Neutrino Mass Models

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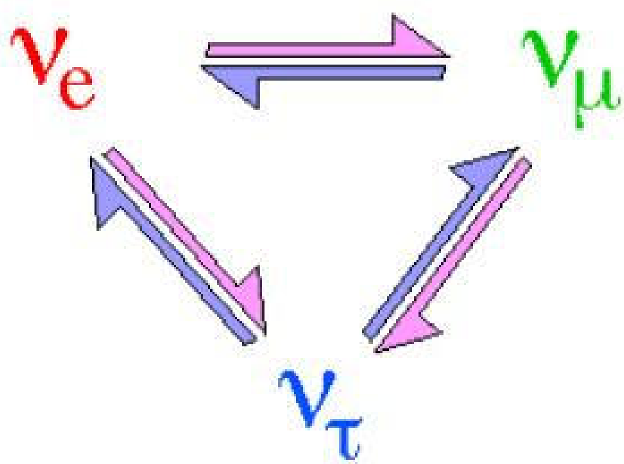
Introduction

- Atmospheric data is well described by active neutrino oscillations $\div_{ö}\ \$\ \div_{ü}$
- Solar data is well described by active neutrino oscillations $\div_{e}\ \$\ \div_{ö;ü}$ (LMA MSW preferred)
- LSND requires sterile neutrino or something non-standard – wait for MiniBooNE
- E.g. Large extra dimension models have infinite tower of sterile neutrinos in the ``bulk'' which could be useful if steriles are required

  (Arkani-Hamed, Dimopoulos,Dvali, March-Russell; Dienes, Dudas, Gherghetta; Dvali,Smirnov; Ioannisian,Valle;Lukas,Ramond,Romanino,Ross; Mohapatra,…many others…)

In this talk we shall focus on the three neutrino paradigm:



The Maki-Nakagawa-Sakata matrix

$$V^{E_L} m_{LR}^{E} V^{E_R \dagger} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

Majorana matrix

$$V^{\nu_L} m_{LL}^{\nu} V^{\nu_L T} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Constructing ⇒ $U_{MNS} = V^{E_L} V^{\nu_L \dagger}$

Three physical phases give CP violation

Parametrising ⇒ $U_{MNS} = R_{23} R_{13} P_1 R_{12} P_2$

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \quad R_{13} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The Large Mixing Angle MNS matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = R_{23} R_{13} R_{12} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

solar LMA MSW

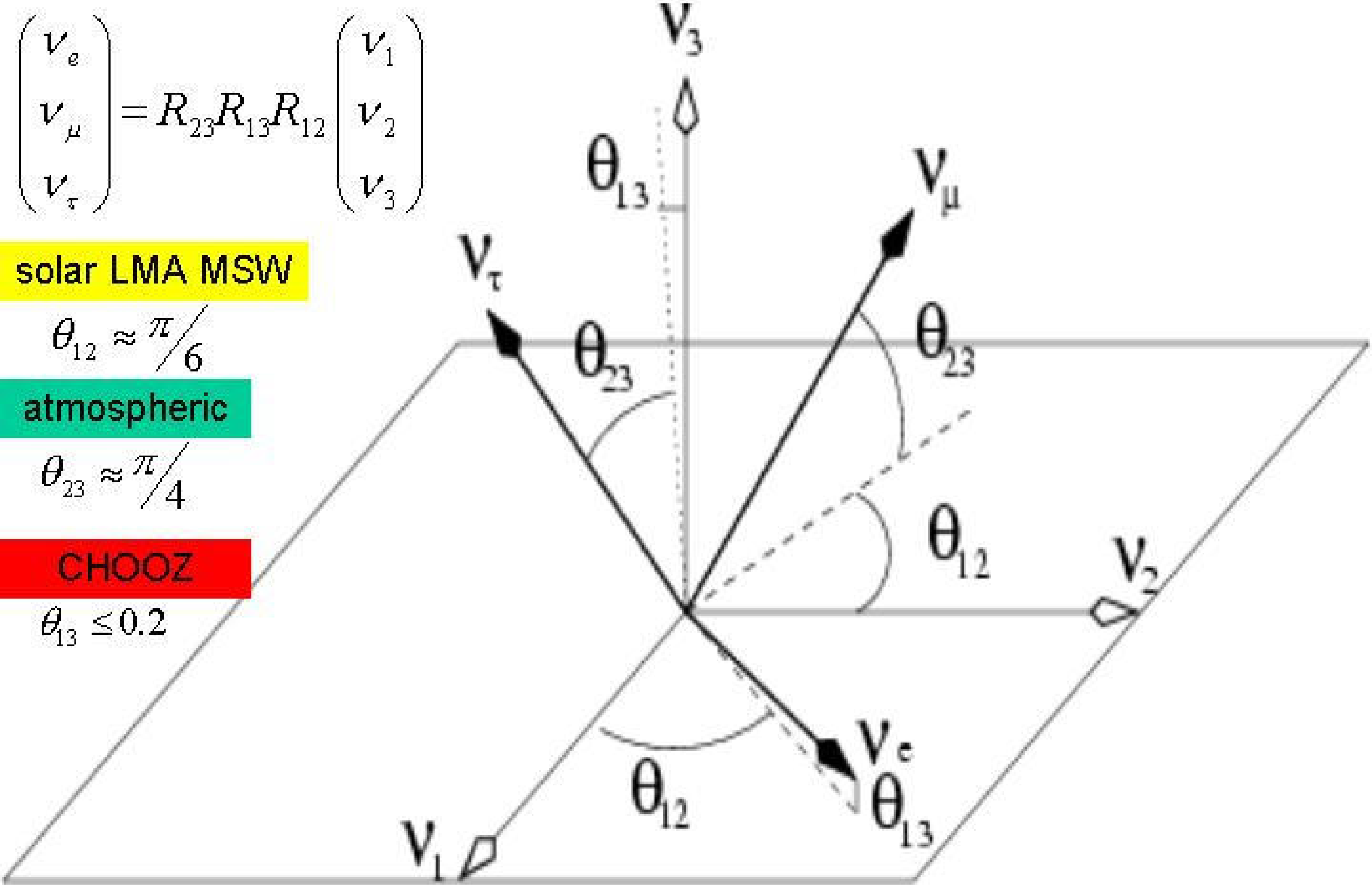
$\theta_{12} \approx \pi/6$

atmospheric

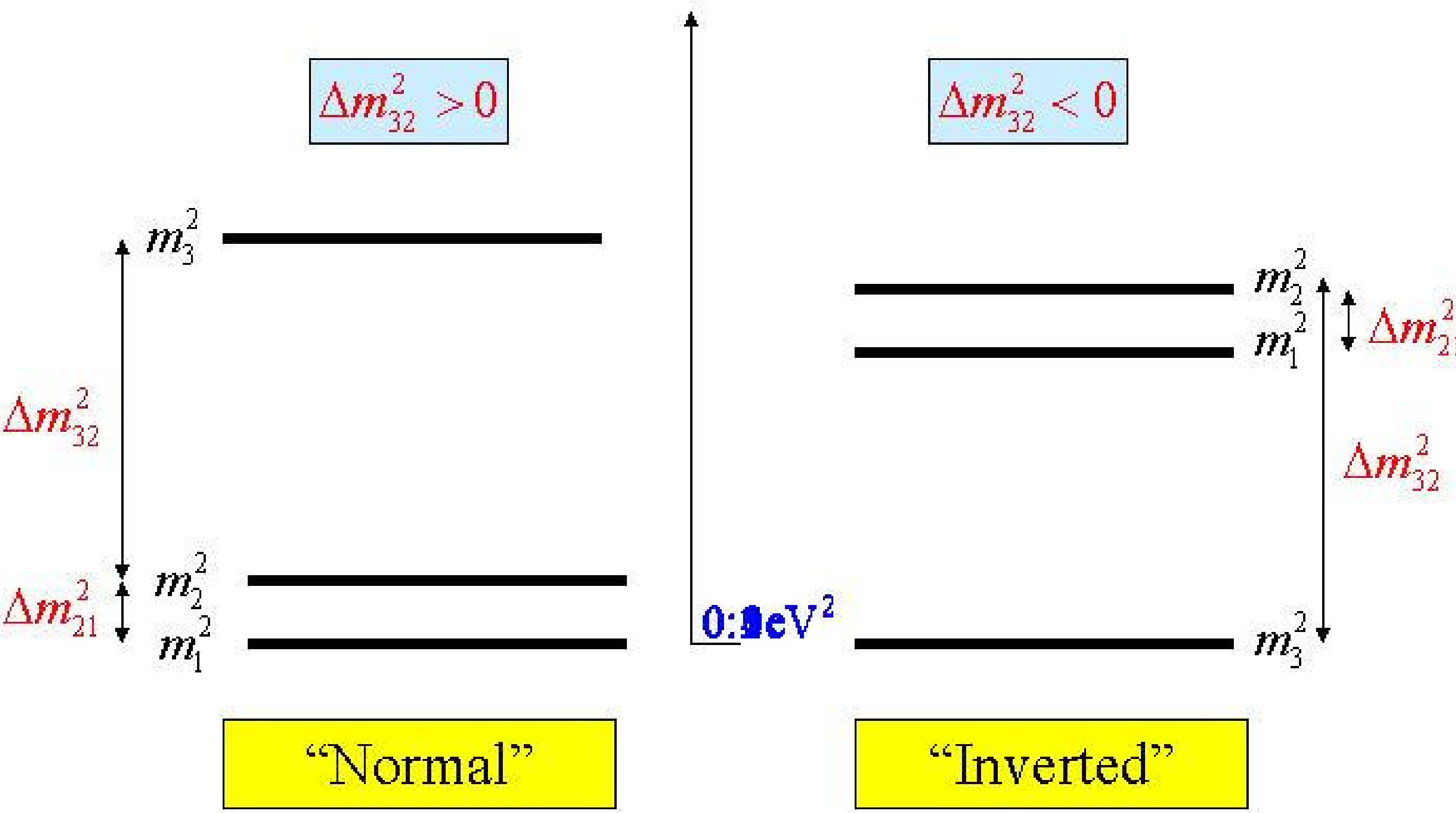
$\theta_{23} \approx \pi/4$

CHOOZ

$\theta_{13} \leq 0.2$



Possible three neutrino mass patterns:



Origin of neutrino mass

In the Standard Model neutrinos are massless, and a neutrino and anti-neutrino are distinguished by a (total) conserved lepton number L.

Majorana or Dirac?

CP conjugate of left-handed neutrino

Majorana mass

$$m_{LL}\overline{\nu}_L\nu_L^c$$

(violates L)

$$M_{RR}\overline{\nu}_R\nu_R^c$$

Right-handed neutrinos

How?

1. Higgs triplets .
2. $\frac{1}{M}$LLHH (Valle talk)
3. See-saw (this talk)

Dirac mass

(conserves L)

$$m_{LR}\overline{\nu}_L\nu_R$$

from Yukawa õ L̈H ÷R

See-saw mechanism

Gell-Mann, Ramond, Slansky and Yanagida ; Mohapatra and Senjanovic, Schechter and Valle,…

Diracmatrix

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Heavy Majorana matrix

Light Majorana matrix

Diagonalise $\rightarrow m_{LL} \overline{\nu}_L \nu_L^c$

$$m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T$$

L is violated by right-handed Majorana mass. L may be global or a part of gauged B-L which is spontaneously broken.

The goal is to reproduce a successful light Majorana matrix

Successful leading order Majorana matrices

Barbieri,Hall,Smith,Strumia,Weiner;Altarelli,Feruglio;many others…

	Type A (zero in 11)	Type B (non-zero 11)
Hierarchy $m_1^2 ; m_2^2$ ü m_3^2	$m_{LL}^{HI} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$	Large neutrinoless double beta decay
Inverted hierarchy m_1^2 ù m_2^2 ý m_3^2	$m_{LL}^{IH(A)} \approx \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{m}{\sqrt{2}}$	$m_{LL}^{IH(B)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} m$
Degenerate m_1^2 ù m_2^2 ù m_3^2	$m_{LL}^{DEG(A)} \approx \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m$	$m_{LL}^{DEG(B1)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m$; $m_{LL}^{DEG(B2)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m$

Pseudo-Dirac

Which Majorana matrix do we shoot for?

Ultimately an experimental question:

- Neutrinoless double beta decay resolves type A from B.
- Neutrino factory can resolve "normal" from "inverted".
- Galaxy structure can constrain "degenerate" mass scale.

In the meantime we have two guiding theoretical principles:

I. Naturalness – want to avoid fine-tuning and produce a light Majorana matrix that is stable under quantum corrections

II. Symmetry - SUSY, GUTs and Family Symmetry are elements of realistic models of quark and lepton masses and mixing angles

I. Naturalness

- Type B matrices involve fine-tuning or carefully broken symmetries to achieve É m^2_{21} → large quantum corrections.
- Degenerate type A involve fine-tuning or carefully broken symmetries to achieve É m^2_{32} → large quantum corrections.
- The most natural possibilities appear to be the hierarchy and Type A inverted hierarchy, but …
- Type A inverted hierarchy has problem that ò$_{12}$ = ù=4 is inconsistent with LMA MNS which requires ò$_{12}$ ù ù=6 Problem can be resolved when charged lepton contributions are included (SFK,Singh;…)

Hierarchy has the puzzle of why m_2 ü m_3 and how to construct the LMA MNS matrix?…focus on this…

Hierarchy

$$m_{LL}^{HI} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$$

Natural eigenvalues are m_2 ø m_3 why m_2 ü m_3 ?

Technically need a small 23 sub-determinant:

$$\det \begin{pmatrix} m_{22} & m_{23} \\ m_{23} & m_{33} \end{pmatrix} << m^2 \qquad \rightarrow m_2 << m_3$$

- But why should the sub-determinant be small ?
- How can we construct the LMA MNS matrix ?

See-saw with diagonal heavy Majorana matrix

Heavy Majorana

$$M_{RR} = \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix}$$

Dirac

$$m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix}$$

Light Majorana

$$m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^{T} = \begin{pmatrix} \left(\frac{a'^2}{X'} + \frac{a^2}{X} + \frac{d^2}{Y}\right) & \left(\frac{a'b'}{X'} + \frac{ab}{X} + \frac{de}{Y}\right) & \left(\frac{a'c'}{X'} + \frac{ac}{X} + \frac{df}{Y}\right) \\ . & \left(\frac{b'^2}{X'} + \frac{b^2}{X} + \frac{e^2}{Y}\right) & \left(\frac{b'c'}{X'} + \frac{bc}{X} + \frac{ef}{Y}\right) \\ . & . & \left(\frac{c'^2}{X'} + \frac{c^2}{X} + \frac{f^2}{Y}\right) \end{pmatrix}$$

Each element has three contributions, one from each right-handed neutrino. If third right-handed neutrino terms (red) dominate then sub-determinant is naturally small!! (SFK 98-)

Right-handed neutrino dominance

SFK hep-ph/0204360

Assume $\frac{|e|^2,|f|^2,|ef|}{Y} \gg \frac{|xy|}{X} \gg \frac{|x'y'|}{X'}$, (sequential dominance)

$x,y \in a,b,c, \quad x',y' \in a',b',c'$ and $|d| \ll |e| \approx |f|$

$m_3 \approx \frac{|e|^2+|f|^2}{Y} \quad m_2 \approx \left| \frac{a^2}{X} + \frac{e^{i\omega}(c_{23}^{\nu} b - s_{23}^{\nu} c e^{i\phi})^2}{X} \right| \quad m_1 \ll m_2 \ll m_3$

Full hierarchy

Maximal atmospheric angle for e=f → $\tan\theta_{23}^{\nu} \approx \frac{|e|}{|f|}$

$$\left[m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \right]$$

Large solar angle for a~b-c → $\tan\theta_{12}^{\nu} \approx \frac{|a|}{c_{23}^{\nu}|b|\cos\beta - s_{23}^{\nu}|c|\cos\gamma}$

Prediction (assuming LMA) → $\theta_{13}^{\nu} \gtrsim \frac{m_2}{m_3} \sim 0.1$

II. Symmetry

Symmetry structure commonly assumed is N=1 SUSY with:

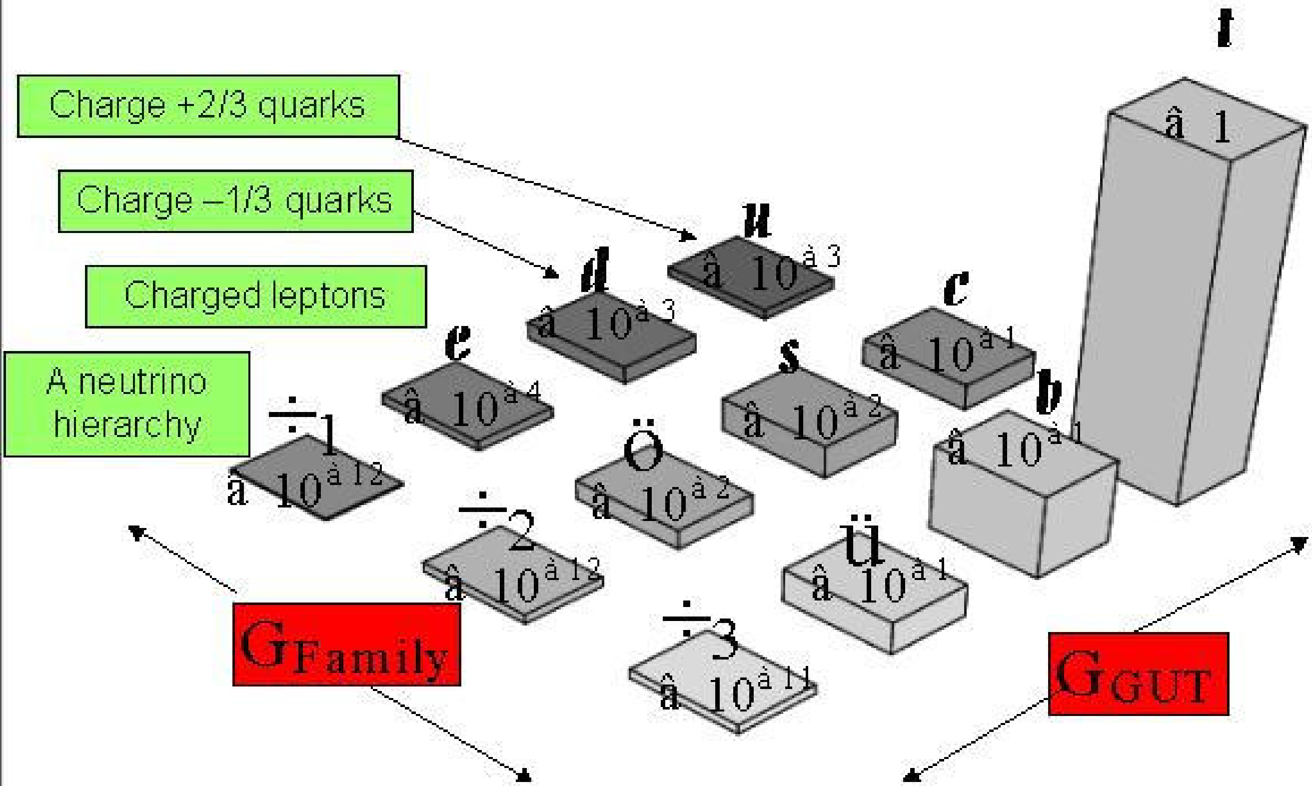
G_{GUT} â G_{Family}

Further model dependence: Higgs/vacuum alignment/orbifold.

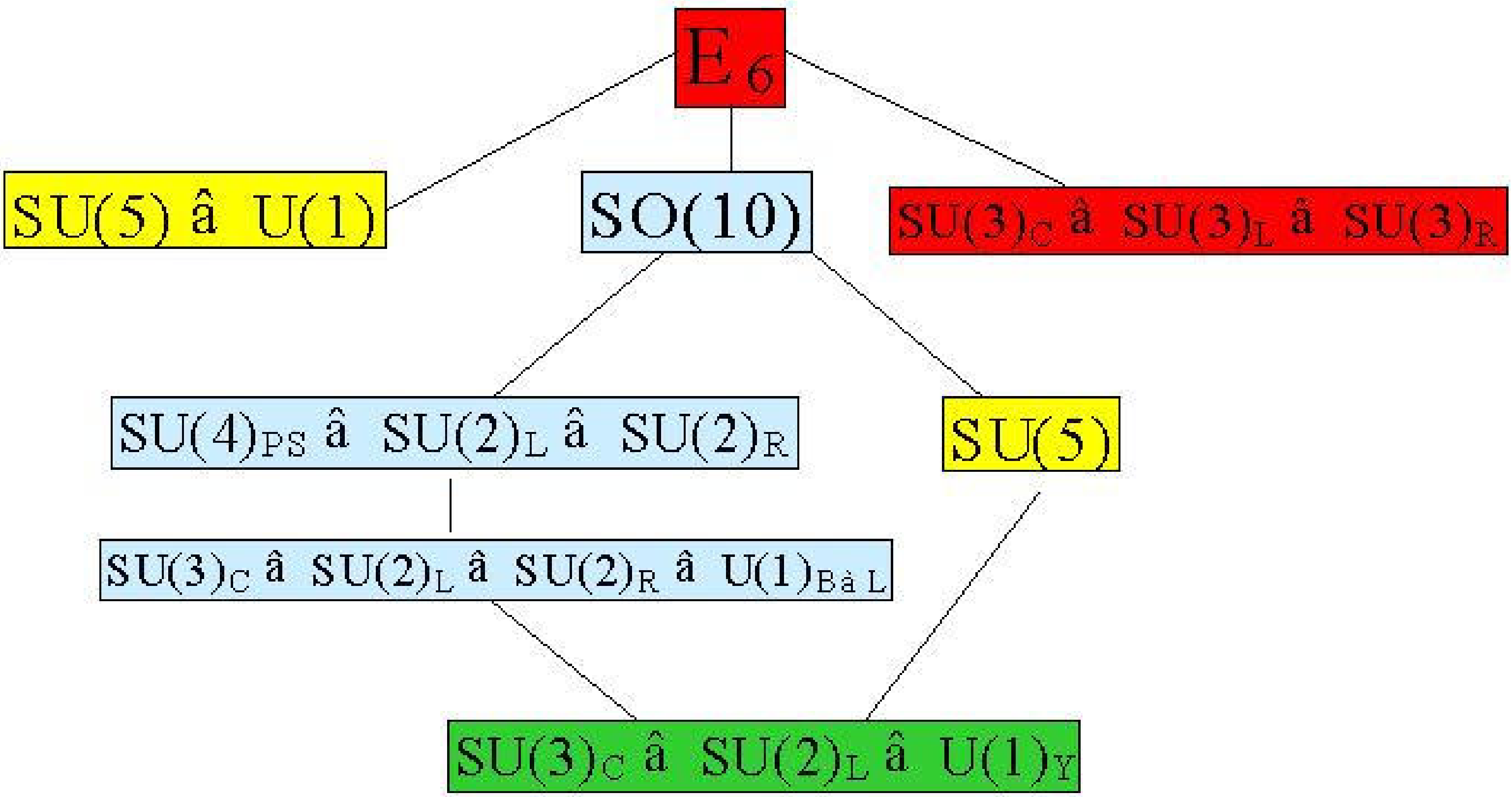
Many many proposals in the literature (with apologies for omissions):

Akhmedov, Albright, Allanach, Altarelli, Anderson, Babu, Baltz, Barbieri, Barr, Barger, Berezhiani, Bijnens, Binetruy, Blazek, Branco, Davidson, Dimopoulos, De Gouvea, Di Clemente, Dudas, Ellis, Feruglio, Froggatt, Fritzsch, Georgi, Glashow, Gibson, Greiner, Grossman, Hall, Harvey, Ibanez, Ibarra, Irges, Jezabek, Joaquim, Joshipura, Kane, Kang, Kaus, Kim, Lavignac, Lola, Leontaris, Lindner, March-Russell, Ma, Masina, Matsuda, Meshkov, Mohapatra, Murayama, Nanopoulos, Nielsen, Nomura, Nussinov, Ohlsson, Pakvasa, Pati, Pokorski, Nielsen, Nir, Raby, Ramond, Reiss, Roberts, Romanino, Ross, Rossi, Roy, Savoy, Seidl, Singh, Shadmi, Shafi, Silva-Marcos, Skadhauge, Smith, Starkman, Stech, Strumia, Sumino, Taminoto, Velasco-Sevilla, Valle, Vergados, Vempati, Volkas, Weiler, Weiner, Wetterich, Whisnant, Wilczek, Wu, Yanagida

The goal is to describe the quark and lepton mass hierarchies



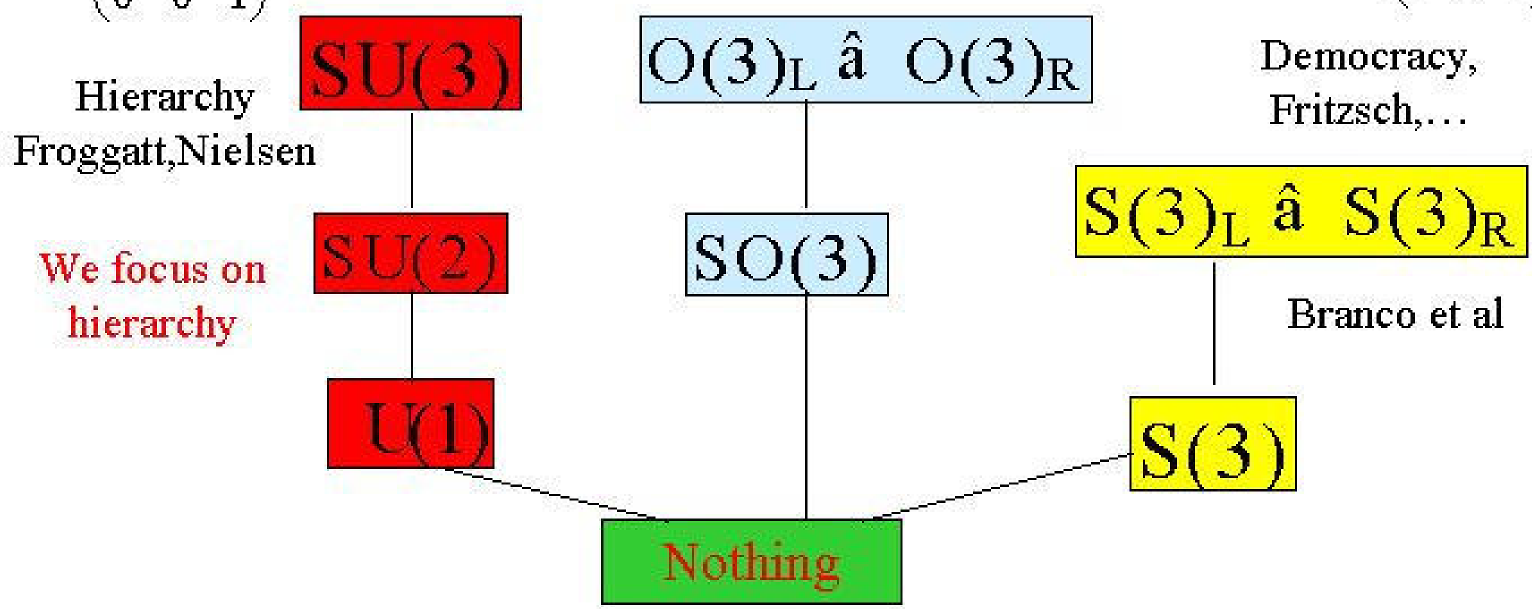
GUTs


E 6
SU(5) â U(1)
SO(10)
SU(3)C â SU(3)L â SU(3)R
SU(4)PS â SU(2)L â SU(2)R
SU(5)
SU(3)C â SU(2)L â SU(2)R â U(1)Bà L
SU(3)C â SU(2)L â U(1)Y

Family Symmetry

$$m^{hier} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m^{dem} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



Natural Models with Symmetry

SU(5) â $U(1)_Q$ Altarelli,Feruglio

$$Q_i^{\bar{5}} = Q_i^{(L;D^c)} = (3;0;0) \qquad Q_i^{10} = Q_i^{(Q;U^c;E^c)} = (3;2;0) \qquad Q_i^{1} = Q_i^{(\nu^c)} = (1;\text{à}\ 1;0)$$

$$m_{LR}^{\nu} = \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^3 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{pmatrix} \qquad M_{RR} = \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix} M \qquad \left[m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \right]$$

$$m_{LR}^{D} = \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & 1 & 1 \end{pmatrix} \qquad m_{LR}^{U} = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

This model has single right-handed neutrino dominance and hence a natural hierarchy.

SMA MSW is predicted. In general many such models predict SMA MSW.

Quarks diagonalised by small left-handed rotations

$SU(4)_{PS} \times SU(2)_L \times SU(2)_R \times U(1)_Q$

SFK, Oliveira

$$F^i_{L,R} = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}^i_{L,R} \qquad m_{LR} = \begin{pmatrix} \varepsilon^5 & \varepsilon^3 & \varepsilon \\ \varepsilon^4 & \varepsilon^2 & 1 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{pmatrix} \qquad M_{RR} = \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{pmatrix} M$$

$$Q_i^F = (1;0;0) \qquad Q_i^{F^c} = (4;2;0)$$

$$m^U_{LR} = \begin{pmatrix} \delta^3\varepsilon^5 & \delta^2\varepsilon^3 & \delta^2\varepsilon \\ \delta^3\varepsilon^4 & \delta^2\varepsilon^2 & \delta^3 1 \\ \delta^3\varepsilon^4 & \delta^2\varepsilon^2 & 1 \end{pmatrix} \quad m^D_{LR} = \begin{pmatrix} \delta\,\varepsilon^5 & \delta^2\varepsilon^3 & \delta^2\varepsilon \\ \delta\,\varepsilon^4 & \delta\,\varepsilon^2 & \delta^2 1 \\ \delta\,\varepsilon^4 & \delta\,\varepsilon^2 & 1 \end{pmatrix} \quad m^E_{LR} = \begin{pmatrix} \delta\,\varepsilon^5 & \delta\,\varepsilon^3 & \delta\,\varepsilon \\ \delta\,\varepsilon^4 & \delta\,\varepsilon^2 & \delta^2 1 \\ \delta\,\varepsilon^4 & \delta\,\varepsilon^2 & 1 \end{pmatrix} \quad m^\nu_{LR} = \begin{pmatrix} \delta^3\varepsilon^5 & \delta\,\varepsilon^3 & \delta\,\varepsilon \\ \delta^3\varepsilon^4 & \delta^2\varepsilon^2 & \delta\,1 \\ \delta^3\varepsilon^4 & \delta^2\varepsilon^2 & 1 \end{pmatrix}$$

$$\left[m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \right]$$

- Gauged B-L predicts three right-handed neutrinos
- Heaviest right-handed neutrino dominates, LMA MSW predicted
- Third family unification with fermion masses from group theory delta's
- Gives acceptable leptogenesis (Hirsch,SFK – Yanagida talk)
- SUSY Pati-Salam appears in type I string constructions

Everett, Kane, SFK, Rigolin, Lian-Tao, Wang

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_F \times Z_2 \times U(1)_R$$ SFK, G.Ross

Symmetric/antisymmetric mass matrices with universal structure

$$m^{\nu}_{LR} = \begin{pmatrix} \varepsilon^8 & \lambda_{\nu}\varepsilon^3 & \lambda_{\nu}\varepsilon^3 \\ -\lambda_{\nu}\varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ -\lambda_{\nu}\varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} \qquad m^{D}_{LR} = \begin{pmatrix} \bar{\varepsilon}^8 & \lambda_{D}\bar{\varepsilon}^3 & \lambda_{D}\bar{\varepsilon}^3 \\ -\lambda_{D}\bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ -\lambda_{D}\bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix} \qquad m^{E}_{LR} = \begin{pmatrix} \bar{\varepsilon}^8 & \lambda_{E}\bar{\varepsilon}^3 & \lambda_{E}\bar{\varepsilon}^3 \\ -\lambda_{E}\bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 3\bar{\varepsilon}^2 \\ -\lambda_{E}\bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 1 \end{pmatrix}$$

ε^{ν} □ $\varepsilon = 0.05$□ $\bar{\varepsilon} = 0.15$

$$m^{\nu}_{LR} = \begin{pmatrix} \varepsilon^8 & \lambda_{\nu}\varepsilon^3 & \lambda_{\nu}\varepsilon^3 \\ -\lambda_{\nu}\varepsilon^3 & a_{\nu}\varepsilon^2 & a_{\nu}\varepsilon^2 + \lambda'_{\nu}\varepsilon^3 \\ -\lambda_{\nu}\varepsilon^3 & a_{\nu}\varepsilon^2 + \lambda'_{\nu}\varepsilon^3 & 1 \end{pmatrix} \qquad M_{RR} = \begin{pmatrix} \varepsilon_{\nu}^4 & 0 & \varepsilon_{\nu}^2 \\ 0 & \varepsilon_{\nu}^3 & \varepsilon_{\nu}^3 \\ \varepsilon_{\nu}^2 & \varepsilon_{\nu}^3 & 1 \end{pmatrix} M \qquad \left[m_{LR} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix} \right]$$

Lightest right-handed neutrino dominates, allows a universal form for mass matrices, predicts LOW solar solution

$$\frac{m_2}{m_3} □ 10^{-3},\ \tan\theta_{23} = 1.0,\ \tan\theta_{12} □ 1,\ \theta_{13} □ 0.03$$

Charged lepton mixing angles

- Charged lepton mixing angles in general contribute to the MNS mixing angles in addition to the neutrino mixing angles (including phases SFK hep-ph/0204360)
- Models exist in which the atmospheric angle originates entirely in the charged lepton sector $SO(10) \times U(1) \times Z_2 \times Z_2$ - predicts small U_e3 and small CPV Albright, Barr
- Inverted hierarchy models have been proposed which are consistent LMA with due to the charged lepton contributions e.g. Ohlsson and Seidl (poster session)

SUSY and LFV

If SUSY is present then neutrino masses inevitably lead to lepton flavour violation due to radiatively generated off-diagonal slepton masses

$$\delta m_{\tilde{L}}^2 \approx -\frac{1}{16\pi^2 v^2} \ln\left(\frac{M_{GUT}^2}{M^2}\right)(3m_0^2 + A^2)\left[m_{LR}^{\nu} m_{LR}^{\nu\dagger}\right]$$

Borzumati, Masiero; Hisano, Moroi, Tobe, Yamaguchi; SFK, Oliveira; Casas, Ibarra; Lavignac, Masina, Savoy; many others…

e.g.

$$m_{LR}^{\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow m_{LR}^{\nu} m_{LR}^{\nu\dagger} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

This will lead to large 23 lepton flavour violation and ü! öi

Blazek, SFK

Final Remarks

- Old models predicted small 23 angle – excluded after Super-K!
- Recent models predicted small 12 angle –excluded after SNO!
- Some current models predict small 13 angle – go measure!
- Naturalness +LMA predicts U_e3 ~0.1
- Naturalness also predicts small neutrinoless double beta decay – go measure!
- See-saw looks good, with right-handed neutrino dominance it looks better – large 23,12 angles from simple relations
- Ideas fit with SUSY, GUTs, family symmetry; most models prefer normal hierarchy not inverted – go measure!
- CP violation is expected in lepton sector and required for leptogenesis; some models predict small CPV – go measure!