# CALCULATION of the ATMOSPHERIC NEUTRINO RATES

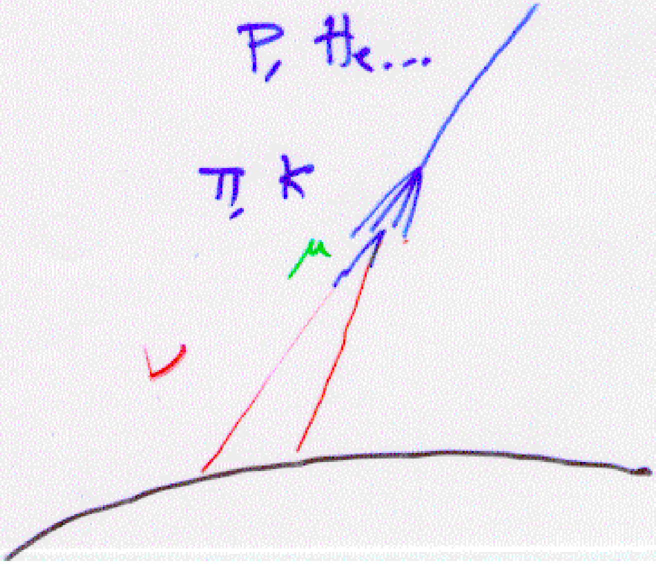
- Primary Cosmic Ray Fluxes
- Geomagnetic Effects
- Hadronic cross sections
- Shower Model
- Neutrino Cross Section.

"Standard Model"

- Detector Acceptance and Efficiency → No comments
- NEW PHYSICS
  beyond the Standard Model

Paolo Lipari

$\nu$–2000, 17-june-2000



Many thanks to my collaborators
in these calculations

Ralph Engel
Tom Gaisser
Todor Stanev
Giuseppe Battistoni
Alfredo Ferrari
M. Honda
Takaaki Kajita

## Calculations of the atmospheric neutrino fluxes

- "HONDA et al."
  M. Honda, T. Kajita, K. Kasahara & S. Midorikawa,
  Phys. Rev. D 52, 4985 (1995).

- BARTOL
  G. Barr, T. K. Gaisser and T. Stanev
  Phys. Rev. D 39 3532 (1989)
  V. Agrawal, T. K. Gaisser, P. Lipari & T. Stanev,
  Phys. Rev. D 53 1314. (1996)

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## Preliminary 3–Dimensional Calculations

- FLUKA
  G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P. R. Sala,
  T. Rancati Astroparticle Physics 12, 315 (2000).

- "Canadian"
  Y. Tserkovnyak, R. Komar, C. Nally and C. Waltham, hep-ph/9907450. ( paper P12 )

- P.L. hep-ph/002282, hep-ph/0003013
  Astroparticle Physics to appear (2000).

No new "certified" calculation" has been made available.

A lot of work has been performed by the different groups
in part in collaboration, in part independently.

QUESTION A:

Is the need for NEW PHYSICS in the description of Atmospheric Neutrinos solidly "ESTABLISHED" ?

QUESTION B:

Are the "allowed intervals" in the $\nu$ oscillation parameter space obtained interpreting the atmospheric $\nu$ data free of systematic biases ?

"The" burning question for LBL $\nu$–beams and the planning of the best strategy for future experimental studies:

**Is the allowed interval in $\Delta m^2$ correctly estimated ?**

Need detailed Study.

# EVIDENCE FOR NEW PHYSICS

THREE sources (Qualitatively)

- UP/DOWN asymmetry of $\mu$–like events

  A "smoking gun"

- $\mu/e$ ratio lower than expectations

  Very robust

- Distortion of the angular distribution of up–going ($\nu$–induced) muons.

  More model dependent

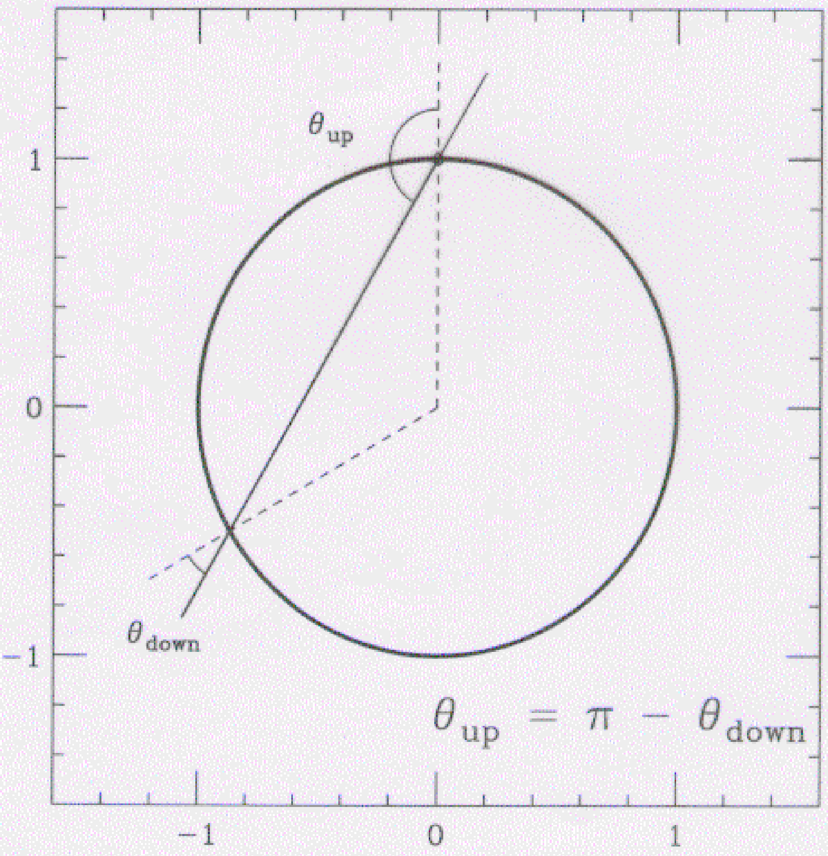
# UP/DOWN asymmetry

An elementary GEOMETRY THEOREM. **If:**

1. The Earth is spherically symmetric
2. The Cosmic Ray fluxes are isotropic

**Then:** The atmospheric neutrino fluxes are Up–Down symmetric.

$$\phi_{\nu_\alpha}(E_\nu, \theta) = \phi_{\nu_\alpha}(E_\nu, \pi - \theta)$$



"All" ν's cross the Earth' surface twice. Down-going Up-going.

# UP/DOWN asymmetry

The prediction is **UNAMBIGUOUS** :

For Super–Kamiokande:

$$\left(\frac{\text{UP}}{\text{DOWN}}\right)_{\text{sub GeV}}^{\text{no osc}} \simeq 1 + \ (\sim 10\%)$$

$$\left(\frac{\text{UP}}{\text{DOWN}}\right)_{\text{multi GeV}}^{\text{no osc}} \simeq 1 + \ (\sim 1\%)$$

The corrections to exact symmetry are:

- Of *opposite* sign with the observed effect
- Small
- Well understood
  - GEOMAGNETIC EFFECTS
  - Mountain above the detector
  - Average atmospheric density profile

The estimate of the "intrinsic asymmetry" will become an important problem with more SK statistics

Can (non $\nu$) BACKGROUND be a problem ?

$$\text{Bkgd(DOWN like)} > \text{Bkgd(UP like)}$$

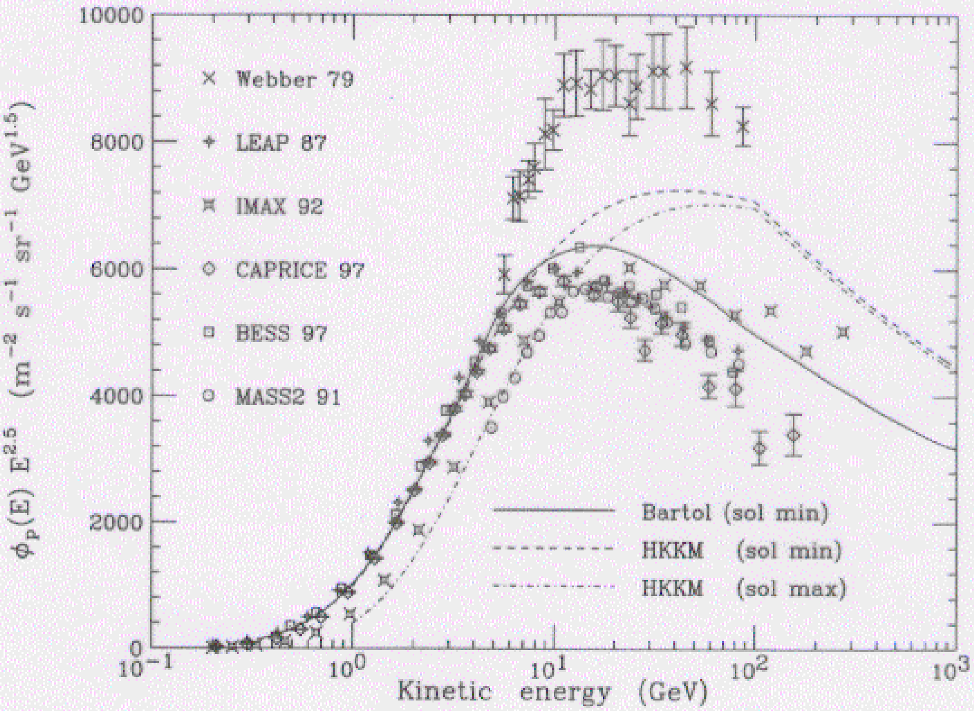
The data is convincing:

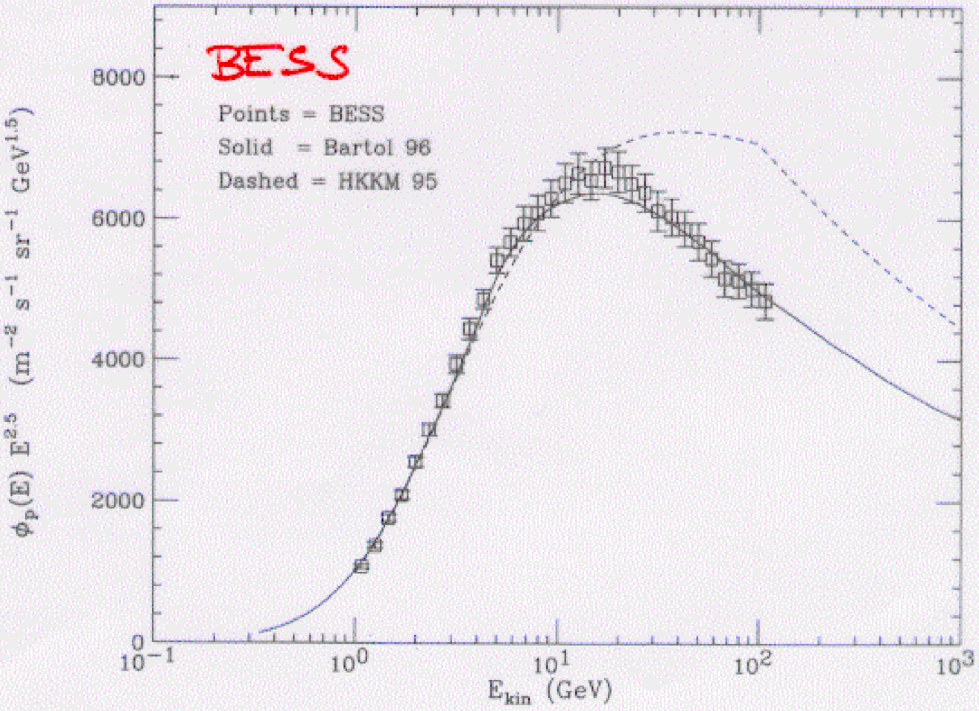
- From the distribution of vertex position it can be estimated that the background is small.
- $e$–like events are consistent with expectations.
- The deformation of the zenith angular distributions of multi–GeV events is different and *stronger* than for sub–GeV events.

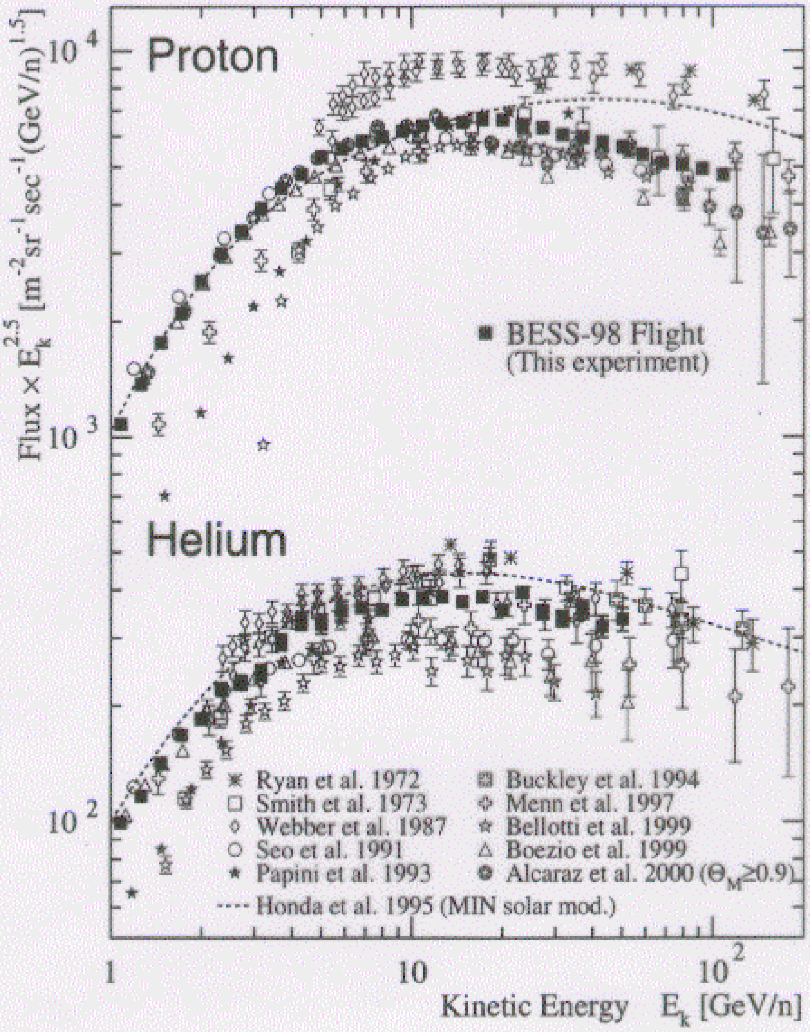
The "Background explanation" requires some peculiar properties:
(i) Mostly $\mu$–like
(ii) Energy dependence

- Oscillations are a good fit

# PROTON FLUX MEASUREMENTS







New Measurement of BESS.

[ Larger discrepancies exist for the Helium Flux ]

The Algorithm used for the primary cosmic ray flux in the atmospheric neutrino calculations:

1. Assume that the primary cosmic ray flux at 1 A.U. from the sun, in the *absence* of the Earth is ISOTROPIC: $\phi_p^\infty(E;\ t)$ $\phi^\infty(E)$

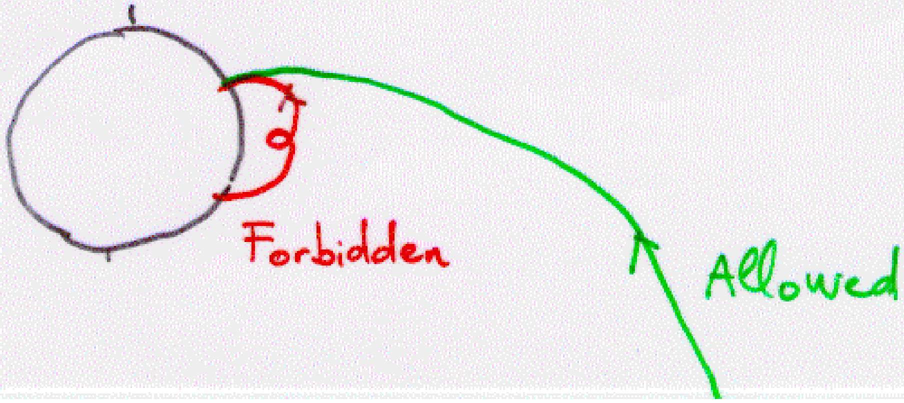
   small time variability connected with "solar modulation"

2. The flux reaching the atmosphere in the position $\vec{x}$
   - Depends on $\vec{x}$
   - Depends on both zenith and azimuth angle $\Theta$, $\varphi$.

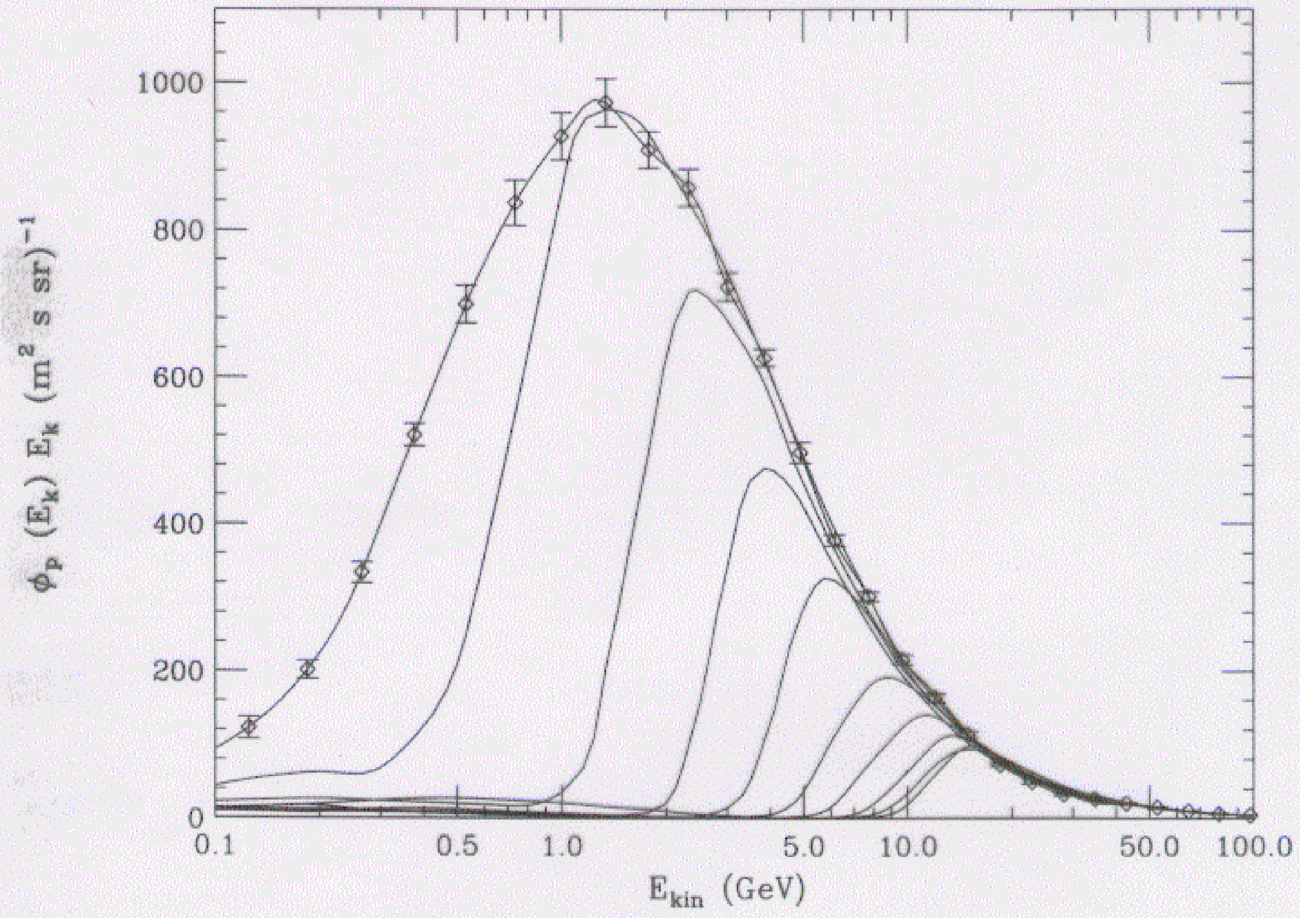
   and is:

$$\phi_p(E;\ \vec{x}, \Omega) = \begin{cases} \phi_p^\infty(E) & \text{for “allowed” trajectories,} \\ 0 & \text{for “forbidden” trajectories.} \end{cases}$$

This Algorithm is based on the **Liouville theorem** and is rigorously valid for: (i) A primary flux isotropic at large distance from the Earth; (ii) Propagation in a purely static magnetic field



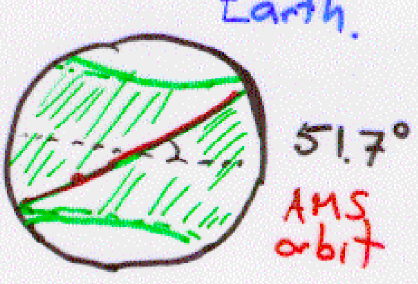
# AMS measurement of the proton flux



Measurement in different regions of Magnetic Latitude.
$E_{\rm kin}({\rm threshold}) \simeq 0.3$ GeV.

$$\Phi({\rm polar}) \simeq 2470\ \ ({\rm m}^2\ {\rm s\ sr})^{-1}$$

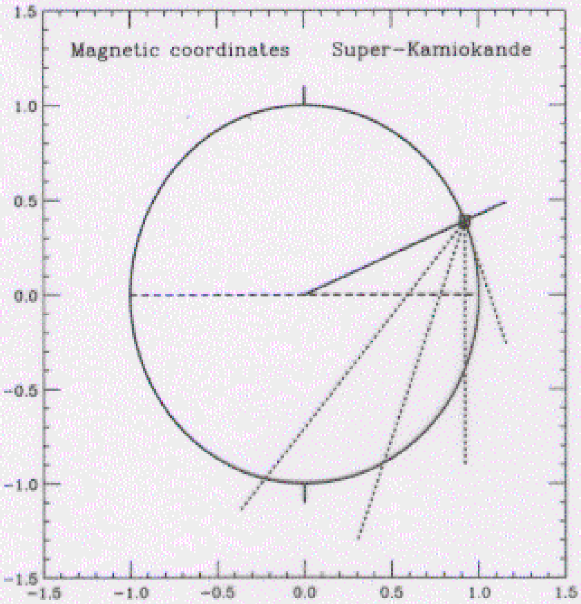
$$\Phi({\rm equator}) \simeq 140\ \ ({\rm m}^2\ {\rm s\ sr})^{-1}$$



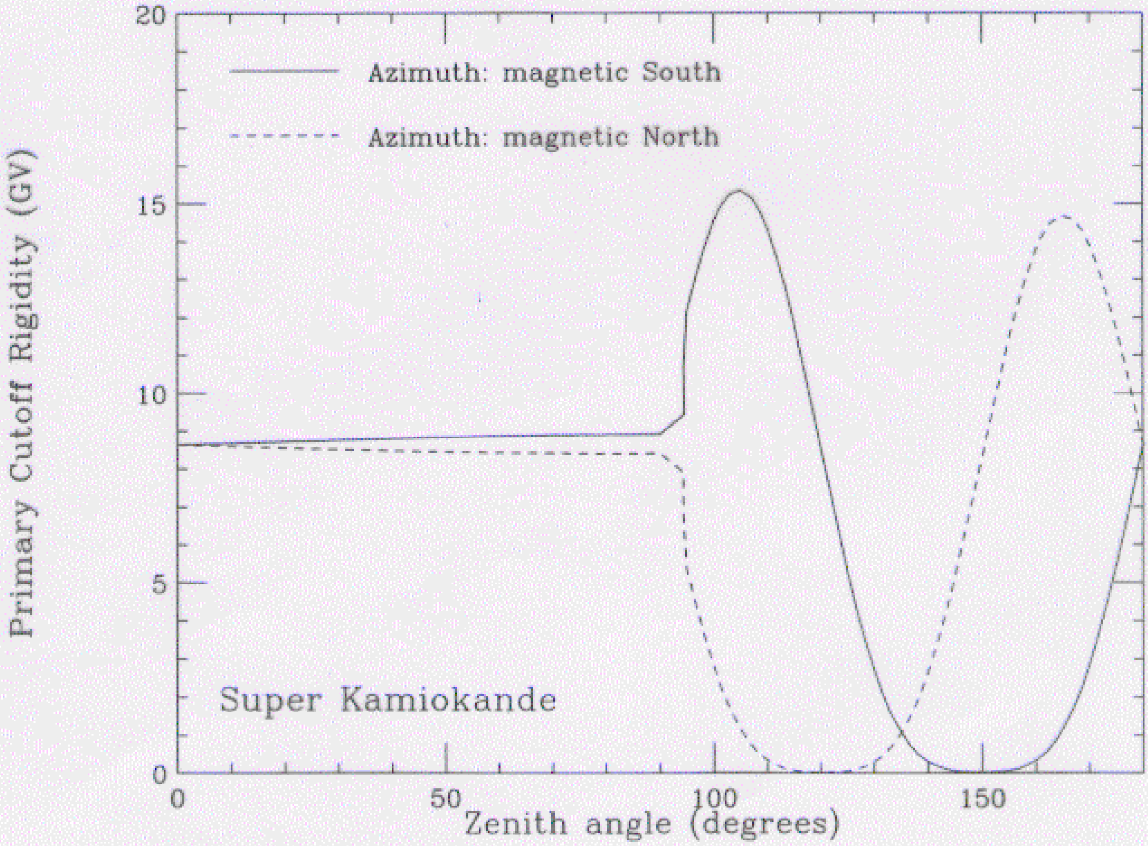
$$\Phi_{\rm primary}({\rm equator}) \simeq 100\ \ ({\rm m}^2\,{\rm s\,sr})^{-1}, \quad \Phi_{\rm albedo}({\rm equator}) \simeq 40\ \ ({\rm m}^2\,{\rm s\,sr})^{-1}$$

# Magnetic Cutoffs for ~~[illegible]~~ Kamioka

magnetic coordinates

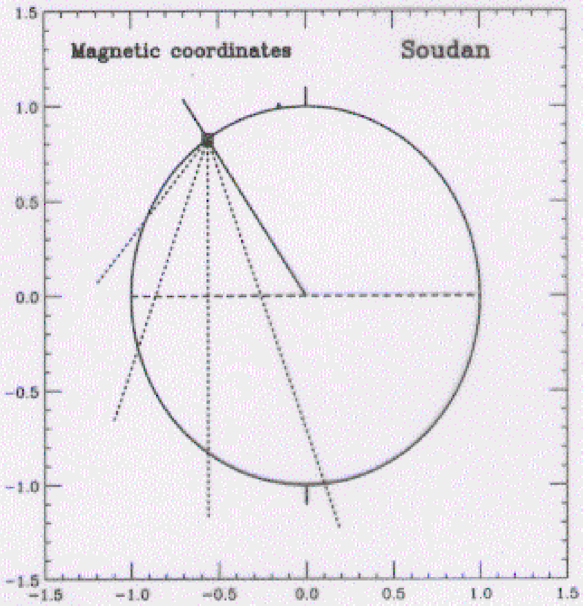


$\lambda_{mag} \sim 26^\circ$

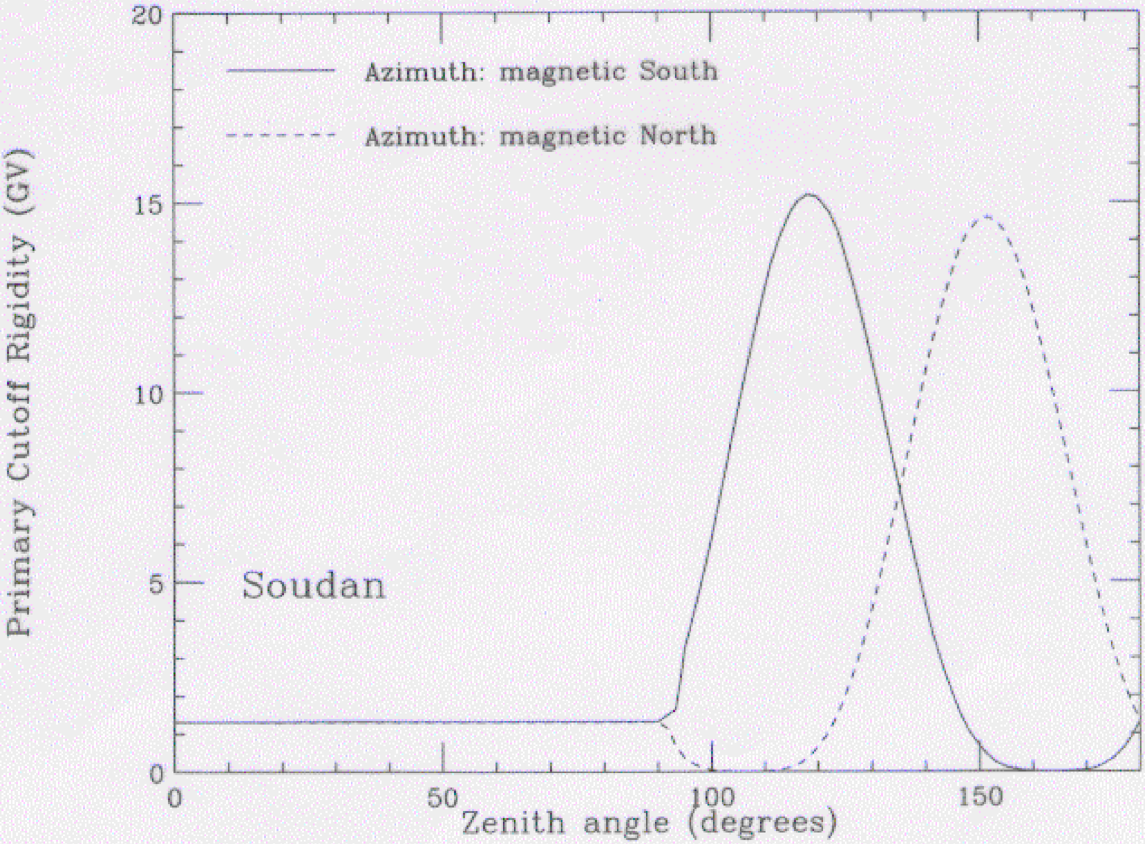


⇒ Kamioka $\boxed{\text{Up > Down.}}$

# Magnetic Cutoffs for Soudan

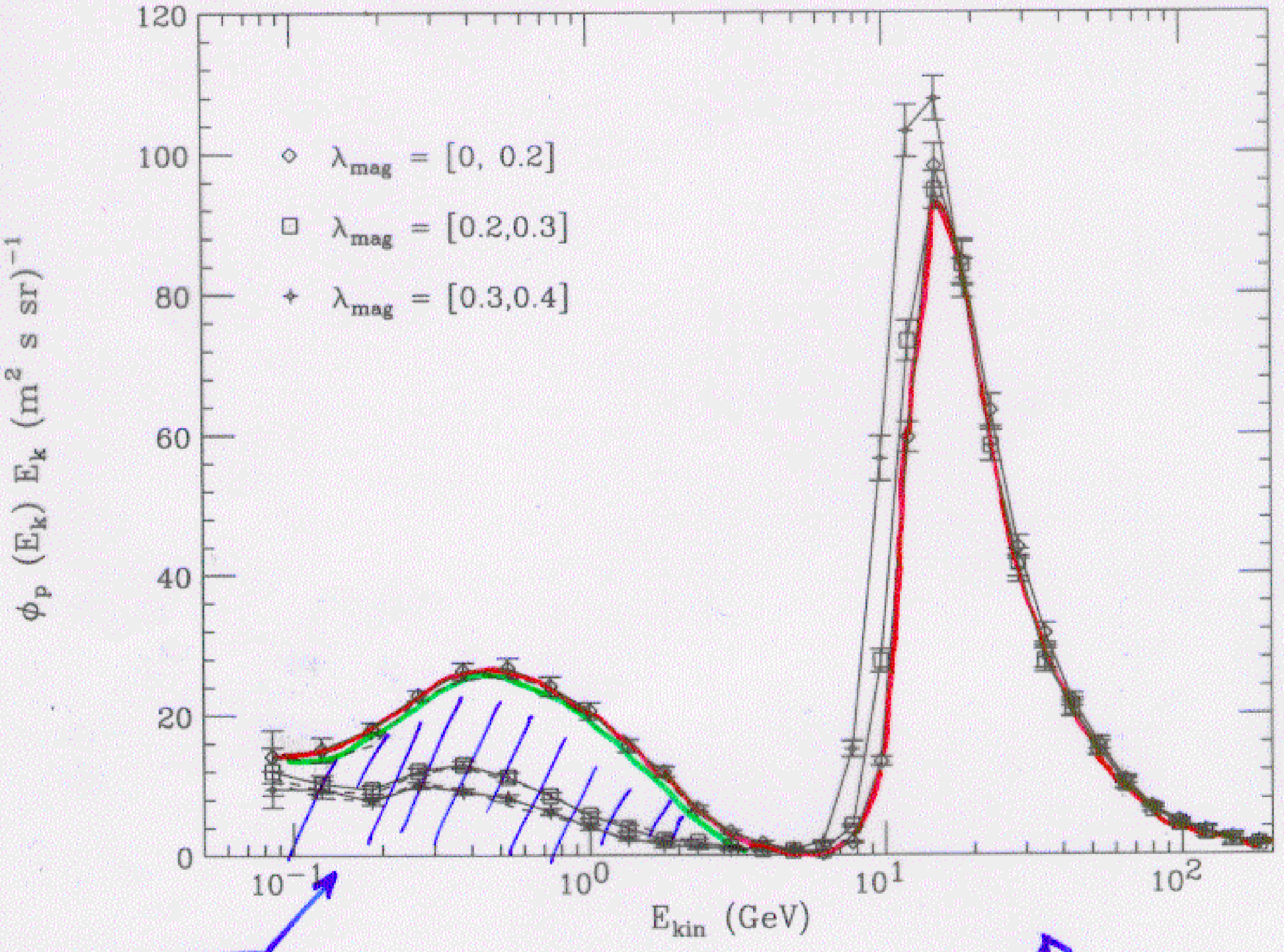


$\lambda_{mag} \sim 56^\circ$



Soudan $\boxed{\text{Up} < \text{Down}}$

# – AMS measurement Equatorial Region



70 $\frac{\text{protons}}{m^2.s.sr}$

"Second Spectrum"

— ↑ up-going

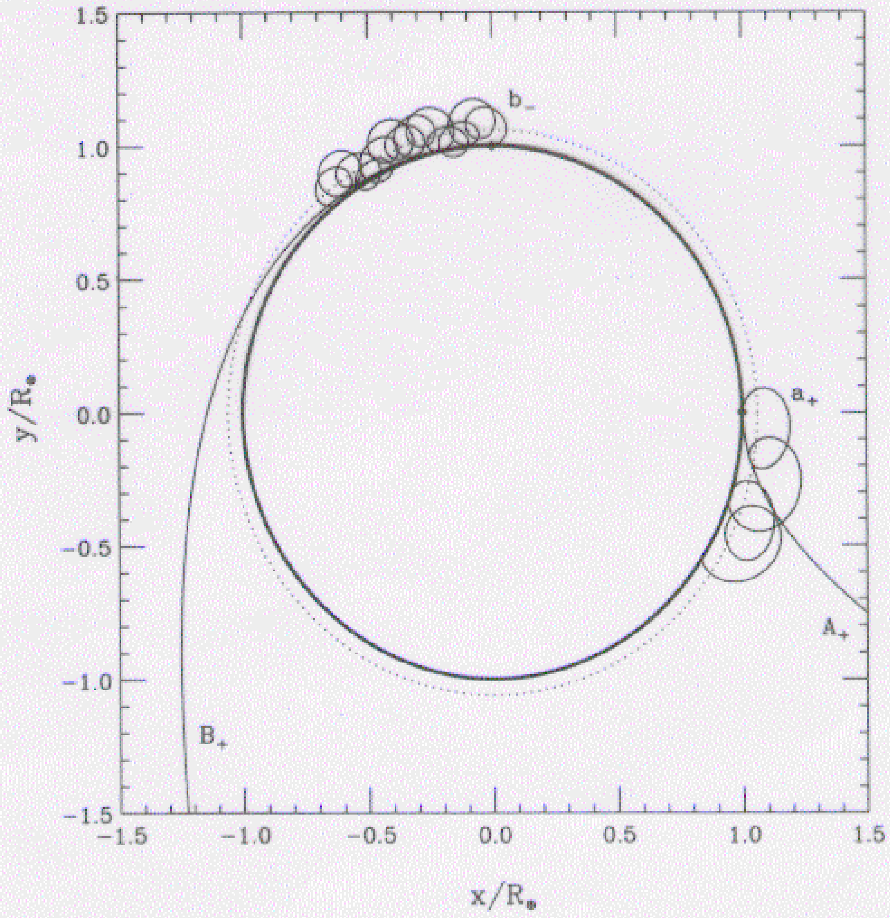
Not predicted (in detail)

"Cutoff"

Primary Spectrum

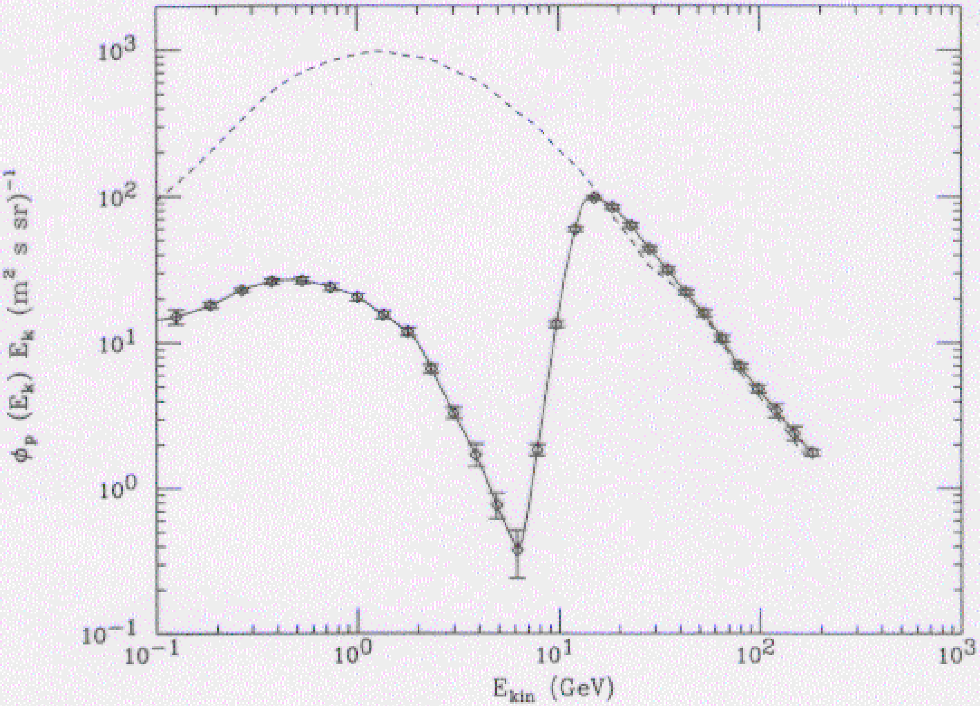
# The AMS data is GOOD AGREEMENT with the description of GEOMAGNETIC effects used in the calculations of atmospheric neutrinos

The proton (and $e^{\pm}$) **second spectra** detected by AMS were **not** expected (or at least not predicted in detail); but *a posteriori* they can be can be well understood as composed of secondary particles generated in hadronic showers of nearly horizontal primaries.



# Importance of ALBEDO PROTONS

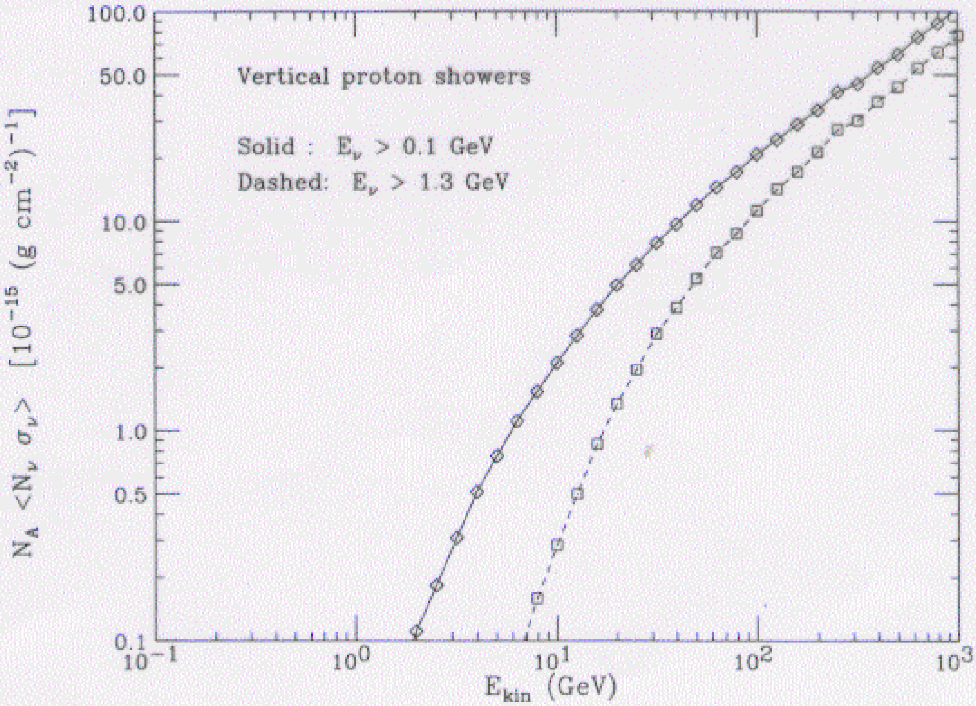
Flux



Pole

Equator

$\langle N_\nu \sigma_\nu \rangle$



$E_\nu > 0.1$

$E_\nu > 1.3$ GeV

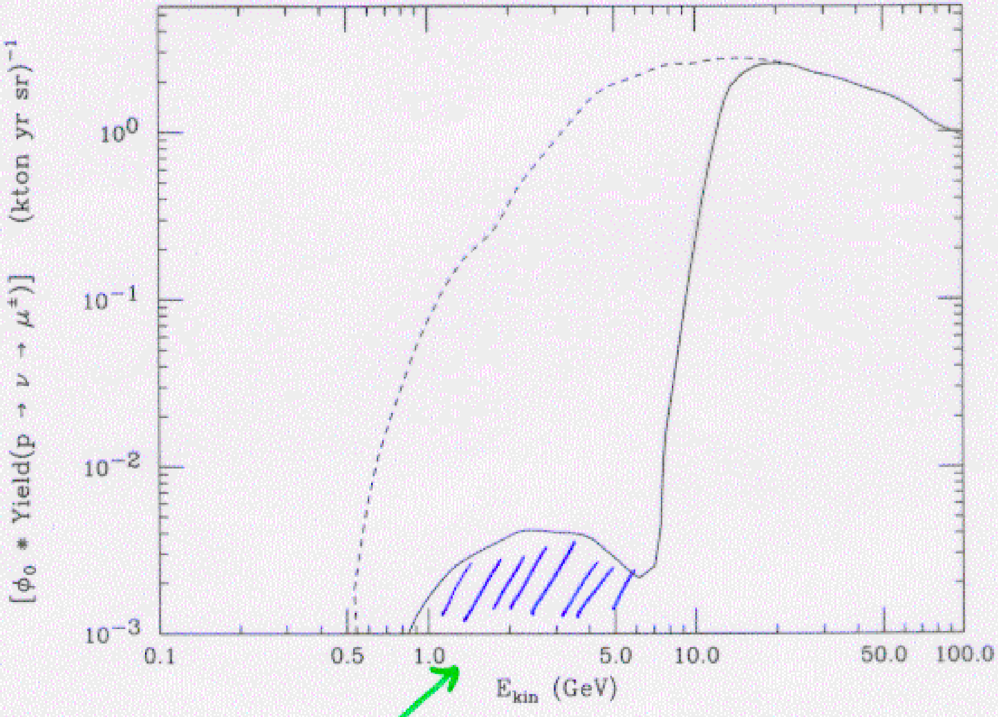
$E_{kin}$

The contribution of ALBEDO ("Second Spectrum") particles is neglibly small ($\lesssim 1\%$).
For $E_\nu \geq 0.1$ GeV:

$$\int dE_0 \, \phi_{\text{Albedo AMS}}^{\text{equator}} \langle N_\nu \sigma_\nu \rangle \simeq 0.06 \ \left(\text{kton year}\ ^{\text{sr}}\right)^{-1} \cdot \text{sr}^{-1}$$

This estimate is actually a large OVERESTIMATE of the effect.
Albedo particles are trapped for a long time $t_{\text{trap}} \simeq 1$–$10$ sec, cross the AMS orbit 100–1000 times.
The contribution of albedo protons should be reduced accordingly.

$\text{Flux} \cdot \langle N_\nu \sigma_\nu \rangle$



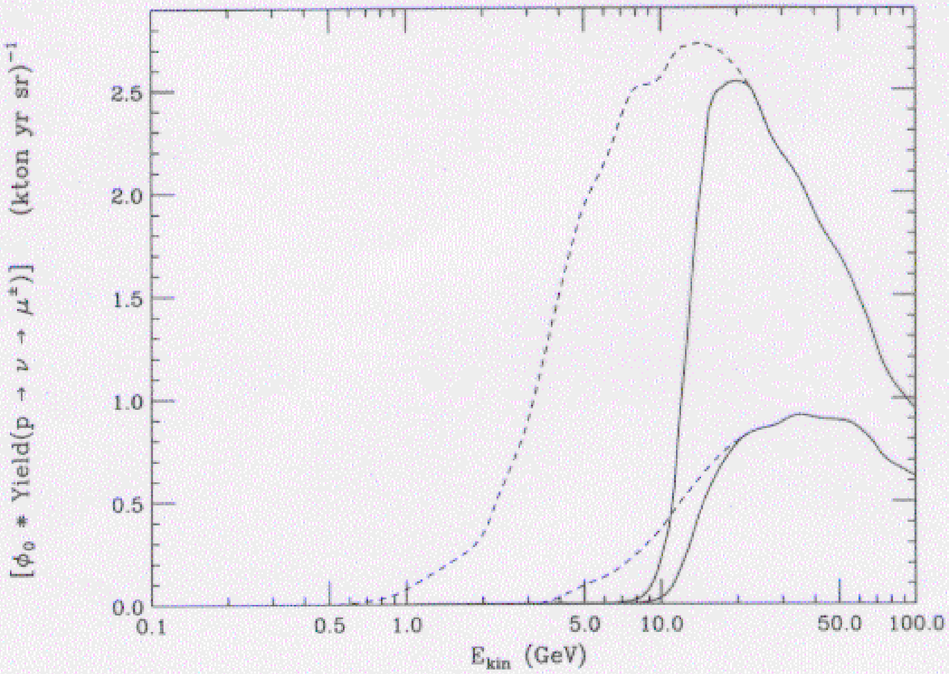
$$\underbrace{0.06 \ (\text{kton yr sr})^{-1}}$$

**Convolution** $\phi_0(E_0) \times \langle N_\nu \; \sigma_\nu \rangle$

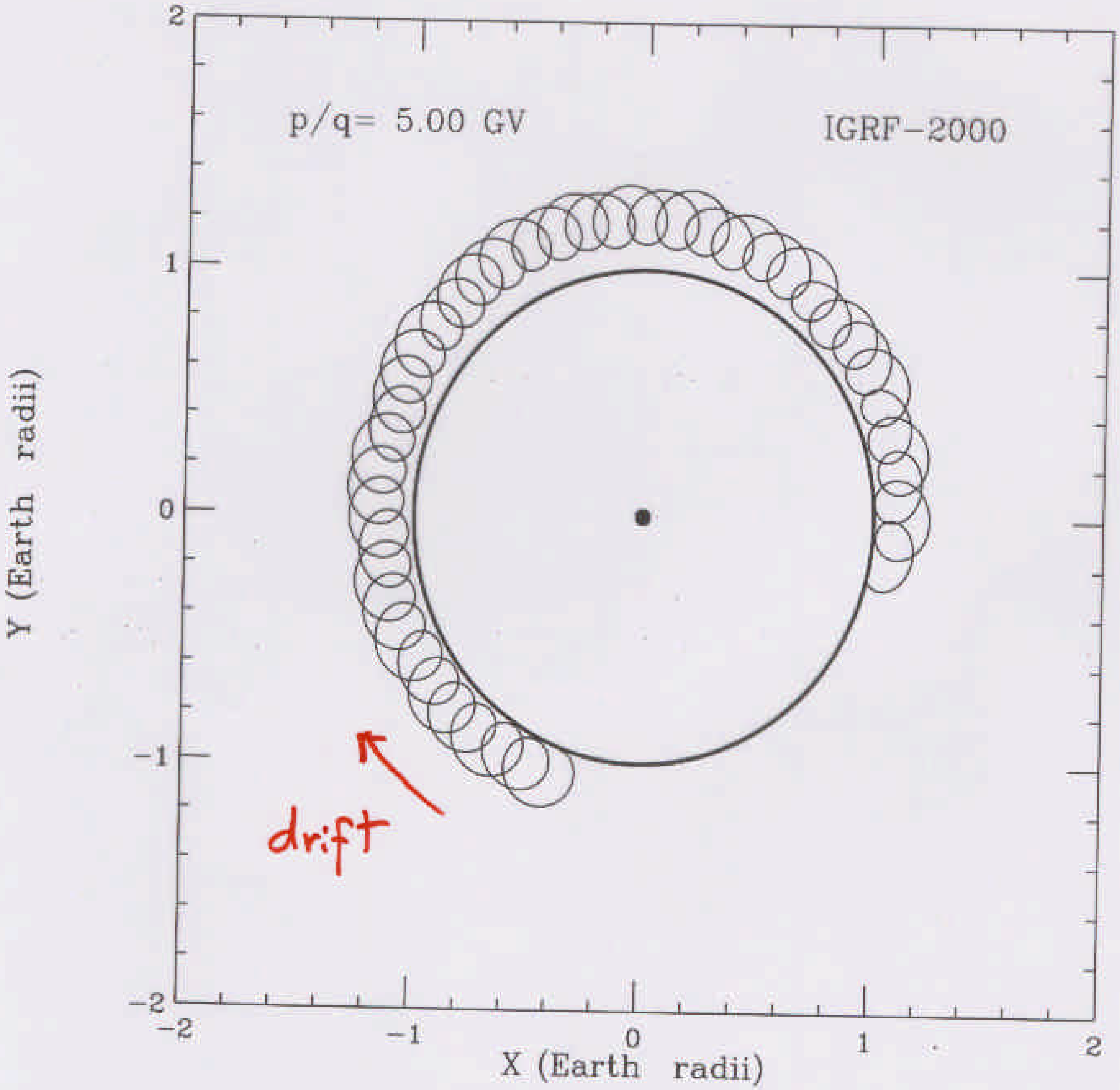
Linear scale

Solid = Magnetic Equator
Dashed = Magnetic Pole

$E_\nu \geq 0.1$ GeV
$E_\nu \geq 1.3$ GeV



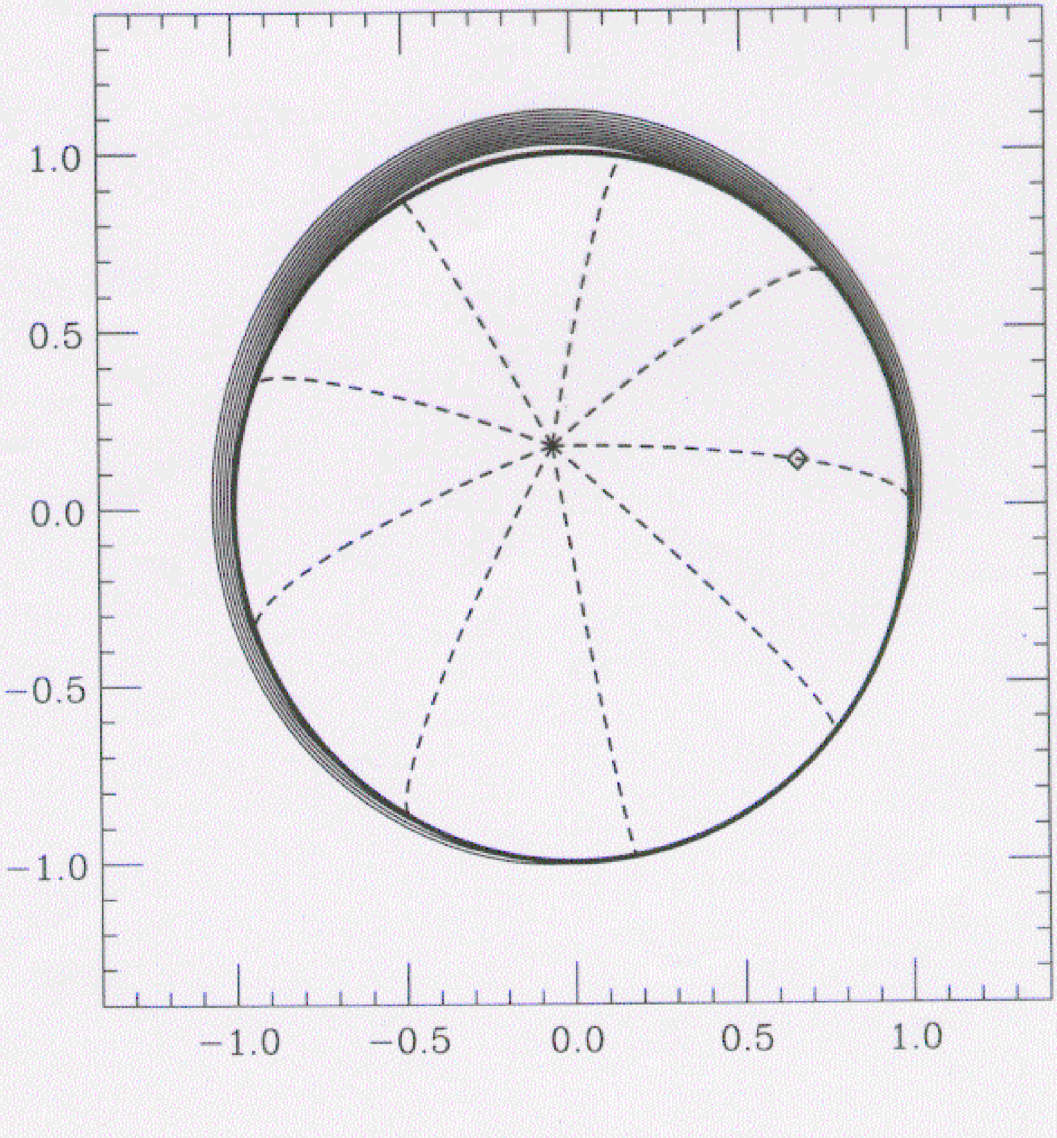
# Example of a second spectrum trajectory



Example of a second spectrum trajectory

The origin of the longitudinal asymmetry of the Long–lived AMS events is the "offset" of the axis of the dipolar component of the geomagnetic field with respect the Earth's center.

Lines of constant $|\vec{B}|$ in the Earth magnetic equatorial plane:



The instruments used for the calculation of atmospheric neutrinos can be applied to compute the AMS "second spectra".

1. A model of the primary c.r. flux reaches the Earth's surface. Geomagnetic effects are essential.
2. The particles interact and generate secondaries.
3. A (small) fraction of the secondary particles is injected into "second spectrum" trajectories.

Preliminary calculations show good quantitative agreement. with the AMS data.

The "qualitative features" of the second spectra:

- Long lived positive particles of the second spectrum originate in a well defined region (not–symmetric in manetic longitude). of the Earth's surface.
- Long lived negative particles originate in a **different** region.
- The "second flux" of positively charged particles ($e^+$) is several times larger than the flux of negative particles ($e^-$).

can be understood with simple arguments.
The deviations of the geomagnetic field from the dipolar form are *essential*.

# 3–DIMENSIONAL EFFECTS

"First generation" One–Dimensional:
$\nu$'s collinear with primary,

Neglect:

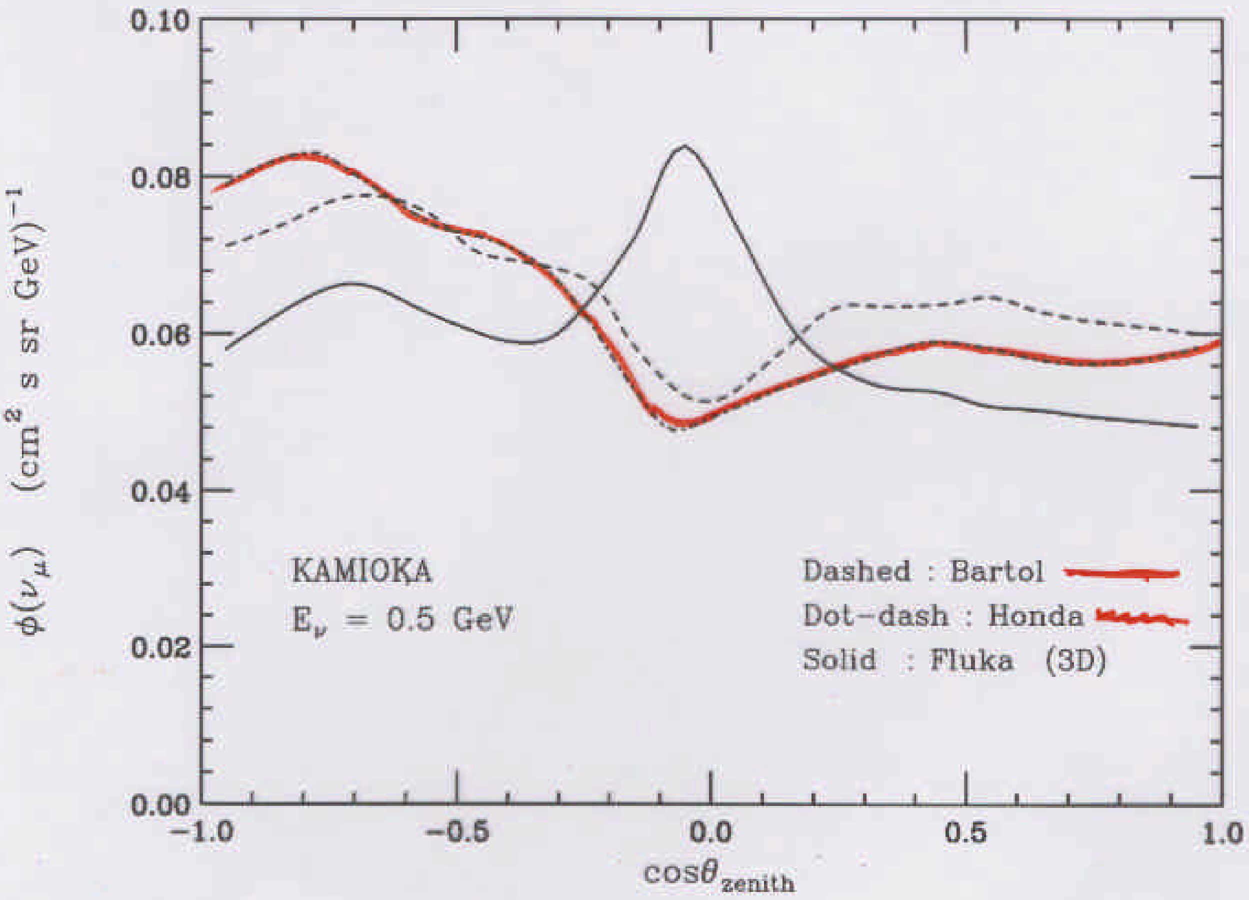
- Tranverse momentum $p_\perp$
- Multiple scattering
- Bending in geomagnetic field $\vec{B}$

**WHY** this approximation ??

CPU time ! More efficient MC calculation

New 3–Dimensional calculations:
New effect seen:

**GEOMETRIC ENHANCEMENT**
**On the HORIZONTAL PLANE**



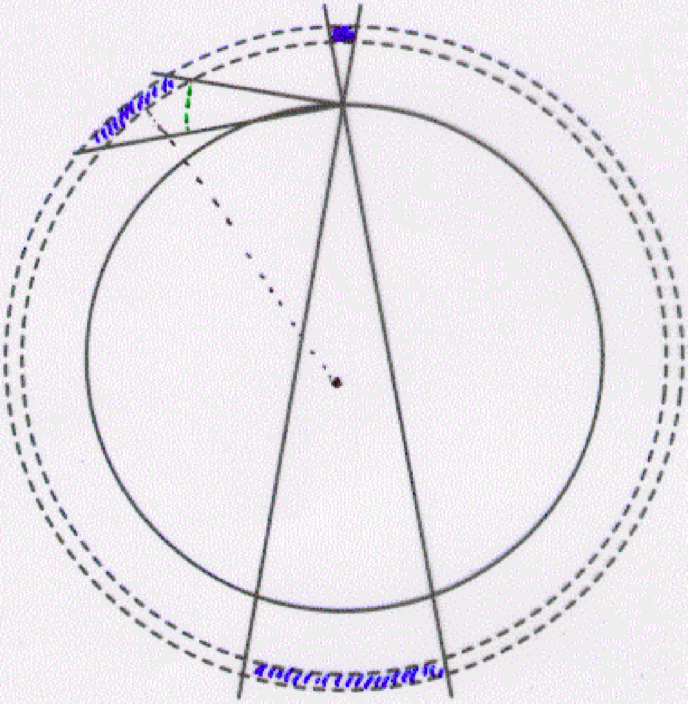
Effect seen first in
G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P. R. Sala, T. Rancati Astroparticle Physics 12, 315 (2000).

Now confirmed independently in other calculations:

1. M. Honda et al
   (private communication)
2. Y. Tserkovnyak, R. Komar, C. Nally and C. Waltham
   (private communication)
3. Critical discussion in: P.L. hep-ph/002282, hep-ph/0003013

# Geometry of Neutrino Production

Source Volume of atmospheric neutrinos

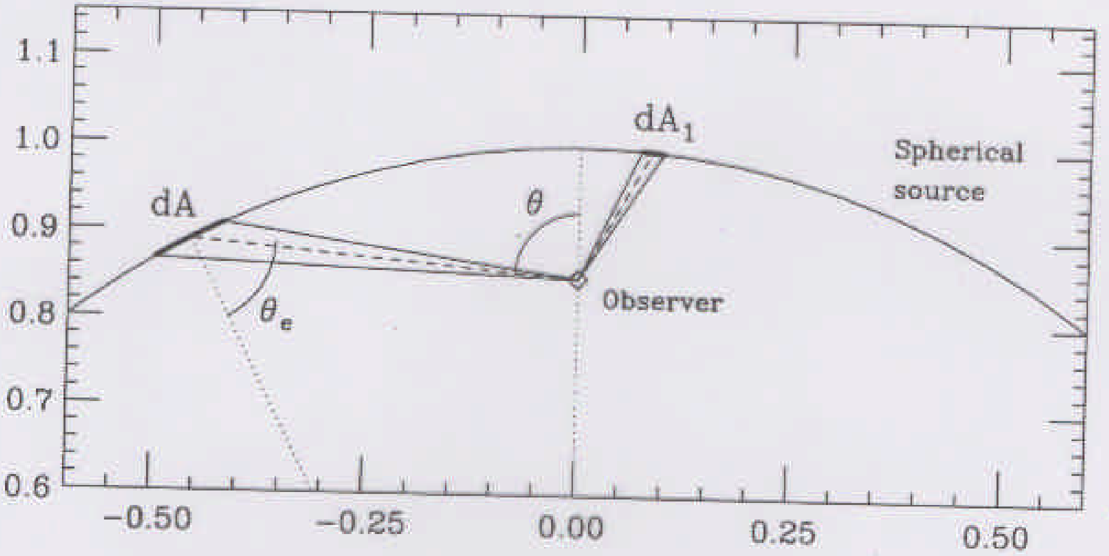


$$V_{\nu\ \mathrm{source}}(\theta+\delta\theta) \propto \frac{\ell^2(\theta)}{\cos\theta_{\mathrm{emission}}}$$

"Gauss theorem" implies Up-Down symmetry.
but **NOT** isotropy.

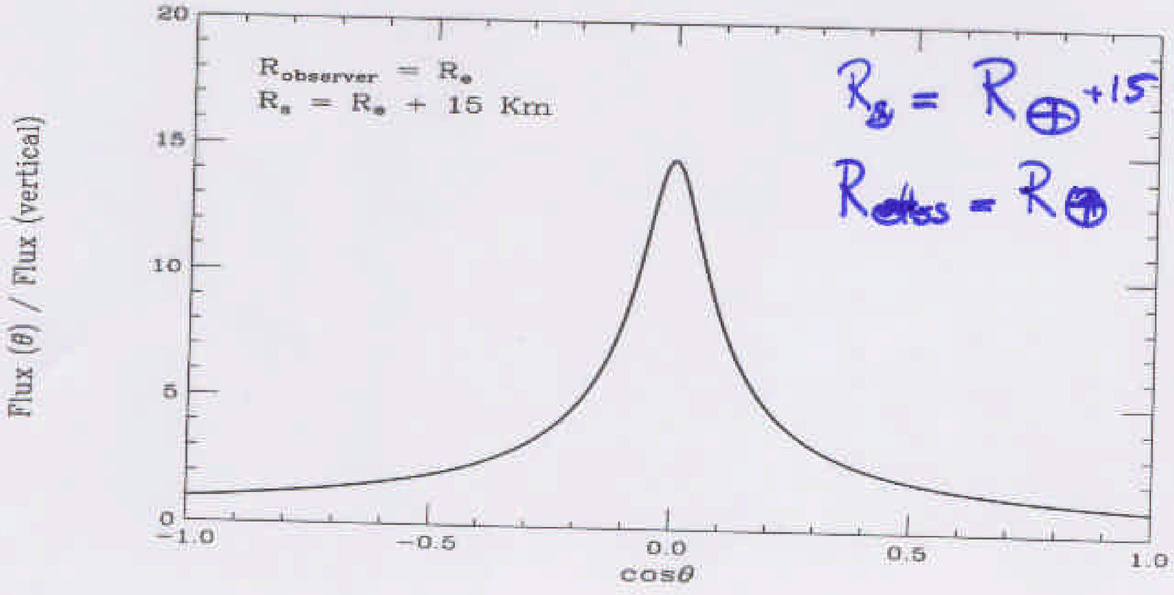
# "Astrophysical" example

Observer inside a thin spherical shell of stars.



What is the angular distribution of the light intensity ?

$$\frac{dL(\Omega)}{d\Omega} = n\,\frac{L_0}{4\pi}\,\frac{1}{\cos\theta_e} = n\,\frac{L_0}{4\pi}\left[1-\left(\frac{R_O}{R_s}\right)^2(1-\cos^2\theta)\right]^{-\frac{1}{2}}$$



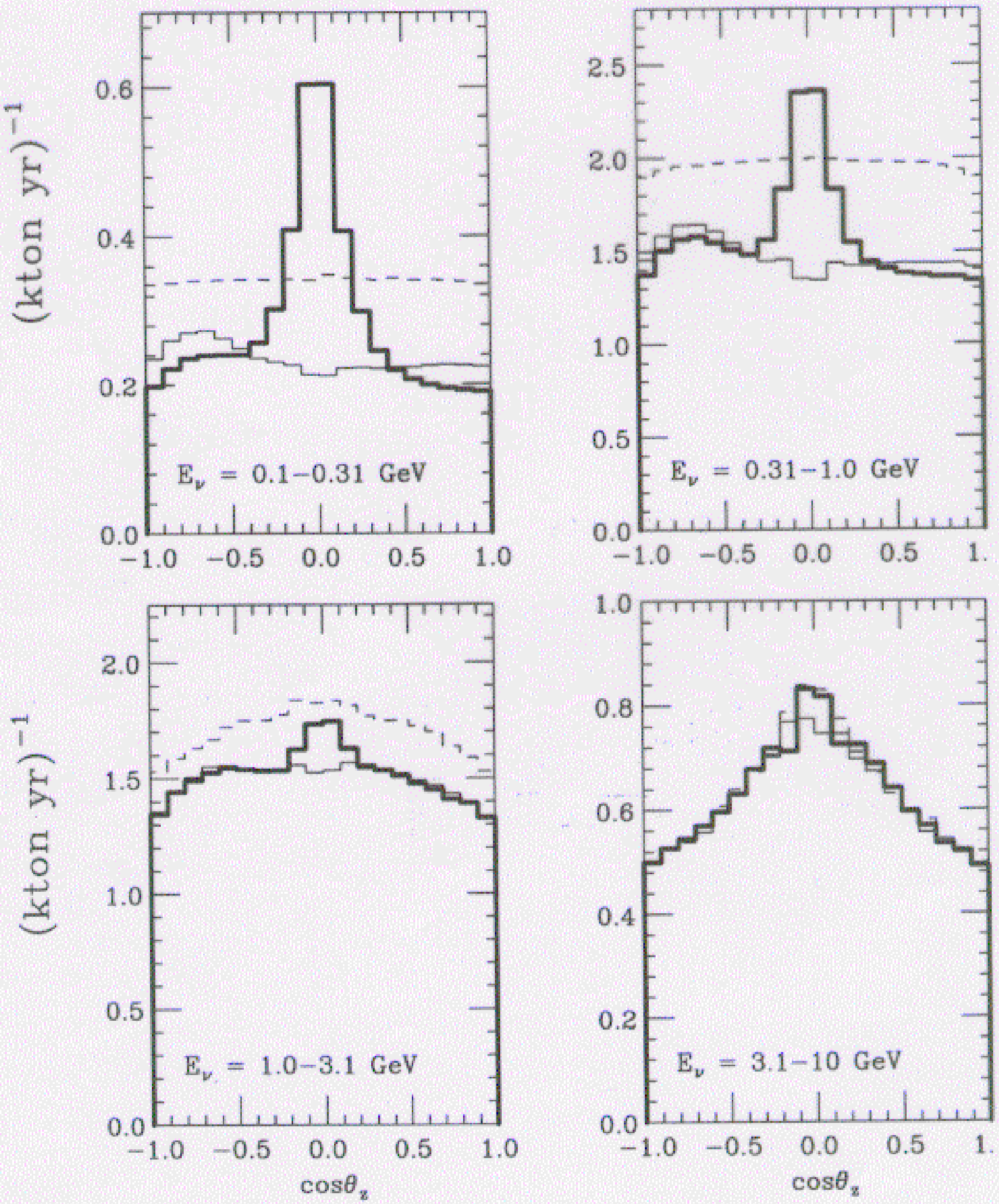
$R_S = R_{\oplus} + 15$ km

$R_{obs} = R_{\oplus}$

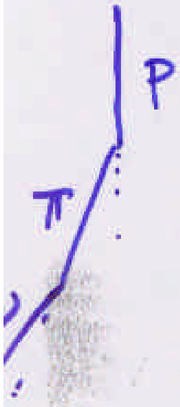
~ Kamioka

(P.L. Astrop. Phys. 2000) hep/ph 0002282

$\mu$ events. Region: $\sin\lambda_{mag} = [0.2,0.6]$



# Sources of Neutrino–Primary angle



$$\theta_{\nu p} = \theta_\pi \oplus \theta_{\pi\nu}$$

$$= \theta_\pi \oplus \theta_{\pi\mu} \oplus \theta_{\mu B} \oplus \theta_{\mu\nu}$$

$$\boxed{\theta_\pi} \sim \frac{p_{\perp\pi}}{p_\pi} \sim \frac{350\ \mathrm{MeV}}{4p_\nu} \sim \boxed{\frac{5^\circ}{p_\nu(\mathrm{GeV})}} \sim \frac{1}{p}$$

$$\theta_{\pi\nu} \sim \frac{1.5^\circ}{p_\nu}$$

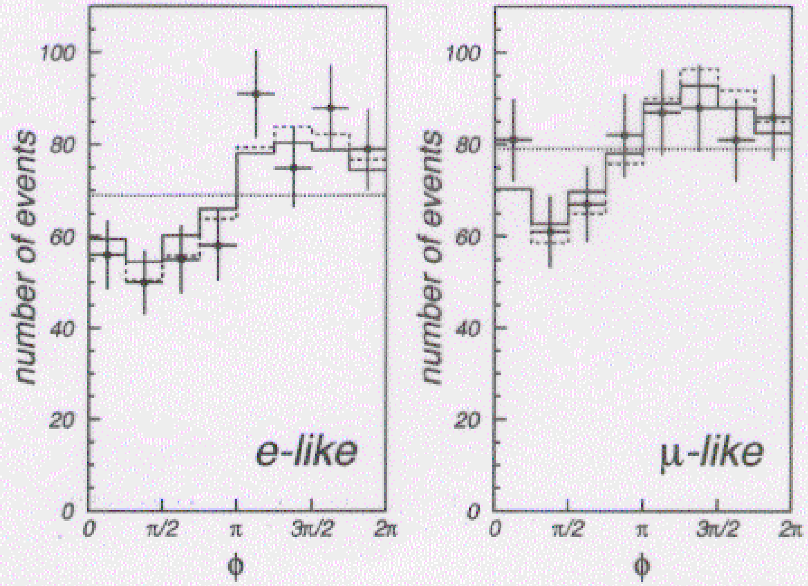
$$\theta_{\pi\mu} \sim \frac{0.5^\circ}{p_\nu}, \quad \theta_{\mu\nu} \sim \frac{2^\circ}{p_\nu}$$

$$\boxed{\theta_{\mu B}} = \frac{L_\mu}{R_{\mathrm{curv.}}(\mu)} = \frac{c\tau_\mu p_\mu/m_\mu}{cp_\perp/B} \sim \boxed{10.7^\circ\ B(\mathrm{Gauss})}$$

Momentum independent

Higher p : Higher rigidity
Longer path.

# The EAST–WEST effect in Super–Kamiokande



$p_{e,\mu} = 0.4$–$3$ GeV; $\cos\Theta_z = [-0.5, -0.5]$

Hint of a discrepancy with the data

$$A_{EW} = \frac{E - W}{E + W}$$

$$A_e^{\rm SK} = 0.21 \pm 0.04, \qquad A_\mu^{\rm SK} = 0.08 \pm 0.04$$

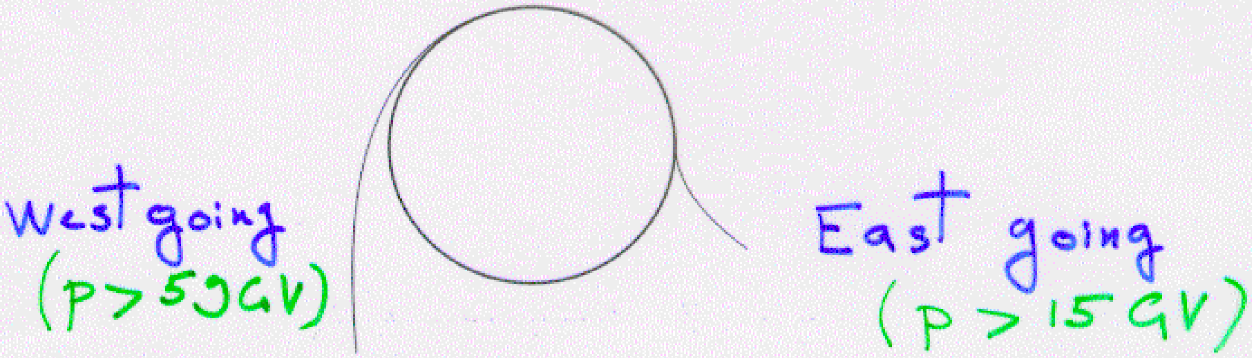
$$A_e^{\rm HKKM} = 0.13 \pm 0.04, \qquad A_\mu^{\rm HKKM} = 0.11 \pm 0.04$$

$$A_e^{\rm Bartol} = 0.17 \pm 0.04, \qquad A_\mu^{\rm Bartol} = 0.15 \pm 0.04$$

# Neutrino Azimuth Distributions

Shaped by two effects:

- The East–West efect of primary cosmic rays
  More positive particle going East than vicevers.



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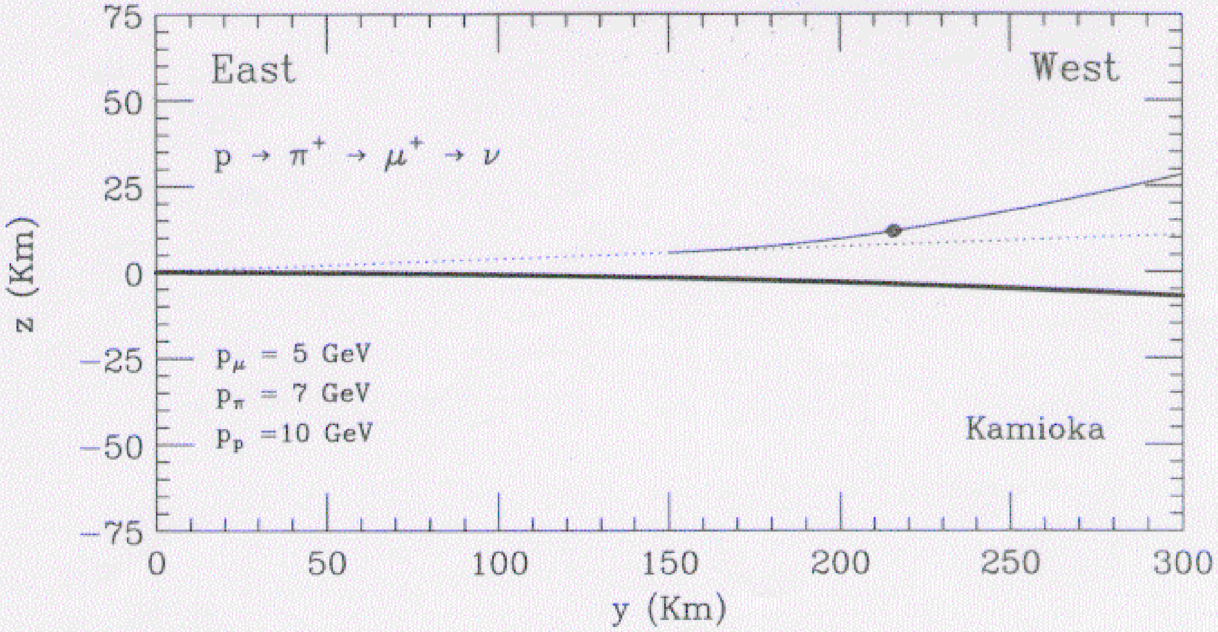
- The Bending of muons (and other charged particles) in the geomagnetic field.



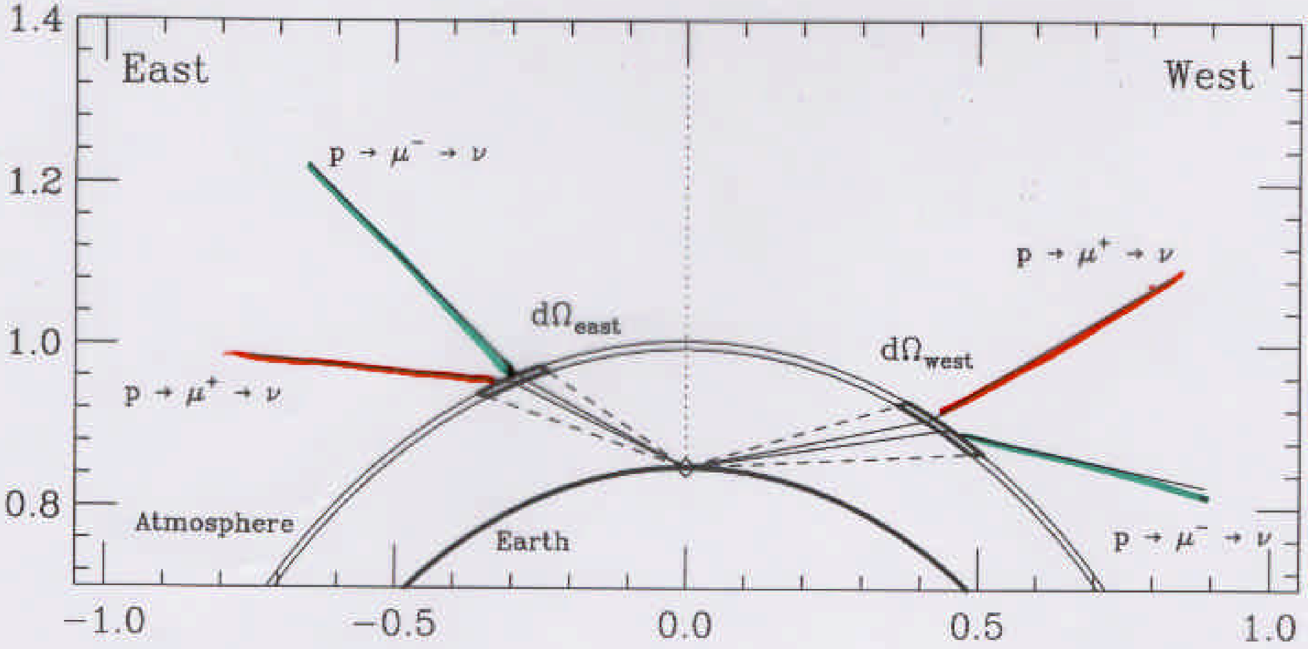
**Enhancement** of East–West effect for neutrinos from
$\mu^+ \to \nu \qquad (\nu_e \overline{\nu}_\mu)$

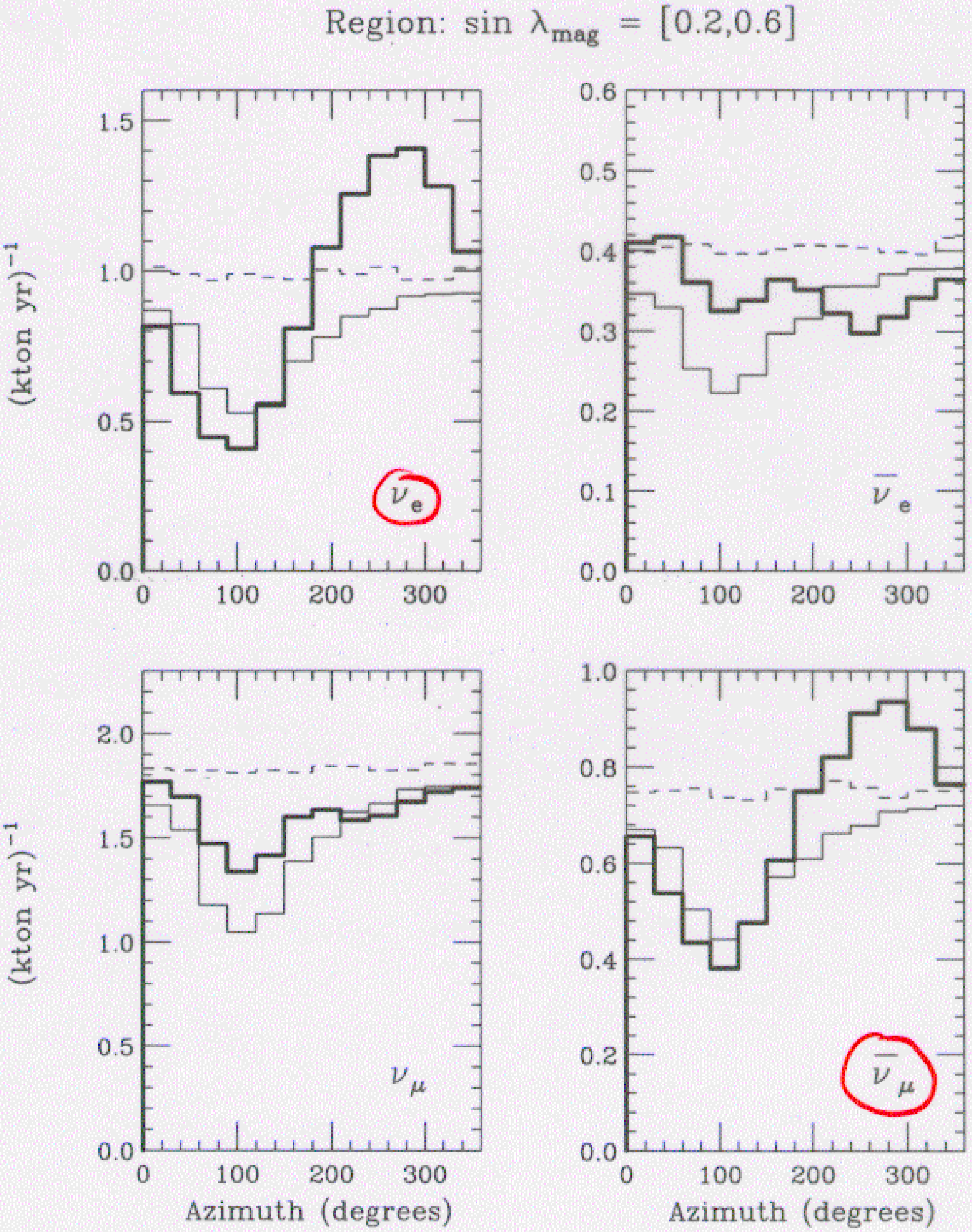
**Suppression** of East–West effect for neutrinos from
$\mu^- \to \nu \qquad (\overline{\nu}_e \nu_\mu)$

Since:
$\phi(\nu_\mu)/\phi(\overline{\nu}_\mu) \simeq 1;$
$\phi(\nu_e)/\phi(\overline{\nu}_e) \simeq \pi^+/\pi^- \simeq 1.2;$

**Enhancement** for $e$–like events

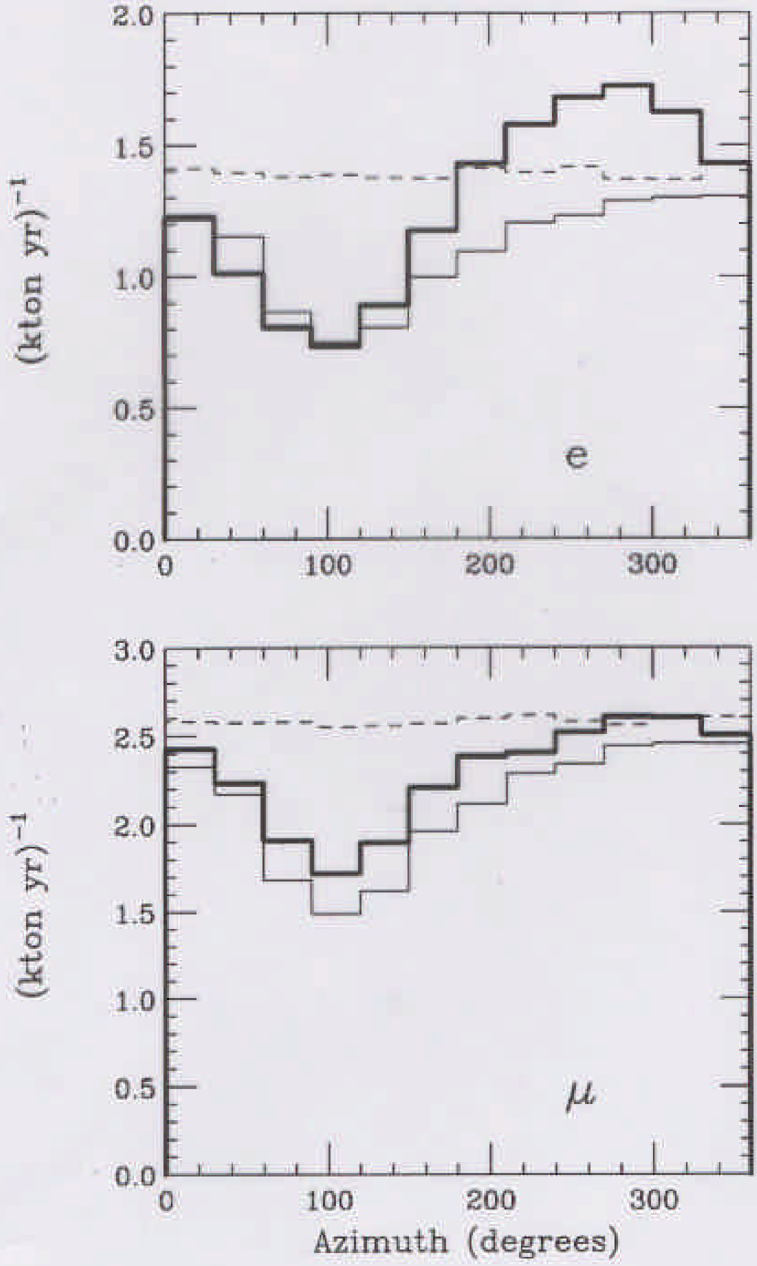
**Suppression** for $\mu$–like





Thin histogram: neglect bending in shower

Dashed: no geomagnetic effects.



Region: $\sin \lambda_{mag} = [0.2,0.6]$

e- like

μ- like

# HADRONIC INTERACTION MODEL

*Preliminary* tables of $\nu$ fluxes calculated with FLUKA
for Kamioka, Soudan, Gran Sasso
available (since june/8/2000) at
`http://www.mi.infn.it/ battist/neutrino.html`

Comparison FLUKA / BARTOL

- The $\mu/e$ ratio remains a ROBUST prediction with little model dependence.

$$\left(\frac{\mu}{e}\right)_{\rm Fluka} \Big/ \left(\frac{\mu}{e}\right)_{\rm Bartol} = 1 \pm (\leq 5\%)$$

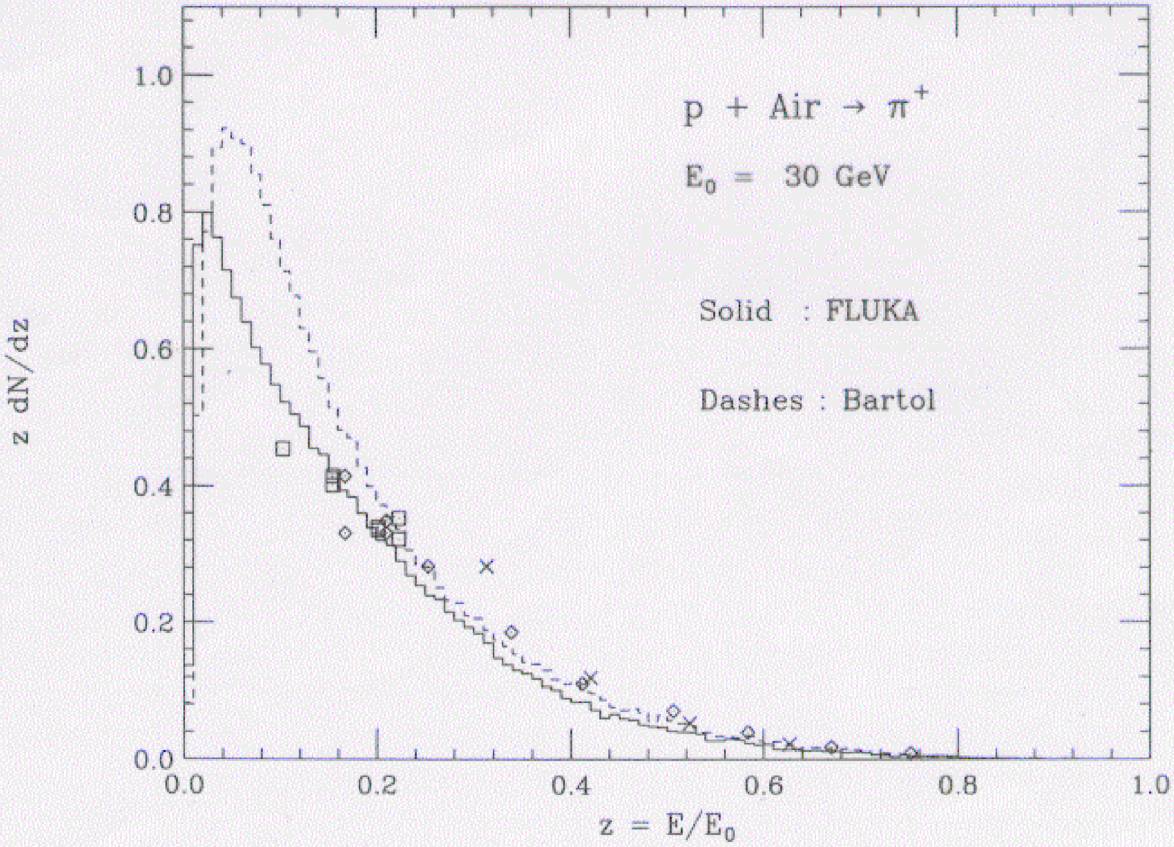
- The ABSOLUTE NORMALIZATION of FLUKA is
**LOWER** by $\sim 15\%$ for $E_\nu \gtrsim 1$ GeV.

- Both models have approximate Feynman scaling for $E_0 \gtrsim 20$ GeV.
It follows that for $E_\nu \gtrsim 1$ GeV the difference between the models is a **constant factor**.

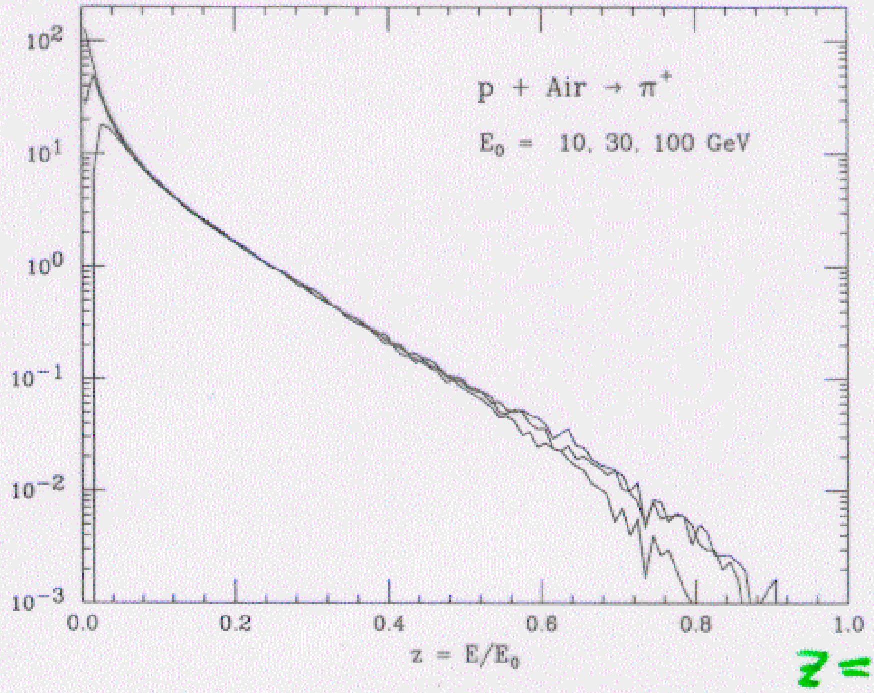
# Hadronic Interactions

The modeling of Hadronic interactions is important in the calculation of the neutrino fluxes:

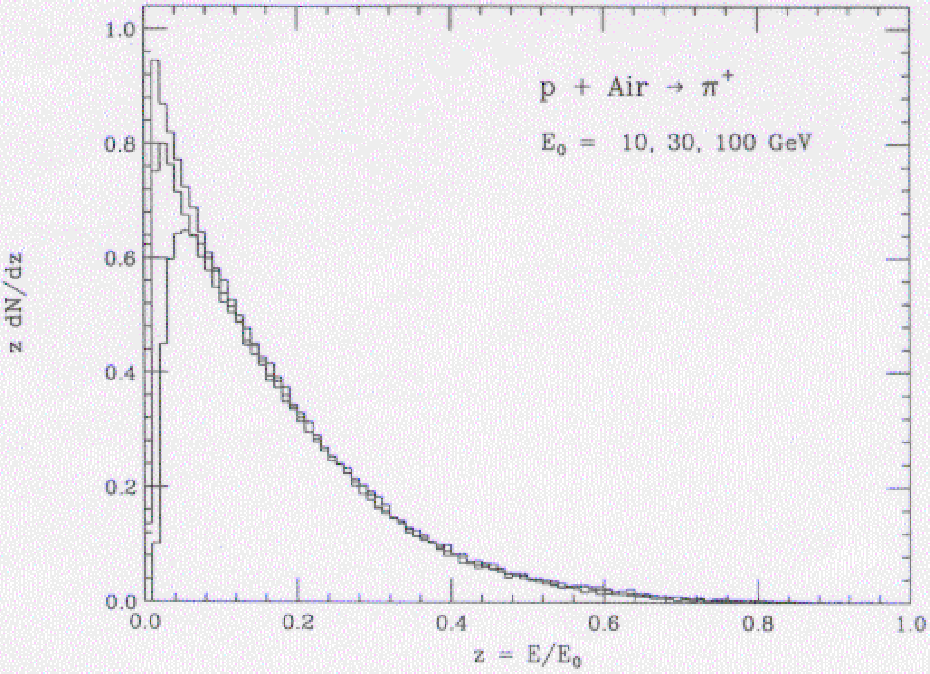
It is important to measure accurately the inclusive spectra of secondary particles of different types: $p$, $\pi^+$, $\pi^-$, $K^+$, $K^-$ in the **entire** kinematical region.



Example of Fluka inclusive distributions:



$z = E_\pi / E_0$



Feynman scaling

Approximate Feynman scaling valid
for projectile energy $E_0 \gtrsim 20$ GeV

[Power Law] $\otimes$ [Feynman scaling] =
[Power Law] × Constant

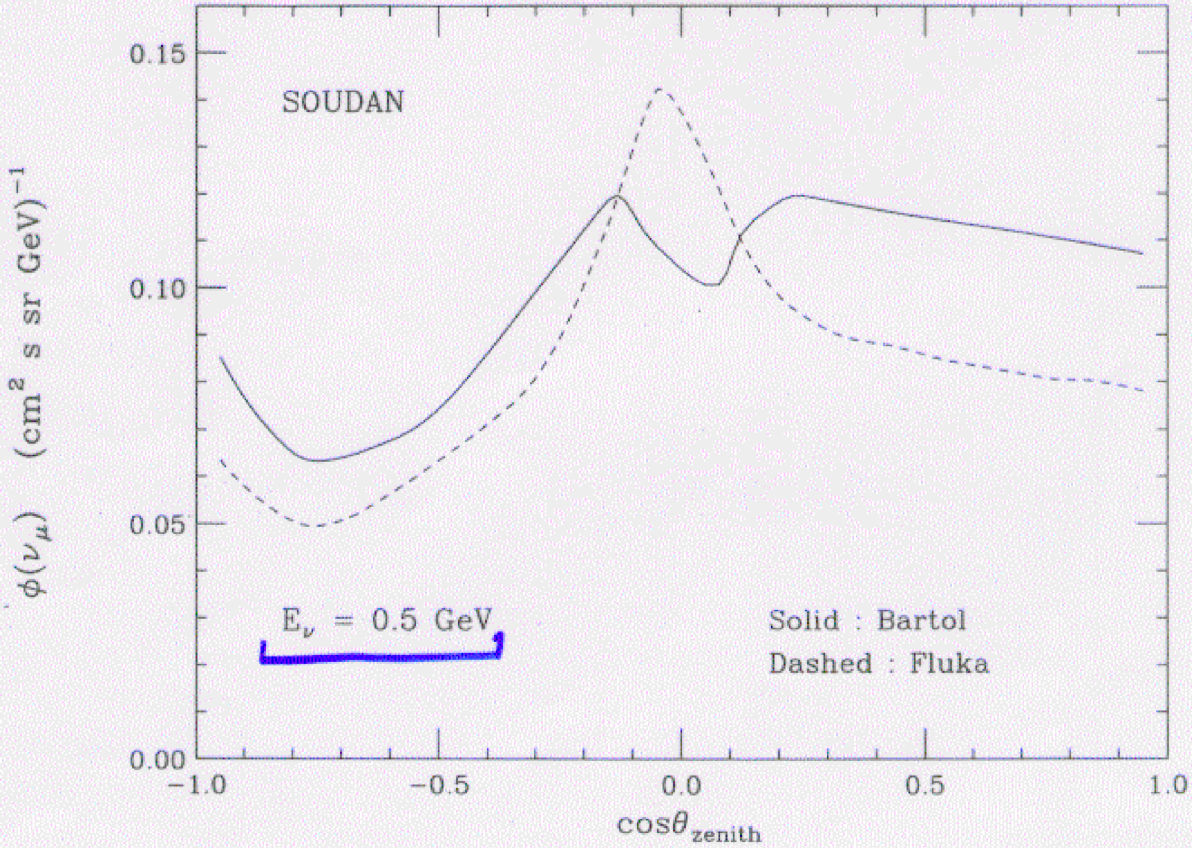
$$\mathrm{Flux}_\pi(E_\pi) = \int_{E_\pi}^{\infty} dE_0 \, [K \; E_0^{-\alpha}] \; \frac{1}{E_0} F\left(\frac{E_\pi}{E_0}\right) =$$

$$K \; E_\pi^{-\alpha} \times \int_0^1 dz \; z^{\alpha-1} \; F(z)$$

For the same primary flux,
models implementing Feynman scaling differ
by a constant factor.

- Larger Difference in the neutrino yield for very low energy primaries ($E_0 \lesssim 10$ GeV).

Illustration:
(calculations use the *same* primary flux)

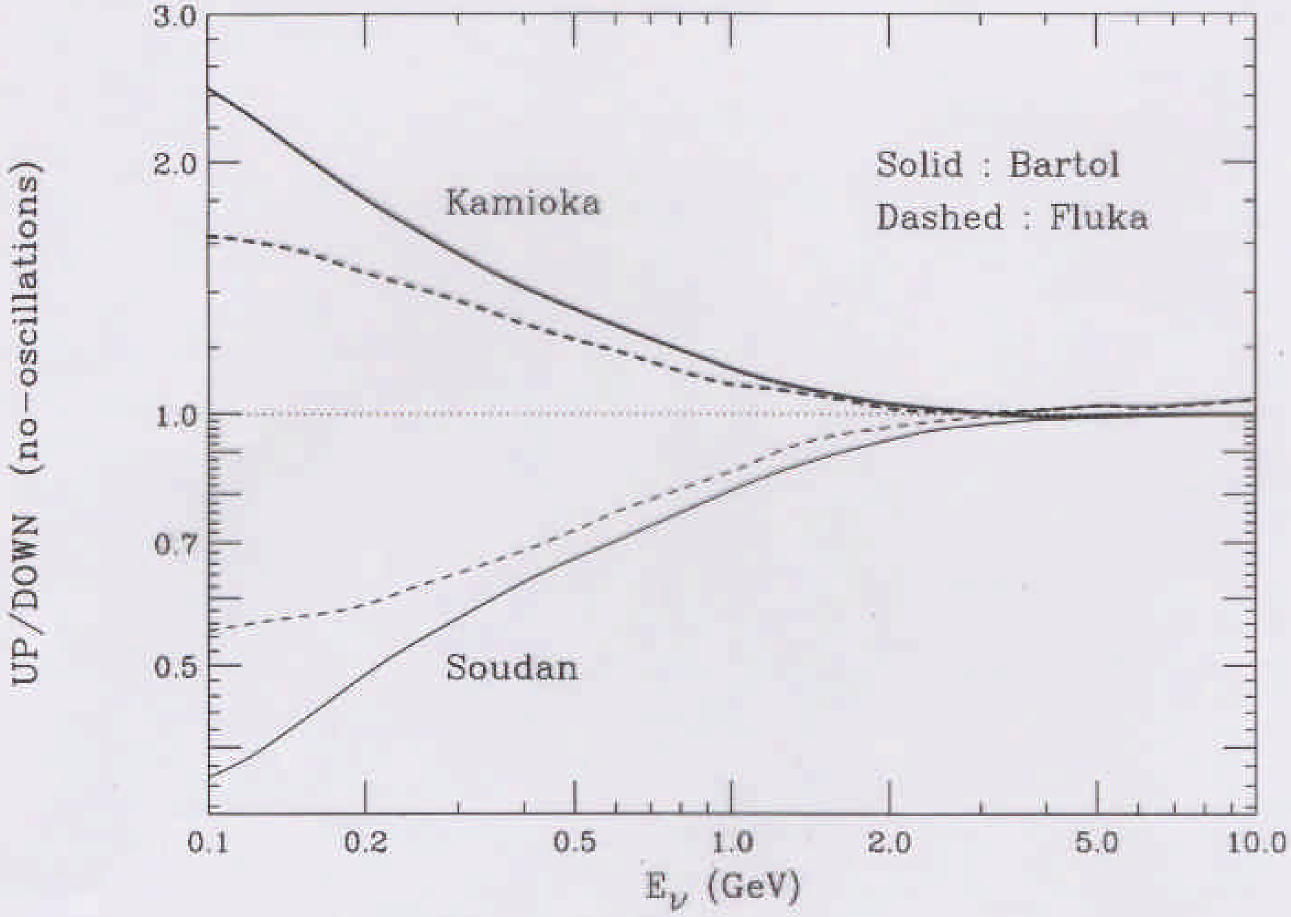


Primary flux relevant for the "DOWN" hemisphere contains a larger number of soft ($E_0 < 10$ GeV) protons.

$$\boxed{\left(\frac{\mathrm{Fluka}}{\mathrm{Bartol}}\right)_{\mathrm{DOWN}} < \left(\frac{\mathrm{Fluka}}{\mathrm{Bartol}}\right)_{\mathrm{UP}}}$$

No Oscillations

# No–Oscillation ASYMMETRY



In the **ABSENCE** of oscillations:

$$(\text{Asymmetry})_{\text{Fluka}} < (\text{Asymmetry})_{\text{Bartol}}$$

Potentially important or the interpretation especially for the **OBSERVED** Up/Down asymmetry in **Soudan**.

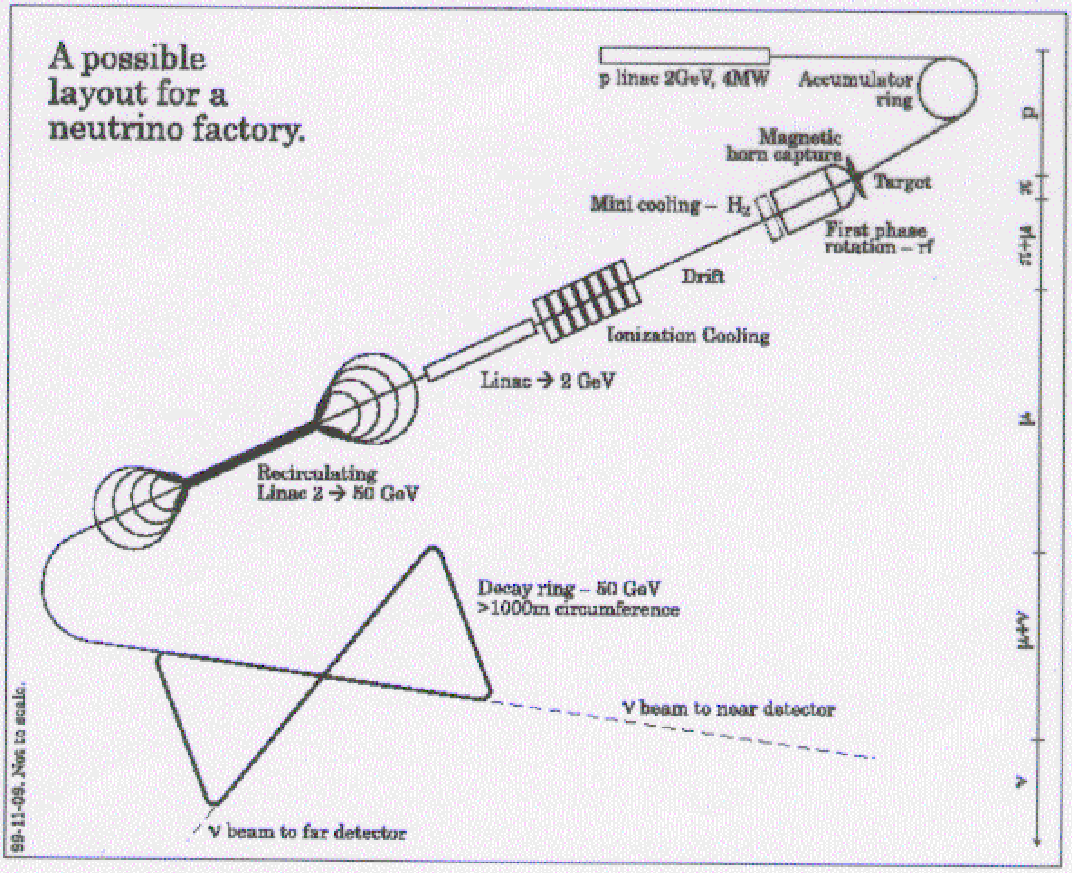
[Low energy, Better angular resolution]

# The HARP experiment at CERN

Physics Motivations:

- Detailed study of $\pi^\pm$ production for the development of the first stage of a $\nu$–Factory,
- Study of hadronic interactions for the calculation of the Atmospheric $\nu$ Fluxes.

LAYOUT of $\nu$–Factory



$p$

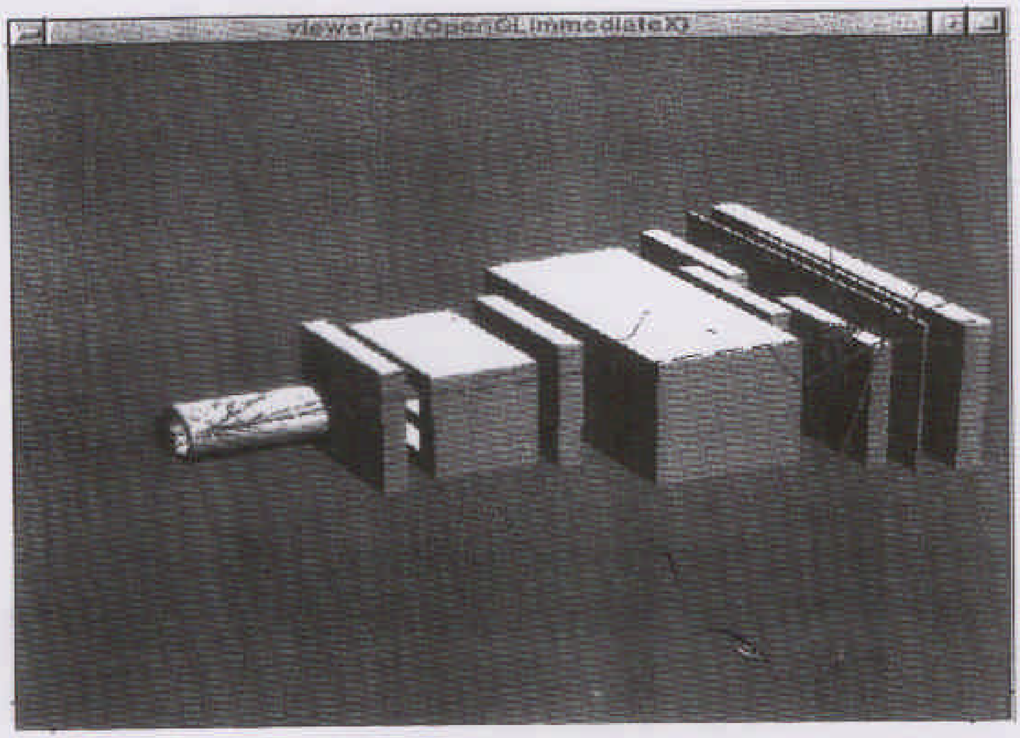
$\pi$

$\pi\mu$

$\mu$

$\mu\nu$

$\nu$



- Proton and Pion beams in the range $p = 2$–$15$ GeV.
- Several THIN targets spanning the full range Be, W. (include Oxygen and Nitrogen). Also thick targets.
- Particle Identification.
- Large phase space coverage
- Aim at a ~ 2% accuracy in the measurement of inclusive cross–sections.

2%

## HIGH ENERGY $\nu$–flux $E_\nu \gtrsim 100$ GeV

Can be studied with "UP–GOING" $\mu$–events

IMPORTANCE:

- Allow the study of the survival probability in a larger region of $\nu$ kinematical variables.

  $P(\nu_\mu \to \nu_\mu) = F(\cos\theta_{zenith}, E_\nu;\ \ \{\text{New Physics Parameters}\})$

- Discrimination with respect to "alternative" models.
  - Standard oscillations: $P = F(L\ E_\nu^{-1})$
  - FCNC ($X$: column density): $P = F(X\ E_\nu^{0})$
  - Violation of Equivalence Principle: $P = F(L\ E_\nu^{+1})$

- Discrimination $\nu_\mu \leftrightarrow \nu_\tau$ versus $\nu_\mu \leftrightarrow \nu_s$.

  Matter effect $\propto E_\nu\ \rho/\Delta m^2$.

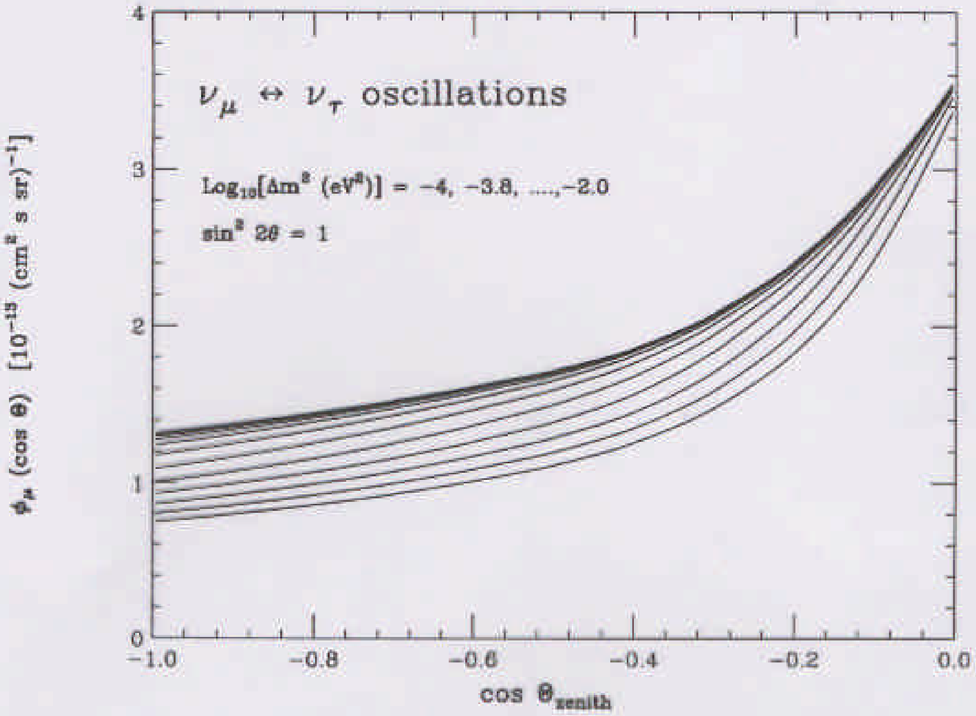
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\sin^2 2\theta + \left(\cos 2\theta \mp \frac{2VE_\nu}{\Delta m^2}\right)^2} \longrightarrow \sin^2 2\theta\ \left(\frac{\Delta m^2}{2VE_\nu}\right)^2$$

$$\ell_m = \ell_0 \sqrt{\sin^2 2\theta + \left(\cos 2\theta \mp \frac{2VE_\nu}{\Delta m^2}\right)^2} \longrightarrow \frac{2\pi}{V}$$

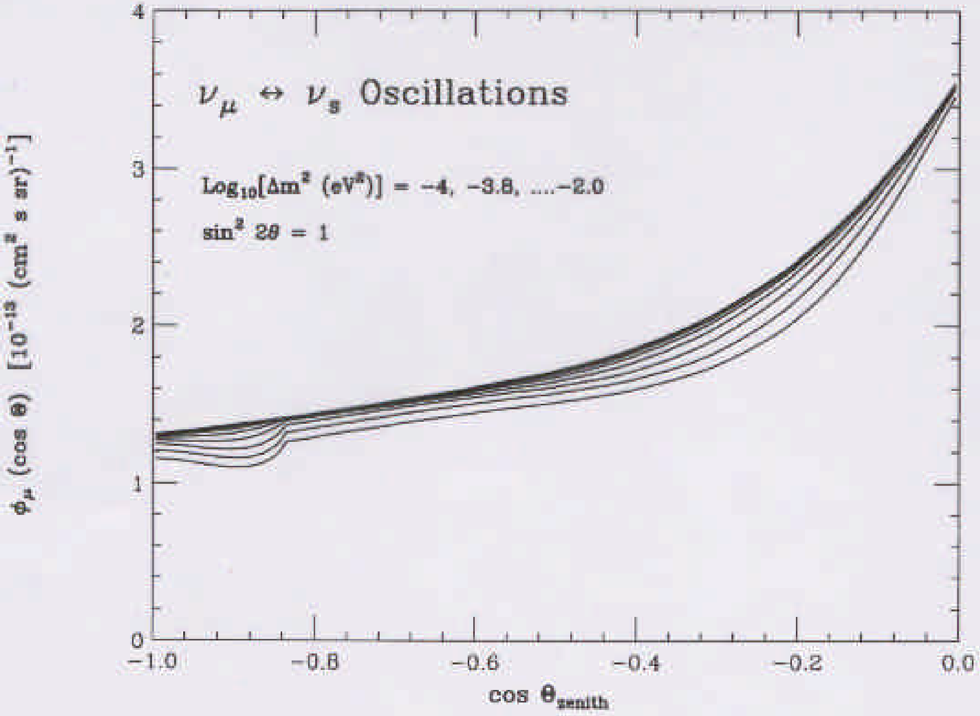
Large $E_\nu$

Suppression of large mixing
Oscillation length → constant value.

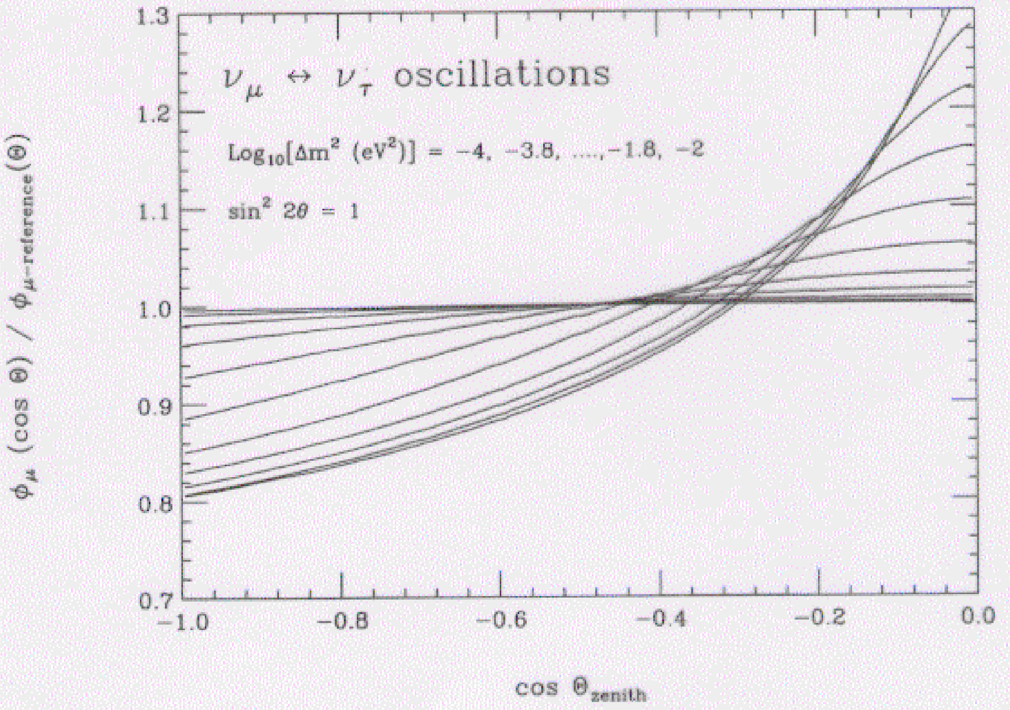
$\left(V = -\frac{\sqrt{2}}{2} G_F N_n\right)$



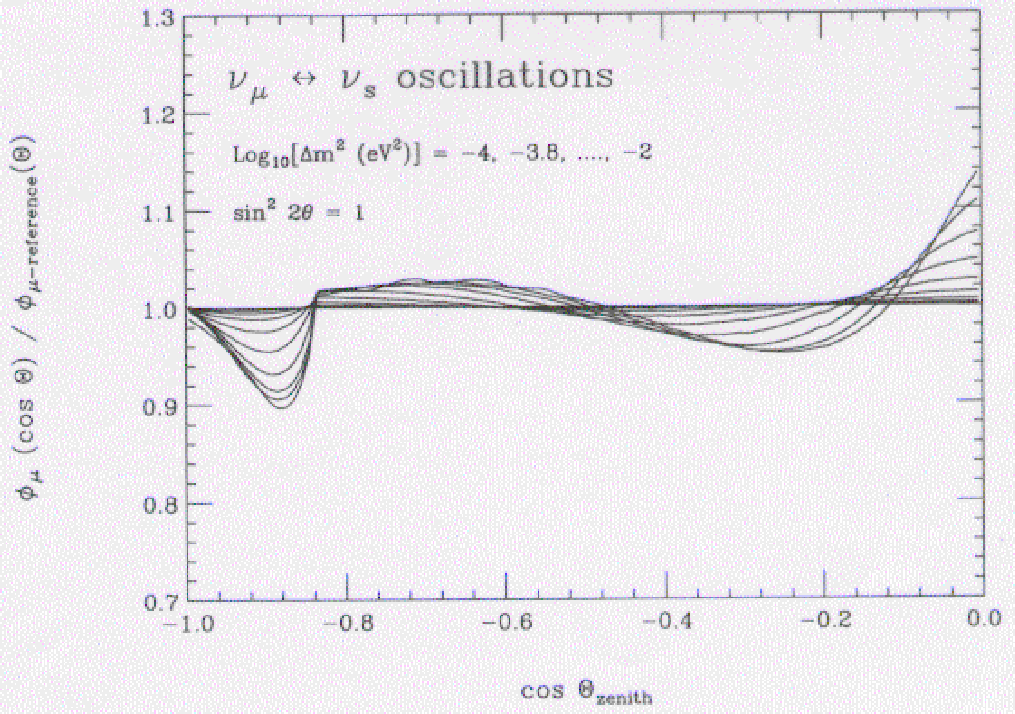
$\nu_\mu \leftrightarrow \nu_\tau$



$\nu_\mu \leftrightarrow \nu_s$



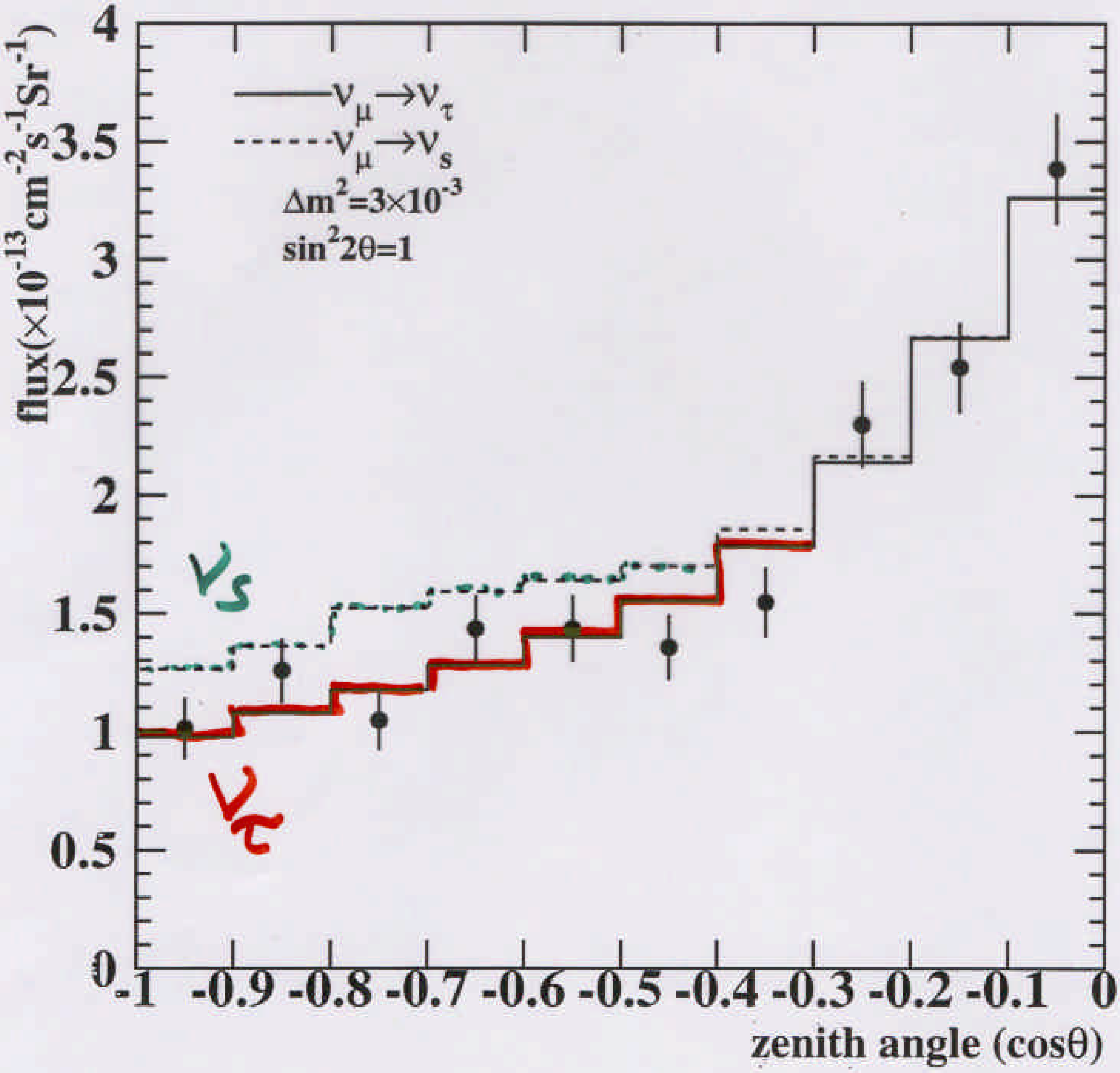
$\nu_\mu \leftrightarrow \nu_\tau$



$\nu_\mu \leftrightarrow \nu_s$

Lipari -45

zenith angle distribution of upward through going μ events (1070days)



Fundamental Problem for Up-going Muons:

- Only $\nu_\mu + \overline{\nu}_\mu$ (No comparison with $\nu_e$)
- Only Up–Going particles.

Need to compare with an **ABSOLUTE** calculation.
Systematic uncertanties more important.

- High Energy Primary c.r. flux less well measured
- Role of $K^\pm$ decay is enhanced

## ANGULAR DISTRIBUTION of Up–Going MUONS

The SHAPE can be reliably predicted.

Why HORIZONTAL $>$ VERTICAL ?

Competition of Interaction and Decay for $\boxed{\pi^\pm.}$

- "Horizontal" $\pi^\pm$ decay more easily that "Vertical" ones
- Effect stronger with increasing energy.
- $\sim$ all $K^\pm$ decay.

$$\begin{aligned} \ell_{\rm int}(\pi^\pm) \simeq \ell_{\rm int}(K^\pm) &\simeq 12.4 \text{ Km} \quad (\text{for } z = 20 \text{ Km}) \\ &\simeq 2.7 \text{ Km} \quad (\text{for } z = 10 \text{ Km}) \end{aligned}$$

$$\ell_{\rm decay}(\pi^\pm) \sim 5.6 \left(\frac{E_\pi}{100 \text{ GeV}}\right) \text{Km}$$

$$\ell_{\rm decay}(K^\pm) \sim 0.75 \left(\frac{E_K}{100 \text{ GeV}}\right) \text{Km}$$

The Shape of the zenith angle distribution
is determined by (dominant effects):

- The energy spectrum of primary radiation
  Harder spectrum $\Longleftrightarrow$ Vertical/Horizontal decreases

- The ratio $K/\pi$ ratio
  Smaller $K/\pi \Longleftrightarrow$ Vertical/Horizontal decreases.

Effect of uncertainty on $K/\pi$ ratio:

$$\frac{\delta(V/H)}{(V/H)_0} \simeq 0.12 \, \frac{\delta(K/\pi)}{(K/\pi)_0}$$

Effect of Cosmic ray spectrum. ($\phi_0 \propto E_0^{-\alpha}$):

$$\frac{\delta(V/H)}{(V/H)_0} \simeq 0.25 \, \delta\alpha$$

$$\text{Combined} = [0.12 \times 0.25] \otimes [0.25 \times 0.05] \simeq 0.033$$

$$\delta\left(\frac{\text{Vertical}}{\text{Horizontal}}\right) \sim 3.3\%$$

# IMPORTANT CONSTRAINT:

# MUON MEASUREMENTS

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

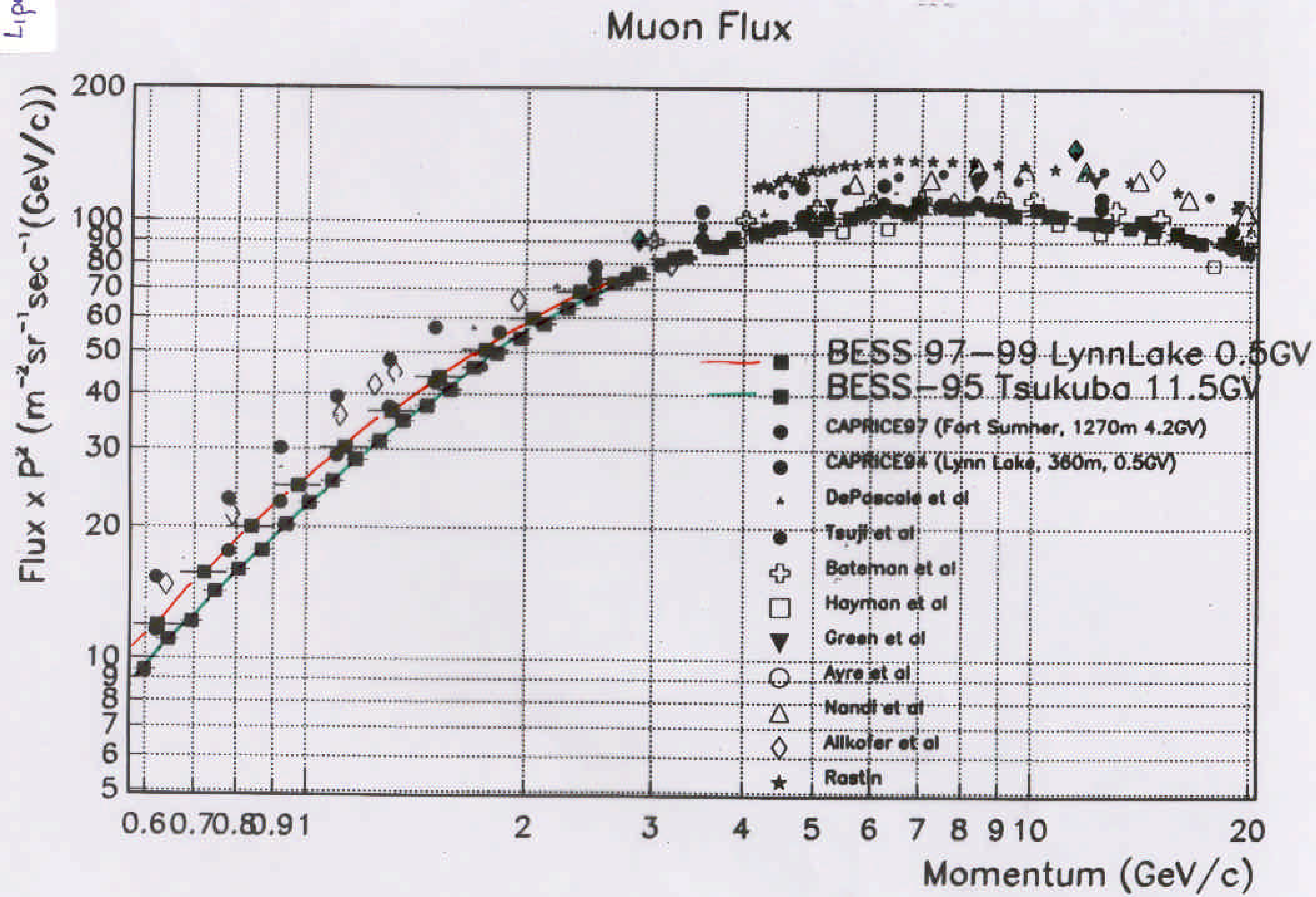
$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

- New Measurements at the ground:
  - CAPRICE
  - BESS

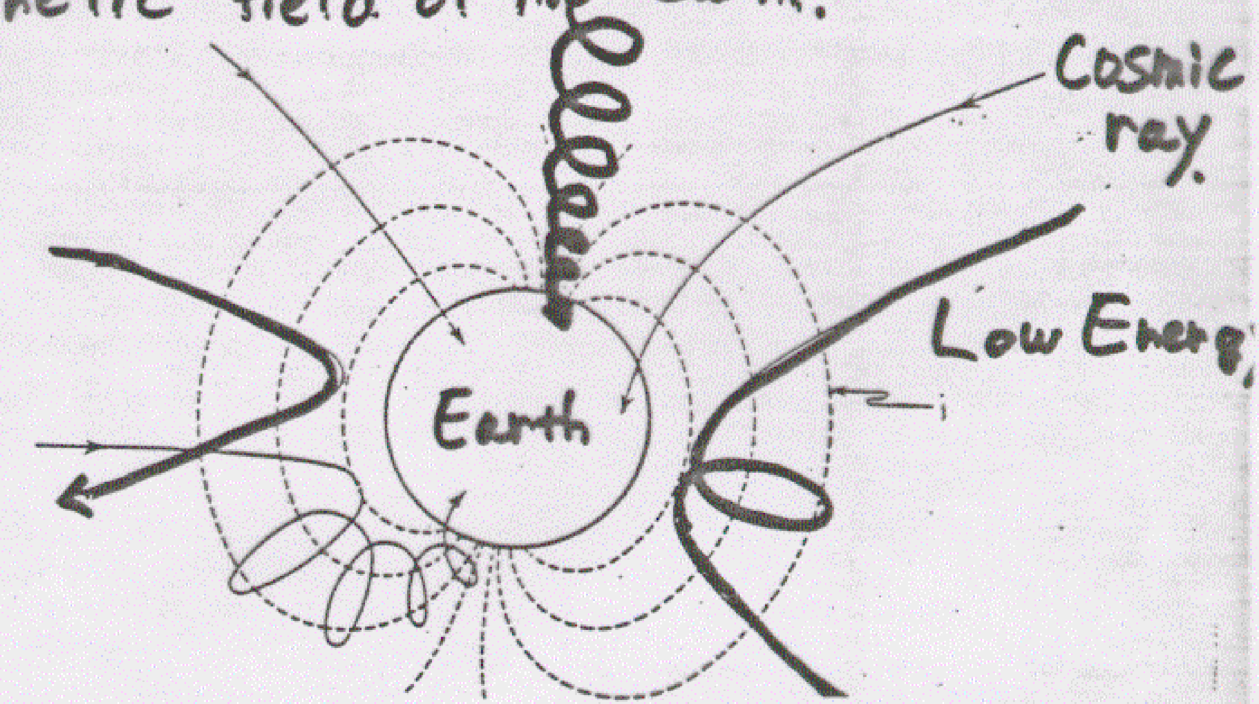
  Results are in agreement with each other (±5%) and **lower** (∼ 15%) than previous results

- Results during balloon ascent ($h \simeq 10$–$30$ km).
  - MASS
  - CAPRICE
  - HEAT
  - BESS (not public)

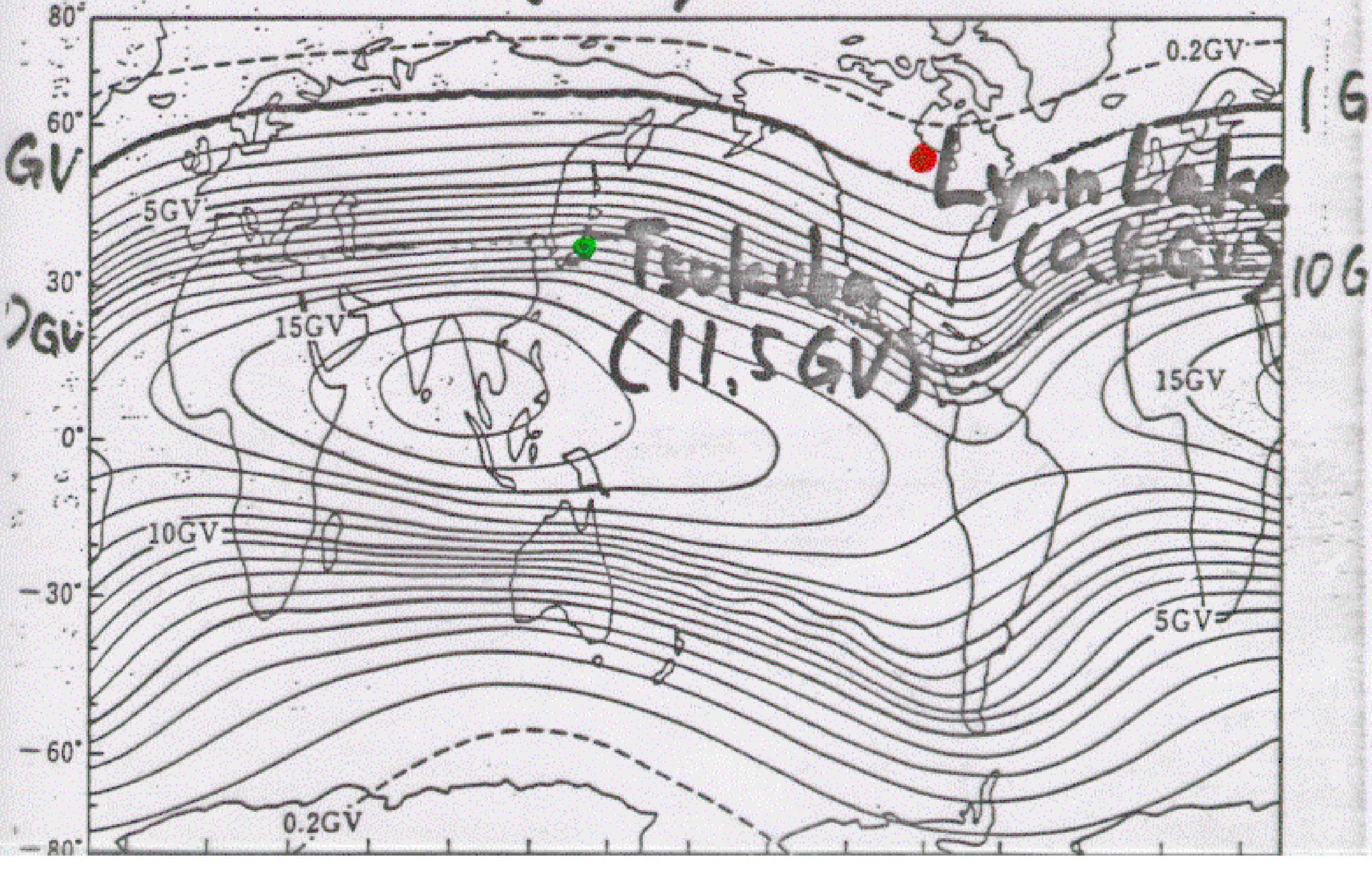
  Difficult measurements.
  VERY sensitive to the details of the calculation.
  3–Dimensional effects very important.
  Discrepancies are present.
  Comprehensive analysis not yet performed.


Muon Flux
Flux x P² (m⁻²sr⁻¹sec⁻¹(GeV/c))
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
0.6
0.7
0.8
0.9
1
2
3
4
5
6
7
8
9
10
20
Momentum (GeV/c)
BESS 97−99 LynnLake 0.5GV
BESS−95 Tsukuba 11.5GV
CAPRICE97 (Fort Sumner, 1270m 4.2GV)
CAPRICE94 (Lynn Lake, 360m, 0.5GV)
DePascale et al
Tsuji et al
Bateman et al
Hayman et al
Green et al
Ayre et al
Nandi et al
Allkofer et al
Rastin

Charged Particle of low energy can arrive at only a polar regeon because of a magnetic field of the Earth.



Cut Off Rigidity.



# THE $\nu$ CROSS SECTION

Energy range $E_\nu \simeq 0.1$–$10$ GeV

- Available DATA is "old" and incomplete.
- The theoretical situation is difficult.
  - **QUASI–ELASTIC reactions**
    $\nu_\mu + n \rightarrow \mu^- + p$
    Nuclear effects
    Fermi gas model (details of implementation ).
    Beyond the Fermi gas model
    (different estimates for the correction)
  - **One–$\pi$ production**
    "Resonance" production,
    "Continuum"
  - **Higher multiplicity**
    Structure functions for $Q^2$ very small:
    evolution is very likely important,
    but no consistent treatment known.

PDF

"GRV94"

The description of $\sigma_\nu$ is a source of systematic uncertainty on the $\nu$ event rates of
**comparable importance** to the description of the neutrino fluxes.

$$\boxed{\text{Event Rate} = \phi_\nu \otimes \sigma_\nu}$$

The different experimental groups

Super–Kamiokande
Soudan (MINOS)
MACRO
ICARUS

are using different **independent**
Montecarlo event generators.

Most of these codes are not publically available, and a critical comparison study has not been performed.

A single "standard" code is not possible
and not desirable, but an open discussion and critical study is very desirable.

MOST DESIRED : **NEW DATA**

**Summary:** Progress in the calculation:

  - High quality Primary Flux measurements.
  - Quantitative measurement of Geomagnetic Effects.
  - Soon new data on hadronic interactions from HARP at CERN.
  - Calculations need to be 3–dimensional including the bending of muons in the atmosphere. Computation becomes "heavier"
  - Very valuable atmospheric Muon data obtained.
  - Theoretical uncertainty on upward going muons are important, but the shape of angular distribution allows a valuable measurement.
  - $\sigma_\nu$ an important source of uncertainty ($\sim 15\%$ in absolute normalization)
  - **New detailed calculations soon available**

- **Evidence for NEW PHYSICS: ROBUST**
- **Parameter determination: No large bias**

(Uncertainty of $\sin^2 2\theta$ mostly from statistics
Uncertainty of $\Delta m^2$ mostly from systematics.