GLOBAL (*and Unified*) ANALYSIS OF SOLAR NEUTRINO DATA

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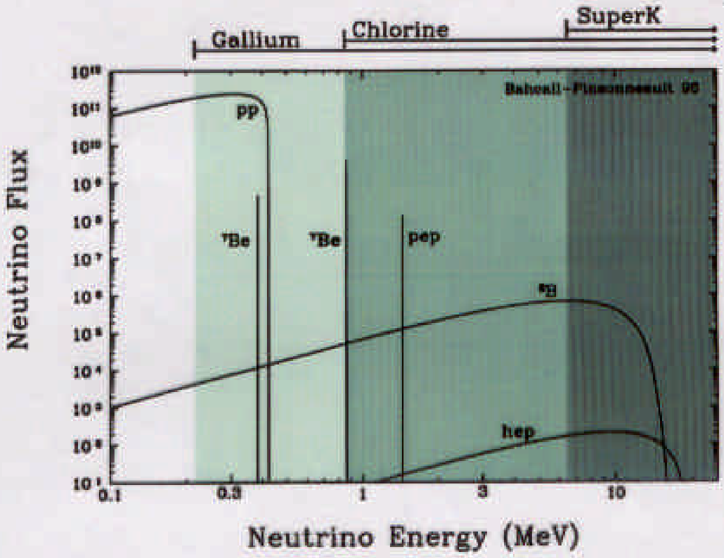
V. Summary

Based on:
M.C.G-G, P.C. de Holanda, C. Peña-Garay, J. Valle, NPB573 (2000)
C. Giunti, M.C.G-G,C. Peña-Garay, PRD62 (2000)
M.C.G-G,C. Peña-Garay, hep-ph/0002186
Works by:
Fogli, Lisi et. al PRD54,2048 (1996)...hep-ph/0005261
A. de Gouvea, A. Friedland, H. Murayama , hep-ph/0002064
A. Friedland, hep-ph/0002063
...

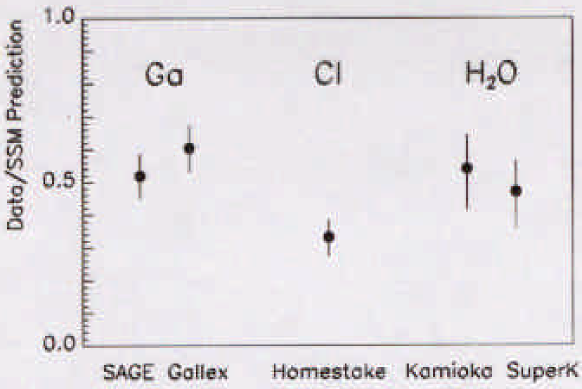
I.Introduction: MSW and vacuum solar-ν oscillations

- The sun emits $\nu_e's$. The Standard Solar Model fluxes



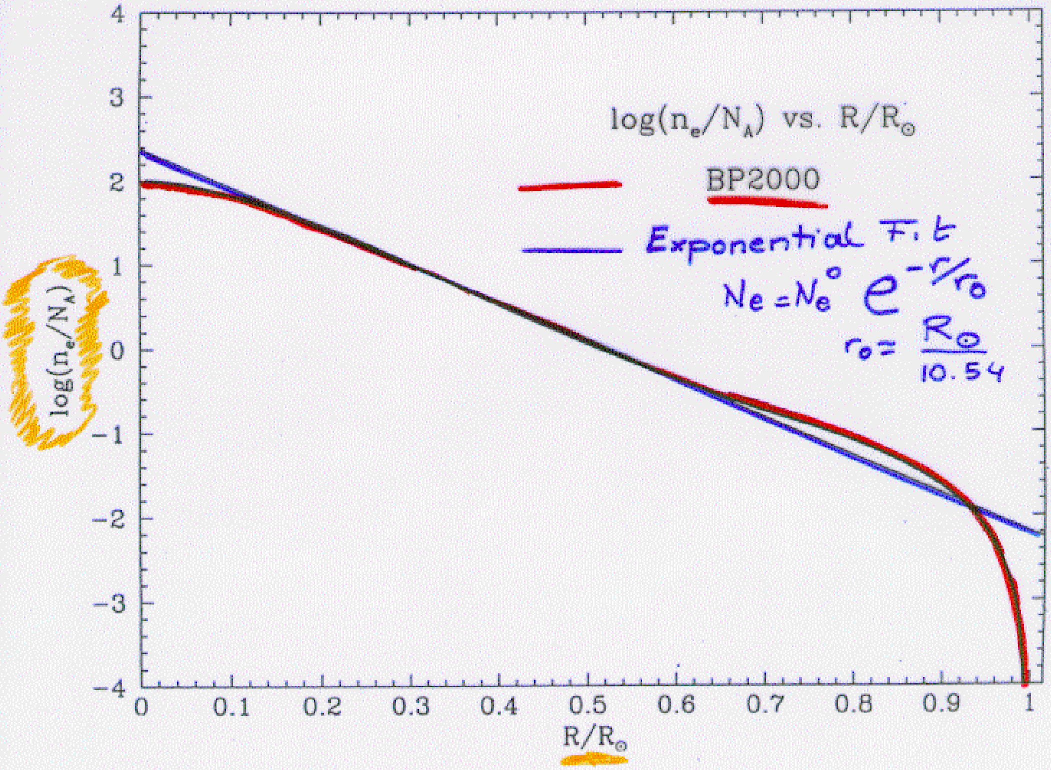
Solar neutrino energy spectrum

- $\nu_e's$ are detected on Earth by
  - Homestake (" Clorine") $\nu_e +^{37} Cl \rightarrow^{37} Ar + e^-$
  - SAGE and GALLEX $\nu_e +^{71} Ga \rightarrow^{71} Ge + e^-$
  - Kamiokande and SuperK $\nu_e e$ scattering in H_2O Target
- The experiments are sensitive to different E_ν
- All experiments observe a deficit...



...Of about 30–60 %

Gonzalez - 03


log(n_e/N_A) vs. R/R⊙
BP2000
Exponential Fit
Ne = Ne° e^{-r/r0}
r0 = R⊙/10.54
log(n_e/N_A)
R/R⊙
4
3
2
1
0
−1
−2
−3
−4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

- The *Standard* Interpretations:

  Oscillation of ν_e into ν_μ, ν_τ (active $\nu's$) , or ν_s (sterile ν)

- Two possible oscillation scenarios:

(a) Vacuum oscillations:

  - The distance between the Sun and the Earth $L \sim 10^{11}$ m is of the order of oscillation wavelength
  - Since $E_\nu \sim$ few MeV$\rightarrow \Delta m^2 \sim \pi/1.27\ E/L \sim 10^{-10}$eV^2
  - Any effects of Sun or Earth matter are neglected

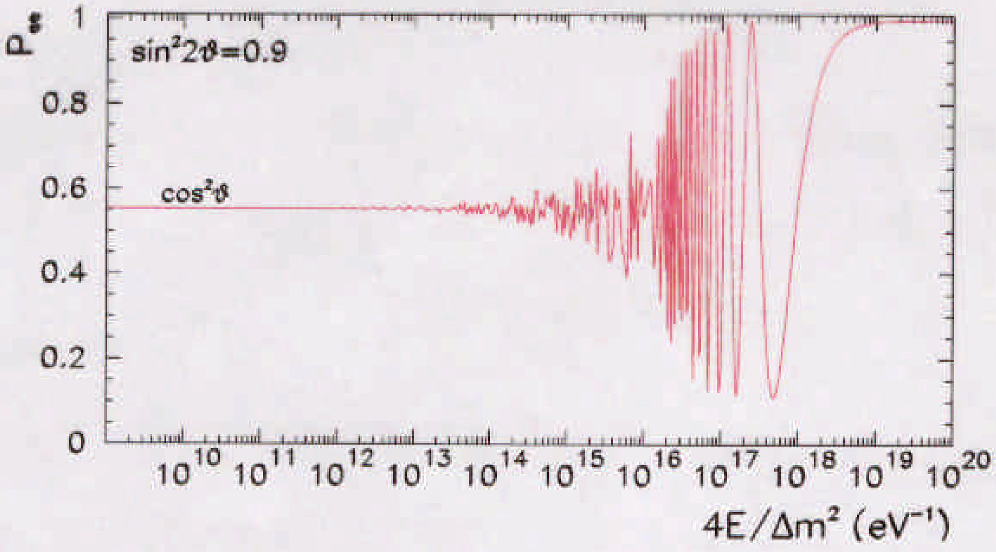
- The elementary survival probability:

$$P_{ee} = 1. - \sin^2(2\theta)\sin^2\left(1.27\Delta m^2 L/E\right)$$

Averaging over Earth Orbit $L(t) = L_0[1 - \varepsilon\cos 2\pi\frac{t}{T}]$:

$$\langle P_{ee}^{vac}\rangle = 1 - \frac{1}{2}\sin^2 2\theta\left[1 - \cos\left(\frac{\Delta m^2 L_0}{2E}\right) J_0\left(\frac{\varepsilon\Delta m^2 L_0}{2E}\right)\right]$$

$\varepsilon \sim$ Earth orbit eccentricity (0.0167).



- P_{ee}^{vac} is symmetric under $\Delta m^2 \rightarrow -\Delta m^2$ or $\theta \rightarrow \theta + \frac{\pi}{4}$

(b) Resonant Oscillations in Matter (MSW effect):

- Neutrinos can interact *coherently* with matter in the sun
- Different flavours have different interactions
- To include this effect: potential in the evolution equation

$$-i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = \begin{pmatrix} V_e + \frac{\Delta m^2}{2E}\cos 2\theta & -\frac{\Delta m^2}{2E}\sin 2\theta \\ -\frac{\Delta m^2}{2E}\sin 2\theta & V_X - \frac{\Delta m^2}{2E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

$$V_e = \frac{\sqrt{2}G_F}{M}(N_e - \frac{1}{2}N_n) \qquad V_s = 0$$

$$V_\mu = V_\tau \frac{\sqrt{2}G_F}{M}(-\frac{1}{2}N_n) \qquad N_{e(n)} \text{ elec (nucl) density}$$

- The approximate solution:

$$P_{e1}^{Sun} = \tfrac{1}{2} + (\tfrac{1}{2} - P_c)\cos(2\theta_{m,0})$$

$*$ $\theta_{m,0}$ is the mixing angle in matter

$$\sin(2\theta_{m,0}) = \frac{\Delta m^2 \sin(2\theta)}{\sqrt{(\Delta m^2\cos(2\theta) - A)^2 + (\Delta m^2\sin(2\theta))^2}}$$

$$A = 2\,E\,(V_e - V_X)$$

–When $\Delta m^2\cos(2\theta) = A$: Resonant conversion → $\sin\theta_m >> \sin\theta$

– For $E \sim$ few MeV → $\Delta m^2 \sim 10^{-4} - 10^{-8}\,\text{eV}^2$

$*$ P_c Level Crossing Probability:

$$P_c = \frac{\exp[-\gamma\sin^2\theta] - \exp[-\gamma]}{1 - \exp[-\gamma]} \qquad \gamma = \pi\frac{\Delta m^2}{E}\left[\left.\frac{d\ln N_e(r)}{dr}\right|_{r=r_{res}}\right]^{-1}$$

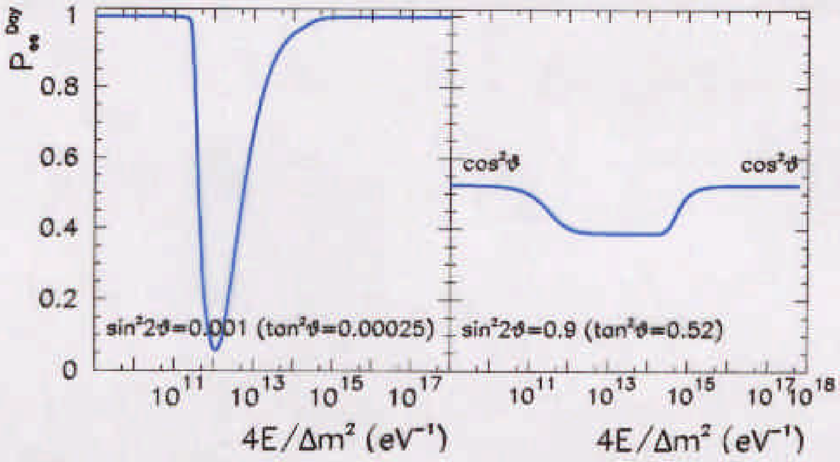
$\gamma = \pi r_0$ *For Exponential Density Profile* $N_e(r) = N_{e0}\exp(-r/r0)$

– Averaging over L-dependent terms in the Sun-Earth Propagation:

$$P_{ee}^{MSW} = P_{e1}^{Sun}P_{1e}^{Earth} + P_{e2}^{Sun}P_{2e}^{Earth}$$

$$P_{2e}^{Earth,Day} = \cos^2\theta = 1 - P_{1e}^{Earth,Day}$$

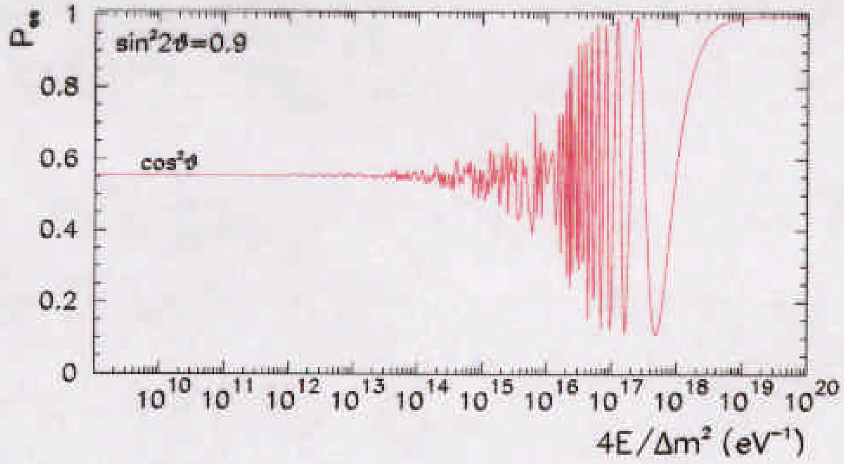
$$P_{ee}^{MSW} = \frac{1}{2} + (\frac{1}{2} - P_c)\cos(2\theta_{m,0})\cos(2\theta)$$



– Due to matter effects P_{ee}^{MSW} is only symmetric under *simultaneous* $(\Delta m^2, \theta) \to (-\Delta m^2, \theta \pm \frac{\pi}{4})$

– For $\Delta m^2 > 0$ resonance possible only for $\theta < \frac{\pi}{4}$ and *Traditionally* MSW solutions are also plotted in $(\Delta m^2, \sin^2(2\theta))$ But in principle solutions are also possible for $\theta > \frac{\pi}{4}$

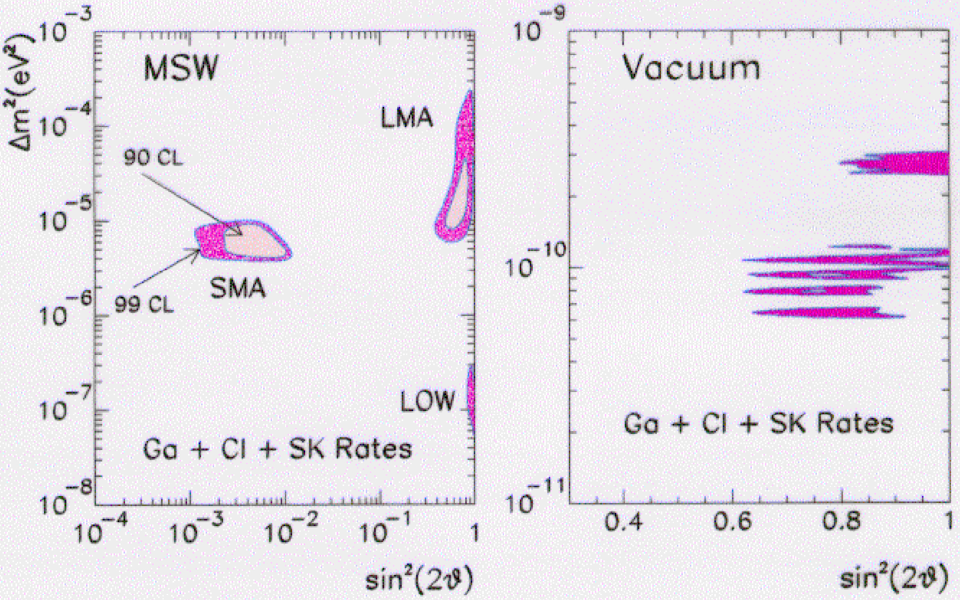
- Comparing P_{ee}^{MSW} with P_{ee}^{vac}



– For $10^{13} \lesssim E/\Delta m^2 \lesssim 10^{18}$ both matter and L dependent effects: Quasi-vacuum oscillations.

– The size of Quasi-vacuum oscillation region depends on how fast the $\cos^2\theta$ asymptotic regime at large $E/\Delta m^2$

Standard 2-ν Oscillation Solutions to the Solar ν Deficit



II.The *Unified* Picture

- From MSW to Vacuum: Quasi-Vacuum Oscillations

– The ν_e survival amplitude after propagation from the Sun to the detector at the Earth:

$$\begin{aligned} A(\nu_e \to \nu_e) = \quad & A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e) \\ + \quad & A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e) \end{aligned}$$

– Where

$$|A_{Sun}(\nu_e \to \nu_1)|^2 = P_{e1}^{Sun} = 1 - |A_{Sun}(\nu_e \to \nu_2)|^2$$

$$|A_{Earth}(\nu_1 \to \nu_e)|^2 = P_{1e}^{Earth} = 1 - |A_{Earth}(\nu_2 \to \nu_e)|^2$$

$$A_{vac}(\nu_i \to \nu_i) = \exp\left(-i\, m_i^2\,(L - R_{Sun})/2E\right)$$

– So in general:

$$\begin{aligned} P_{ee} = \quad & P_{e1}^{Sun} P_{1e}^{Earth} + (1 - P_{e1}^{Sun})(1 - P_{1e}^{Earth}) \\ + \quad & 2\sqrt{P_{e1}^{Sun}(1 - P_{e1}^{Sun}) P_{1e}^{Earth}(1 - P_{1e}^{Earth})} \cos\left(\frac{\delta m^2 L}{2E} + \delta\right) \end{aligned}$$

δ small phase $\sim 10^{-3}$.

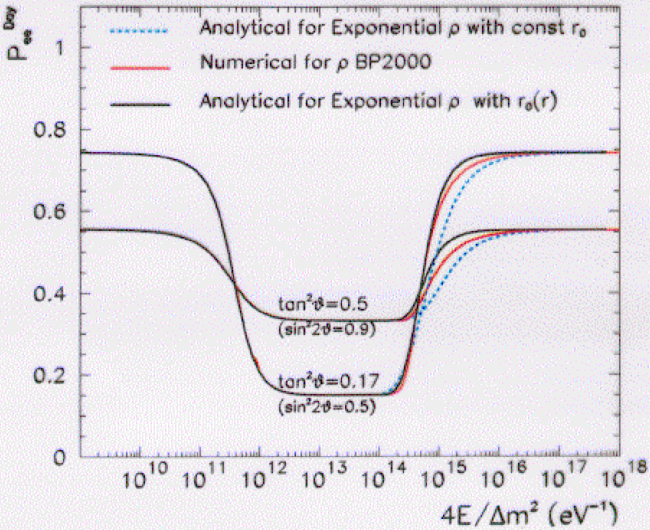
– The size of the Quasi-vacuum Oscillation Region depends on how fast the first line acquires the asymptotic vacuum value $\cos^2\theta$.

– The First Piece at the Day side of the Earth:

$$P_{ee}^{Day} = P_{e1}^{Sun}\cos^2\theta + (1 - P_{e1}^{Sun})\sin^2\theta$$

– Comparing the numerical result with analytical approximations:

P_{ee}^{Day}

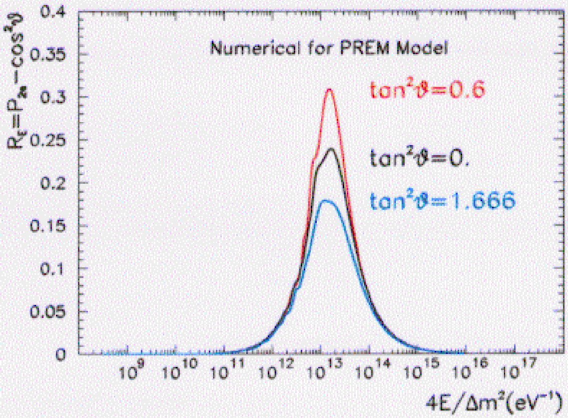


– Quasi-Vacuum oscillations are slightly over estimated in the approximate analytical solution for constant exponential density profile

– Quasi-Vacuum oscillations are slightly under-estimated in the approximate analytical solution for variable exponential density profile

– How about Earth Matter effects?:

$P_{2e} - \cos^2\theta$



– Earth Matter effects are small in the Quasi-Vacuum region

- Matter effects at $\theta > \frac{\pi}{4}$: The dark side?

– The First Piece at the Day side of the Earth:

$$P_{ee}^{Day} = P_{e1}^{Sun} \cos^2\theta + (1 - P_{e1}^{Sun}) \sin^2\theta$$

– How to compute P_{e1}^{Sun} for $\theta > \frac{\pi}{4}$?:

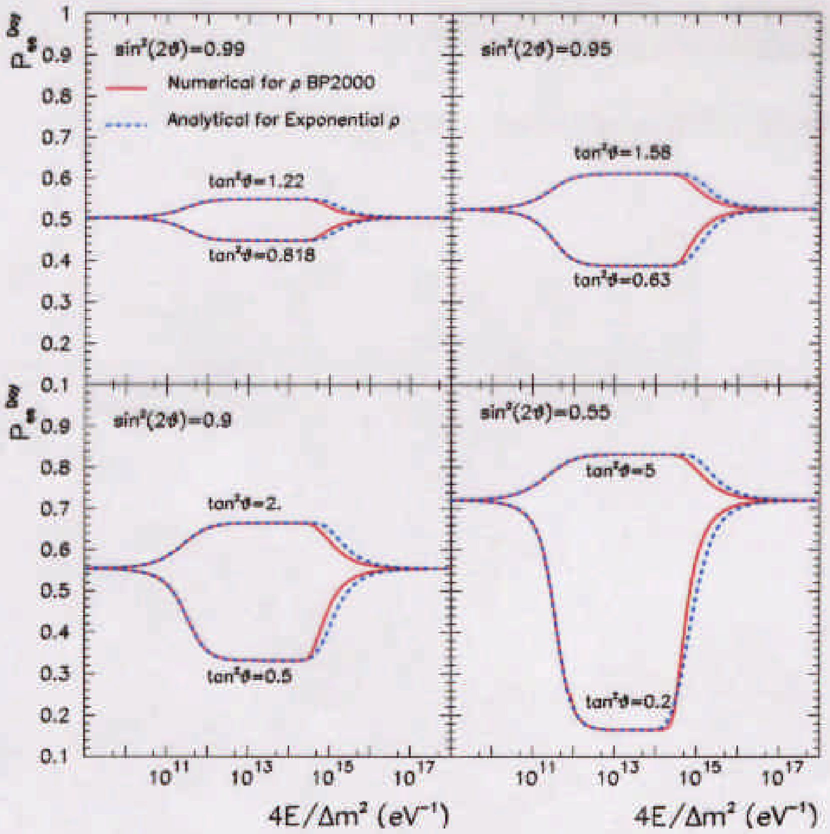
∗ For Exponential Profile $N_e(r) = N_{e0} \exp -r/r0$:

Trivial analytical continuation to the second octant using:

$$P_{e1}^{Sun} = \frac{1}{2} + (\frac{1}{2} - P_c)\cos(2\theta_{m,0})$$

with P_c and $\cos(2\theta_{m,0})$ computed for $\sin^2\theta > \frac{1}{2}$

∗ Numerically

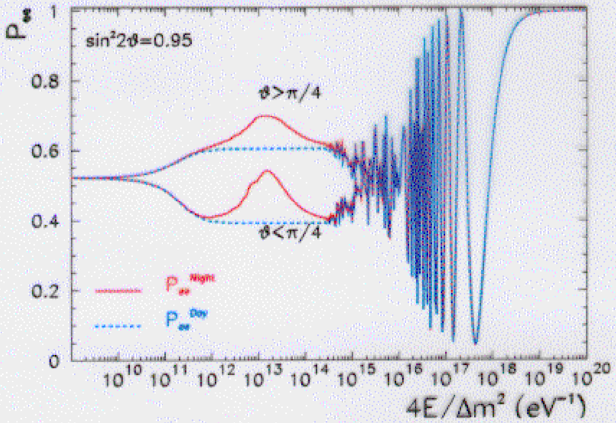


P_{ee}^{DAY}

– For angles close to $\theta = \frac{\pi}{4}$ the survival Probabilities are *mirror–symmetric*

– Again differences in the Quasi-vacuum region

– In what follows we will use numerical probabilities valid on full parameter space

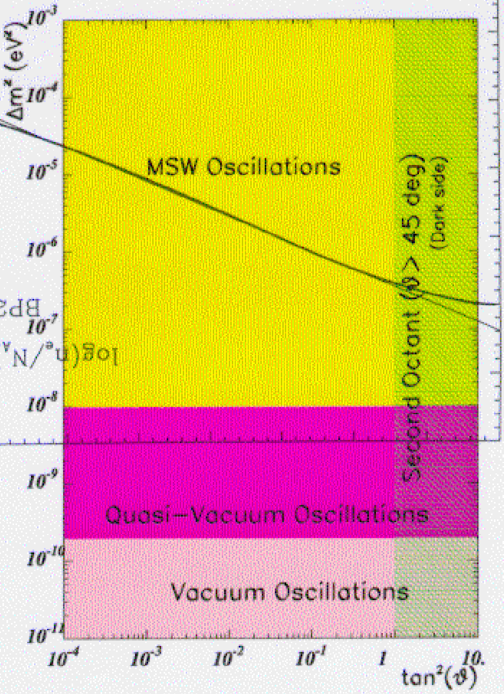


Pee

— Night

··· Day

– And plot the allowed regions in the *Unified Oscillation Parameter Plot*:



III.Global Analysis

We include in the analysis the data on:

∗ Total Rates:

$$\chi^2_R = \sum_{i,j=1,3} (R_i^{th} - R_i^{exp})\sigma_{ij}^{-2}(R_j^{th} - R_j^{exp})$$

σ_{ij} contains theoretical uncertainties and
the experimental systematic and statistical errors

∗ Zenith Angle Distribution: Day + 5 Nights data points

$$\chi^2_Z = \sum_{i=1,6} \frac{(\alpha_z \frac{R_i^{th}}{R_i^{\mathrm{BP98}}} - R_i^{exp})^2}{\sigma_i^2}$$

α_z free normalization factor.

∗ Recoil Electron Energy Spectrum: 18 bins

$$\chi^2_S = \sum_{i,j=1,18} (\alpha_{sp} \frac{R_i^{th}}{R_i^{\mathrm{BP98}}} - R_i^{exp})\sigma_{ij}^{-2}(\alpha_{sp} \frac{R_j^{th}}{R_i^{\mathrm{BP98}}} - R_j^{exp})$$

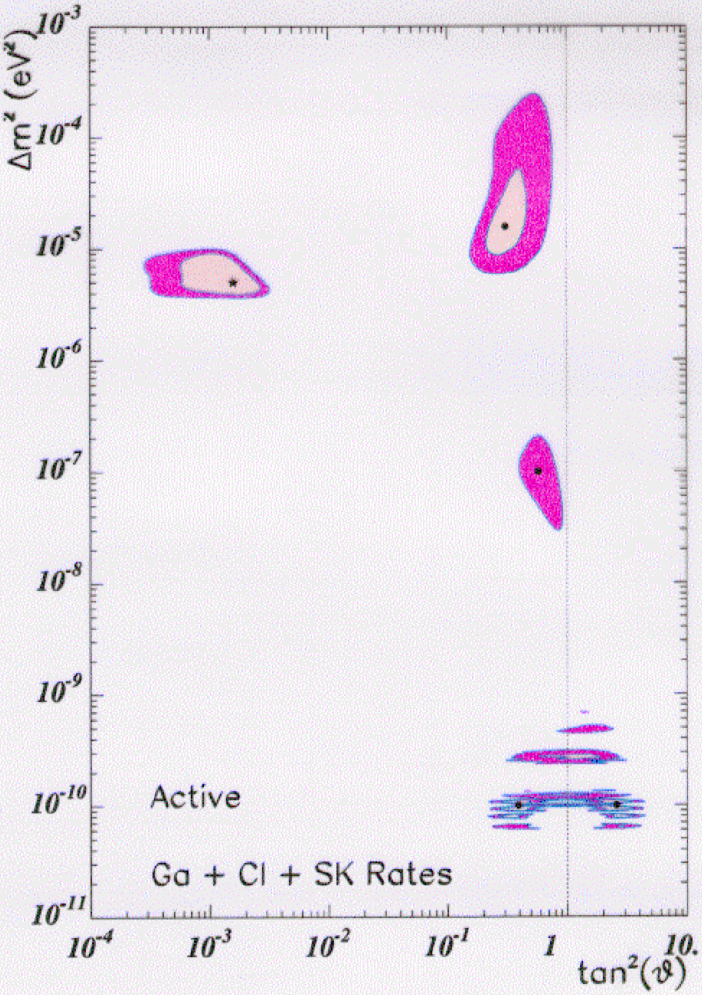
α_{sp} free normalization factor.

$$\sigma_{ij}^2 = \delta_{ij}(\sigma_{i,stat}^2 + \sigma_{i,uncorr}^2) + \sigma_{i,exp}\sigma_{j,exp} + \sigma_{i,cal}\sigma_{j,cal} \quad (1)$$

A total of 25 independent data points

Solutions for $\nu_e \to \nu_{active}$

Allowed regions from Rates:

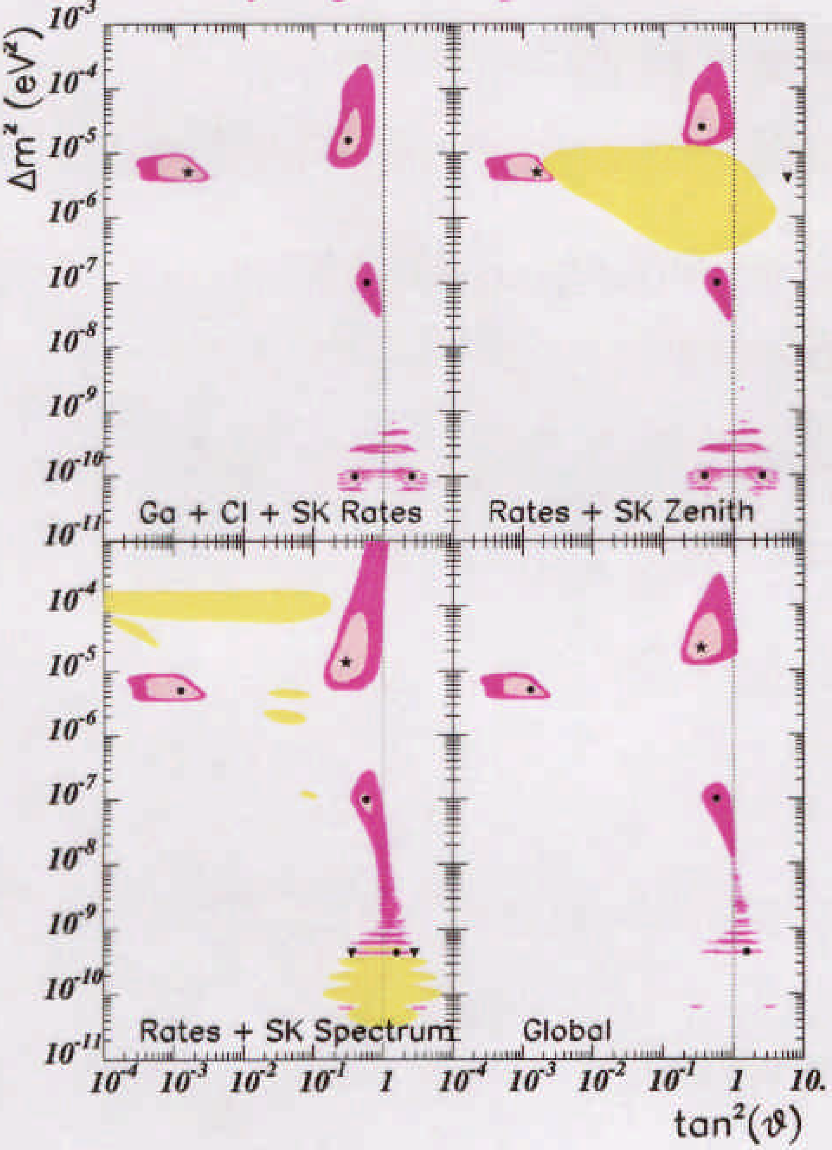


Observables	Probability (%)			
	SMA	LMA	LOW	Vac
Rates	54	9	1	3.4

– For higher *Vacuum* solutions matter effects break symmetry

Solutions for $\nu_e \to \nu_{active}$

Effect of Day-Night and Spectrum data:

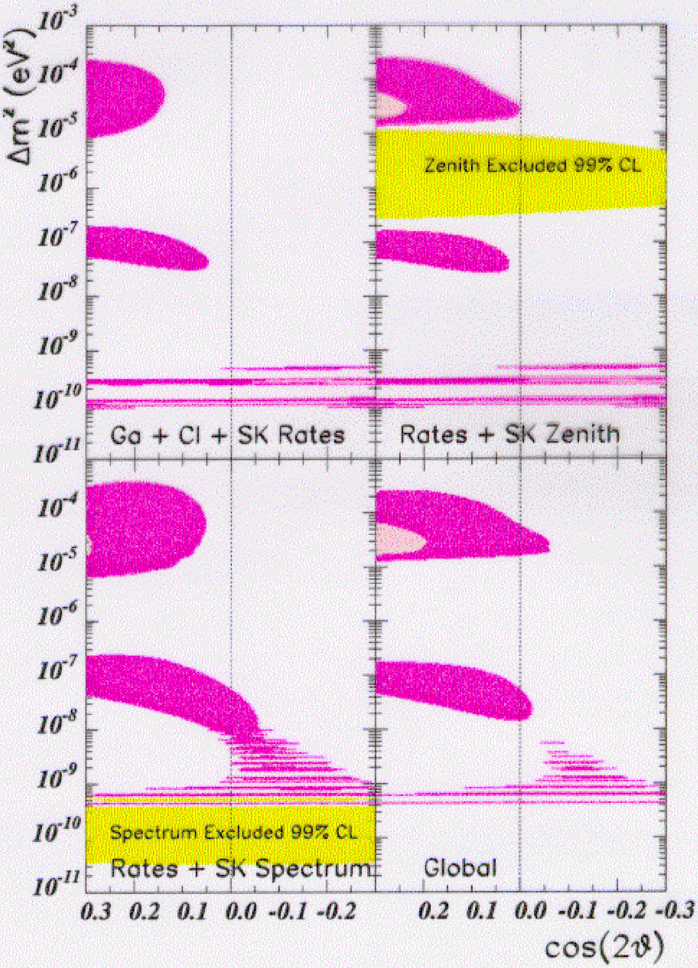


Observables	Probability (%)			
	SMA	LMA	LOW	Vac
Rates	50	8	1	3.4
Rates + Zenith +Spectrum	18	26	11	9

- Best fit point for zenith distribution in second octant
- LOW and Vacuum regions connected in the second octant

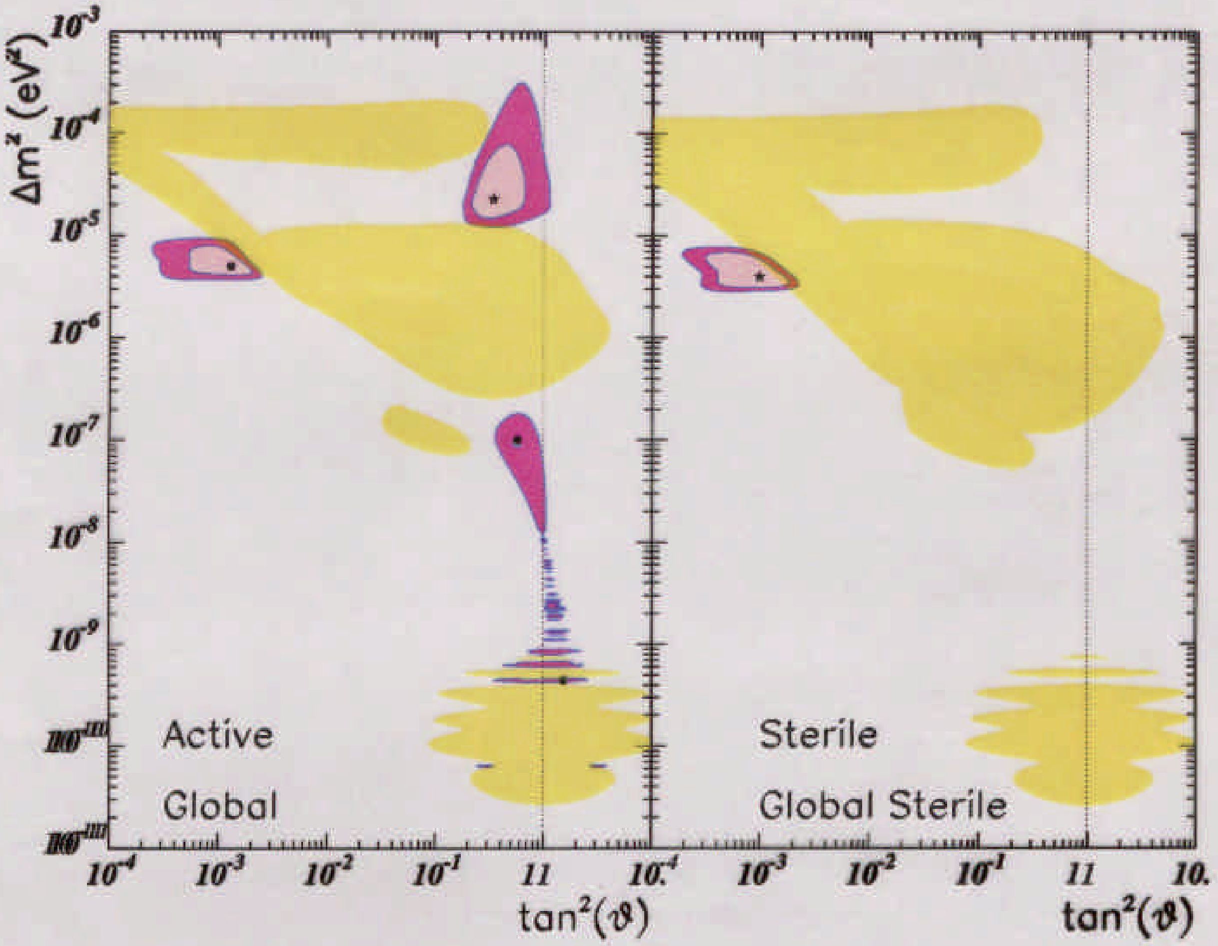
Solutions for $\nu_e \to \nu_{active}$

Blow up around Maximal mixing



- Inclusion of Day-Night data pushes LMA towards second octant
- Inclusion of Spectrum data pushes LOW towards second octant
- At 99 % CL New set of alowed islands in the Second Octant in Quasi-Vacuum Region

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$



- Why differences for oscillations into active or sterile?

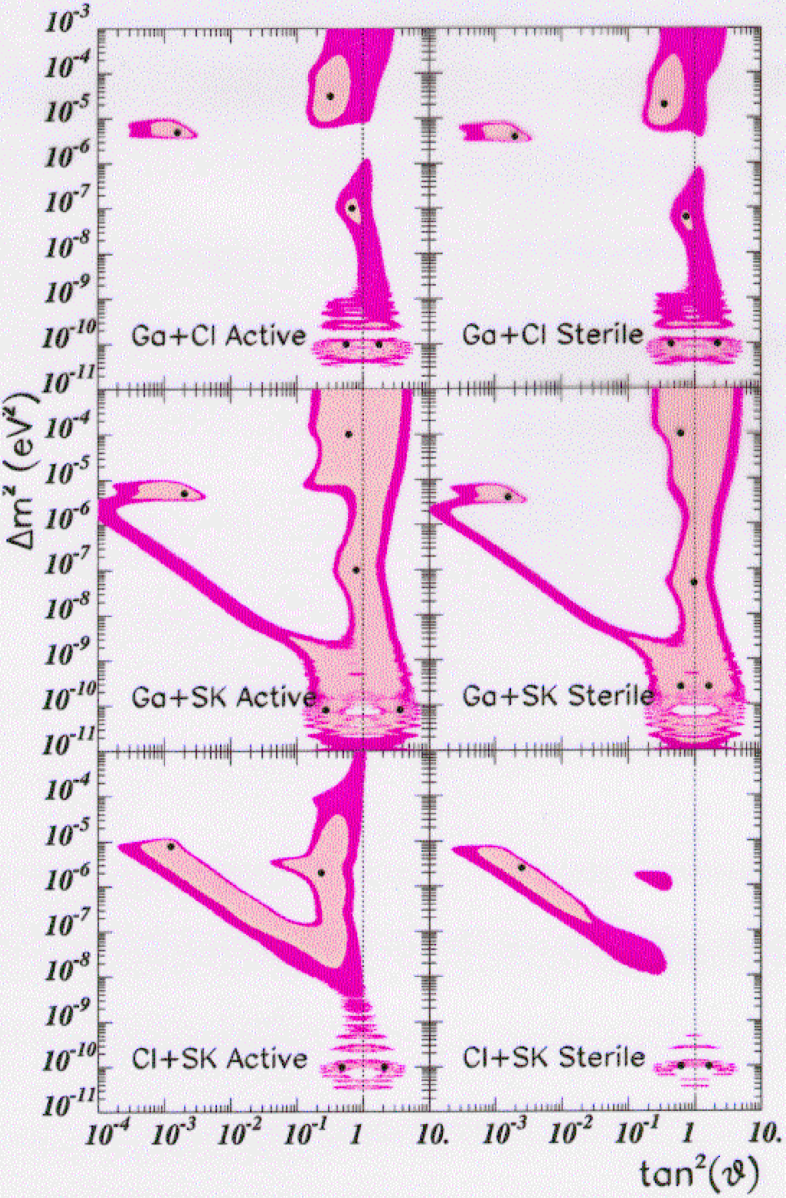
Main effect is different contribution to event rates in SuperK

$\nu_{\mu(\tau)} + e \to \nu_{\mu(\tau)} + e \to$ NC events in SuperK

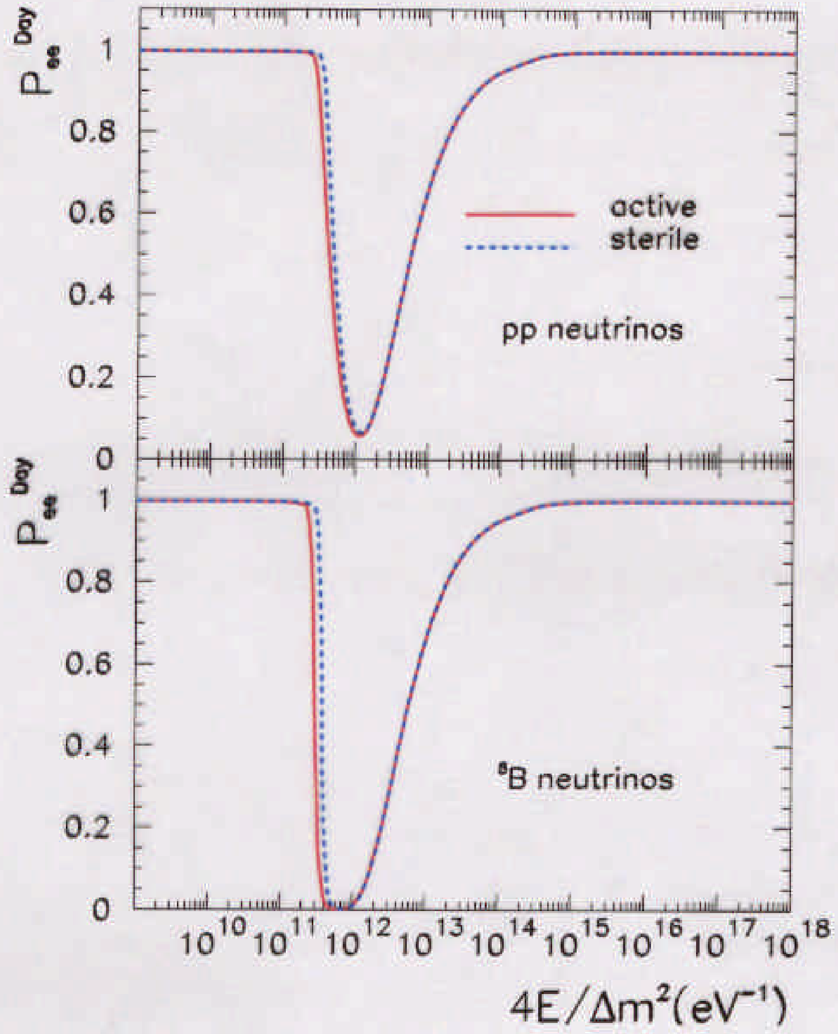
$\nu_s + e \not\to \nu_s + e \to$ no NC events in SuperK

Also slightly different survival probabilities in the sun

Regions with Only Two Experiments



Survival Probabilities For Active and Sterile Neutrinos



Due to smaller matter potential the SMA region for sterile is slightly shifted down

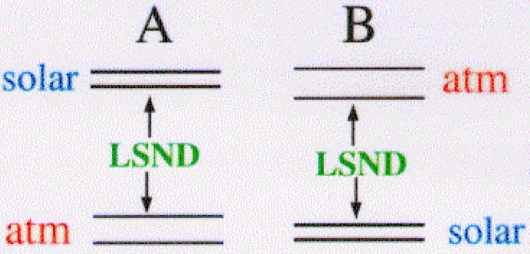
$$\Delta V_{active} = \frac{\sqrt{2}\, G_F}{M} N_e$$

$$\Delta V_{sterile} = \frac{\sqrt{2}\, G_F}{M} \left(N_e - \frac{1}{2} N_n \right)$$

IV. *Unifying* Active and Sterile Oscillations: Four-neutrino Oscillations

- To fit solar, atmospheric and LSND → 4 $\nu's$
- But LEP data implies only 3 neutrino flavours → 4th ν sterile
- The mixing matrix U: 6 mixing angles
- Limits from Accelerator and Reactor:

→ Only two possible mass spectra



→ Mixings between e and heavy states neglegeable : Only 4 angles

$$\begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12}c_{23}c_{24} & c_{12}c_{23}c_{24} & s_{23}c_{24} & s_{24} \\ s_{12}(c_{23}s_{24}s_{34}+s_{23}c_{34}) & -c_{12}(s_{23}c_{34}+c_{23}s_{24}s_{34}) & c_{23}c_{34}-s_{23}s_{24}s_{34} & c_{24}s_{34} \\ s_{12}(c_{23}s_{24}c_{34}-s_{23}s_{34}) & c_{12}(s_{23}s_{34}-c_{23}s_{24}c_{34}) & -(c_{23}s_{34}+s_{23}s_{24}c_{34}) & c_{24}c_{34} \end{pmatrix}$$

- $c_{23}c_{24} = 0$ → pure $\nu_e \to \nu_{active}$ oscillations with mixing θ_{12}
- $c_{23}c_{24} = 1$ → pure $\nu_e \to \nu_{sterile}$ oscillations with mixing θ_{12}
- Intermediate cases $0 < c_{23}c_{24} < 1$ also possible

– In the general case of simultaneous $\nu_e \to \nu_s$ and $\nu_e \to \nu_a$ oscillations:

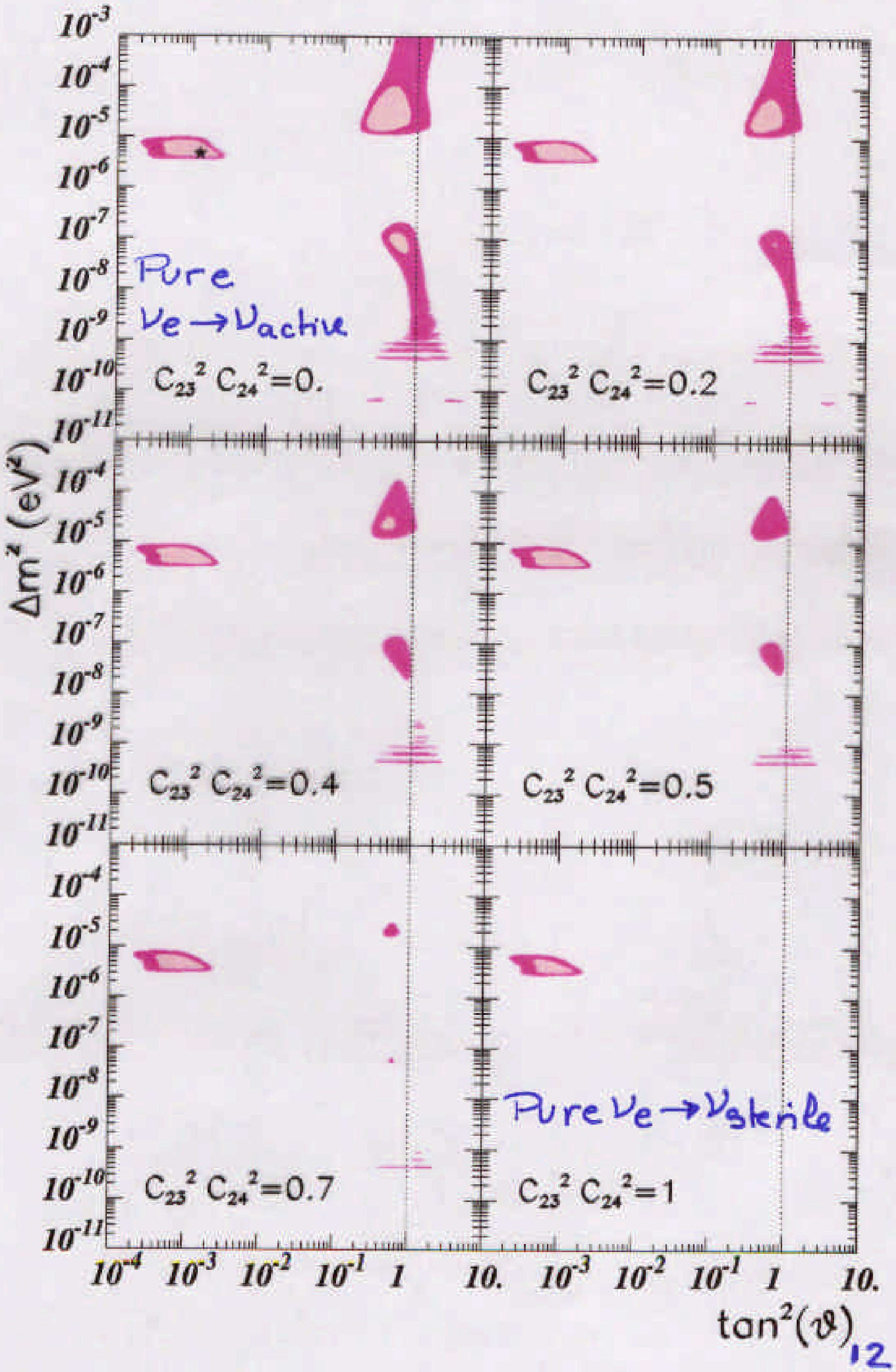
$$P_{ee}^{4\nu,Sun} = P_{ee}^{2\nu,Sun}$$

$$P_{es}^{4\nu,Sun} = c_{23}^2 c_{24}^2 \left(1 - P_{ee}^{Sun}\right)$$

but $P_{ee}^{2\nu,Sun}$ is computed for a matter potential $A \equiv A_{CC} + c_{23}^2 c_{24}^2\, A_{NC}$

– Same analysis but with 3 parameters: Δm_{12}^2,θ_{12}, and $c_{23}^2 c_{24}^2$

Solutions for Four-neutrino Oscillations



- LMA, LOW and VAC solutions can have a subdominant $\nu_e \rightarrow \nu_s$ component

IV. Summary

- Global Analysis of Solar Neutrino Data in the *Full Parameter Space for Oscillations*:
  - Including Matter Effects in Quasi-Vacuum Oscillation Region
  - Including MSW Transitions for $\theta > \frac{\pi}{4}$
- For oscillations $\nu_e \to \nu_{active}$:

  The best fit points:

	Δm^2	$\tan^2\theta$	g.o.f
LMA	2.2×10^{-5}	0.34	26 %
SMA	4.9×10^{-6}	0.0013	18 %
LOW	9×10^{-8}	0.49	11 %
QVO	4.4×10^{-10}	1.75	9 %

  - At 99% CL: *New set of allowed islands in the Second Octant in Quasi-Vacuum Region*
  - Best Fit for Zenith angle distribution *in the Second Octant*
  - At 99% CL: LMA and LOW regions extend to $\theta > \frac{\pi}{4}$: *Maximal Mixing is Allowed*
- For oscillations $\nu_e \to \nu_{sterile}$:
  - Only SMA ($\Delta m^2 = 3.8 \times 10^{-6}$ $\tan^2\theta = 0.001$) 13%
- In Four-ν mixing:
  - Continuous mixture of simultaneous $\nu_e \to \nu_{sterile}$ and $\nu_e \to \nu_{active}$ oscillations
  - LMA, LOW and VAC solutions can have a subdominant $\nu_e \to \nu_{sterile}$ component