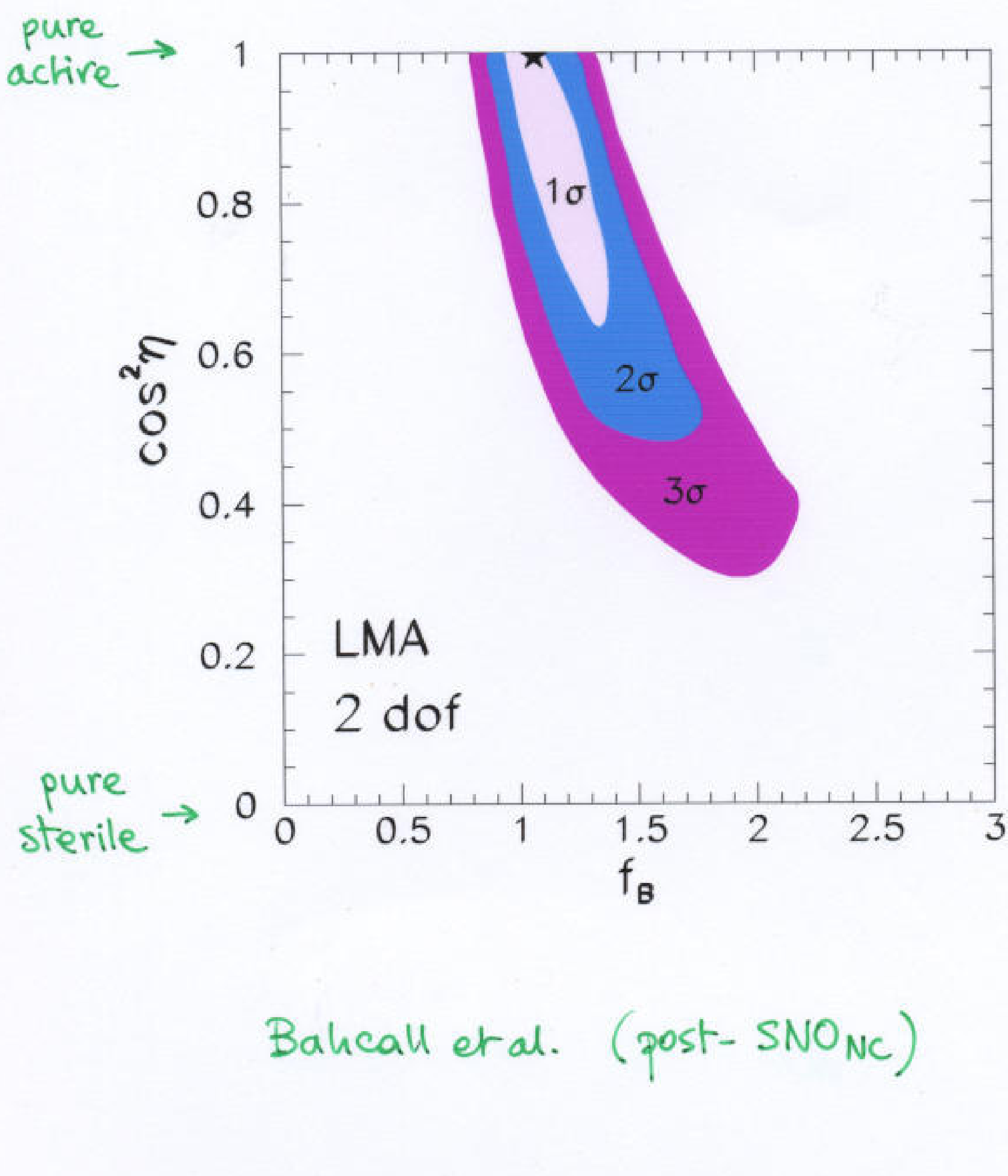
... BUT DATA BREAK UP REVEALS

3+1 : tension between acc/reac data
2+2 : " " solar/atm data
3+0 : " " LSND and all other data

Then one has more elements to accept/reject!

* Giunti et al; Gonzalez-Garcia, Peña-Garay, Bahcall; Yasuda; Barger & al.; Valle & al.; Peres & Smirnov; Fogli & al., ...

E.g. : tension between
sterile ν fraction
in solar ν oscillations ....



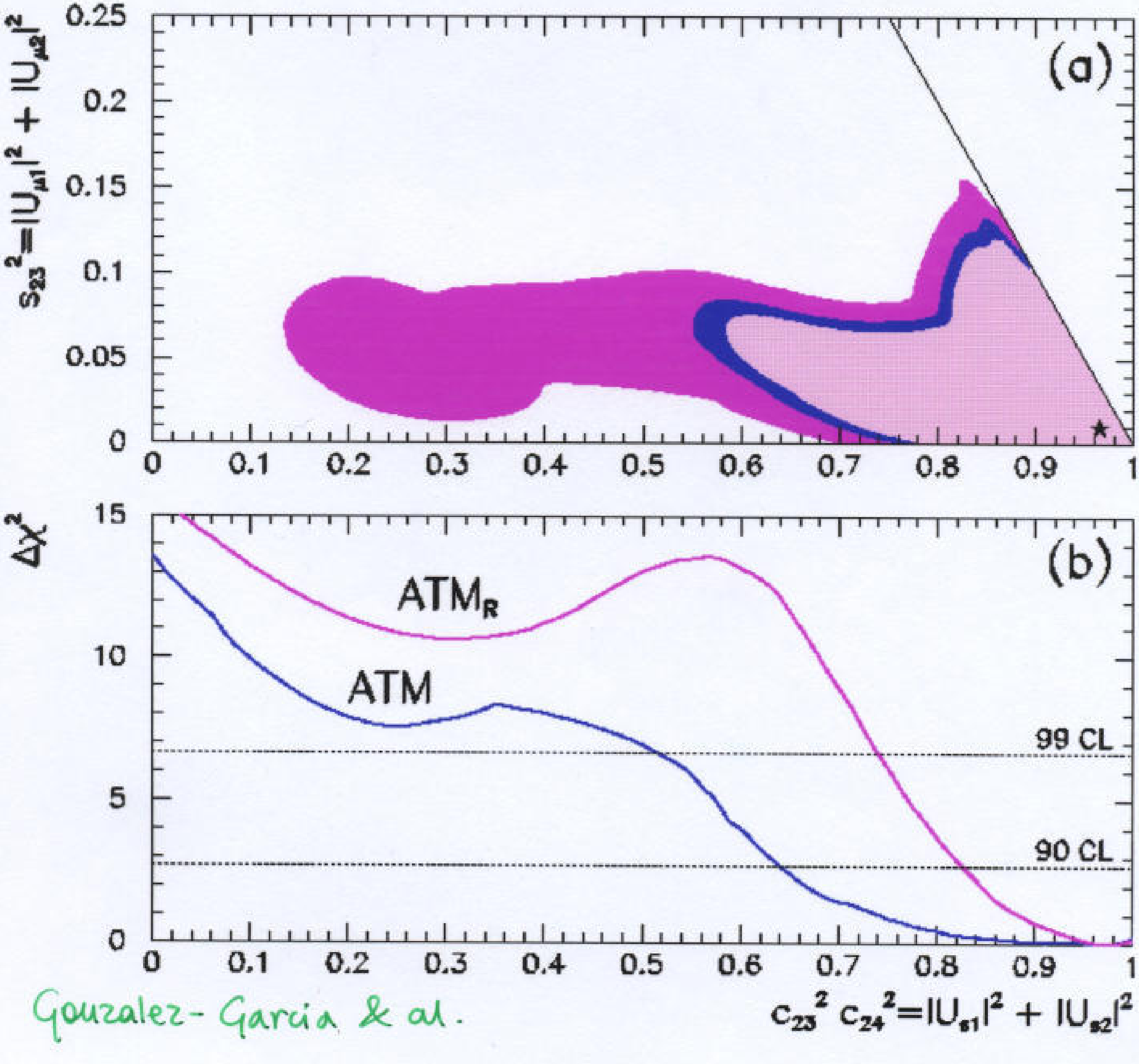
Bahcall et al. (post- SNO_{NC})

... and atmospheric ν oscillations.

ν_s is "repelled" by both data sets.

pure sterile ↓

pure active ↓



Gonzalez-Garcia & al.

In general, analysis of data subsets useful to understand global results

---

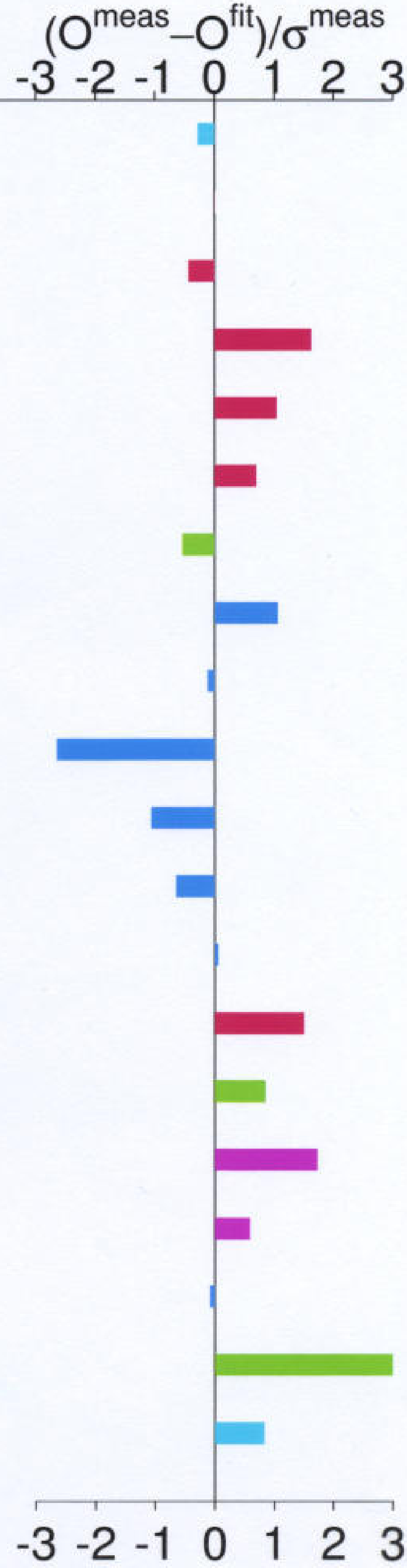
Maximum of information reached when individual components are resolved

---

→ Situation already realized in other areas of physics (e.g., electroweak precision fits)

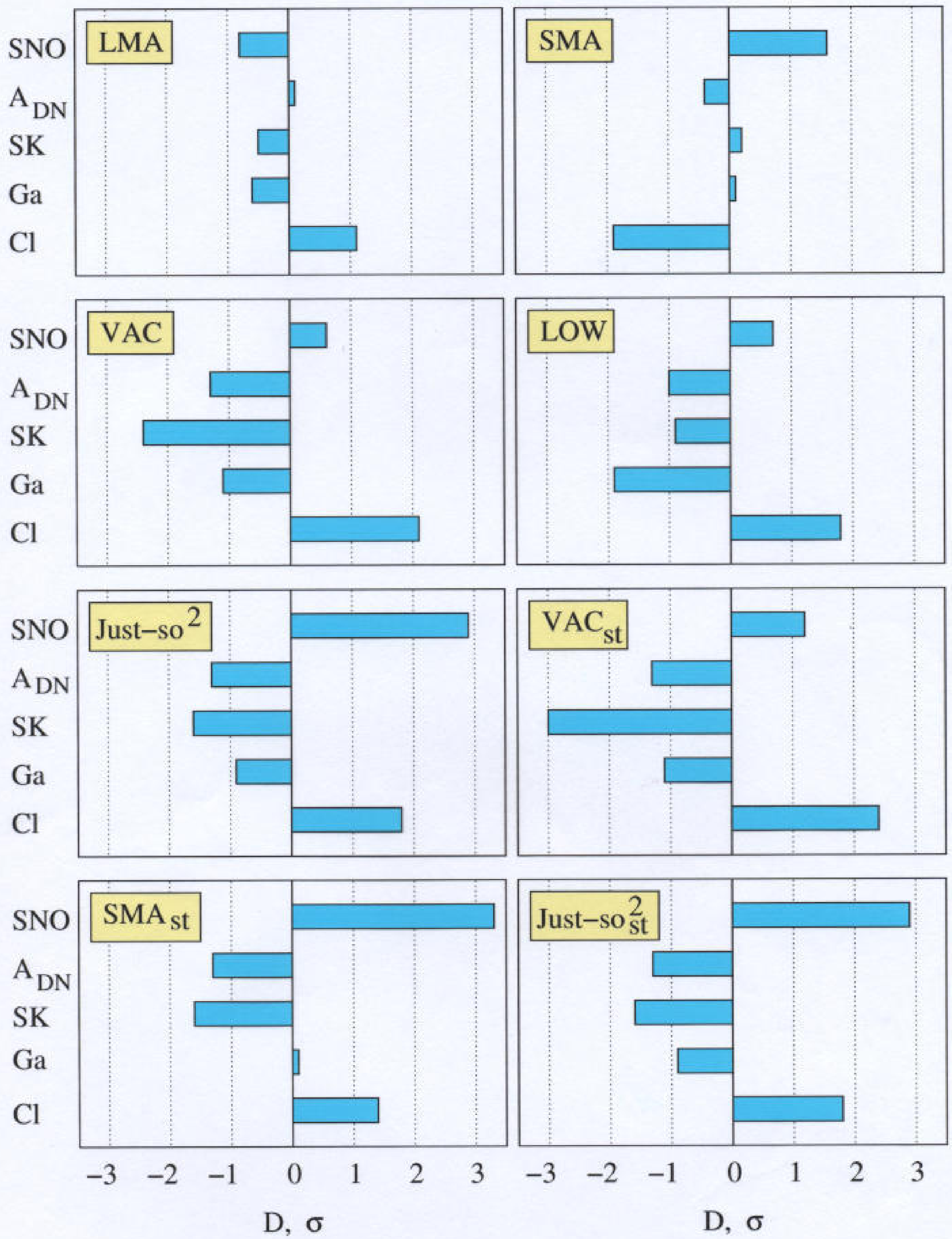
Winter 2002

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-.27
m_Z [GeV]	91.1875 ± 0.0021	.01
Γ_Z [GeV]	2.4952 ± 0.0023	-.42
σ^0_{had} [nb]	41.540 ± 0.037	1.63
R_l	20.767 ± 0.025	1.05
$A^{0,l}_{fb}$	0.01714 ± 0.00095	.70
$A_l(P_\tau)$	0.1465 ± 0.0033	-.53
R_b	0.21646 ± 0.00065	1.06
R_c	0.1719 ± 0.0031	-.11
$A^{0,b}_{fb}$	0.0994 ± 0.0017	-2.64
$A^{0,c}_{fb}$	0.0707 ± 0.0034	-1.05
A_b	0.922 ± 0.020	-.64
A_c	0.670 ± 0.026	.06
A_l(SLD)	0.1513 ± 0.0021	1.50
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	.86
m_W [GeV]	80.451 ± 0.033	1.73
Γ_W [GeV]	2.134 ± 0.069	.59
m_t [GeV]	174.3 ± 5.1	-.08
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	3.00
$Q_W(Cs)$	-72.39 ± 0.59	.84



LEP ElectroWeak working group

... and a first application to solar ν
(Krastev & Smirnov)



Also : A. Yu. Smirnov @ this conference

However, previous examples of "pull" analyses do not represent true χ^2 decompositions, since:

$$\chi^2 \neq \sum (\text{pull})^2 ,$$

due to correlations among uncertainties.
(Issue of correlations is becomming more and more crucial for current and prospective analyses).

Q. Is it possible to get a real χ^2 decomposition ("diagonalization") of the kind

$$\chi^2_{\text{pull}} \equiv \sum (\text{pull})^2 \quad ?$$

A. YES.

This is a <u>general</u> result, that we apply to a novel analysis of solar ν data as a first application. ← Hot topic!
(we plan to do the same for atm ν).

Detailed proofs and results will appear in a forthcoming preprint:

" Getting the most from the statistical analysis of solar neutrino oscillations"

by G.L. Fogli, E.L., A. Marrone, D. Montanino, A. Palazzo

→ Some results

BASICS

Global statistical analyses need the following ingredients:

- A set of N experimental observables

  $\{R_n^{exp}\}$ $n = 1, \ldots, N$

- A set of N theoretical predictions

  $\{R_n^{theo}\}$ $n = 1, \ldots, N$

- A set of N errors, uncorrelated among different observables (e.g., statistical)

  $\{u_n\}$ $n = 1, \ldots, N$

- A set of $\leq N \cdot K$ correlated errors

  $\{c_n^k\}$ $\begin{matrix} k = 1, \ldots, K \\ n = 1, \ldots, N \end{matrix}$

  induced by K systematic error sources on the N observables.

  Tipically: $c_n^k = \dfrac{\partial \ln R_n^{theo}}{\partial \ln X_k} \Delta \ln X_k$

  X_k = syst. error source

TWO SEEMINGLY DIFFERENT χ^2's

"**Covariance**" approach:

- Build $\sigma^2_{nm} = \delta_{nm} u_n u_m + \sum_k c_n^k c_m^k$
- Calculate $\chi^2_{covar} = \sum_{n,m} (R_n^{expt} - R_n^{theo}) \sigma^{-2}_{nm} (R_m^{exp} - R_m^{theo})$

"**Pull**" approach:

- Shift: $R_n^{theo} \to R_n^{theo} + \sum_k \xi_k c_n^k$

  with $\langle \xi_k \rangle = 0$, $\langle \xi_k^2 \rangle = 1$
- Add penalty function and minimize:

$$\chi^2_{pull} = \min_{\{\xi_k\}} \sum_n \left(\frac{R_n^{exp} - R_n^{theo} - \sum_k \xi_k c_n^k}{u_n} \right)^2 + \sum_k \xi_k^2$$

Is there a relation between χ^2_{covar} and χ^2_{pull}?

It turns out that...

$$\boxed{\chi^2_{pull} \equiv \chi^2_{covar}}$$

This is a powerful and useful statistical result. Proof given in forthcoming paper.

We have recently learned that this theorem has been previously and independently found in the context of parton distribution fitting, where it is now routinely used in global analyses; see the electronic proceedings of the March 2002 workshop on "Advanced Statistical Techniques in Particle Physics" (Durham, UK), talk by R. Thorne.

Although $\chi^2_{covar} \equiv \chi^2_{pull}$, the use of χ^2_{pull} provides many advantages, e.g.:

- Minimization over ξ_k's in χ^2_{pull} is analytical, and requires only $K \times K$ matrix inversion – to be contrasted with $N \times N$ covariance inversion, $N \gg K$.

- At minimum ($\xi_k \equiv \bar{\xi}_k$), χ^2_{pull} is "diagonalized":

$$\chi^2_{pull} = \sum_{n=1}^{N} \left(\frac{R_n^{exp} - R_n^{theo} - \sum_k \bar{\xi}_k c_n^k}{u_n} \right)^2 + \sum_{k=1}^{K} \bar{\xi}_k^2$$

$$= \sum_{n=1}^{N} (\text{pull})^2 + \sum_{k=1}^{K} (\text{pull})^2$$

pulls of observables

pulls of systematics

→ GET ULTIMATE χ^2 DECOMPOSITION, EVEN IN PRESENCE OF CORRELATED ERRORS

→ We have completely redesigned our solar ν statistical analysis in terms of pulls

APPLICATION TO SOLAR ν

N = 81 observables included:

1	Cl absolute rate
1	Ga absolute rate (SAGE+GALLEX/GNO)
1	Ga W-S difference (GALLEX/GNO)*
44	SK energy-nadir absolute rates
34	SNO day-night energy spectrum rates
81	total

Associate uncorrelated errors u_n: statistical only, except for Cl and Ga where

$$u_n^2 = (u_n^{stat})^2 + (u_n^{cross\text{-}sec})^2 + (u_n^{syst})^2$$

* included for the first time

$K = 31$ sources of systematic correlated uncertainties included:

S_{11}
S_{33}
$S_{1,14}$
$S_{1,7}$
C_{Be}
* S_{hep}
Luminosity
Z/X
Age
Opacity
Diffusion

← 12 SSM systematics X_k, propagated to SSM fluxes Φ_i as:

$$\Phi_i \to \Phi_i \left(1 + \sum_{k=1}^{12} \xi_k \alpha_{ik} \Delta \ln X_k\right)$$

SSM log-deriv. from Bahcall & al

** ^8B ν shape ← ^8B ν spectrum shape uncert.

SK scale
SK resolution
SK offset
+8 SK E-bin syst.

← 11 SK sources of systematics, correctly propagated to all 44 bins

---

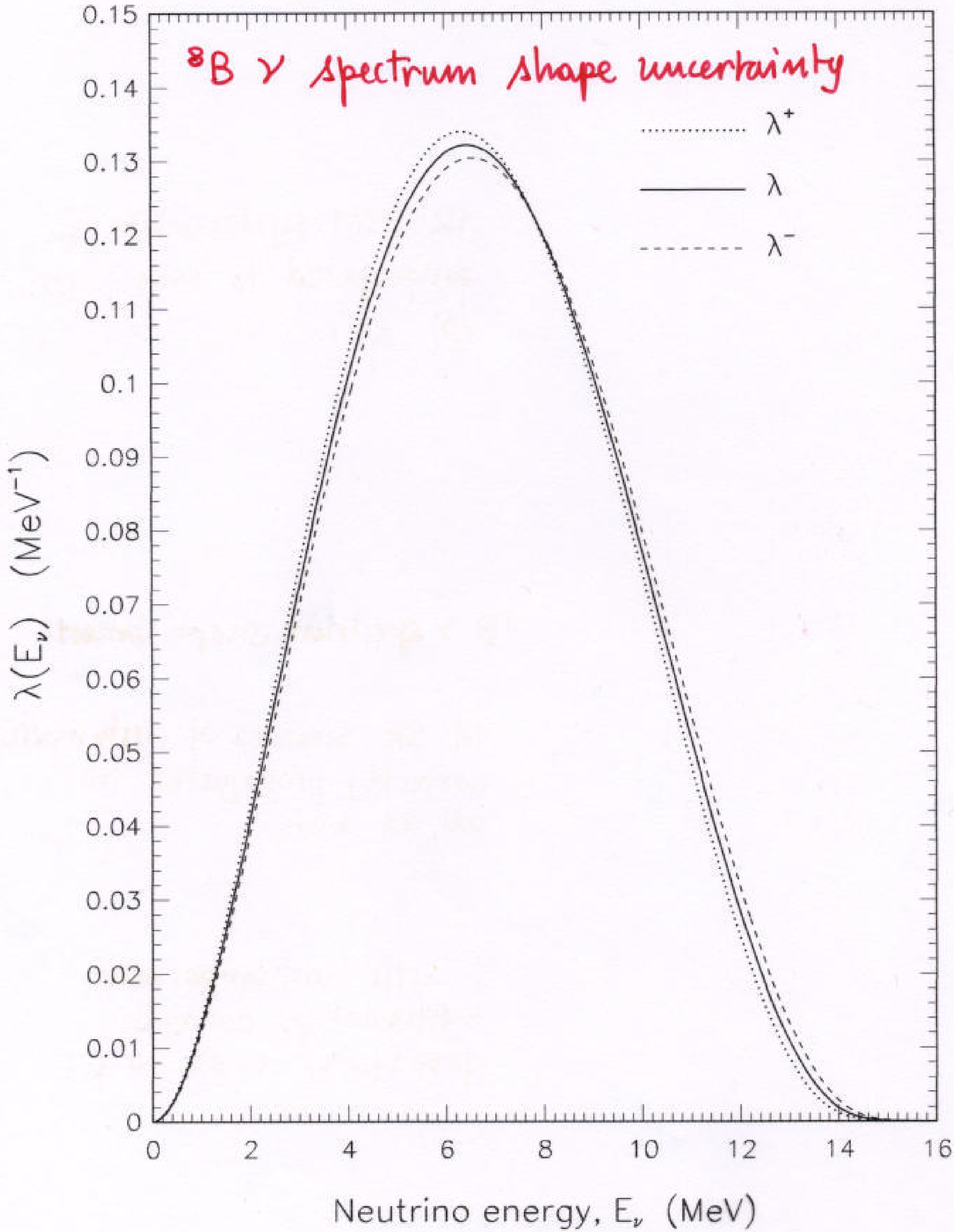
SNO scale
SNO resolution
SNO vertex accur.
SNO n capture
SNO n backg.
SNO LE backg.
SNO cross sect.

← 7 SNO most important systematics, correctly propagated to all 34 bins

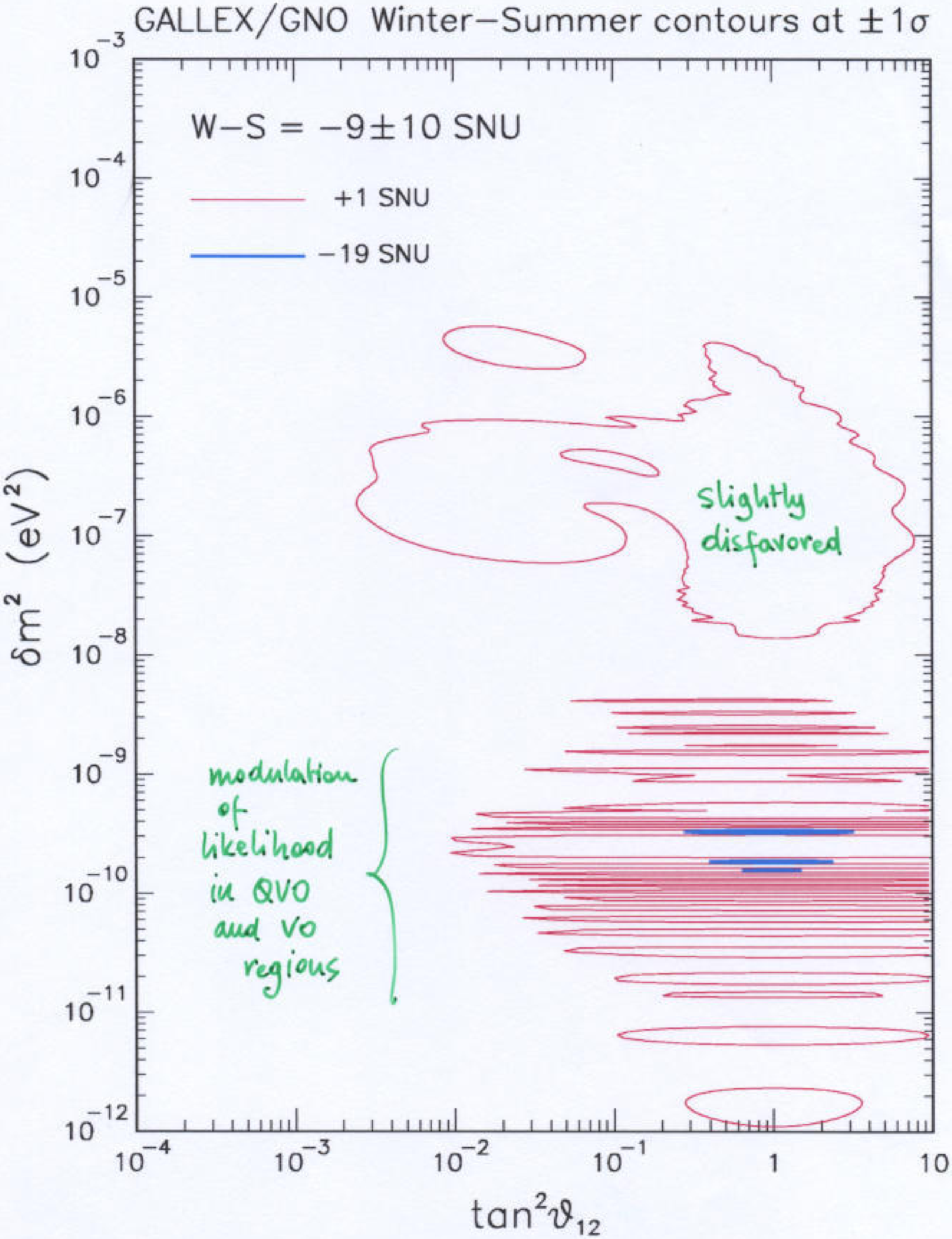
---

* $\Delta \ln S_{hep} = 30\%$ assumed

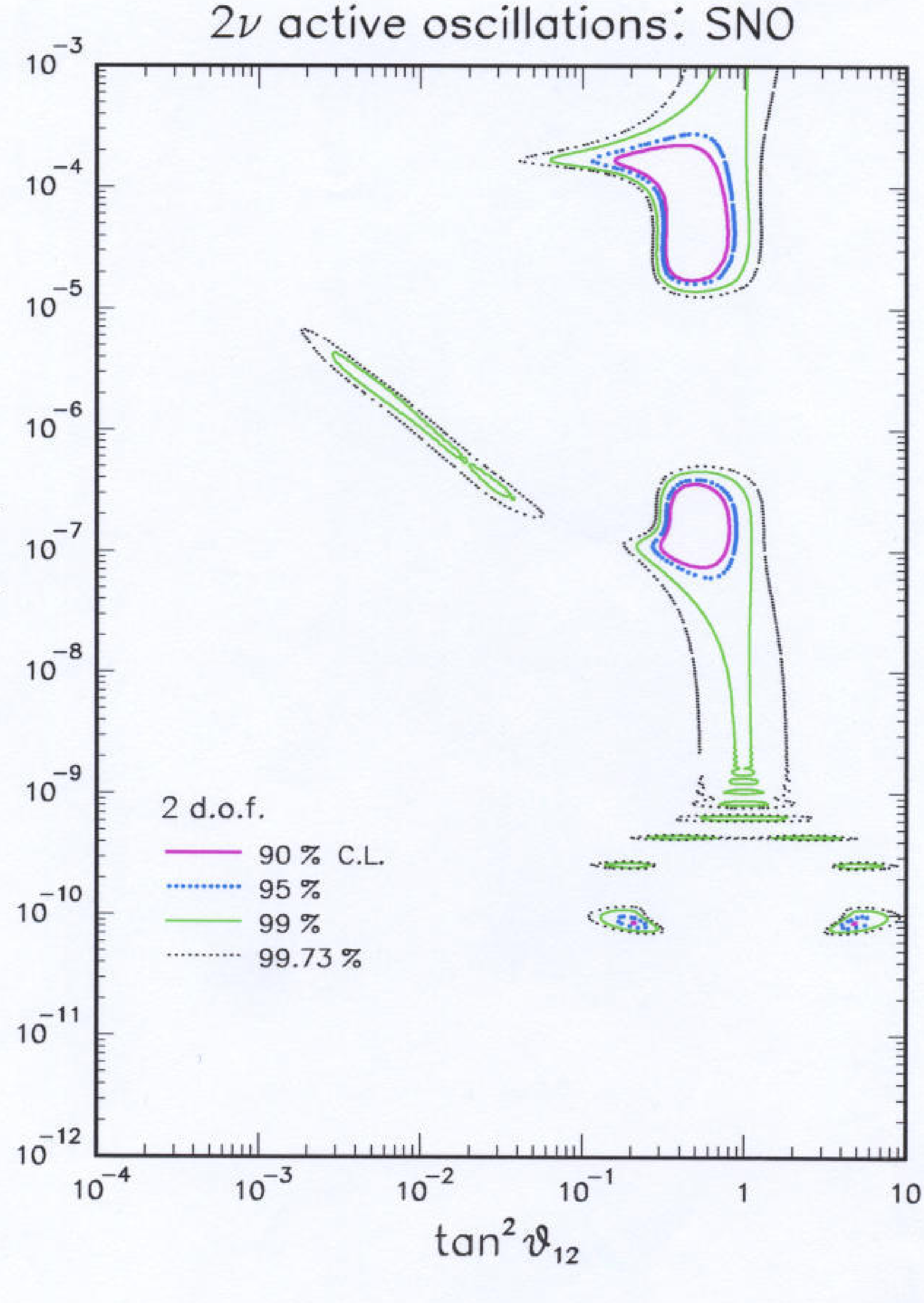
** Propagated for the first time also to Ga, Cl, and SNO spectrum ($\equiv$ all observables)



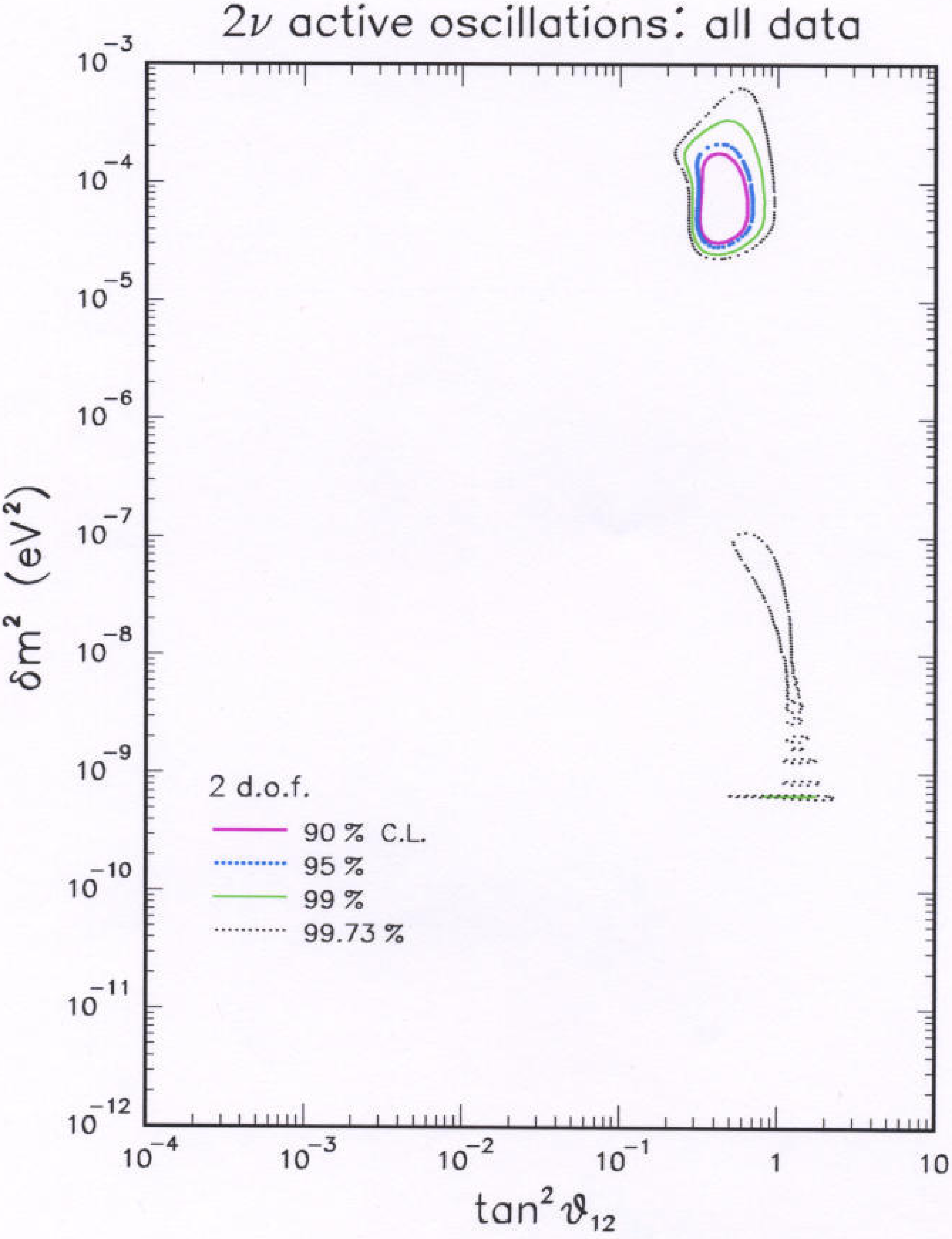
Correlated in SK <u>and</u> SNO



$(W\text{-}S)_{Ga}$ mainly generated by low-energy ν
$\rightarrow$ can be compatible with $(W\text{-}S)_{SK} \sim 0$



2ν active oscillations: SNO
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
δm^2 (eV^2)
2 d.o.f.
90 % C.L.
95 %
99 %
99.73 %
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
$\tan^2\vartheta_{12}$



Global results

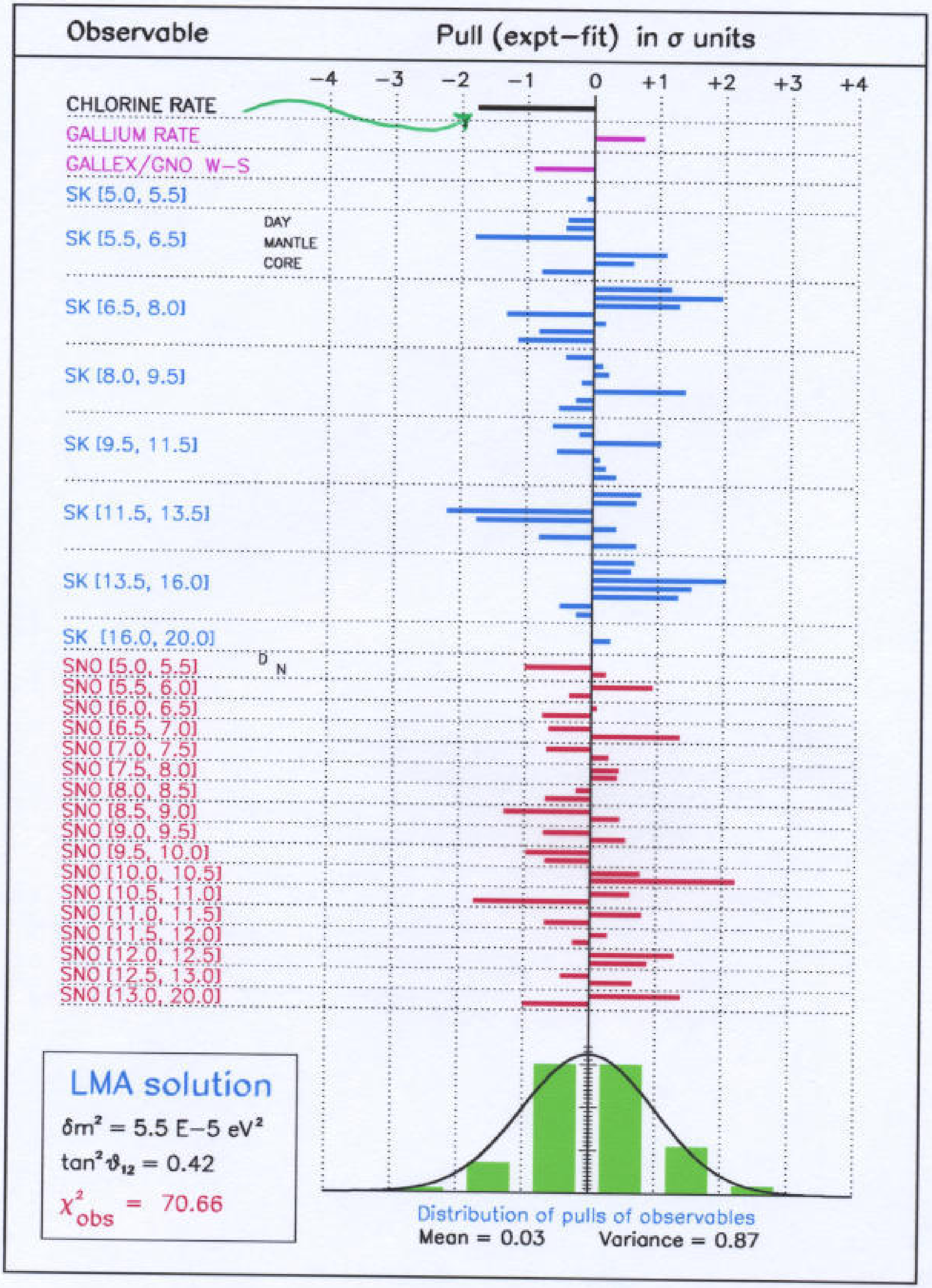
- What do we learn from obs. + sys. pull decomposition, $\chi^2_{pull} = \chi^2_{obs} + \chi^2_{sys}$?

	Δm^2 (eV^2)	$\tan^2\theta_{12}$	χ^2_{obs}	χ^2_{sys}	χ^2_{pull}	$\Delta\chi^2_{pull}$
LMA	5.5×10^{-5}	0.42	70.7	2.1	72.8	—
LOW	6.5×10^{-7}	0.75	77.9	4.8	82.7	+9.9
QVO	6.5×10^{-10}	1.3	75.0	6.0	81.0	+8.2
SMA	5.8×10^{-6}	1.1×10^{-3}	81.5	13.7	95.2	+22.4

- LMA solution minimizes both χ^2_{obs} and χ^2_{sys}. In particular, χ^2_{sys} is very small → no evidence for miscalibrated correlated systematics in LMA (either SSM or experimental)

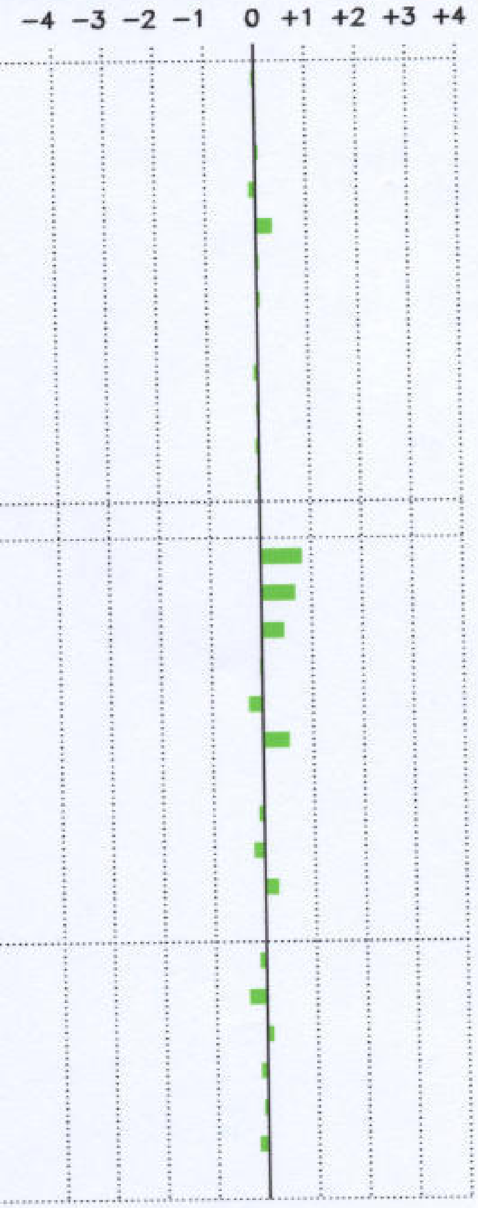
- All other solutions (LOW, QVO, SMA) give "evidence" for some "miscalibration" either in the SSM or in SK / SNO

- More info from individual pulls →



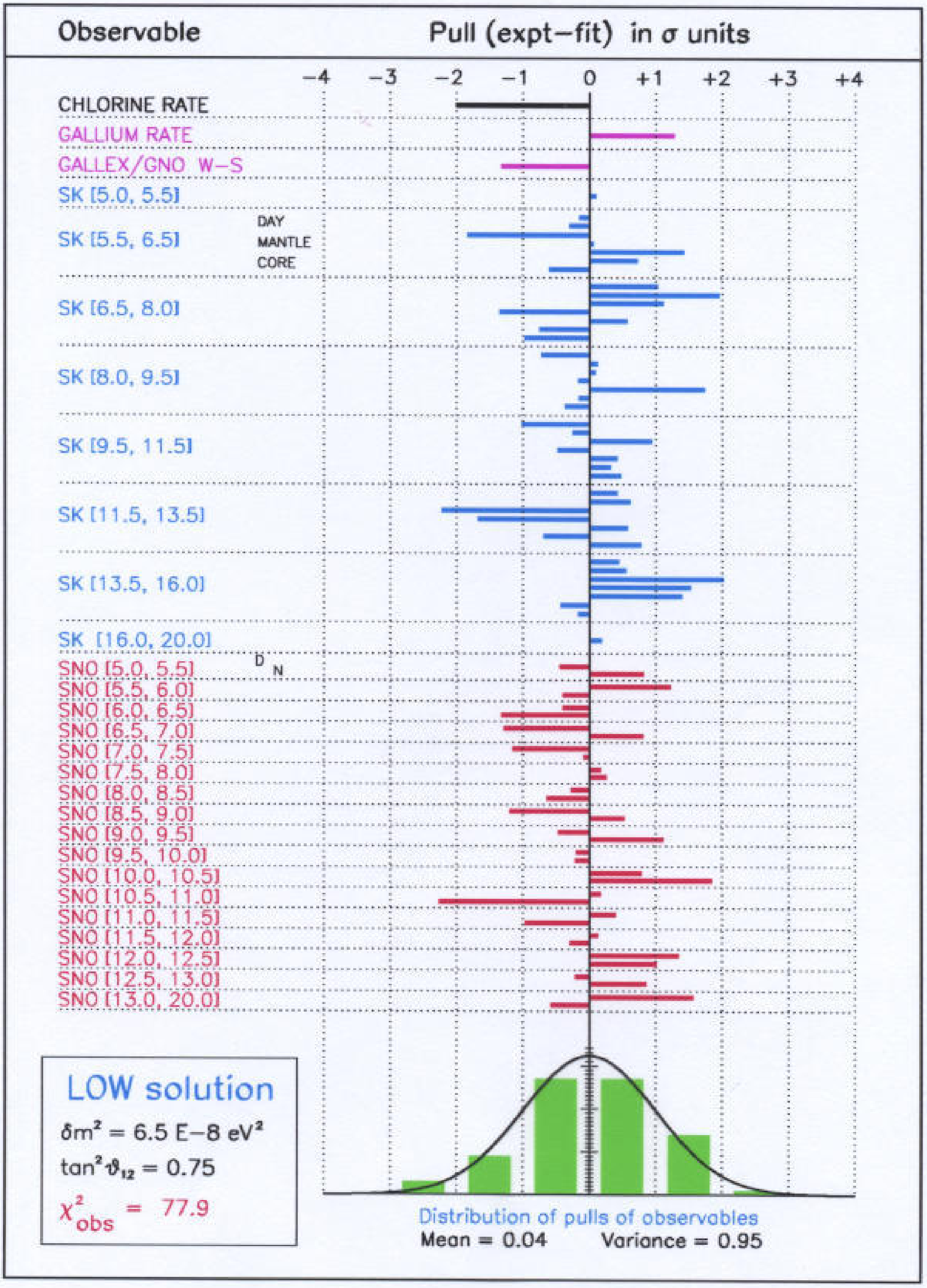
- No anomalously large pulls of observables
- Pull distribution ~OK but variance a bit small, mainly due to SNO

Systematics	Pulls (σ) for LMA solution
S11	−0.05
S33	−0.02
S34	+0.06
S1,14	−0.12
S17	+0.33
Luminosity	+0.05
Z/X	+0.06
Age	+0.01
Opacity	−0.07
Diffusion	−0.04
CBe	−0.06
Shep	−0.03
8B shape	+0.03
SK scale	+0.81
SK resol.	+0.68
SK offset	+0.44
SK [5.0, 5.5]	−0.04
SK [5.5, 6.5]	−0.26
SK [6.5, 8.0]	+0.52
SK [8.0, 9.5]	+0
SK [9.5, 11.5]	−0.1
SK [11.5, 13.5]	−0.2
SK [13.5, 16.0]	+0.26
SK [16.0, 20.0]	+0.01
SNO scale	−0.12
SNO resol.	−0.34
SNO vertex	+0.12
SNO n capture	−0.12
SNO n bkgd	−0.07
SNO LE bkgd	−0.17
SNO cross sec.	+0.03



$\chi^2_{sys} = 2.11$

- No indications for offsets of correlated systematics, except perhaps some SK ones (fit likes more SK events, especially at high E)
- Very close to "ideal statistical solution" with zero offsets, $\xi_k \equiv 0$
- Note exceptional agreement with SSM

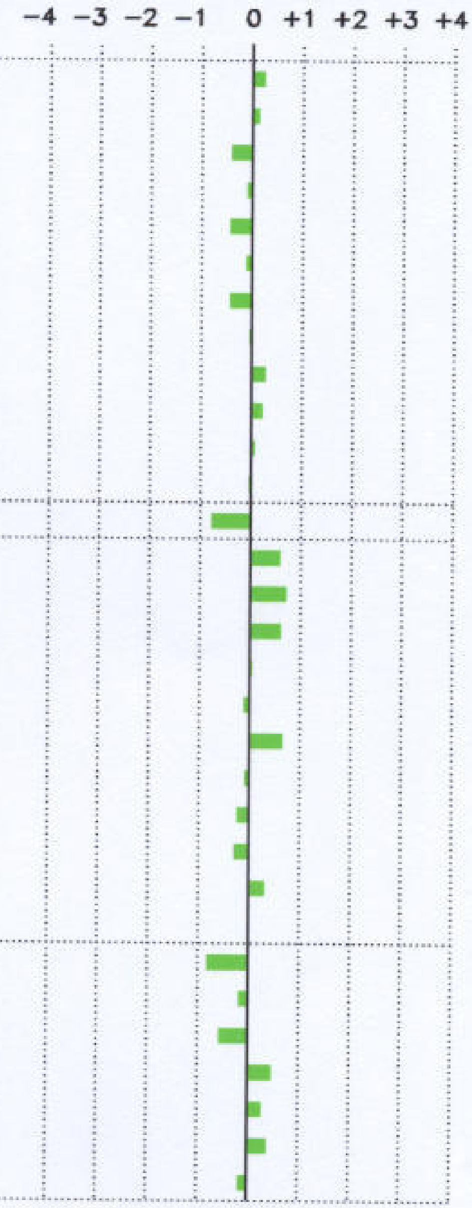


Pull distribution of observables ~OK

but....

Systematics Pulls (σ) for LOW solution

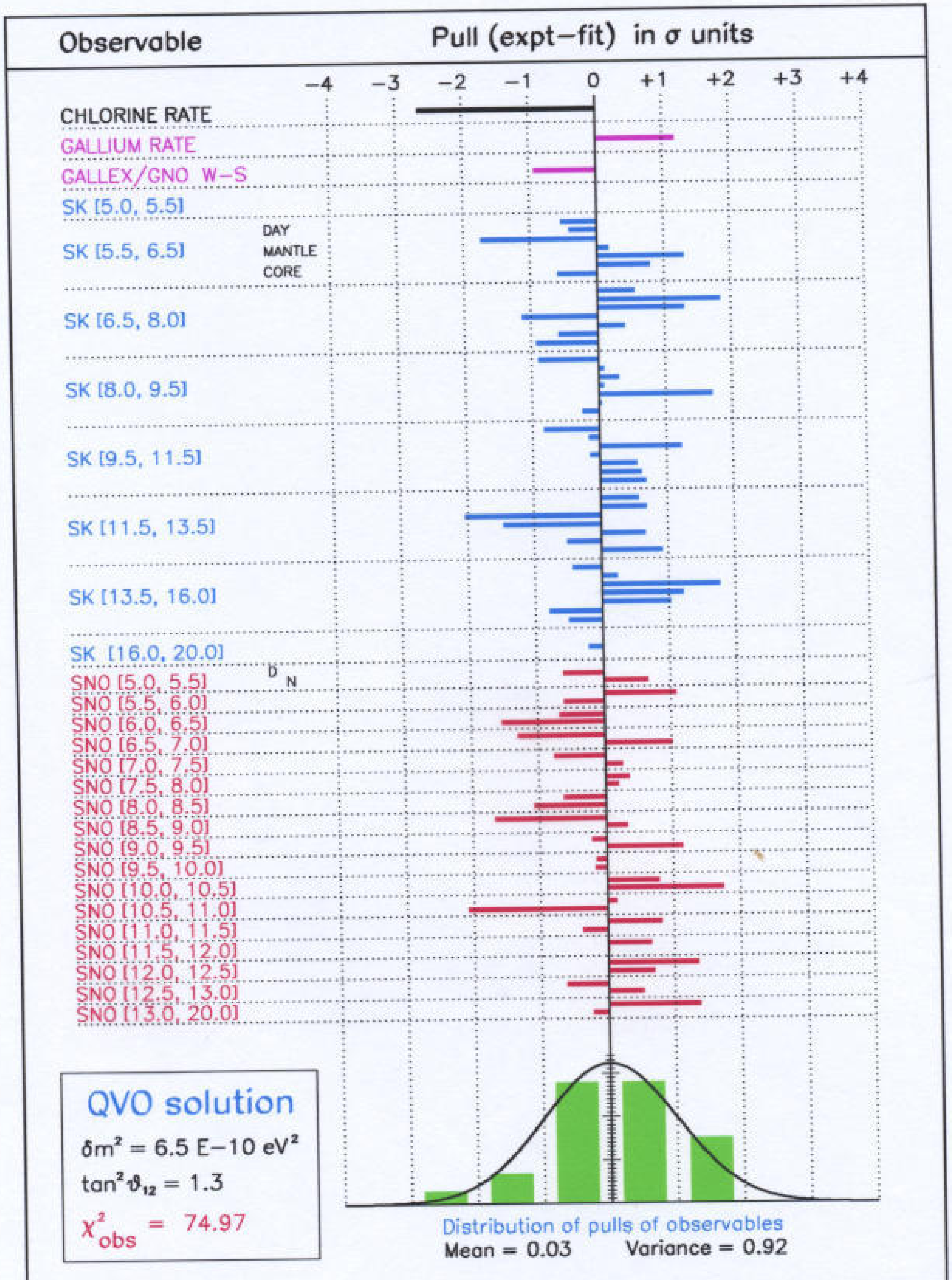
Systematics	Pulls (σ)
S11	+0.24
S33	+0.13
S34	−0.41
S1,14	−0.1
S17	−0.44
Luminosity	−0.11
Z/X	−0.44
Age	−0.03
Opacity	+0.28
Diffusion	+0.22
CBe	+0.08
Shep	−0.03
8B shape	−0.77
SK scale	+0.59
SK resol.	+0.72
SK offset	+0.62
SK [5.0, 5.5]	+0.05
SK [5.5, 6.5]	−0.11
SK [6.5, 8.0]	+0.67
SK [8.0, 9.5]	−0.08
SK [9.5, 11.5]	−0.22
SK [11.5, 13.5]	−0.29
SK [13.5, 16.0]	+0.31
SK [16.0, 20.0]	+0.01
SNO scale	−0.81
SNO resol.	−0.18
SNO vertex	−0.56
SNO n capture	+0.47
SNO n bkgd	+0.28
SNO LE bkgd	+0.39
SNO cross sec.	−0.17



$\chi^2_{sys} = 4.81$

... more pronounced preference for nonzero offsets of systematics.

"Recalibration" of systematics in opposite direction would enhance the LOW likelihood



Pull distribution ~ok but one large ($> 2.5\sigma$) outlier = Chlorine.

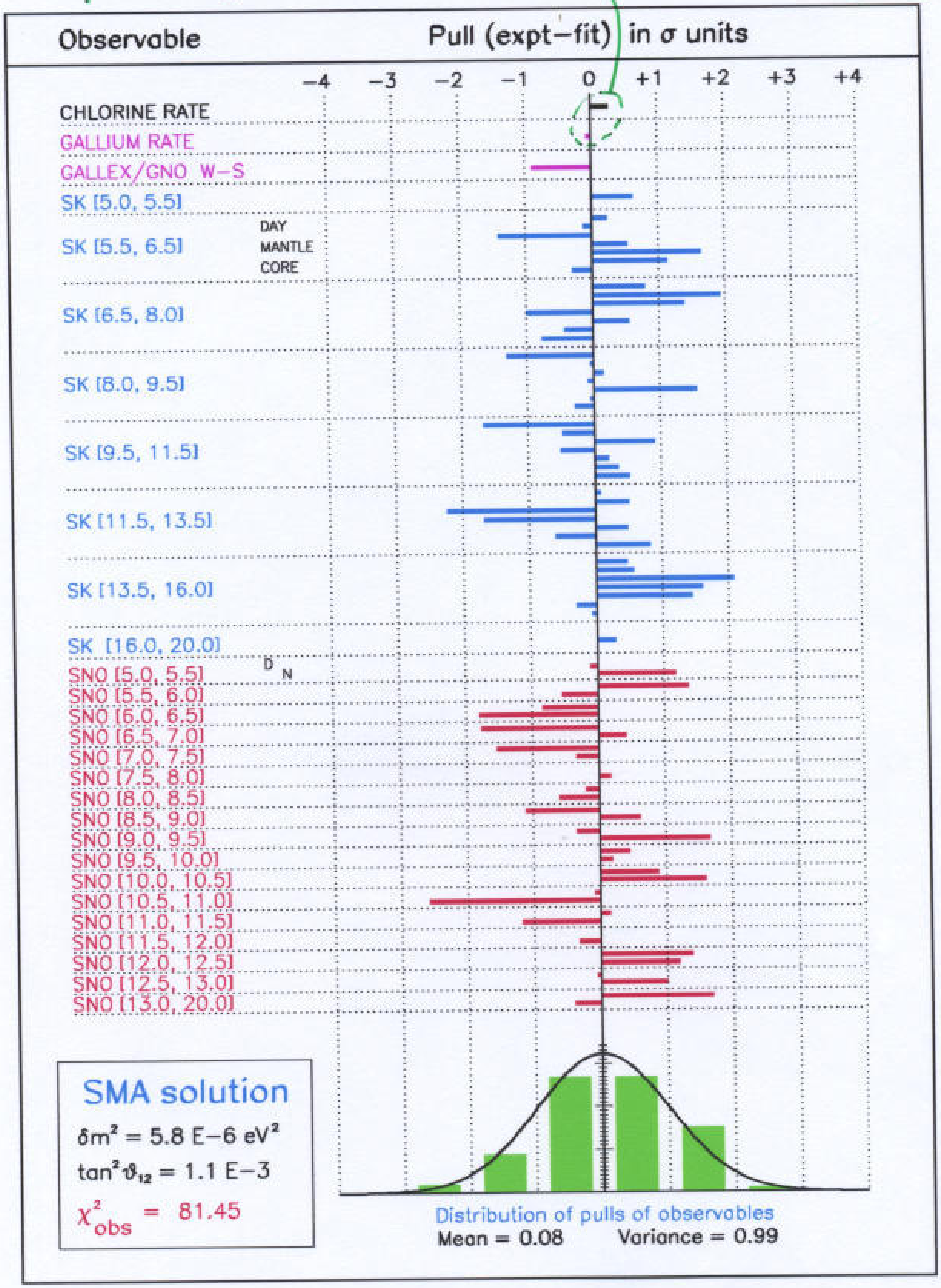
Moreover...

Systematics	Pulls (σ) for QVO solution
S11	+0.35
S33	+0.22
S34	−0.67
S1,14	−0.18
S17	−0.54
Luminosity	−0.19
Z/X	−0.66
Age	−0.05
Opacity	+0.41
Diffusion	+0.34
CBe	+0.1
Shep	−0.1
8B shape	−0.92
SK scale	+0.47
SK resol.	+0.05
SK offset	+0.74
SK [5.0, 5.5]	−0.01
SK [5.5, 6.5]	−0.27
SK [6.5, 8.0]	+0.52
SK [8.0, 9.5]	−0.02
SK [9.5, 11.5]	+0.17
SK [11.5, 13.5]	−0.13
SK [13.5, 16.0]	+0.1
SK [16.0, 20.0]	−0.02
SNO scale	−1.12
SNO resol.	−0.73
SNO vertex	−0.34
SNO n capture	+0.38
SNO n bkgd	+0.23
SNO LE bkgd	+0.33
SNO cross sec.	−0.08

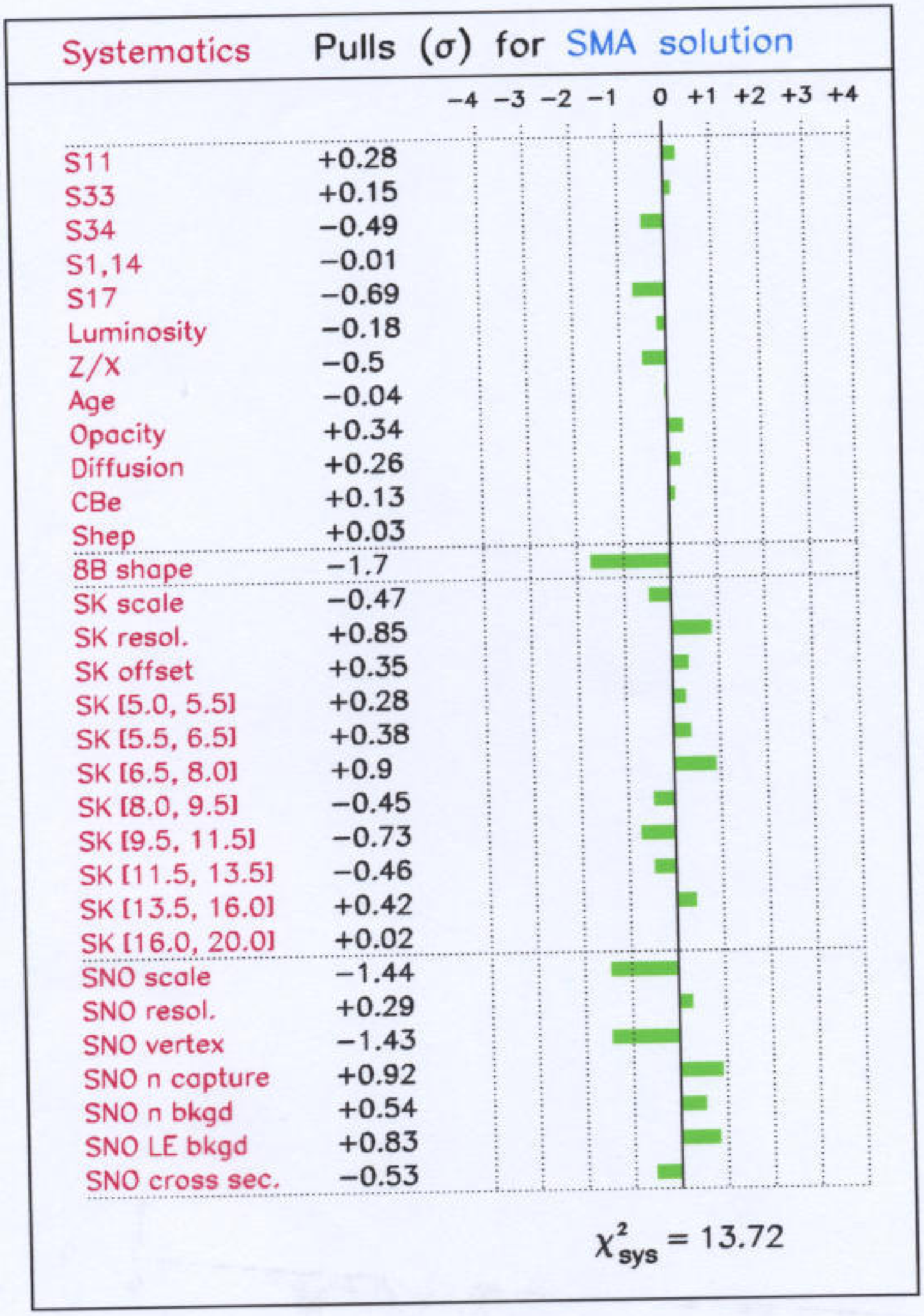
$\chi^2_{sys} = 5.95$

... indications for nonzero offsets (different from LMA, LOW cases)

Enthusiasm for these small pulls responsible for 15-year detour on small mixing physics....

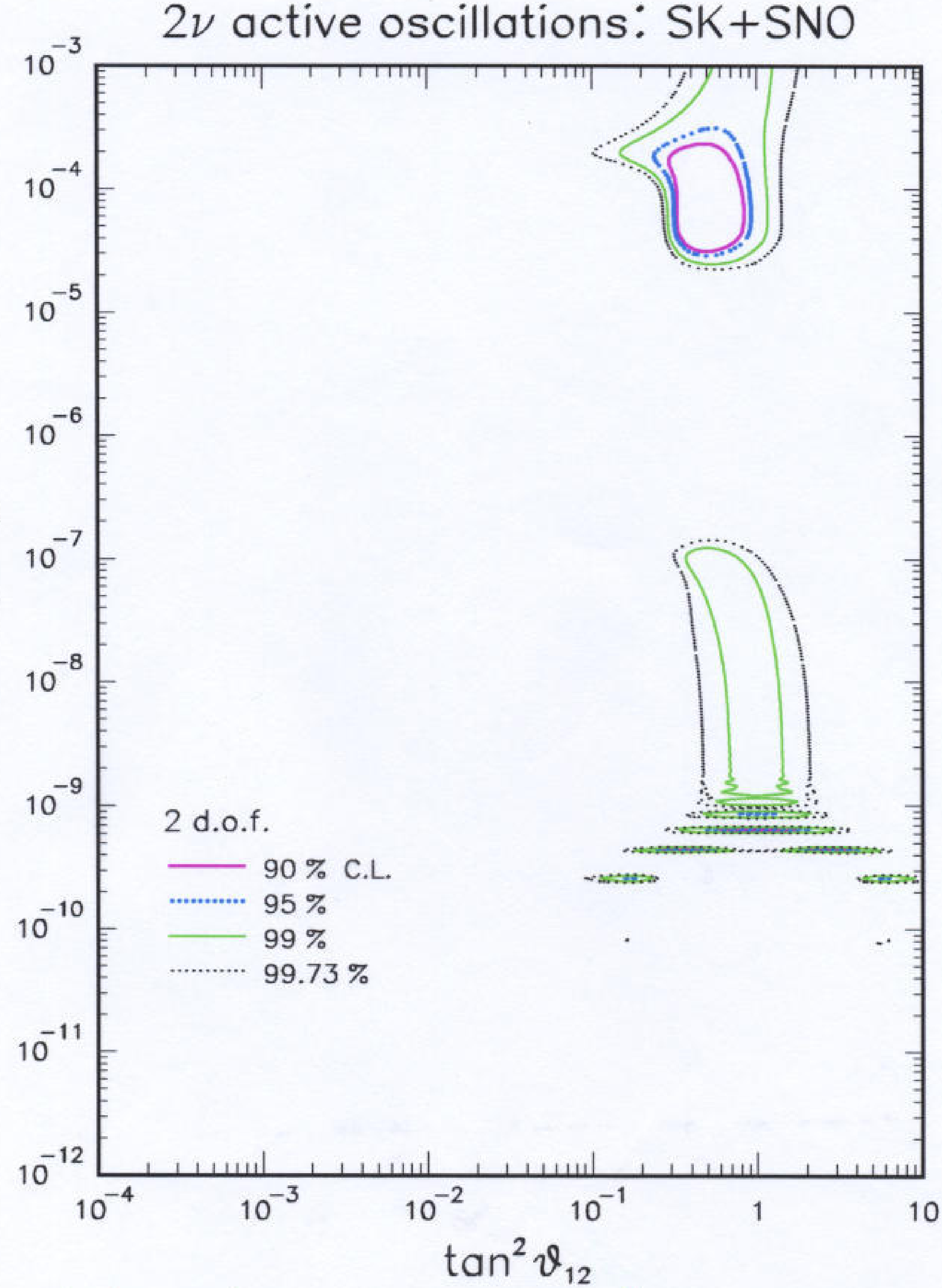


DISTRIBUTION OF PULL OF OBSERVABLES STILL ACCEPTABLE BUT....

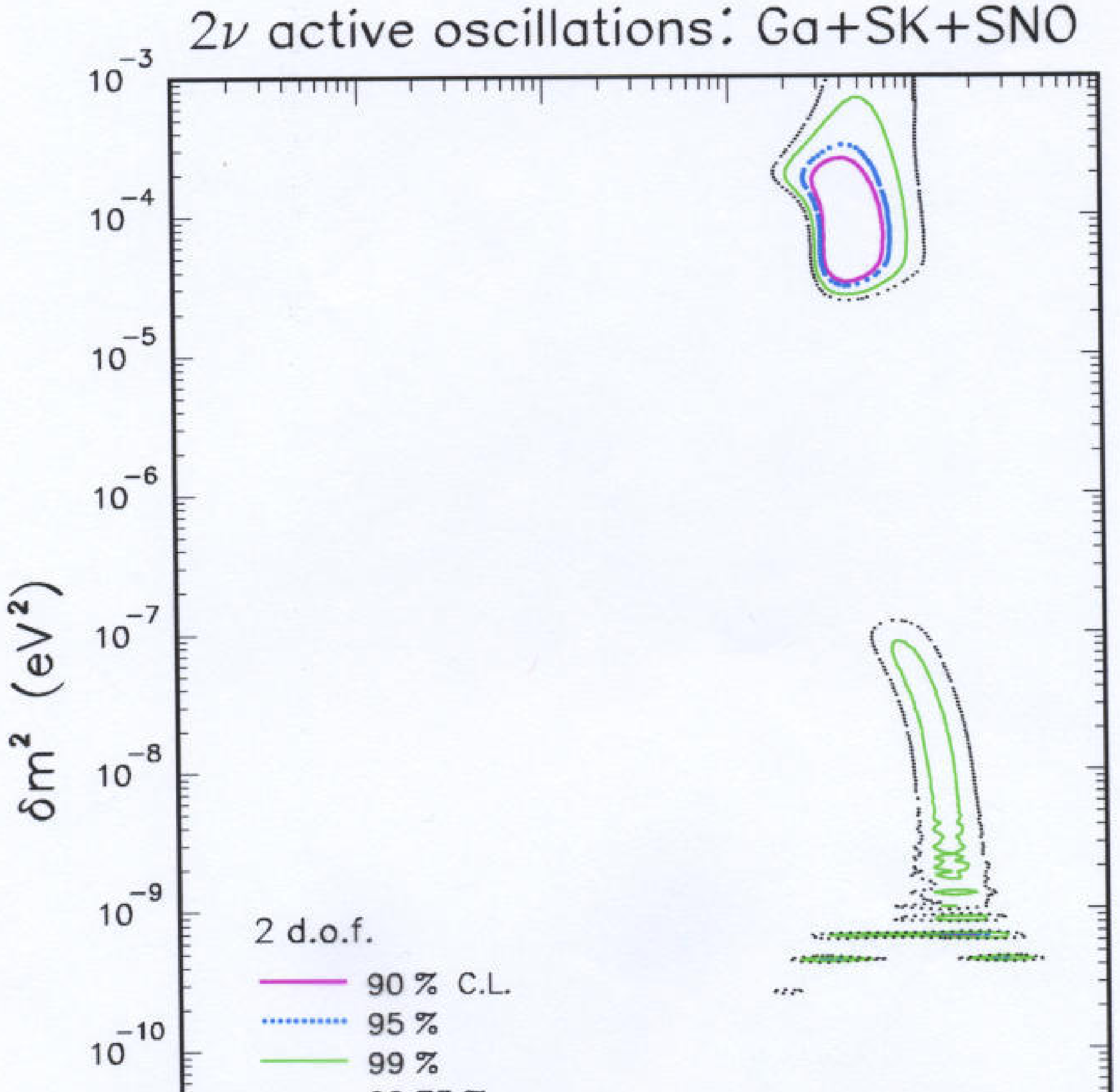


... FIT REQUIRES RELATIVELY LARGE AND SPECIFIC OFFSETS IN SSM, SK AND SNO SYSTEMATICS.

LARGE OFFSET also for ^8B ν shape



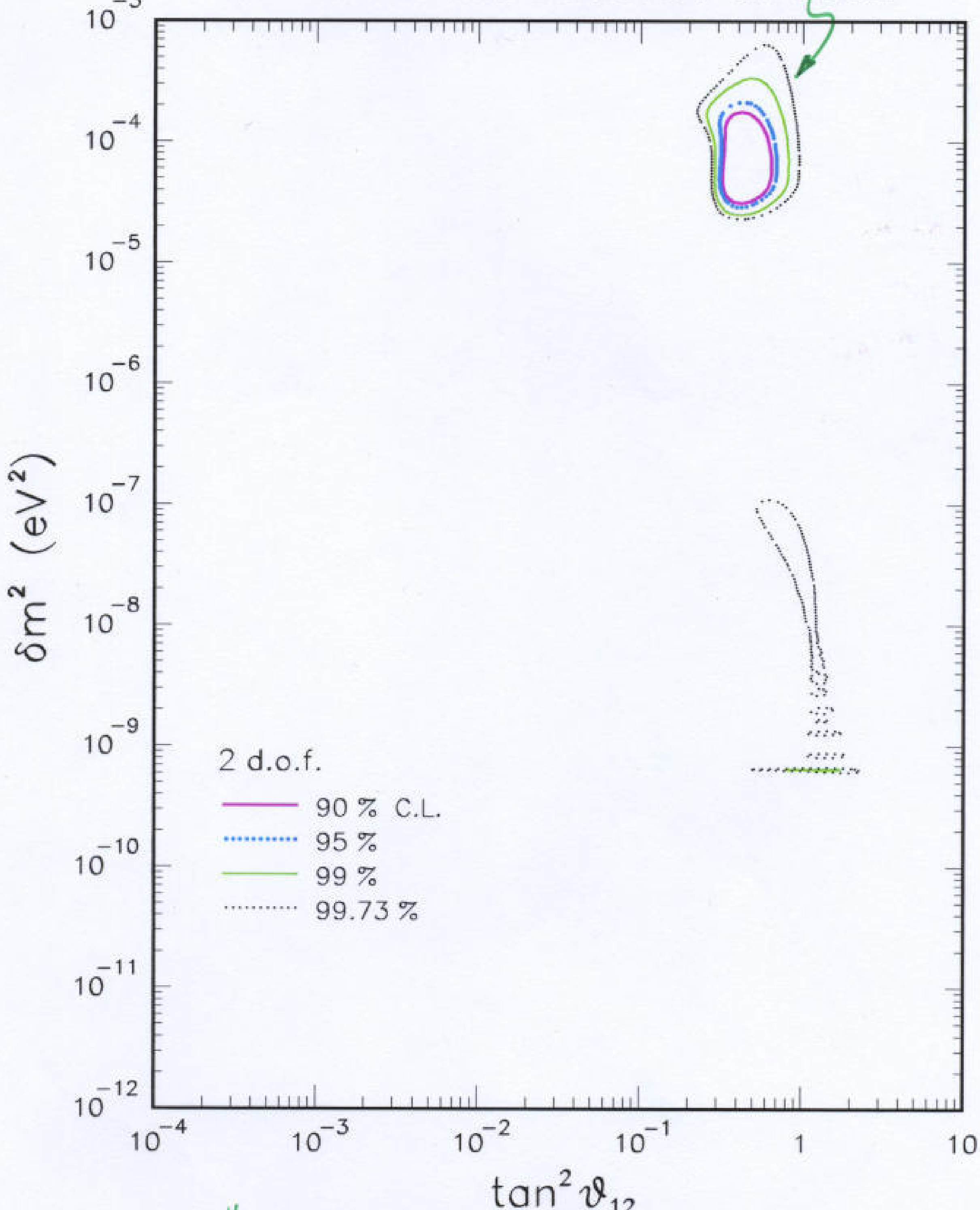
Under "time reversal" (ν expts. before radiochemical ones), interest in SMA would have been much smaller ....



... Ga data would then shrink θ_{12} range

Cl relevant for δm^2_{max}

2ν active oscillations: all data



... and "final" Cl data would still be decisive.
However: Cl pull in remaining solutions is $\gtrsim 2\sigma$. Statistical fluctuation?

WHAT DOES IT IMPLY FOR THE SSM ?

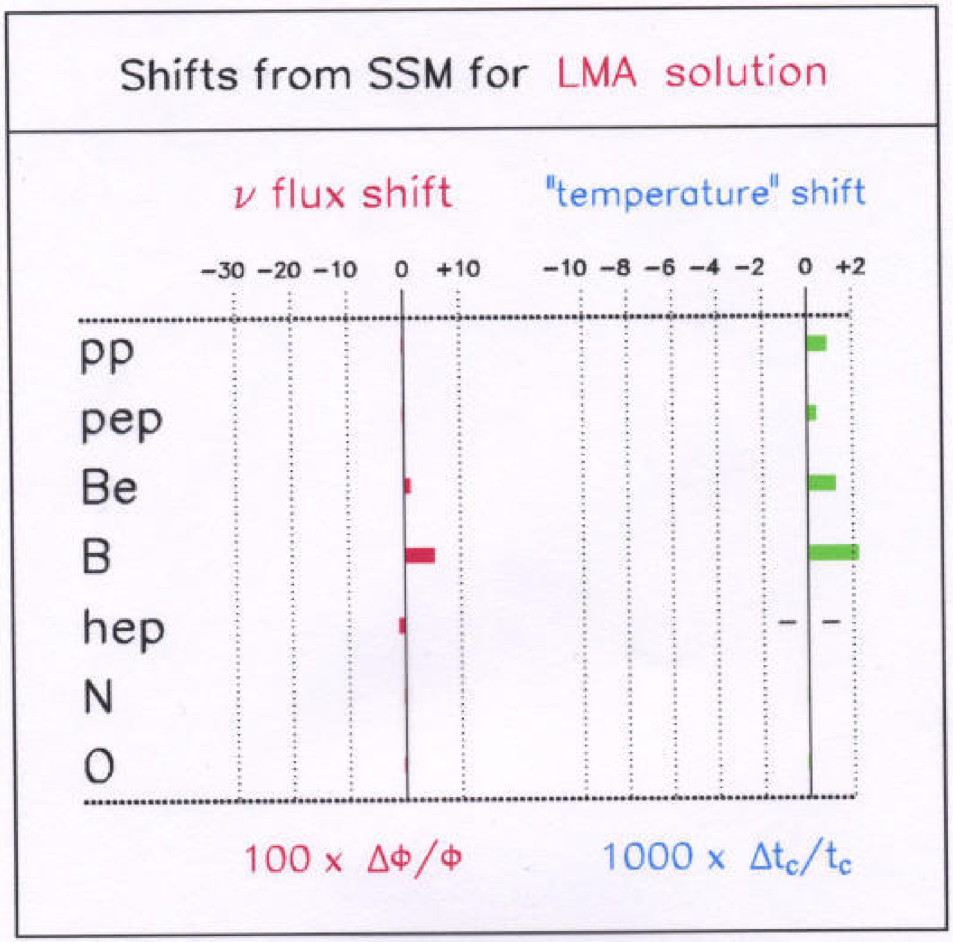
- Remember that shifts $\bar{\xi}_k$ of SSM systematics induce shifts of SSM fluxes:

$$\frac{\Delta\phi_i}{\phi_i} = \sum_{k=1}^{12} \bar{\xi}_k \, \alpha_{ik} \, \Delta \ln X_k$$

- It can be proved that such shifts automatically satisfy the luminosity constraint (bonus of pull analysis)

- They can be related to "effective temperature" shifts through power-laws $\phi_i \propto (t_c)^{\beta_i}$

$$\left(\frac{\Delta t_c}{t_c}\right)_i = \frac{\Delta\phi_i}{\phi_i} \Big/ \beta_i$$

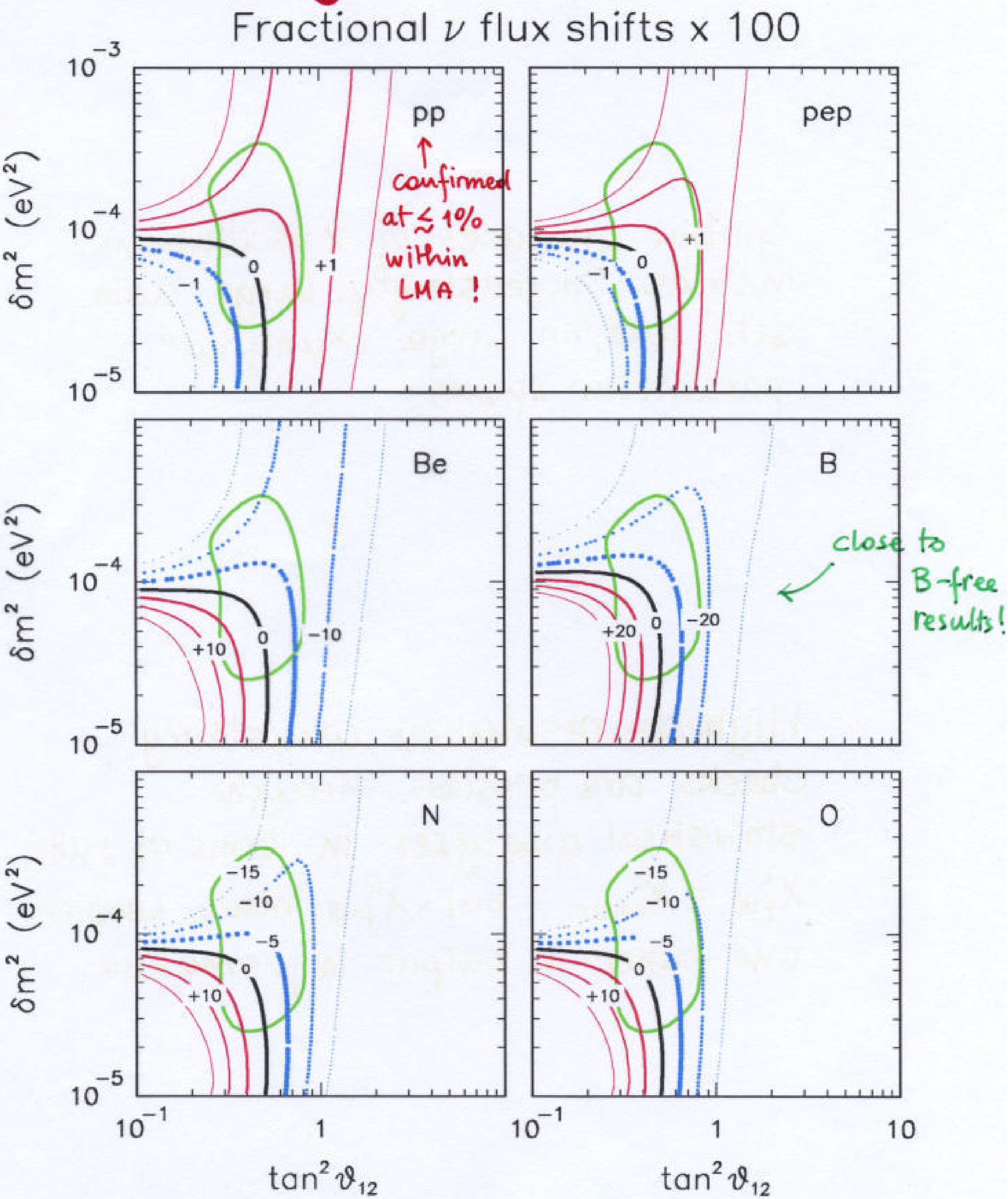


At LMA best-fit point:

SSM "temperature" confirmed at $\lesssim 2‰$ level, consistent with helioseismology

Non-LMA solutions indicate instead small negative $\Delta t_c / t_c$ (up to ~1%)

Zooming in on the LMA



PULL ANALYSIS INCORPORATES SSM FLUX SHIFTS! → and tell you where they come from!

CONCLUSIONS

- Global analyses of ν oscillations include increasingly large data sets and/or large ("unified") parameter spaces
- From such analyses, we have already learned a lot about 2ν, 3ν, and 4ν oscillation physics, and about consistency of data sets
- Highest-resolution consistency checks are possible through statistical analyses in terms of pulls. $\chi^2_{pull} = \chi^2_{covar}$, but χ^2_{pull} much simpler and richer in output information.
- Application to solar ν allows to localize specific SSM or experimental problems of non-LMA solutions, and to probe the SSM in a novel way

  → a good candidate to become the "standard tool" for any kind of global analyses