Volkas - 01

LEPTON ASSYMETRIES FROM NEUTRINO OSCILLATIONS

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1. BRIEF OVERVIEW

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}$$ can be generated by $\nu_\alpha \leftrightarrow \nu_s$ & $\bar{\nu}_\alpha \leftrightarrow \bar{\nu}_s$ oscillations at $T \sim$ few 10's of MeV provided that

$\Delta m^2 < 0$ and θ_0 small

$$|\nu_\alpha\rangle = \cos\theta_0 |\nu_a\rangle + \sin\theta_0 |\nu_b\rangle$$
$$|\nu_s\rangle = -\sin\theta_0 |\nu_a\rangle + \cos\theta_0 |\nu_b\rangle$$
$$\Delta m^2 \equiv m_b^2 - m_a^2 < 0 \Rightarrow \text{"}m_{\nu_s}\text{"} < \text{"}m_{\nu_\alpha}\text{"}$$

Seeded by CP asymmetric background (i.e. baryon/electon asym.)

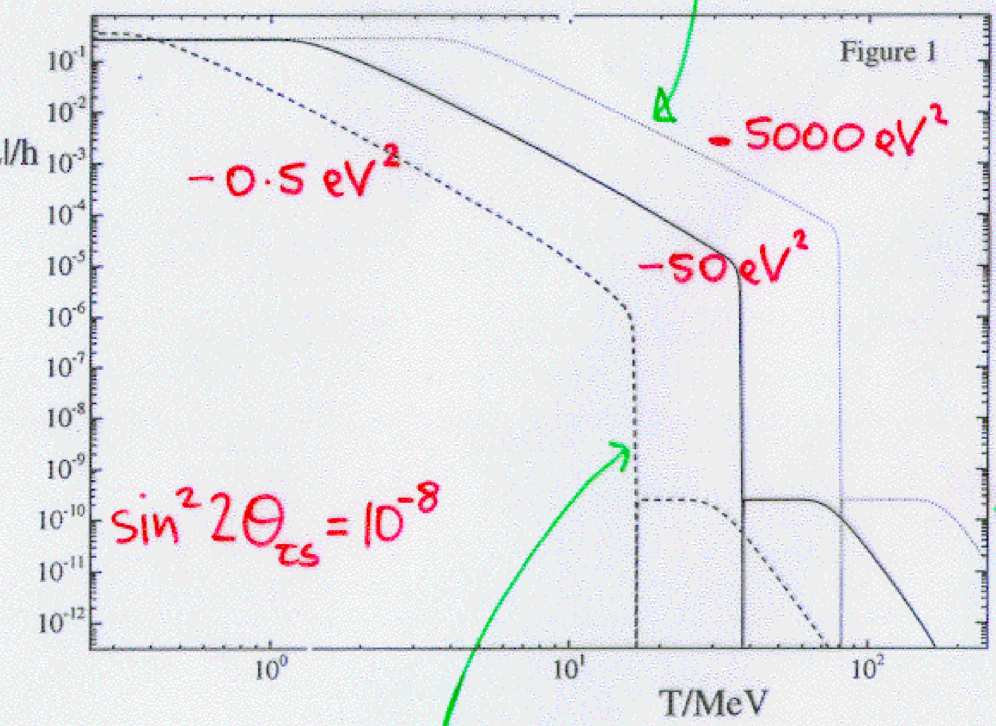
"runaway positive feedback"

(Foot, Thomson, RV)

Taken from R. Foot, Astropart. Phys. (1999)

power law

$$L \equiv \frac{n_{\nu_\tau} - n_{\bar{\nu}_\tau}}{n_\gamma}$$



← B asym.

exponential

27

Why ν_s?

* 2-fold maximal mixing
  (mirror or pseudoDirac)

* solar + atmos + LSND

Large enough L_{ν_α} suppresses ν_s production

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (b - a - \cos 2\theta_0)^2}$$

$$a \equiv -\frac{2p}{\Delta m^2} V_{Wolfenstein}$$

$$V_{Wolf} = -\frac{2\sqrt{2}\,\zeta(3)}{\pi^2} T^3 L^{(\beta)}$$

B, e asym.

$$L^{(\beta)} \equiv L_{\nu_\beta} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + O(10^{-10})$$

large $L^{(\beta)} \Rightarrow \sin^2 2\theta_m$ small

$$b \equiv -\frac{2\sqrt{2}\,\zeta(3)}{\pi^2} A_\beta \frac{G_F}{m_W^2} p T^4$$

finite T term – important in early universe

(Notzold-Raffelt)

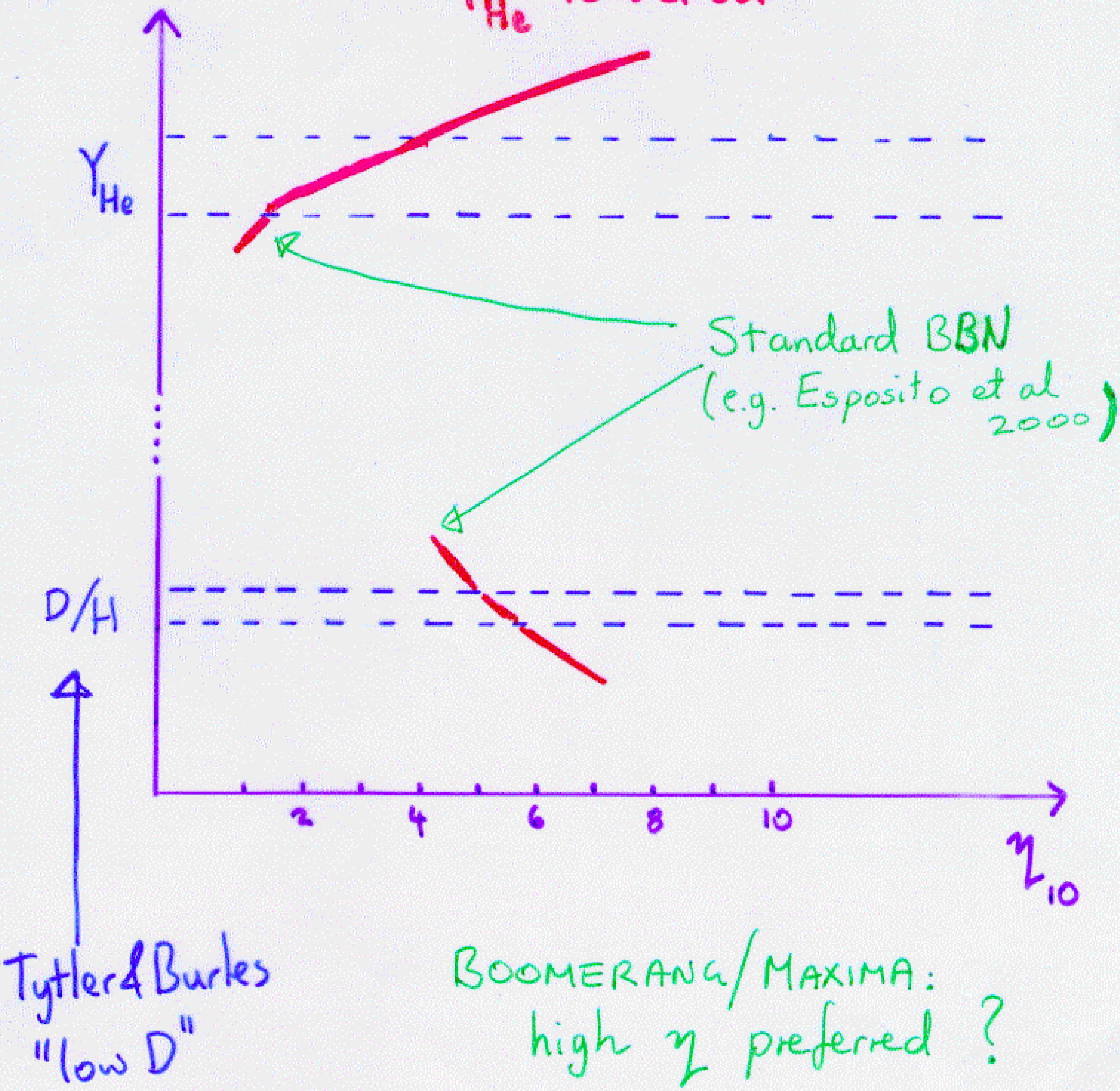
L_{ν_e} and Helium abundance

$\frac{n}{p}$ at freeze-out $\longrightarrow$ Helium abundance

$\bar{\nu}_e p \leftrightarrow e^+ n \quad , \quad \nu_e n \leftrightarrow e^- p$

$L_{\nu_e} > 0 \Rightarrow \frac{n}{p}$ lowered

& Y_{He} lowered



BOOMERANG/MAXIMA: high η preferred ?

Lepton domains?

Inhomogeneous baryon density can seed L_{ν_e} of different signs in different domains

$\Rightarrow$ inhomogeneous BBN (P. Di Bari)

Chaoticity?

Is L_ν generation sensitive to initial conditions?

(Foot, Thomson, RV ; Shi ; Shi+Fuller)

For large region in $(\Delta m^2, \sin^2 2\theta_0)$ space answer is No. (Di Bari + Foot)

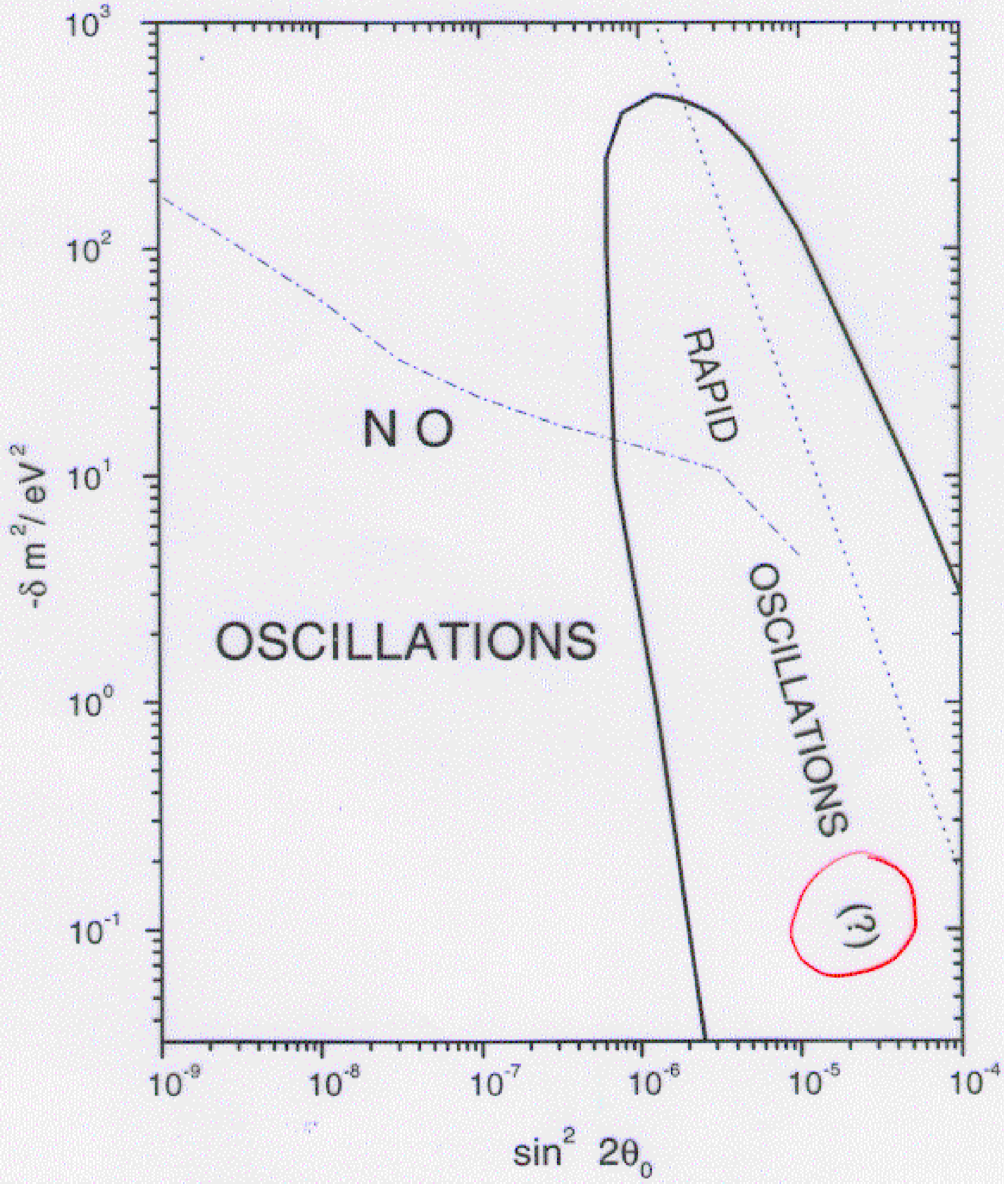
Remaining region unclear:

rapid L_ν oscs. vs numerical error

$\Rightarrow$ Open question.

Di Bari & Foot PRD (~~in press~~, 2000)

figure 7



2. Quantum Kinetic Equations

reduced density matrix:

$$\rho(p) \equiv \frac{1}{2}\left[P_0(p) + \mathbb{P}(p)\cdot\sigma\right] = \frac{1}{2N^{eq}}\begin{pmatrix} 2N_\alpha & P_x - iP_y \\ P_x + iP_y & 2N_s \end{pmatrix}$$

$N_f(p,T)$, distribution fn, $n_f = \int N_f \, dp$.

$P_{x,y} \equiv$ coherences.

Similarly $\bar{\rho}$ for $\bar{\nu}$ system.

N^{eq} = FD dist. with $\mu = 0$.

$$\boxed{\begin{aligned} \frac{\partial \mathbb{P}(p)}{\partial t} &= \mathbb{V}(p) \times \mathbb{P}(p) - D(p)\,\mathbb{P}_T(p) + R(p)\,\hat{z} \\ \frac{\partial P_0(p)}{\partial t} &= R(p) \simeq \Gamma(p)\left[\frac{N^{eq}(p,\mu)}{N^{eq}(p,0)} - \frac{1}{2}\left[P_0(p) + P_z(p)\right]\right] \end{aligned}}$$

QKEs

$$\mathbb{V}(p) = \beta(p)\,\hat{x} + \lambda(p)\,\hat{z}$$

$$\beta \equiv \frac{\Delta m^2}{2p} \sin 2\theta_0$$

$$\lambda \equiv V_{eff} - \frac{\Delta m^2}{2p} \cos 2\theta_0$$

$$V_{eff} = V_{Wolf} + V_T \equiv \frac{\Delta m^2}{2p}(-a + b)$$

prev. defined

$\mathbb{V} \times \mathbb{P}$ term $\rightarrow$ coherent non-linear MSW evolution

$D(p) = \frac{1}{2}\Gamma(p)$; $\Gamma(p) \simeq G_F^2 T^5 \frac{p}{\langle p \rangle}$

total coll. rate for ν_α's of momentum p.

$\mathbb{P}_T \equiv P_x \hat{x} + P_y \hat{y}$ coherences

$D\mathbb{P}_T$ term $\rightarrow$ collisional decoherence
tries to exp. damp P_x, P_y

R term $\rightarrow$ repopulation of ν_α dist. from the background

$R \sim 0$ apparently good approx. when repop. rapid ($T > T_{\nu\,\text{decoupling}}$)

QKEs + conservation of $\alpha + s$ lepton no.

$$\Rightarrow \boxed{\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{2n_\gamma} \int \beta (P_y - \bar{P}_y) N^{eq}(p,0)\, dp}$$

* redundant but num. useful b/c $L = (\text{large}\ \#) - (\text{large}\ \#)$

* non-linearity & feedback b/c V_{eff} depends on L_{ν_α}

3. ANALYTIC INSIGHT & WHY $L_{final} \simeq \frac{3}{8}$

QKEs: $$\frac{\partial}{\partial t}\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} \simeq \begin{pmatrix} -D & -\lambda & 0 \\ \lambda & -D & -\beta \\ 0 & \beta & 0 \end{pmatrix}\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}$$

or $$\frac{\partial}{\partial t}\mathbb{P} \simeq \mathbb{K}\,\mathbb{P}$$

Instantaneous diagonal basis: $\mathbb{Q} = \mathbb{U}\,\mathbb{P}$

$$\frac{\partial}{\partial t}\mathbb{Q} \simeq \mathbb{K}_{diag}\,\mathbb{Q} - \mathbb{U}\frac{\partial \mathbb{U}^{-1}}{\partial t}\mathbb{Q} \simeq \begin{pmatrix} k_1 & \cdot & \cdot \\ \cdot & k_2 & \cdot \\ \cdot & \cdot & k_3 \end{pmatrix}\mathbb{Q}$$

in adiabatic limit

$$\Rightarrow \mathbb{P}(t) \simeq \mathbb{U}^{-1}(t)\begin{bmatrix} e^{\int_0^t k_1 dt'} & \cdot & \cdot \\ \cdot & e^{\int_0^t k_2 dt'} & \cdot \\ \cdot & \cdot & e^{\int_0^t k_3 dt'} \end{bmatrix}\mathbb{U}(0)\,\mathbb{P}(0) \quad \cdots (*)$$

where
$$k_1 = k_2^* = -d + i\omega$$
$$k_3 = -\,\beta^2 D / d^2 + \omega^2$$
$$d = D + \frac{k_3}{2}$$
$$\omega^2 = \lambda^2 + \beta^2 + k_3 D + \frac{3}{4}k_3^2$$

(Bell, RV, Wong PRD 1999)

(RV+ Wong, in prep.)

Roughly 2 regimes:

higher T & collision-dominated

$D \sim T^5$, $\lambda \sim T^5$, $\beta \sim T^{-1}$

$d \simeq D$, $\omega \simeq \lambda$, $k_3 \simeq -\dfrac{\beta^2 D}{D^2+\lambda^2+\beta^2} \ll D$

Put $e^{\int k_{1,2}} = 0$ "collision dominance"

Then, $P_y(t) = \dfrac{k_3}{\beta} P_z(t)$ from eqn. (*),

& $\dfrac{dL}{dt}$ eqn on p6 becomes

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{2n_\gamma} \int \frac{\sin^2 2\theta_0 \, \Gamma_\alpha \, a \left[\cos 2\theta_0 - b\right] \left(N_\alpha + \bar{N}_\alpha - N_s - \bar{N}_s\right)}{\left[x + (c_{2\theta_0} - b + a)^2\right]\left[x + (c_{2\theta_0} - b - a)^2\right]} dp \qquad (**)$$

Boltzmann-like rate eqn.

For $\Delta m^2 < 0$, $\cos 2\theta_0 \simeq 1$ the $\cos 2\theta_0 - b$ term changes sign:

$$\frac{dL}{dt} \sim -L \xrightarrow[\text{to}]{\text{evolves}} \frac{dL}{dt} \sim +L$$

destruction $\qquad$ exponential growth

IMPORTANT: When $D=0$ (no collisions), low T

$$\text{Eqn}(**) \Rightarrow \frac{dL}{dt} = 0$$

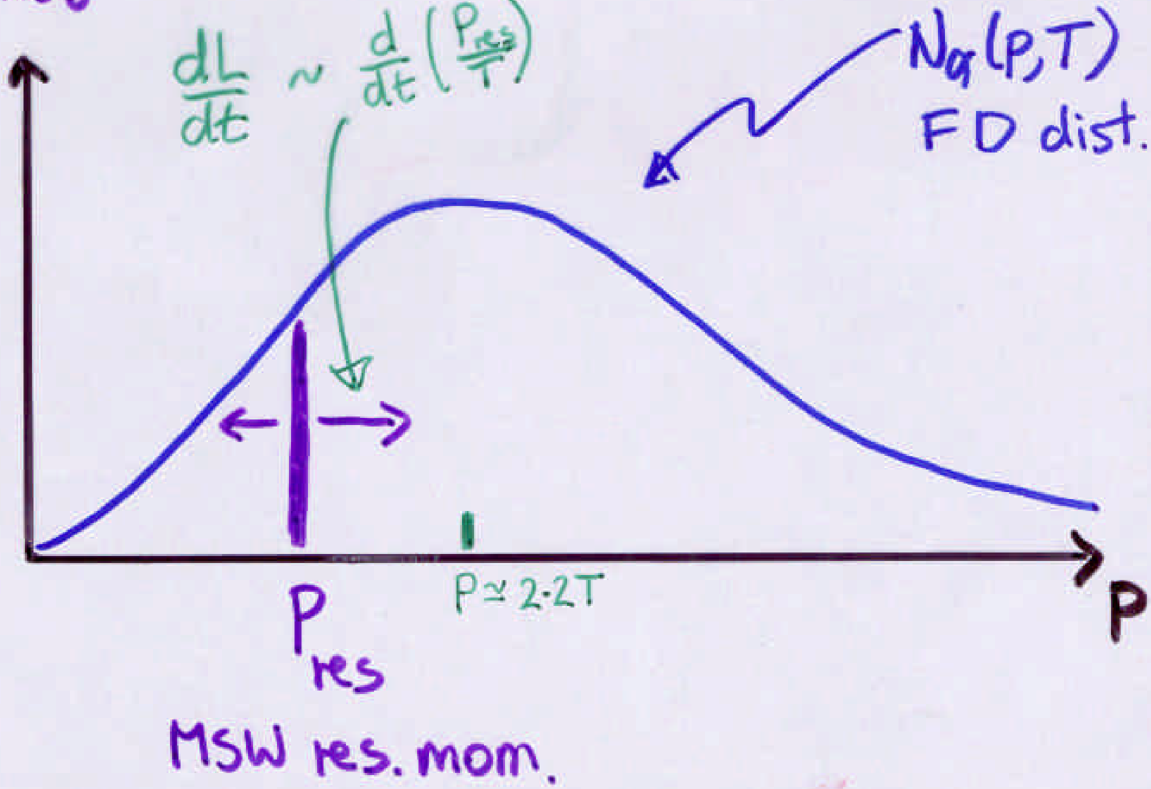
BUT ...

lower T & MSW-dominated

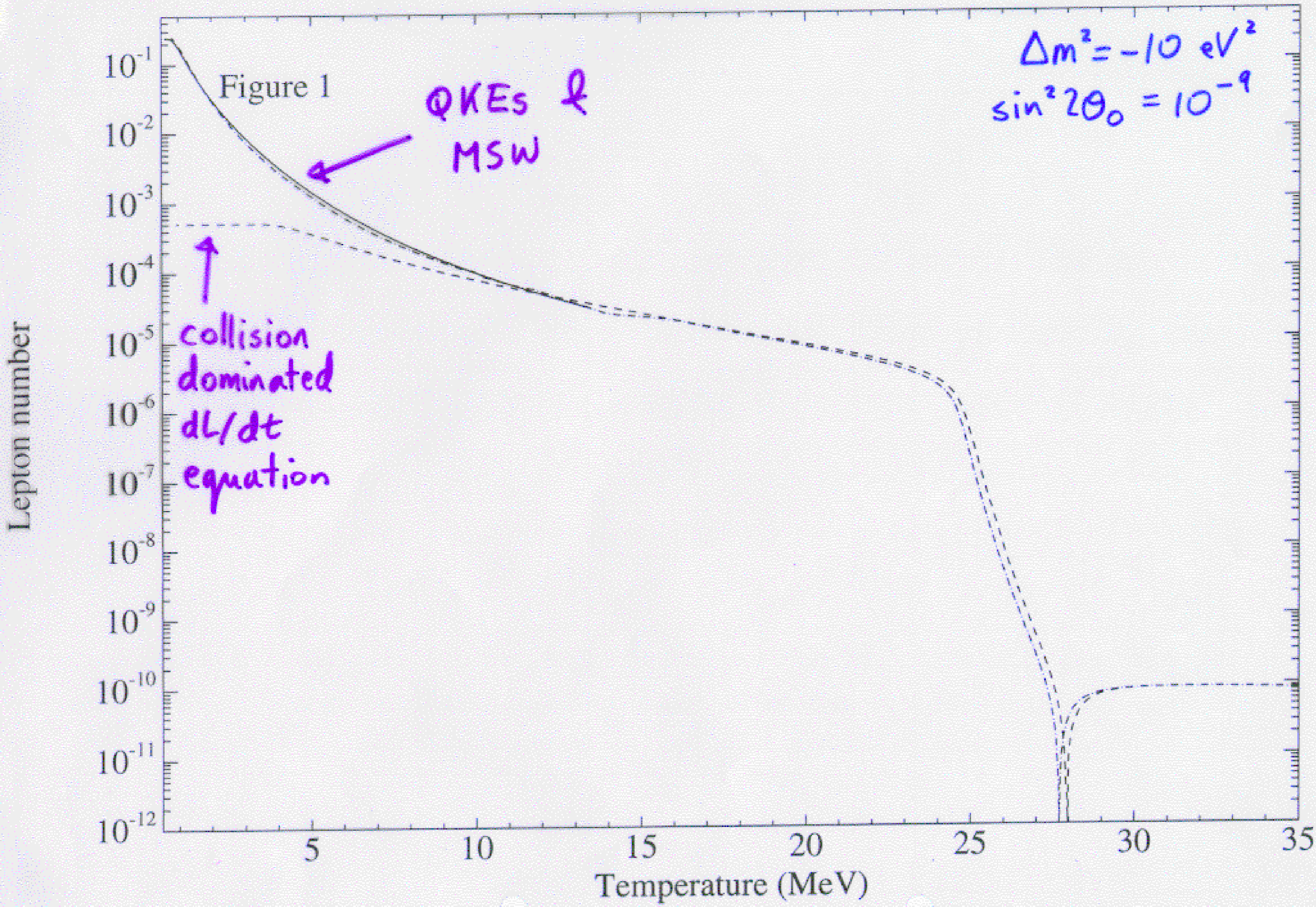
when D small, assumptions behind (**) no longer hold. In fact at $D=0$:

$$k_3 = 0\,, \quad k_1 = k_2^* = i\sqrt{\lambda^2+\beta^2}$$

& Eqns (*) become just the early universe (non-linear) adiabatic MSW Eqns!



Foot & RV, PRD56, 6653(1997)



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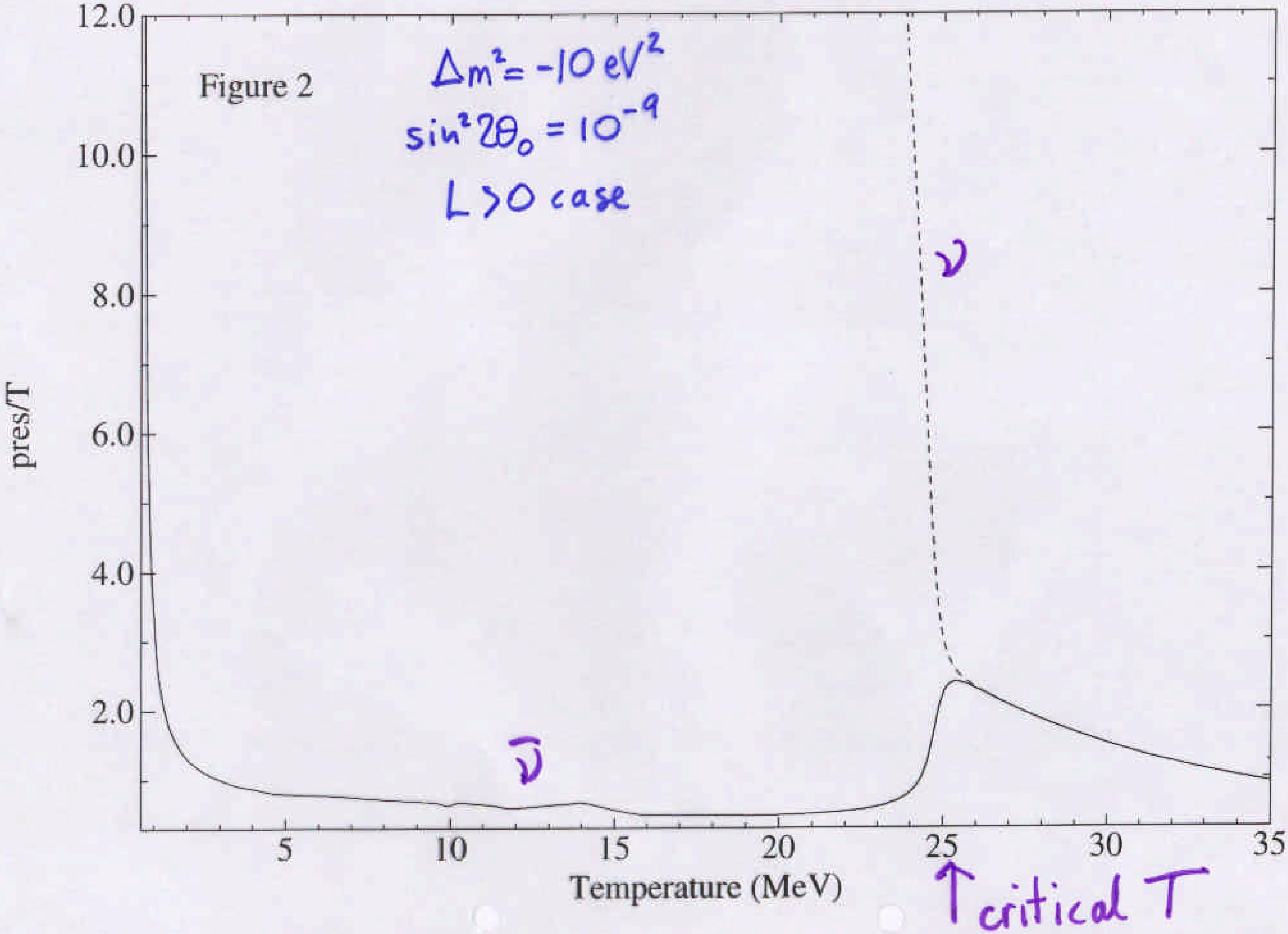

Bell, RV, Wong: adiabatic, narrow res. width

MSW $\Rightarrow$

$$\frac{dL_{\nu_\alpha}}{dT} = -\left[\frac{\bar{N}_\alpha(P_{res}) - \bar{N}_s(P_{res})}{n_\gamma}\right] T \left| \frac{d}{dT}\left(\frac{P_{res}}{T}\right) \right|$$

(first derived heuristically in Foot, RV 1997)

resonance conditions:

$$\left(\frac{P_{res}}{T}\right)_\nu = \frac{\alpha + \sqrt{\alpha^2 + 4\beta}}{2\beta}$$

$$\left(\frac{P_{res}}{T}\right)_{\bar{\nu}} = \frac{-\alpha + \sqrt{\alpha^2 + 4\beta}}{2\beta}$$

$a \equiv \alpha \frac{P}{T}$

$b \equiv \beta \left(\frac{P}{T}\right)^2$

$\beta \ll \alpha$ & $\alpha \sim L \Rightarrow \left(\frac{P_{res}}{T}\right)_\nu \longrightarrow \infty$ as L grows

$\beta \sim T^6$

But $\left(\frac{P_{res}}{T}\right)_{\bar{\nu}} \simeq \dfrac{-\alpha + \alpha\left[1 + \frac{1}{2}\cdot\frac{4\beta}{\alpha^2}\right]}{2\beta} = \dfrac{1}{\alpha}$

i.e. $\left(\frac{P_{res}}{T}\right)_{\bar{\nu}} = \dfrac{\pi^2 |\Delta m^2|}{8\sqrt{2}\,\zeta(3)\,G_F} \dfrac{1}{L_{\nu_\alpha} T^4}$

$\frac{P}{T}$ const $\Rightarrow L \sim T^{-4}$

pwr law phase

Evaluate $\frac{d}{dT}\left(\frac{P_{res}}{T}\right)_{\bar{\nu}}$ & use $\frac{dL}{dT}$ eqn to get

$$\frac{dL_{\nu_\alpha}}{dT} = -\frac{4X\left(\frac{P_{res}}{T}\right)_{\bar{\nu}}}{T\left[1 + \frac{X}{L_{\nu_\alpha}}\left(\frac{P_{res}}{T}\right)_{\bar{\nu}}\right]}$$

$$X \equiv \frac{I}{n_\gamma}(\bar{N}_\alpha - \bar{N}_s)$$

$$L_{\nu_\alpha} \ll 1 \Rightarrow \frac{dL_{\nu_\alpha}}{dT} \simeq -\frac{4L_{\nu_\alpha}}{T}$$

$$\Rightarrow L_{\nu_\alpha} \sim T^{-4}$$

MSW effect keeps L_{ν_α} growing!

Eventually, $\left(\frac{P_{res}}{T}\right)_{\bar{\nu}} \longrightarrow \infty$ adiabatically converting the FD-distributed $\bar{\nu}_\alpha$'s into steriles.

So ...

$$L_{final} \sim \frac{1}{4\zeta(3)} \int_{(p/T)_{initial}}^{\infty} \frac{x^2\,dx}{1+e^x} \simeq \frac{3}{8}$$

4. CONCLUSIONS

* Active-sterile oscs. will produce asymmetries $\sim \frac{3}{8}$ in appropriate osc. parameter range.
* Applications: ν_s suppression prior to BBN
  ν_e asym & He abundance
  lepton domains?
  chaoticity/rapid L oscs?
* Numerical soln to full QKEs has been understood analytically/physically.