Neutrino Oscillations Beyond Two Flavours

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3 neutrino flavours are known to exist – ν_e, ν_μ, ν_τ

If LSND is right $\Rightarrow$ light $\nu_{sterile}$

But:

Until recently – All analyses in 2-flavour framework

Reasons:

(1) Simplicity

(2) Hierarchy of Δm^2

2f analyses of solar and atm. ν data a good first approximation – a consequence of smallness of $|U_{e3}|$ and $\Delta m^2_{\odot} \ll \Delta m^2_{\rm atm}$

These days:

- The data more accurate
- LMA favoured – the hierarchy of Δm^2 may be not too strong
- Effects specific to ≥ 3 – flavour ν oscillations widely discussed

3f (4f) analyses becoming a must!

Some theoretical issues pertaining to ≥ 3 – flavour neutrino oscillations

- 3-flavour oscillations in matter – approximate analytic descriptions
- Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
- 3f effects in atmospheric, solar, and reactor ν oscillations and in LBL experiments
- 3f effects in oscillations of supernova neutrinos
- $\not{CP}$ and $\not{T}$ in ν oscillations in vacuum
- $\not{CP}$ and $\not{T}$ in ν oscillations in matter
- The problem of U_{e3}
- 4f oscillations

Lepton mixing and neutrino oscillations in vacuum

$$\nu_a = U_{ai}\,\nu_i$$

ν_a – flavour eigenstates, ν_i – mass eigenstates

Transition probability:

$$P(\nu_a \to \nu_b; t) = \left| \sum_i U_{bi}\, e^{-iE_i t}\, U_{ai}^* \right|^2$$

Can be obtained from the evolution equation

$$i\frac{d}{dt}\begin{pmatrix} \nu_a \\ \nu_b \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} = U \begin{pmatrix} E_1 & & & & \\ & E_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{pmatrix} U^\dagger \begin{pmatrix} \nu_a \\ \nu_b \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

2-flavour case:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_a \to \nu_b; t) = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m_{ij}^2}{4E}\,L\right)$$

Neutrino oscillations in matter (3f)

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\left[U\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}U^\dagger+\begin{pmatrix}V(t) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$E_i=\sqrt{p^2+m_i^2}\simeq p+\frac{m_i^2}{2p}\,; \qquad t\simeq r$$

$$U=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_1}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_1} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$V(t)=[V(\nu_e)]_{CC}=\sqrt{2}G_F N_e(t)$$

$[V(\nu_e)]_{NC}=[V(\nu_\mu)]_{NC}=[V(\nu_\tau)]_{NC}$ – do not contribute

But: Radiative corrections induce a tiny $\nu_\mu-\nu_\tau$ potential difference $\simeq\ 10^{-5}\,V$ – may be important for supernova neutrinos! (Botella, Lim & Marciano, 1987)

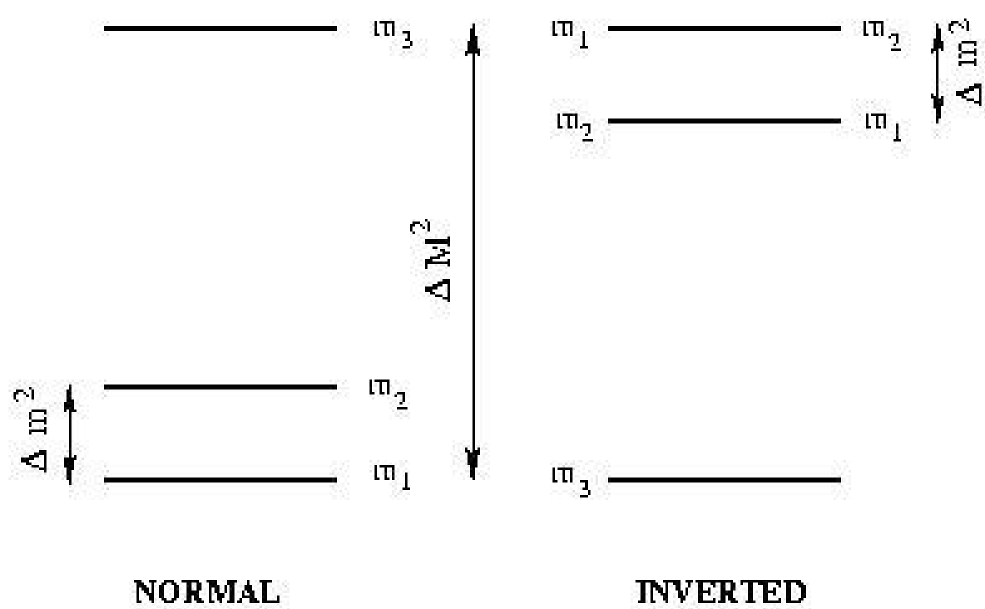
For constant-density matter closed form solutions can be found (Barger, Whisnant, Pakvasa & Phillips, 1980; Zaglauer & Schwartzer, 1988; Ohlsson & Snellman, 1999; Xing, 2000; Kimura, Takamura & Yokomakura, 2002)

But: Expressions rather complicated and not easily tractable
For a general $N_e \neq const$ no closed form solutions exist
Approximate analytic solutions desirable

Two kinds of approximations, use

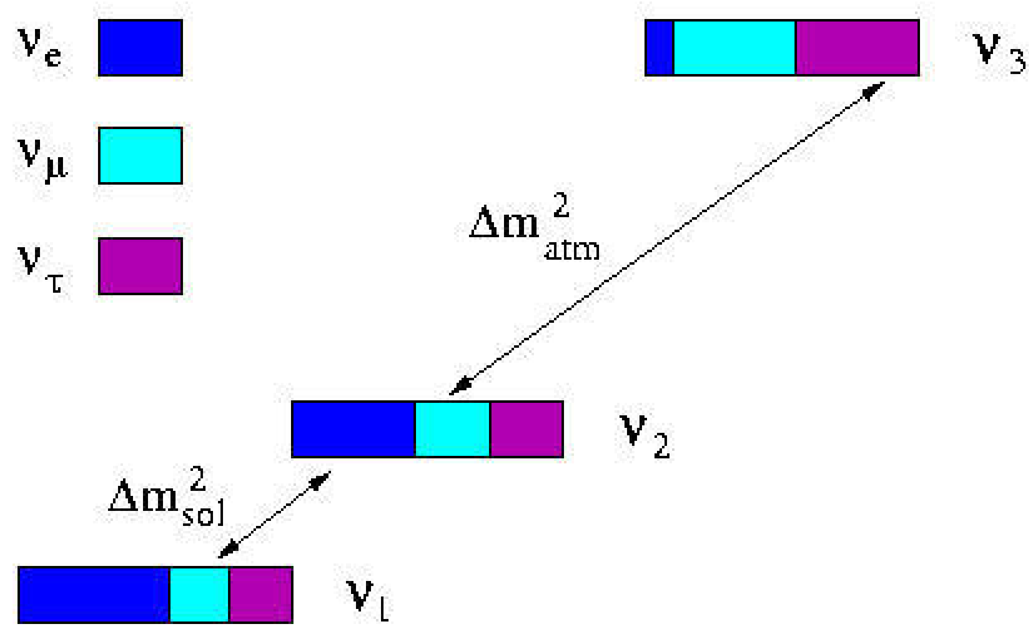
$$(1) \quad \frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \lesssim 0.1$$

$$(2) \quad |U_{e3}| = |\sin\theta_{13}| \lesssim 0.2 \quad \text{(CHOOZ)}$$

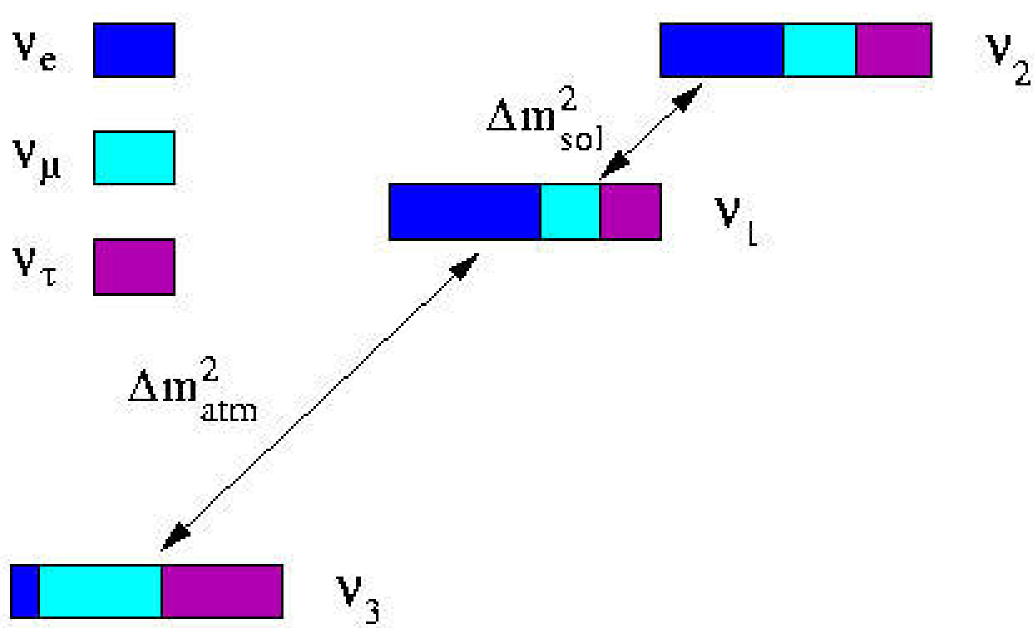


$\Delta m^2_{21} = 0$ or $U_{13} = 0$ – effective 2f limits

Normal hierarchy:



Inverted hierarchy:



Constant-density matter

(a) Expansion in $\alpha \equiv \Delta m^2_\odot / \Delta m^2_{\rm atm}$ (Yasuda, 1999; Freund, Lindner, Petcov & Romanino, 1999; Freund, 2001; Freund, Huber & Lindner, 2001; Mocioiu & Shrock, 2001)

(b) Expansion in both α and $\sin\theta_{13}$ (Cervera *et al.*, 2000)

$$P(\nu_e \leftrightarrow \nu_\mu) \sim s^2_{23} P_2(\Delta m^2_{31}, \theta_{13}) + c^2_{23} P_2(\Delta m^2_{21}, \theta_{12})$$
$$+ \text{ interference term}$$

Interf. term (linear in α and $\sin\theta_{13}$) – genuine 3-flavourness!

Matter of constant density – a good first approximation for LBL experiments (neutrinos traverse Earth's mantle). Not very useful for solar, atmospheric and supernova neutrinos

Different approach: matter with arbitrary density profile, reduce the problem to an effective 2-flavour one + easily calculable 3f corrections

(a) $\alpha \ll 1$ (E.A., Dighe, Lipari & Smirnov, 1998)

(b) $|\sin\theta_{13}| \ll 1$ (Peres & Smirnov, 1999; E.A., Huber, Lindner & Ohlsson, 2001; Peres & Smirnov, 2002)

Matter with arbitrary density profile, adiabatic approximation (Kuo & Pantaleone, 1987; Ohlsson & Snellman, 2001)

3f effects in oscillations of solar neutrinos

- What do the solar ν_e oscillate to?

From $|U_{e3}| \ll 1$: $\quad \nu_3 \simeq s_{23}\,\nu_\mu + c_{23}\,\nu_\tau$

$\Rightarrow$ From unitarity of U: Solar ν oscillations between

ν_e and $\nu' = c_{23}\,\nu_\mu - s_{23}\,\nu_\tau$

$\Rightarrow$ Solar ν_e oscillate into a superposition of ν_μ and ν_τ with almost equal weights

$\longleftarrow - \longrightarrow$

- Oscillation probability

At low E ν_μ and ν_τ experimentally indistinguishable $\Rightarrow$ all observables depend just on $P(\nu_e \to \nu_e)$

Averaging over fast oscillations due to large $\Delta m^2_{\rm atm} = \Delta m^2_{31}$:

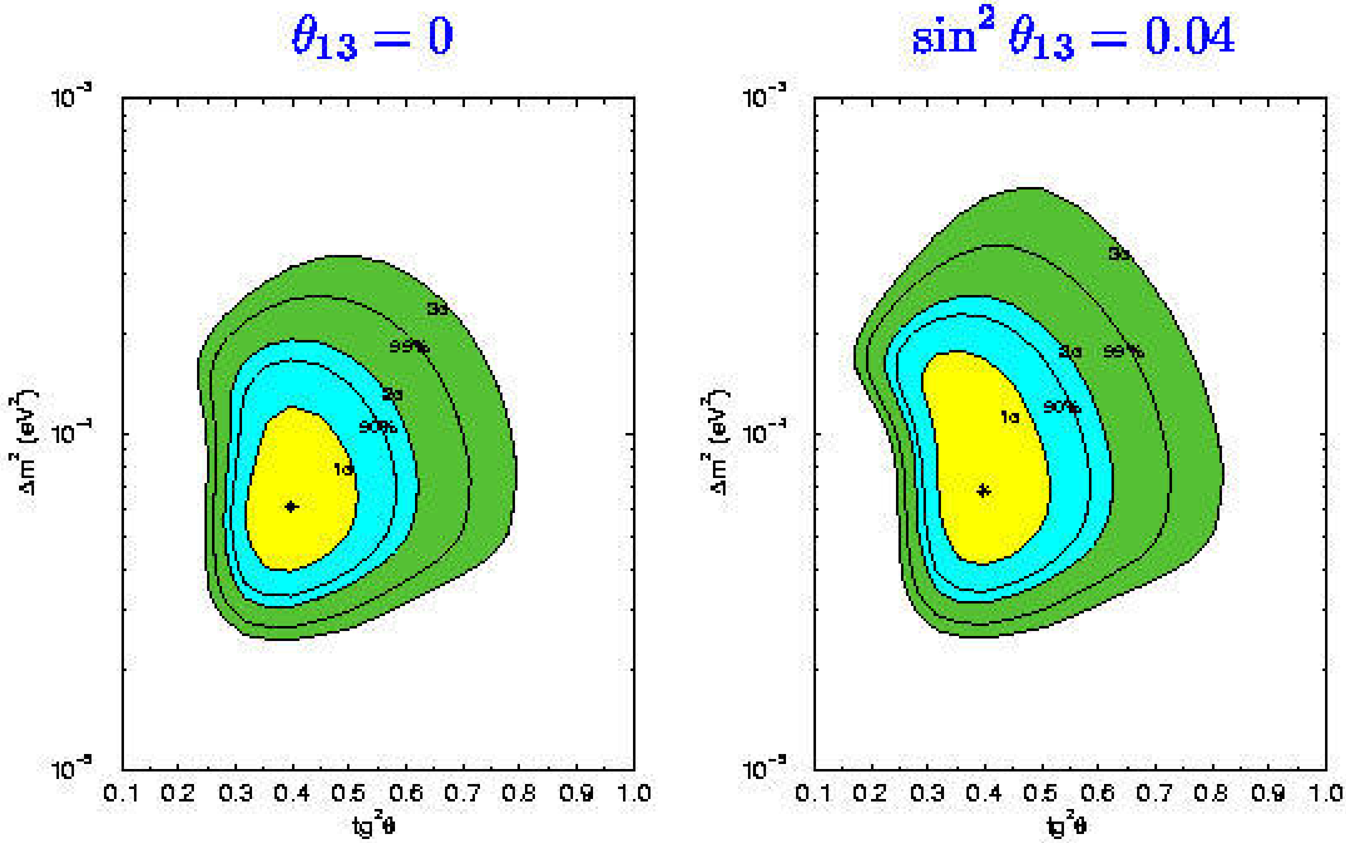
$$P(\nu_e \to \nu_e) \simeq c_{13}^4 \tilde{P}_{2ee}(\Delta m^2_{21}, \theta_{12}, V_{\rm eff}) + s_{13}^4 \,,$$

$$V_{\rm eff} = c_{13}^2\, V_{CC} \quad \text{(Lim, 1987)}$$

$s_{13}^4 \lesssim 10^{-3}$ – negligible. But: c_{13}^4 may differ from 1 by as much as $\sim 5 - 10\,\%$ (E - independent suppression) – with high precision solar data must be taken into account!

LMA allowed region (Δm^2_{21} , $\tan^2\theta_{12}$)

(de Holanda & Smirnov, 2002)



3f effects in atmospheric neutrino oscillations

(1) Dominant channel $\nu_\mu \leftrightarrow \nu_\tau$

In 2f case – no matter effects (neglecting tiny $V_{\mu\tau}$ caused by rad. corrections). Independent from the sign of Δm^2_{31} (direct vs inverted hierarchy). In 3f case – weak sensitivity to matter effects, sign of Δm^2_{31}

(2) Subdominant channels $\nu_e \leftrightarrow \nu_{\mu,\tau}$

Contribution to μ – like events: subleading, difficult to observe In 2f limits – suppression of oscillation effects on e-like events:

- $\Delta m^2_{21} \to 0$ (E.A., Dighe, Lipari & Smirnov, 1998) :

$$\frac{F_e - F_e^0}{F_e^0} = \tilde{P}_2(\Delta m^2_{31}, \theta_{13}, V_{CC}) \cdot (r\, s^2_{23} - 1)$$

- $s_{13} \to 0$ (Peres & Smirnov, 1999):

$$\frac{F_e - F_e^0}{F_e^0} = \tilde{P}_2(\Delta m^2_{21}, \theta_{12}, V_{CC}) \cdot (r\, c^2_{23} - 1)$$

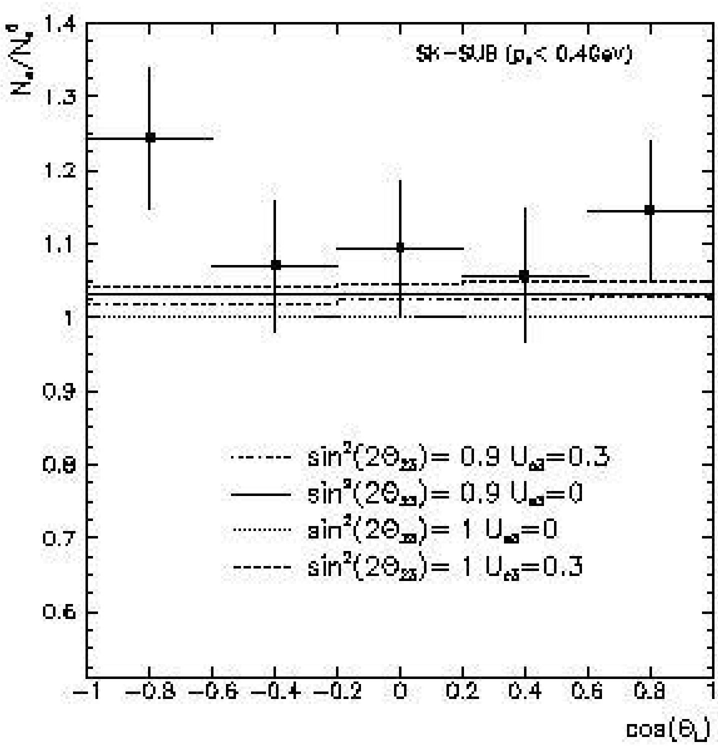
At low energies $r \equiv F^0_\mu / F^0_e \simeq 2$; also $s^2_{23} \simeq c^2_{23} \simeq 1/2$ – a conspiracy to hide oscillation effects on e-like events! Results from a peculiar flavour composition of the atmospheric ν flux.

Breaking the conspiracy – 3f effects in $\nu_{\rm atm}$ oscillations

(Peres & Smirnov, 2002)

$$\frac{F_e - F_e^0}{F_e^0} \simeq \tilde{P}_2(\Delta m_{31}^2, \theta_{13}) \cdot (r\, s_{23}^2 - 1)$$
$$+ \ \tilde{P}_2(\Delta m_{21}^2, \theta_{12}) \cdot (r\, c_{23}^2 - 1)$$
$$- \ 2 s_{13}\, s_{23}\, c_{23}\, r\, \mathrm{Re}(\tilde{A}_{ee}^*\, \tilde{A}_{\mu e})$$

Interference term not suppressed by the flavour composition of the $\nu_{\rm atm}$ flux; may be responsible for observed excess of upward-going sub-GeV e-like events



3f effects in oscillations of reactor antineutrinos

- CHOOZ, Palo Verde, ... ($L \lesssim 1$ km)

$$\overline{E} \sim 2 \text{ MeV}; \qquad \frac{\Delta m_{31}^2}{4E} L \sim 1; \qquad \frac{\Delta m_{21}^2}{4E} L \underset{?}{\ll} 1$$

One mass scale dominance (2f) approximation:

$$P(\overline{\nu}_e \to \overline{\nu}_e; L) = 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right)$$

But: for LMA solution, at high C.L. Δm_{21}^2 can be comparable to Δm_{31}^2 $\Rightarrow$ Full 3f analyses necessary.

$\Rightarrow$ **Constraints on $|U_{e3}|$ from CHOOZ – slightly more stringent** (Gonzalez-Garcia, Maltoni, Peña-Garay & Valle, 2000; Bilenky, Nicolo & Petcov, 2001; Gonzalez-Garcia & Maltoni, 2002) – less likely with new SNO data (large Δm_{21}^2 disfavoured)

- KamLAND ($\overline{L} \simeq 170$ km)

$$\frac{\Delta m_{31}^2}{4E} L \gg 1; \qquad \frac{\Delta m_{21}^2}{4E} L \gtrsim 1 \quad \text{(for LMA)}$$

Averaging over fast oscillations due to $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$:

$$P(\overline{\nu}_e \to \overline{\nu}_e) = c_{13}^4 P_{2\,\overline{e}\,\overline{e}}(\Delta m_{21}^2, \theta_{12}) + s_{13}^4$$

Can differ from 2f probability by as much as $\sim 10\%$.

3f effects in LBL experiments

- 3f corrections to ν_μ disappearance probability up to $\sim 10\%$

$$\sin^2(2\theta_{\mu\tau})_{\rm eff} = c_{13}^4 \sin^2 2\theta_{23}$$

  Also, subdominant $\nu_\mu \to \nu_e$ contribution; small matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ – similar to effects in $\nu_{\rm atm}$

- No suppression of $\nu_e \leftrightarrow \nu_{\mu,\tau}$ due to flavour composition of the original flux

- 3f effects especially important for precision measurements (ν factories!)

- For $3 \cdot 10^{-3} \lesssim \theta_{13} \lesssim 3 \cdot 10^{-2}$ – competition between two channels ($\Delta m_{31}^2, \theta_{13}$ and $\Delta m_{21}^2, \theta_{12}$) in $P(\nu_e \leftrightarrow \nu_\mu)$

- Dependence on CP-violating phase $\delta_{\rm CP}$ (both $\sim \sin\delta_{\rm CP}$ and $\sim \cos\delta_{\rm CP}$) comes from the interference term – pure 3f effect!

$\longleftarrow$ – $\longrightarrow$

Matter effect in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations:

- Pure 3f effect (neglecting $V_{\mu\tau}$); vanishes only when both Δm_{21}^2 and U_{e3} vanish

3f effects in supernova neutrino oscillations

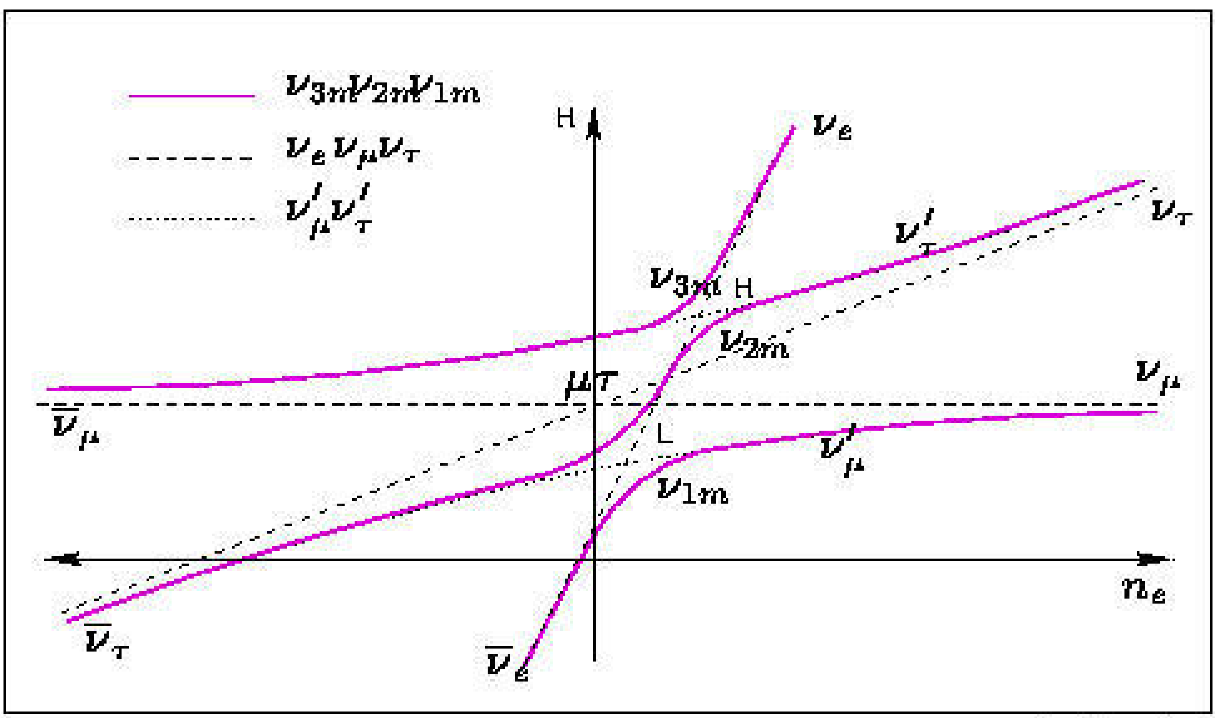
Matter density varies in a wide range – conditions for three MSW resonances satisfied ($V_{\mu\tau} \neq 0$)

Hierarchy of Δm^2 – approximate factorization of transition dynamics at the resonances: Effectively 2f transitions

But: observable effects depend on transitions between all 3 neutrino species

Earth matter effects on SN neutrinos can be used to measure $|U_{e3}|$ to a very high accuracy ($\sim 10^{-3}$) and to determine the sign of Δm^2_{31} (Lunardini & Smirnov, 2000, 2001; Minakata & Nunokawa, 2000; Barger, Marfatia & Wood, 2002)

The transitions due to tiny $V_{\mu\tau}$ caused by rad. corrections may have observable consequences if originally produced ν_μ and ν_τ fluxes are not exactly the same (E.A., Lunardini & Smirnov, 2002)



$\not{CP}$ and $\not{T}$ in ν oscillations in vacuum

$\nu_a \to \nu_b$ oscillation probability:

$$P(\nu_a, t_0 \to \nu_b; t) \;=\; \left|\sum_i U_{bi}\, e^{-iE_i(t-t_0)}\, U_{ai}^*\right|^2$$

(1) CP: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b} \quad \Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

(2) T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_a \leftrightarrow \nu_b$

$\Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

T-reversed oscillations ("backwards in time") $\Leftrightarrow$ oscillations between interchanged initial and final flavours

$\not{CP}$ and $\not{T}$ – absent in 2f case, pure $N \geq 3$f effect!

(3) CPT: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b} \quad \& \quad t \rightleftarrows t_0 \quad (\nu_a \leftrightarrow \nu_b)$

$$P(\nu_a \to \nu_b) \to P(\bar{\nu}_b \to \bar{\nu}_a)$$

$\not{CP} \Leftrightarrow \not{T}$ – consequence of CPT

Measures of $\not{CP}$ and $\not{T}$ – probability differences:

$$\Delta P_{ab}^{\rm CP} \equiv P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)$$

$$\Delta P_{ab}^{\rm T} \equiv P(\nu_a \to \nu_b) - P(\nu_b \to \nu_a)$$

From CPT:

$$\Delta P^{\rm CP}_{ab} = \Delta P^{\rm T}_{ab}; \qquad \Delta P^{\rm CP}_{aa} = 0$$

$\leftarrow - \rightarrow$

3f case

One $\not{C}\not{P}$ Dirac-type phase $\delta_{\rm CP}$ (NB: Majorana phases do not affect ν oscillations!) $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

$$\Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
$$\times \left[\sin\left(\frac{\Delta m_{12}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{23}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{31}^2}{2E}L\right)\right]$$

Vanishes when

- At least one $\Delta m_{ij}^2 = 0$
- At least one $\theta_{ij} = 0$ or $90°$
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

$\not{CP}$ and $\not{T}$ in ν oscillations in matter

(1) CP: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b}$ $\quad\Rightarrow\quad$ $U_{ai} \to U_{ai}^*$ $\quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

$V(r) \to -V(r)$

(2) T: $\quad t \rightleftarrows t_0$ $\quad\Leftrightarrow\quad$ $\nu_a \leftrightarrow \nu_b$

$\Rightarrow\quad U_{ai} \to U_{ai}^*$ $\quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

$V(r) \to \tilde{V}(r)$

$$\tilde{V}(r) = \sqrt{2} G_F \tilde{N}(r)$$

$\tilde{N}(r)$: corresponds to interchanged positions of ν source and detector

Symmetric matter density profiles: $\tilde{N}(r) = N(r)$

The very presence of matter violates C, CP and CPT!
[assuming (# of particles) $\neq$ (# of antiparticles)]

$\Rightarrow$ Fake (extrinsic) $\not{CP}$ which may complicate the study of fundamental (intrinsic) $\not{CP}$

◊ $\not{CP}$ in matter

Exists even in 2f case (in $\geq$ 3f case exists even when all $\{\delta_{\rm CP}\} = 0$) due to matter effects:

$$P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$$

E.g., MSW effect can enhance $\nu_e \leftrightarrow \nu_\mu$ and suppress $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ or vice versa.

Survival probabilities are not CP-invariant:

$$P(\nu_a \to \nu_a) \neq P(\bar{\nu}_a \to \bar{\nu}_a)$$

To disentangle fundamental $\not{CP}$ from the matter induced one in LBL experiments – need to measure energy dependence of oscillated signal or signal at two baselines – a difficult task

(Difficult) alternatives:

- Low-E LBL experiments ($E \sim$ 0.1 – 1 GeV, $L \sim$ 100 – 1000 km) (Koike & Sato, 1999; Minakata & Nunokawa, 2000, 2001);
- Indirect measurements:
  (A) CP-even terms $\sim \cos\delta_{\rm CP}$ (Lipari, 2001)
  (B) Area of leptonic unitarity triangle (Farzan & Smirnov, 2002; Aguilar-Saavedra & Branco, 2000; Sato, 2000)

$\not{CP}$ cannot be studied in SN ν experiments because of experimental indistinguishability of low-energy ν_μ and ν_τ

◇ $\not{T}$ in matter

CPT not conserved in matter $\Rightarrow$ $\not{CP}$ and $\not{T}$ are not related!

- Matter does not necessarily induce $\not{T}$ (only asymmetric matter with $\tilde{N}(r) \neq N(r)$ does)
- There is no $\not{T}$ (either fundamental or matter induced) in 2f case – a consequence of unitarity:

$$P_{ee} + P_{e\mu} = 1$$
$$P_{ee} + P_{\mu e} = 1$$
$$\Rightarrow \qquad P_{e\mu} = P_{\mu e}$$

- In 3f case – only one T-odd probability difference for ν's (and one for $\bar{\nu}$'s) irrespective of matter density profile – a consequence of unitarity in 3f case (Krastev & Petcov, 1988):

$$\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$$

Matter-induced $\not{T}$:

(1) An interesting, pure 3f matter effect; absent in symmetric matter (e.g., $N(r) = const$)

(2) Does not vanish in the regime of complete averaging

(3) May fake fundamental $\not{T}$ and complicate its study

(4) Vanishes when either $U_{e3} = 0$ or $\Delta m^2_{21} = 0$ (2f limits) $\Rightarrow$ doubly suppressed by both these small parameters

$\Rightarrow$ Perturbation theory can be used to get analytic expressions

General structure of T-odd probability differences:

$$\Delta P^T_{e\mu} \;=\; \underbrace{\sin\delta_{CP}\cdot Y}_{\text{fundam. } \not{T}} \;+\; \underbrace{\cos\delta_{CP}\cdot X}_{\text{matter-ind. } \not{T}}$$

In adiabatic approximation: $X = J_{\rm eff}\cdot(\text{oscillating terms})$,

$$J_{\rm eff} \;=\; s_{12}\,c_{12}\,s_{13}\,c_{13}^2\,s_{23}\,c_{23}\,\frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

(E.A., Huber, Lindner & Ohlsson, 2001)

Compare with the vacuum Jarlskog invariant:

$$J \;=\; s_{12}\,c_{12}\,s_{13}\,c_{13}^2\,s_{23}\,c_{23}\,\sin\delta_{CP}$$

$$\Rightarrow \qquad \sin\delta_{CP} \;\Leftrightarrow\; \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

To extract fundamental $\not{T}$ need to measure:

$$\Delta P_{ab} \;\equiv\; P_{\rm dir}(\nu_a\to\nu_b) - P_{\rm rev}(\nu_b\to\nu_a) \;\propto\; \sin\delta_{CP}$$

Even survival probabilities P_{aa} $(a=\mu,\tau)$ can be used!
(Fishbane & Kaus, 2000)

$$P_{\rm dir}(\nu_a\to\nu_a) - P_{\rm rev}(\nu_a\to\nu_a) \sim \sin\delta_{CP} \qquad (a\neq e)$$

In 3f case P_{ee} does not depend on δ_{CP} (Kuo & Pantaleone, 1987; Minakata & Watanabe, 1999) – not true if $\nu_{\rm sterile}$ is present!

Matter-induced $\not{T}$ in LBL experiments due to imperfect sphericity of the Earth density distribution cannot spoil the determination of δ_{CP} if the error in δ_{CP} is $> 1\%$ at 99% C.L. (E.A., Huber, Lindner & Ohlsson, 2001)

$\Rightarrow$ No need to interchange positions of ν source and detector!

Experimental study of $\not{T}$ difficult because of problems with detection of $e^{\pm}$

Matter-induced $\not{T}$:

- Negligible effects in terrestrial experiments
- Cannot be observed in supernova ν oscillations due to experimental indistinguishability of low $- E$ ν_μ and ν_τ
- Can affect the signal from ~GeV neutrinos produced in annihilations of WIMPs inside the Sun (de Gouvêa, 2000)

The problem of U_{e3}

- The least known of leptonic mixing parameters
- Discriminates between various neutrino mass models
  (Barr & Dorsner, 2000; Tanimoto, 2001)
- Unexplained smallness (related to $\Delta m^2_{\odot}/\Delta m^2_{\rm atm}$?)
- The (likely) bottleneck for studying fundamental $\not{CP}$ and $\not{T}$ effects and matter-induced $\not{T}$ in neutrino oscillations
- Important for measuring the sign of Δm^2_{31} in future LBL experiments (neutrino factories!) – direct vs inverted ν mass hierarchy
- Governs subdominant oscillations of atmospheric neutrinos
- Governs the Earth matter effects on supernova neutrino oscillations
- The only opportunity to see the "canonical" MSW effect (strong matter enhancement of small mixing)?

4f neutrino oscillations

If LSND is right $\Rightarrow$ 3 different Δm^2 necessary:

$$\Delta m^2_\odot , \qquad \Delta m^2_{\rm atm} , \qquad \Delta m^2_{\rm LSND}$$

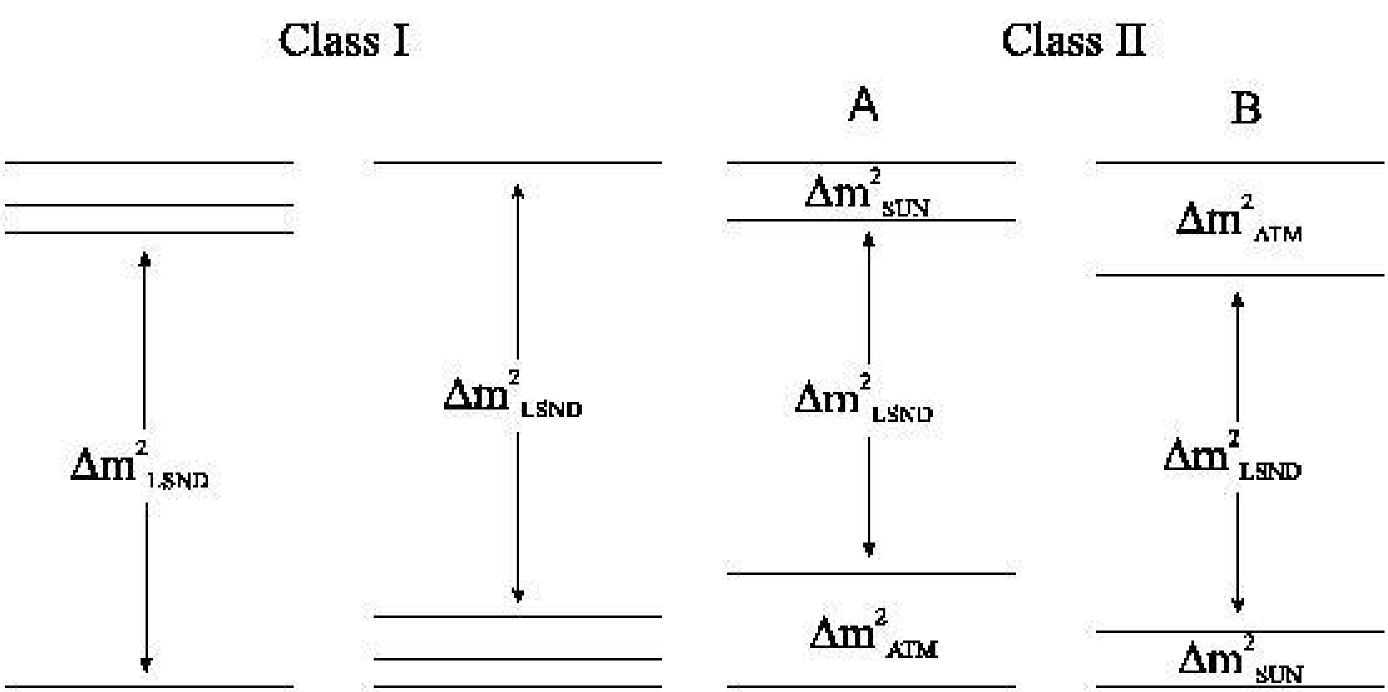
$\Rightarrow$ 4 light neutrino species: $\nu_e, \; \nu_\mu, \; \nu_\tau, \; \nu_s$

(An alternative: strong CPT violation in neutrino sector, $(\Delta m^2)_{\nu\nu} \neq (\Delta m^2)_{\overline{\nu\nu}}$ – Murayama & Yanagida, 2000; Barenboim, Borissov, Lykken & Smirnov, 2001; Barenboim, Beacom, Borissov & Kayser, 2002)

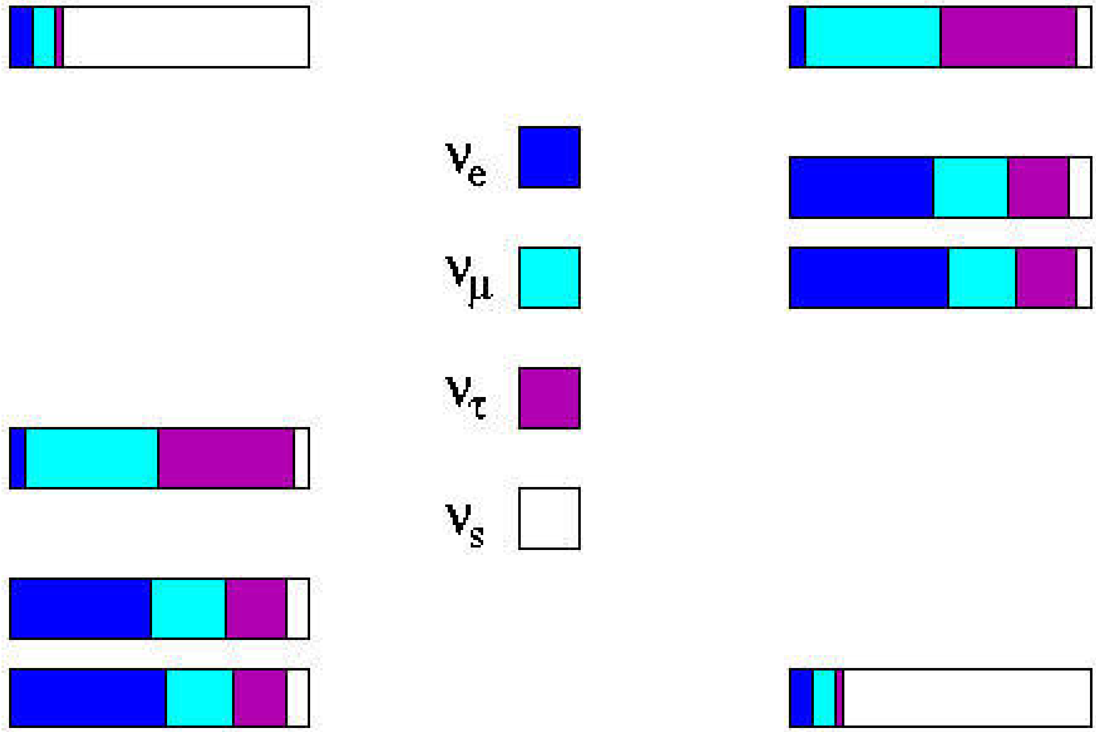
4 flavours $\Rightarrow$ 6 mixing angles θ_{ij} , 3 Dirac-type $\not{CP}$ phases

A simplification: Only 2 classes of 4f schemes can fit the data,

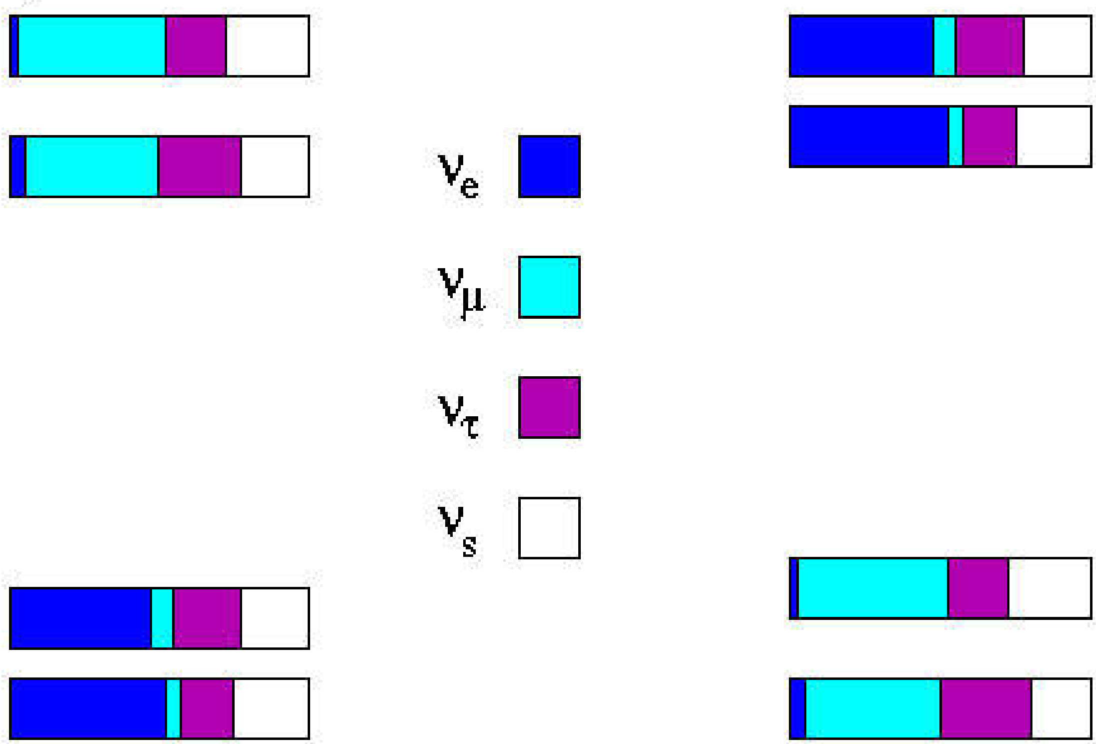
(3+1) and (2+2)



(3 + 1):



(2 + 2):



$(3+1)$ schemes:

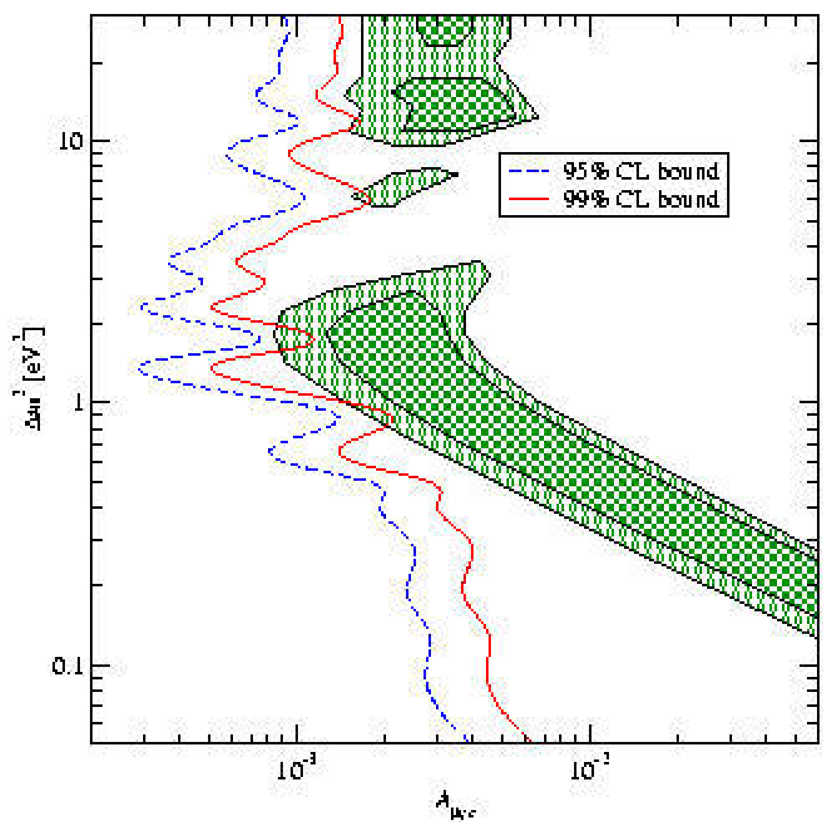
$$\nu_4 \simeq \nu_s + \mathcal{O}(\epsilon)\cdot(\nu_e,\ \nu_\mu,\ \nu_\tau)\,, \qquad \epsilon \ll 1$$

$(\nu_1,\ \nu_2,\ \nu_3)$ – usual linear combinations of $(\nu_e,\ \nu_\mu,\ \nu_\tau)$ + small ($\sim \epsilon$) admixtures of ν_s.

$$\sin^2 2\theta_{\rm LSND} = 4\,|U_{e4}\,U_{\mu 4}|^2 \sim \epsilon^4$$

Strong upper bounds on $|U_{e4}|$ and $|U_{\mu 4}|$ from $\bar{\nu}_e$ and ν_μ disappearance experiments $\Rightarrow$ difficult to fit LSND data

LSND-allowed and SBL-excluded regions on $(\sin^2 2\theta_{\rm LSND},\ \Delta m^2_{\rm LSND})$ plane (Maltoni, Schwetz & Valle, 2001):



But: different stat. analysis gives bigger overlap

$(2+2)$ schemes:

ν_e predominantly in the pair responsible for $\nu_\odot$ oscillations, ν_μ in the pair responsible for $\nu_{\rm atm}$ oscillations:

$$\nu_{\rm atm}\ \text{osc.:}\quad \nu_\mu \leftrightarrow \nu', \qquad \nu' \simeq c_\xi\,\nu_\tau + s_\xi\,\nu_s + \mathcal{O}(\epsilon)\cdot\nu_e\,,$$

$$\nu_\odot\ \text{osc.:}\quad \nu_e \leftrightarrow \nu'', \qquad \nu'' \simeq -s_\xi\,\nu_\tau + c_\xi\,\nu_s + \mathcal{O}(\epsilon)\cdot\nu_\mu\,,$$

$$\sin^2 2\theta_{\rm LSND} \sim \epsilon^2$$

Involvements of ν_s in $\nu_\odot$ and $\nu_{\rm atm}$ oscillations – sum rule (Peres & Smirnov, 2000):

$$|\langle\nu_s|\nu''\rangle|^2 + |\langle\nu_s|\nu'\rangle|^2 = c_\xi^2 + s_\xi^2 = 1$$

SK atm. data: $\sin^2\xi < 0.25$ (90% C.L); < 0.36 (99% C.L.) (Messier, 2001)

(Pre-SNO NC) solar ν data:

$\sin^2\xi > 0.3$ (Lisi, 2000; Giunti, 2000);
$\sin^2\xi > 0.7$ (90% C.L.); $\sin^2\xi > 0.48$ (99% C.L.) for LMA (Gonzalez-Garcia, Maltoni & Peña-Garay, 2001; Maltoni *et al.*, 2001)

$\Rightarrow$ (2+2) scenarios strongly disfavoured

$\longleftarrow$ – $\longrightarrow$

Matter effects on 4f oscillations: (Dooling, Giunti, Kang, Kim, 1999)
$\not{CP}$: several observables; large effects possible (in general, no suppression due to small $\Delta m^2_\odot$); also large $\not{T}$ effects possible

Conclusions

- Solar and atm. ν data imply oscillations between 3 neutrino flavours, ν_e, ν_μ and ν_τ; with LSND $\Rightarrow$ ν_s should exist
- 2f analyses give a good first approximation due to $|U_{e3}| \ll 1$, $\Delta m^2_{21} \ll \Delta m^2_{31}$. But: increasing accuracy of the data makes $\geq$ 3f description necessary
- 3f effects in solar, atm., reactor and LBL accel. experiments may be quite important $\Rightarrow$ up to $\sim 10\%$ corrections to oscillation probabilities + specific $\geq$ 3f effects
- Manifestations of ≥ 3 flavours in neutrino oscillations:
  - ◇ Fundamental $\not{CP}$ and $\not{T}$
  - ◇ Matter-induced $\not{T}$
  - ◇ Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
  - ◇ Specific CP and T conserving interference terms in oscillation probabilities
- U_{e3} plays a very special role
- In 4f case large $\not{CP}$ and (both fundamental and matter-induced) $\not{T}$ effects possible. But: 4f scenarios disfavoured by the data