New Directions for New Dimensions: From Strings to Neutrinos to Axions to ... ?

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Introduction

Extra spacetime dimensions have been discussed since the original work of Kaluza & Klein in the 1920's...

- original motivation to unify gravity and electromagnetism
- 4D gauge invariance derived from 5D general coordinate invariance!

More recently, extra dimensions have emerged in the context of string theory...

- best candidate for unifying *all* fundamental forces
- at least *six* extra dimensions required

In fact, two kinds of extra dimensions are possible in string theory...

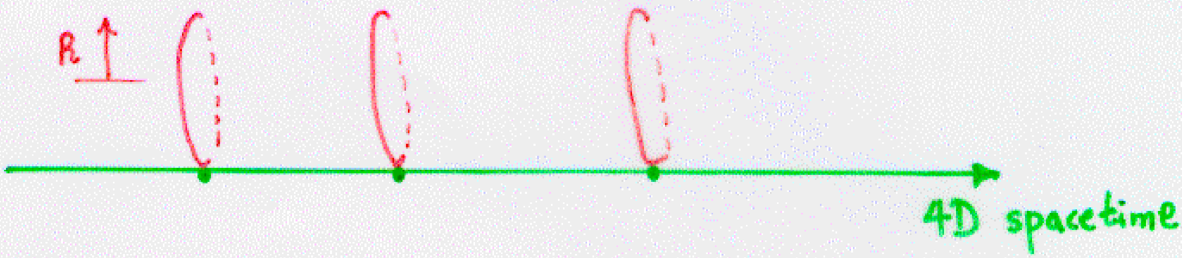
"Universal" extra dimensions

These extra dimensions would be experienced by *all* forces
$\Longrightarrow$ such extra dimensions would therefore be accessible via accelerator experiments

$$R^{-1} \geq \mathcal{O}(700 \text{ GeV}) \qquad R \leq \mathcal{O}(10^{-19} \text{ m})$$

Extra dimensions and the MSSM

We imagine that at every point in spacetime,
there are extra *compactified* dimensions of radius R:



Why don't we see this experimentally?

If $R^{-1} \geq \mathcal{O}(10^2 \text{ GeV})$, then too small to observe!

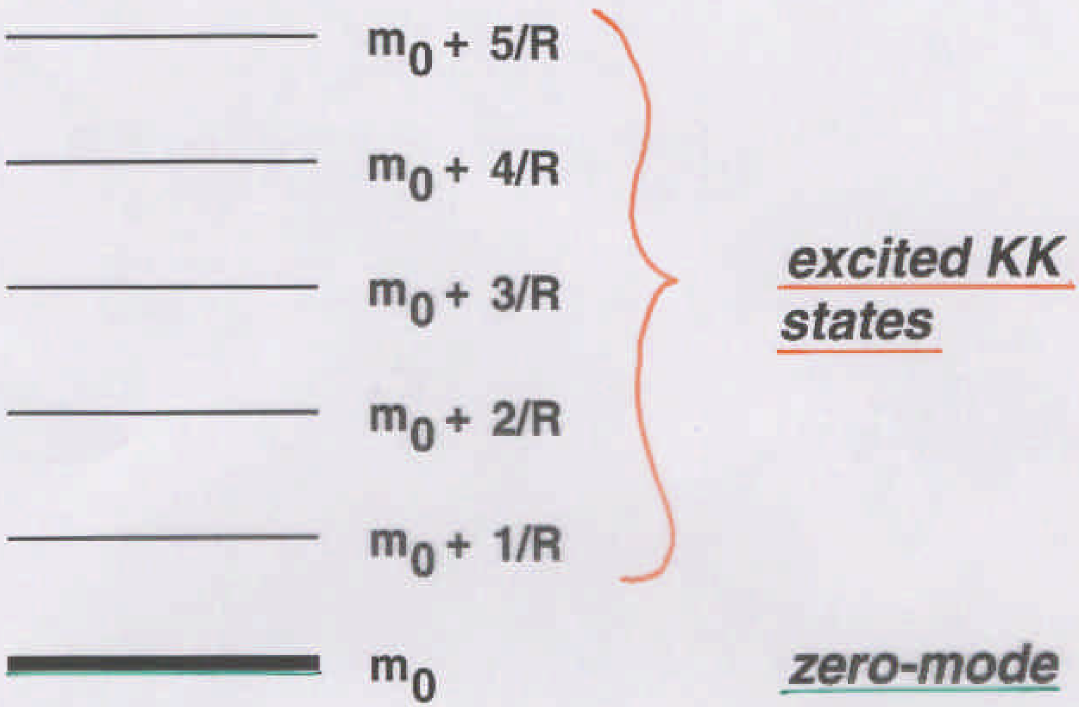
Technically, all wavefunctions Φ must be *periodic* under $y \to y + 2\pi R$:

$$\Phi \;=\; \sum_{n=0}^{\infty} \phi_n(\mathbf{x}) \, \exp(in\, y/R)$$

$\phi_n(\mathbf{x})$ are the "Kaluza-Klein modes", $n \in \mathbb{Z}$.

$$m_n^2 \;=\; m_0^2 \;+\; \frac{n^2}{R^2}$$

Thus, if $m_0 \ll R^{-1}$,
we have an infinite tower of Kaluza-Klein modes...



If accessible energy scale is much less than R^{-1}, then we cannot excite higher KK modes!

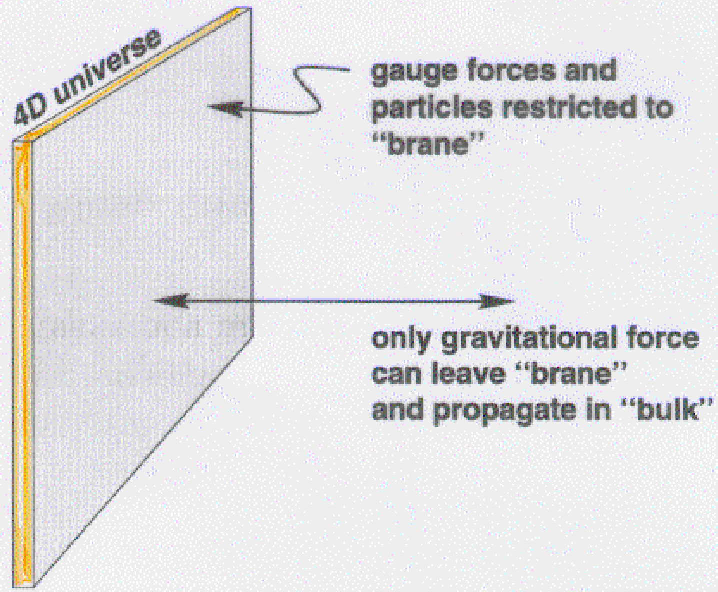
$\Longrightarrow$ Only see KK ground state (zero-mode)! This is the usual four-dimensional quantum state.

$\Longrightarrow$ Threshold for extra dimensions is $\mu_0 \equiv R^{-1}$.

$\Longrightarrow$ Extra dimensions are manifested by appearance of Kaluza-Klein modes at higher energy scales $\mu \geq \mathcal{O}(R^{-1})$.

"Gravity-only" extra dimensions

Recent developments in non-perturbative string theory predict the existence of solitonic "membranes" to which various forces can be restricted — *e.g.*,



$\Longrightarrow$ *Only gravity feels this kind of extra dimension*!
$\Longrightarrow$ Cannot be tested via accelerator experiments!
$\Longrightarrow$ Instead, must test gravity itself...

- tests of gravitational inverse-square law (*e.g.*, Cavendish-type experiments)
- astronomical and cosmological constraints.

$$R^{-1} \geq \mathcal{O}(10^{-4}\,\text{eV}) \qquad R \leq \mathcal{O}(\text{millimeter})\ !$$

Despite these bounds, extra dimensions have traditionally been considered to be at the Planck scale: $R \approx 10^{-33}$ cm.

However, this has changed dramatically during the past year. People began to wonder...

- what if the extra dimensions are not so small...?
- what if the corresponding KK states are *light*...?
- what effects would this have on physics beyond the SM...?

Surprisingly, the answer turned out to be —

Large extra spacetime dimensions have the power to alter the fundamental high energy scales of physics!

These developments have primarily come in three "flavors"...

• KRD, Dudas, Gherghetta 1998

(1) Extra dimensions to lower the GUT scale

- These are δ *universal* extra dimensions ("in the brane").
- They are felt by *gauge* forces and change the running of the *gauge* couplings $\alpha_i(\mu)$.
- Unification is *preserved*, but unification scale is lowered!
- No large hierarchy needed: $M'_{\text{GUT}}/R^{-1} \leq 20$.

• Arkani-Hamed, Dimopoulos, Dvali 1998

(2) Extra dimensions to lower the Planck scale

- These are n *gravity-only* extra dimensions ("off the brane").
- They change the running of *gravitational* coupling $G_N(\mu)$.
- This lowers the Planck scale (where $G_N \sim \mathcal{O}(1)$).
- A larger hierarchy is needed: $M'_{\text{Planck}}/r^{-1} \leq 10^{16}$.

• Witten 1996; Lykken 1996

(3) Extra dimensions to lower the string scale

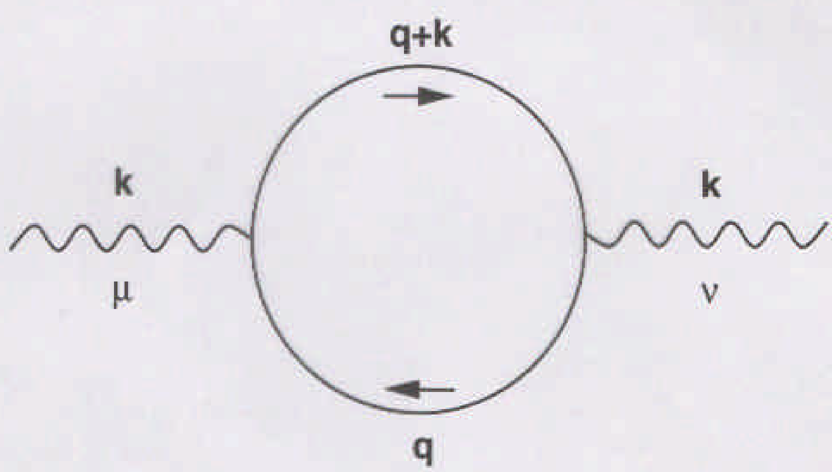
- These are mixtures of both kinds of extra dimensions (depending on M-theory or Type I realizations).
- The string scale is decoupled from the usual GUT scale and Planck scale, and can be adjusted arbitrarily!

Extra Dimensions and Gauge Coupling Unification

Recall: *Without* extra dimensions, gauge couplings have usual logarithmic running:

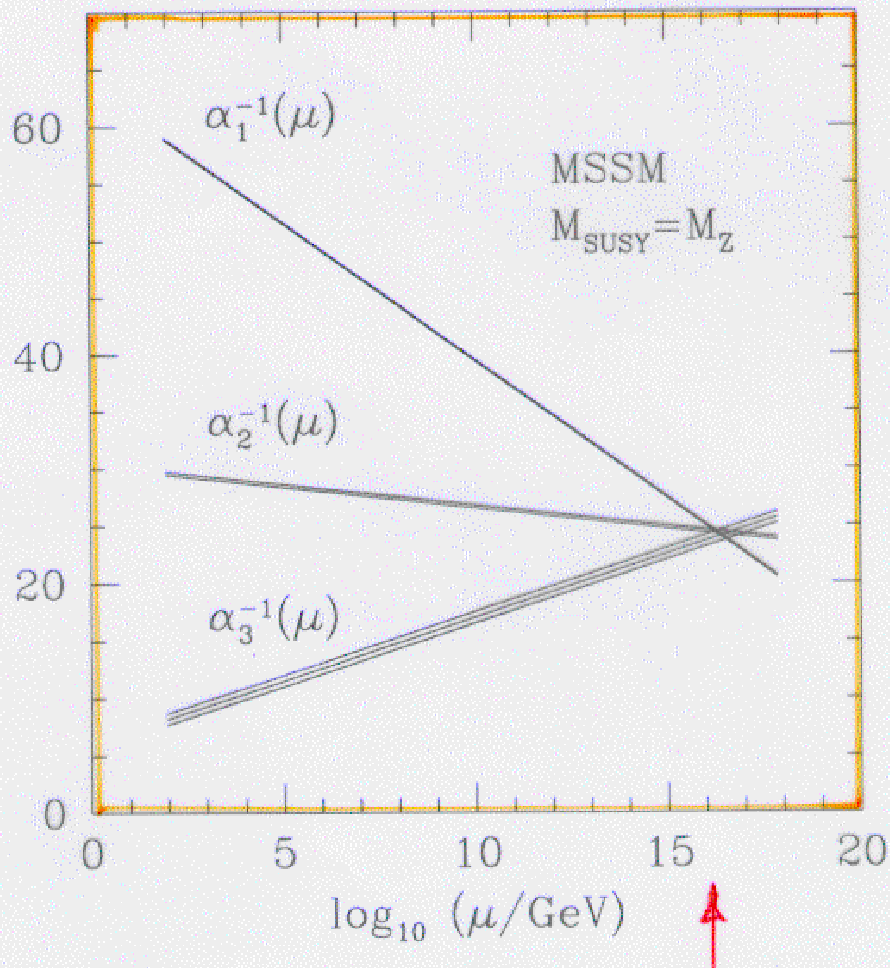
$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}$$

...result of evaluating the vacuum polarization diagram:



There is even a bit of “experimental” evidence for SUSY grand unification:

gauge coupling unification!



Thus, unification scale is $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV.

Now, imagine $\delta \equiv D - 4$ extra dimensions at scale R^{-1}.
Must also include Kaluza-Klein states in loop!

$\Longrightarrow$ Below R^{-1}, no appreciable effect.
Above R^{-1}, couplings now evolve according to:

$$\alpha_i^{-1}(\mu) \approx \alpha_i^{-1}(R^{-1}) - \frac{b_i - \tilde{b}_i}{2\pi} \ln(R\mu) - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[(R\mu)^\delta - 1\right]$$

- T. Taylor and G. Veneziano, *Phys. Lett.* **B212** (1988) 147
- KRD, E. Dudas, and T. Gherghetta, *Phys. Lett.* **B436** (1998) 55
  Nucl. Phys. **B537** (1999) 47

where

- $(b_1, b_2, b_3) \equiv (33/5, 1, -3)$
- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \equiv (3/5, -3, -6)$
- $X_\delta \equiv \dfrac{2\,\pi^{\delta/2}}{\Gamma(\delta/2)}$

Power-law behavior is the consequence of extra dimensions!

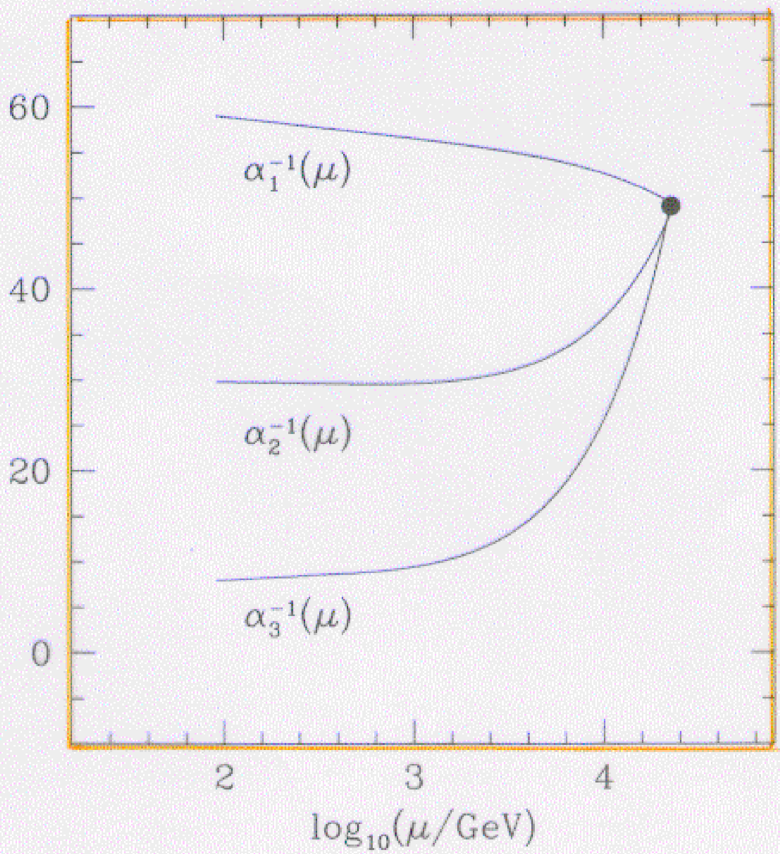
But how does this affect gauge coupling unification?

• KRD, E. Dudas, and T. Gherghetta, *Phys. Lett.* **B436** (1998) 55
Nucl. Phys. **B537** (1999) 47

Let us choose

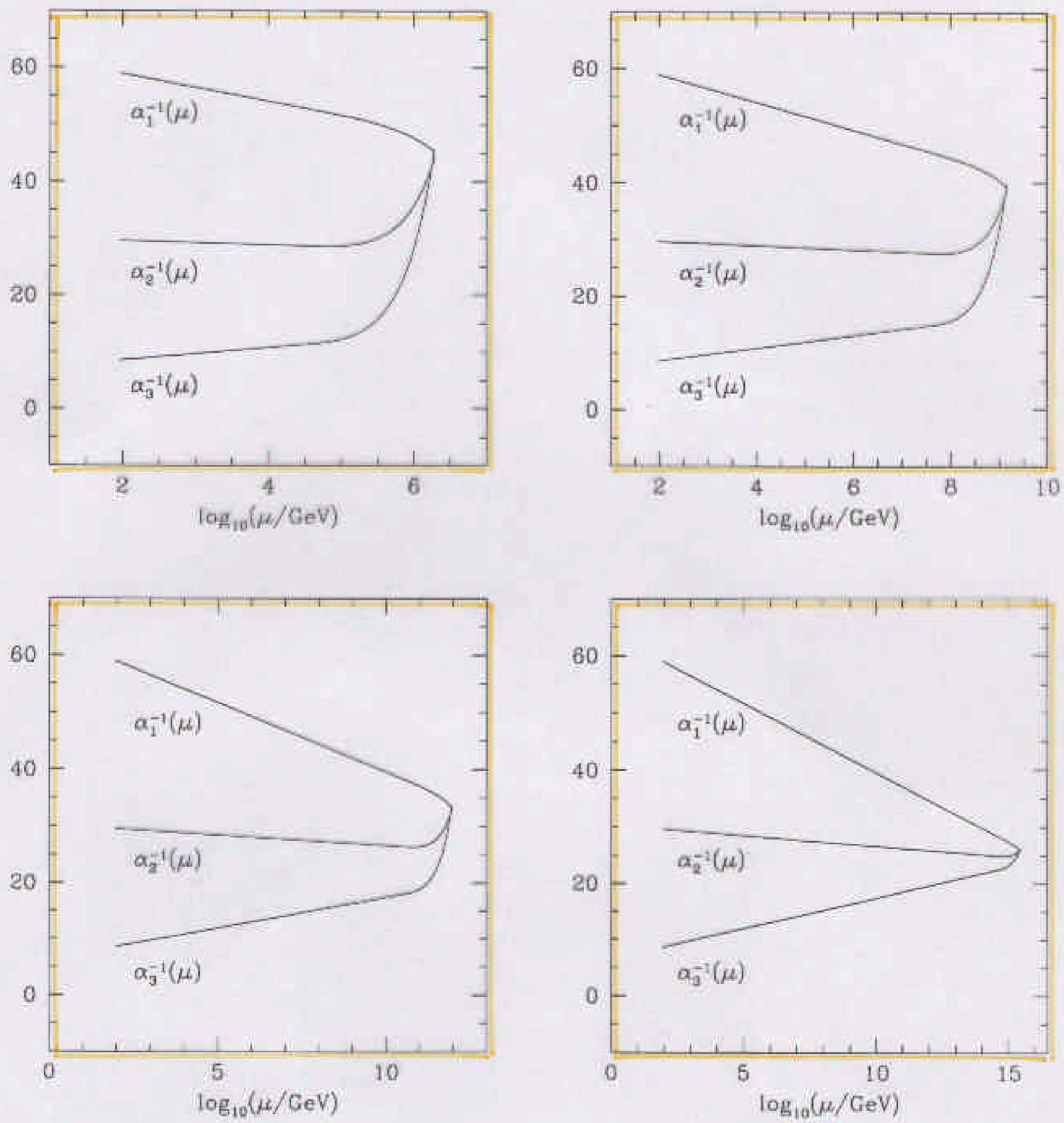
- $R^{-1} = 1$ TeV (the most *extreme* case)
- $\delta = 1$ (one extra dimension)

We then find...



- Evolution is dramatically altered,
  but gauge couplings still unify!
- Potential new scale for grand unification!

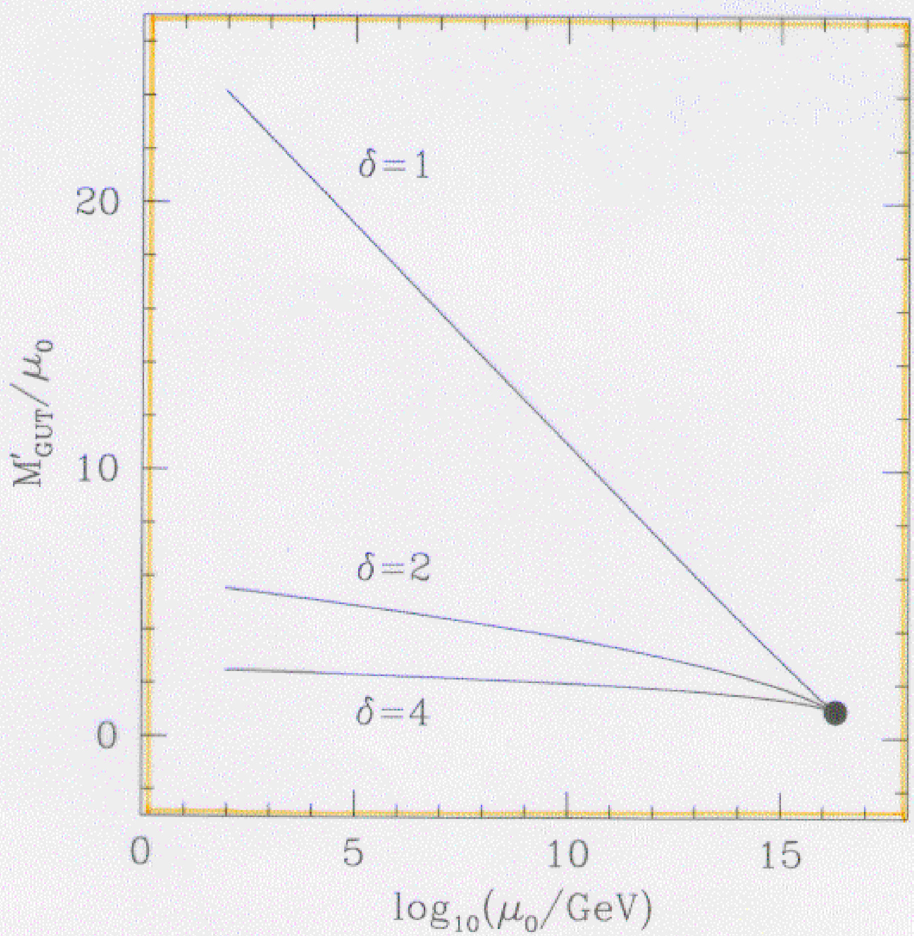
How does this depend on the chosen radius?



Unification is always preserved!

How does this depend on the number of extra dimensions?

Increasing δ increases the power-law exponent!
Preserves unification, and accelerates it even further!



Thus, we see that extra large dimensions $\Longrightarrow$

- power-law evolution for gauge couplings
- gauge couplings continue to unify
- unification is weakly coupled and generally perturbative
- unification scale is *flexible* and can be lowered
  (even all the way to TeV, if desired!).

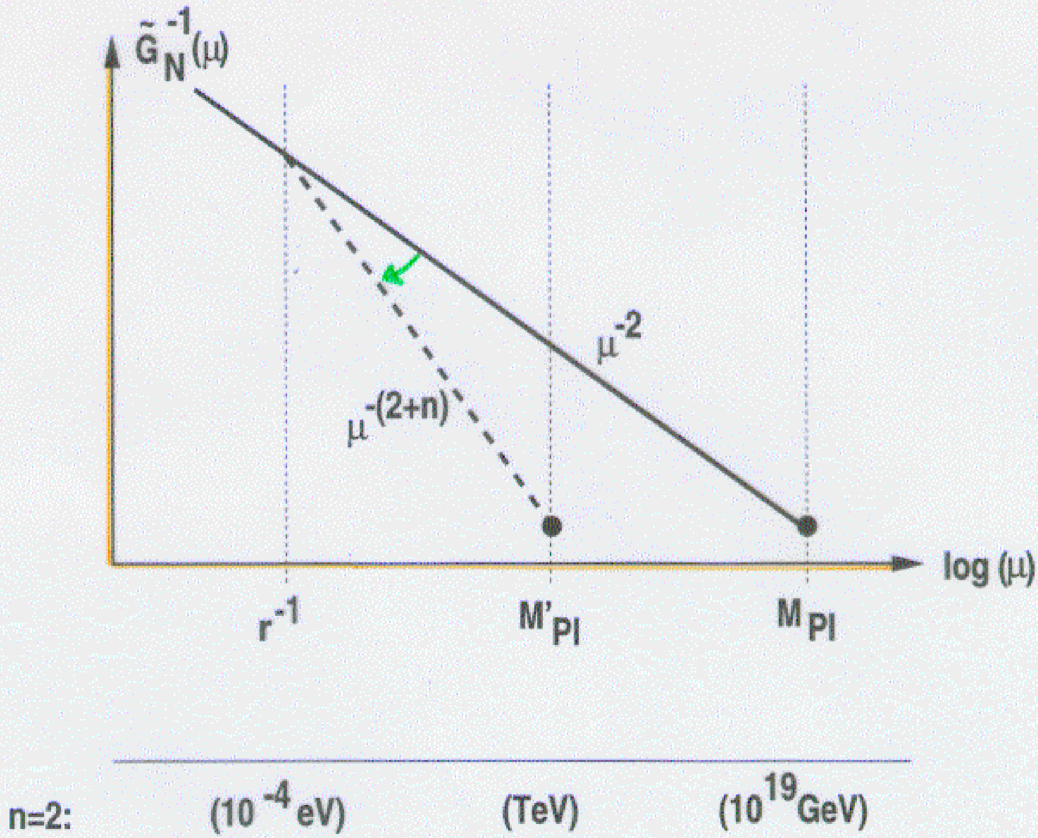
This scenario also has other nice phenomenological features...

- cancellation of leading proton-decay diagrams due to *higher-dimensional* momentum conservation (Kaluza-Klein selection rules)
- possible explanation of fermion mass hierarchy
  — small flavor dependence at high scales is *amplified* by power-law running to yield hierarchy at low scales
- possible unification even without SUSY.

Lowering the Planck scale via extra dimensions

- N. Arkani-Hamed, S. Dimopoulos, G. Dvali, hep-ph/9803315

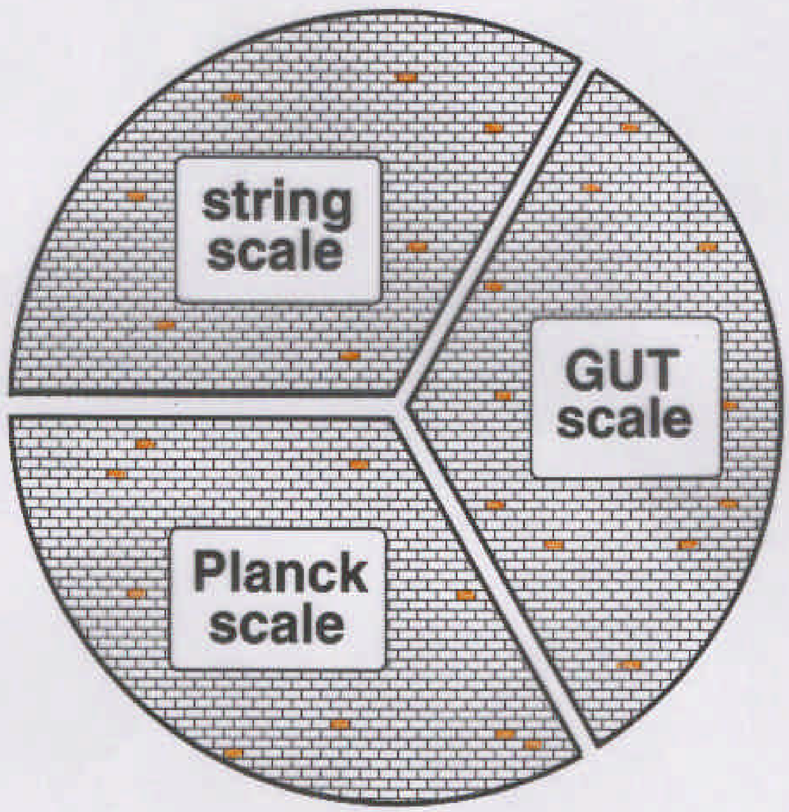
$$\tilde{G}_N(\mu) = \mu^2 G_N$$



$\Longrightarrow$ *fundamental Planck scale is reduced!*
Required radii in range mm $\leq r \leq$ fermi *for* $n \geq 2$.

How do these three scenarios fit together?

Together, they form an inter-related "circle of ideas"...



But how and why are these scenarios combined?

Embeddings into String Theory

Just as with any GUT theory, we seek an eventual embedding into string theory.

- String theory is automatically consistent with gauge coupling unification.
- String theory provides unification with gravity.
- String theory is *finite* — resolves issues of non-renormalizability.
- Many other compelling phenomenological features...

But now our fundamental GUT scale is very low!

$\Longrightarrow$ Requires a very low string scale!

Is this possible?

- Usually, the string scale is tied to the Planck scale:

$$M_{\rm string} \sim g_{\rm string} M_{\rm Planck}$$

- But this holds only for *weakly coupled heterotic strings*!

  As the ten-dimensional coupling increases, this behavior changes due to non-perturbative effects.

  $\Longrightarrow$ Often, the resulting theory is better described as a Type I (open) string theory!

- For open strings, relation between scales is different:

$$M_{\rm string} \sim e^{\langle\phi\rangle/2} g_{\rm gauge} M_{\rm Planck}$$

  where
  - — ϕ is the ten-dimensional *dilaton* field
  - — $g_{\rm gauge}$ is the Type I gauge coupling.

Thus, for Type I strings, it is possible to separate the string scale from the Planck scale!

After eliminating the dilaton dependence, one finds that the separation between scales ultimately depends on the volume of compactification:

$$M_{\text{string}} \sim \sqrt{\frac{1}{\alpha_{\text{GUT}} M_{\text{Planck}}}}\, V^{-1/4}$$

Now, in our scenario,

- R and $\delta \equiv D - 4$ are input parameters.
- M'_{GUT} and α'_{GUT} are predicted.
- We choose to set $M_{\text{string}} = M'_{\text{GUT}}$

$\Longrightarrow$ uniquely fixes the volume V !

Thus, if we write

$$V \sim R^{\delta}\, r^{6-\delta}$$

we can actually *solve* for the radii of the remaining dimensions! This is the remaining radius needed to lower M_{string} to M'_{GUT}!

Most interesting case:

One extra dimension at $R \approx 0.5$ TeV

$\Longrightarrow$ $M'_{\text{GUT}} \approx 10$ TeV

$\Longrightarrow$ remaining five dimensions must have radius

$$r \approx (10 \text{ MeV})^{-1} \approx 10 \text{ fermi}$$

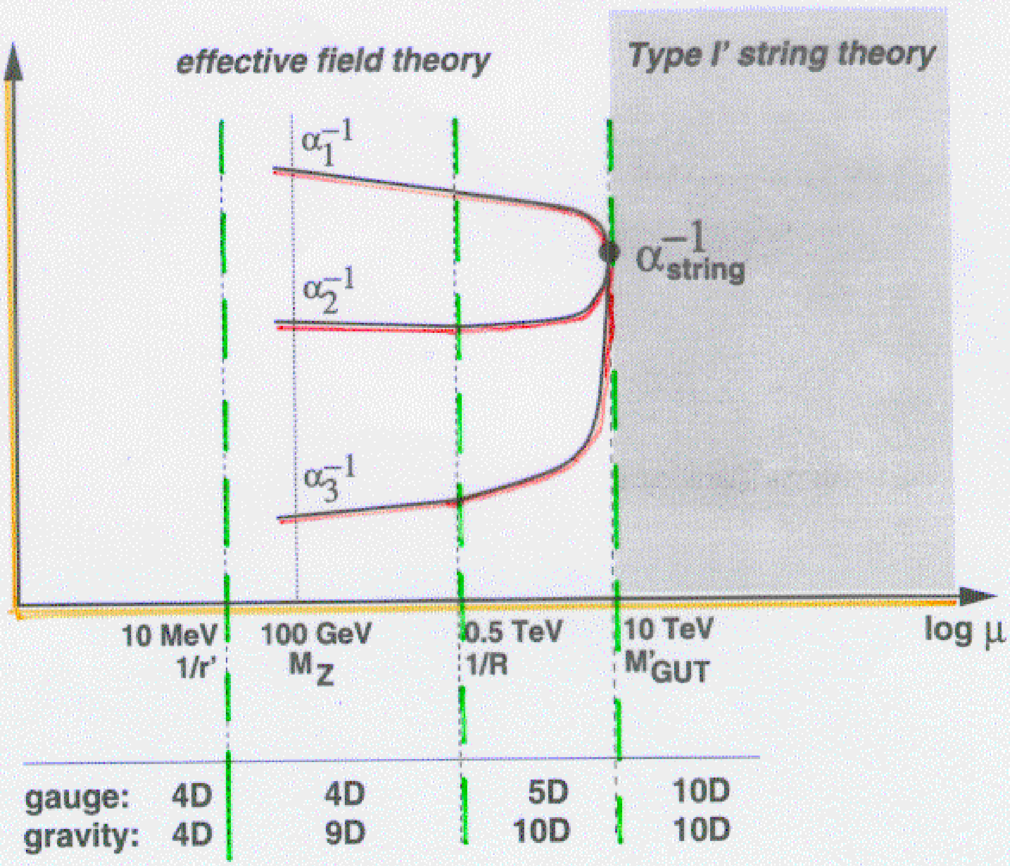
So large! Seems impossible!

But these extra dimensions are felt *only by gravity*!
(Gauge forces are immune to their effects.)

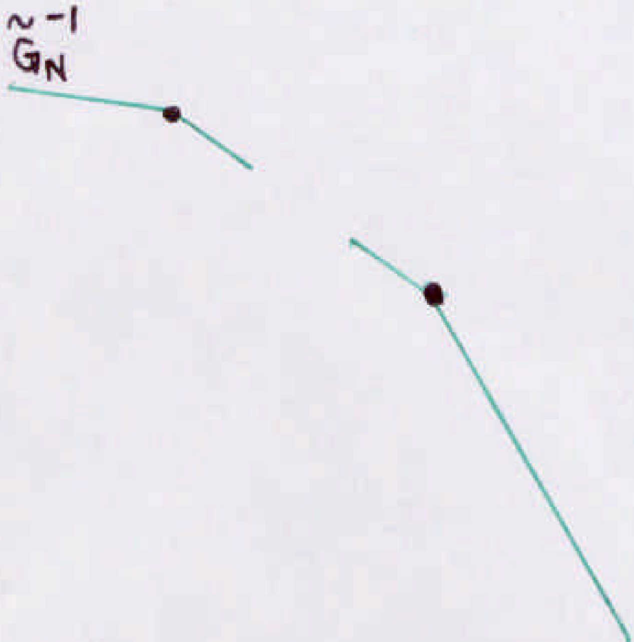
$\Longrightarrow$ Not excluded by any laboratory, astrophysical, or cosmological constraints!

- N. Arkani-Hamed, S. Dimopoulos, G. Dvali, hep-ph/9803315

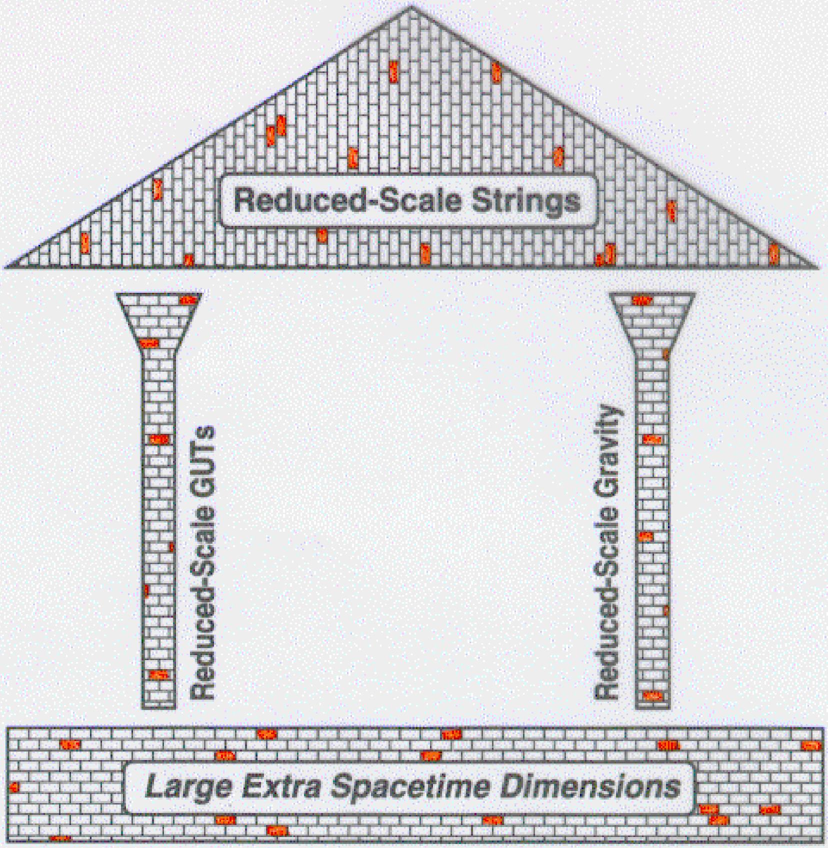
Thus, we are led to a unified embedding into string theory:



$\Longrightarrow$ *All of the fundamental scales (GUT, string, Planck) have been lowered simultaneously!*


$\tilde{G}_N^{-1}$

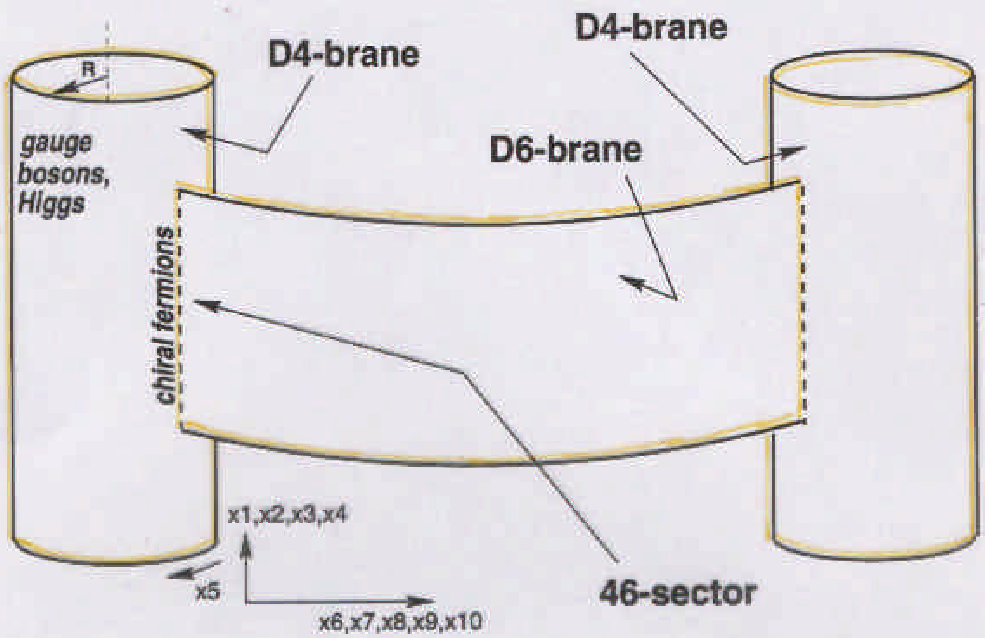
This makes sense, since string theory is a theory of both gauge forces and gravitational forces...



- This reduced-scale "structure" survives *regardless* of the amount by which the fundamental energy scales are lowered.

Thus, we see that extra spacetime dimensions have succeeded in lowering all of the fundamental high-energy scales of physics in a self-consistent way...

This then leads to a new view of spacetime:



- At low energies, the (MS)SM lives on the four-dimensional intersection of different "branes".
- At higher energies, the gauge forces feel the extra dimension of radius $R \implies$ world becomes a cylinder! *This lowers GUT scale, ensures gauge coupling unification.*
- Remaining orthogonal directions felt only by gravitational interactions! *This lowers the Planck scale, explains weakness of gravity.*
- Lowered string scale: *ensures consistency, calculability.*

This also opens up whole new vistas that await exploration!

Particle Theory:

Many aspects of physics beyond the SM must now be considered in a new light!

- SUSY
- SUSY-breaking
- GUT physics
- String theory

} role of extra large dimensions? effects of changed energy scales?

Collider Experiments:

If scales are extrapolated to TeV-range, many striking signals are possible at future colliders!

- direct detection of Kaluza-Klein states
- decays of KK states into (s)fermions
- Drell-Yan production of KK states in $p\bar{p}$ collisions
- new effective four-fermi contact interactions via exchange of gauge-boson KK states
- direct detection of GUT particles, Regge states, quantum gravity...?

$\Longrightarrow$ Can probe properties of GUTs and strings *experimentally*!
$\Longrightarrow$ A new *experimental* direction for string phenomenology?

Cosmology:

Profound effects for cosmology!
Fundamental energy scales have changed!

- role of light stable KK states
- possible new dark-matter candidates
- reheating and thermal regeneration of KK states
- inflation in higher dimensions
- phase transitions in higher dimensions
- topological defects
- density perturbations
- cosmological methods for *generating* large radii?
- possible explanation for dimensionality of spacetime??
- ... even other wilder ideas ...

Lots to think about!

However, the important point is that

- extra dimensions can be taken seriously as physical entities
- they can have measurable, significant effects

**The "fundamental"
high energy scales of physics
are not immutable.**

**The "parameter space" of ideas
for physics beyond the Standard Model
may be significantly broader than
we previously expected.**

These are undoubtedly the most valuable lessons.

Neutrinos

ν masses

"So, moral of the story seems to be

HIGH FOUR-DIMENSIONAL SCALES

are replaced by

LOWER HIGHER-DIMENSIONAL SCALES!

But there are still other scales
eg. seesaw scale!

→

Generating Light Neutrino Masses

Recall usual $SO(10)$ seesaw mechanism...

- $SO(10) \implies$ right-handed neutrino N
- Assume Yukawa coupling $y_\nu \nu_L H_+ N$
  $\implies$ Dirac mass $m \equiv y_\nu \langle H_+ \rangle \approx \mathcal{O}(10^2)$ GeV.
- Also assume Majorana mass for N
  (can arise from GUT breaking via **126** vevs)
  $\implies$ $M \approx \mathcal{O}(10^{16})$ GeV.
- Total mass terms then take form

$$(\,\nu_L \;\; N\,)\,\mathcal{M}\begin{pmatrix}\nu_L\\ N\end{pmatrix} \qquad \text{where} \quad \mathcal{M} \equiv \begin{pmatrix}0 & m\\ m & M\end{pmatrix} .$$

- Diagonalizing to find mass eigenvalues, we obtain

$$\lambda_- \approx -\frac{m^2}{M}, \qquad \lambda_+ \approx M .$$

— Lightest eigenvalue "seesaws" against heavy mass scale!
— $m_\nu \approx 10^{-2}$ eV $\implies$ $M \approx 10^{14-16}$ GeV!

Thus, light neutrino masses seem to provide further evidence for a high fundamental GUT scale!

Light Neutrinos without Heavy Mass Scales

- KRD, E. Dudas, & T. Gherghetta, hep-ph/9811428.
- N. Arkani-Hamed *et al*, hep-ph/9811448.

Fundamental observation:

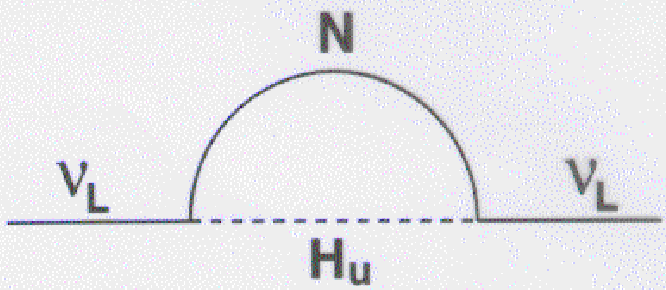
Unlike all other fields, right-handed neutrino is SM singlet!

$\Longrightarrow$ not restricted to brane like other SM particles
$\Longrightarrow$ can propagate in higher-dimensional "bulk"
$\Longrightarrow$ can have infinite tower of KK excitations!

This can have many effects on the resulting neutrino mass...

- KRD, E. Dudas, & T. Gherghetta, hep-ph/9811428.

(1) *Power-law running of neutrino Yukawa couplings...*



⟹ neutrino Yukawa coupling driven to very small values over short energy interval

⟹ neutrino mass is power-law suppressed relative to masses of all other fermions!

- KRD, E. Dudas, & T. Gherghetta, hep-ph/9811428.
- N. Arkani-Hamed *et al*, hep-ph/9811448.

(2) *Suppression by bulk volume factor...*

— if N field is in bulk, its wavefunction must be renormalized by bulk volume factor
— suppresses Dirac coupling m by factors $(RM_s)^n$.

Both of these mechanisms suppress m_ν by suppressing m.

But is there a higher-dimensional analogue of the seesaw mechanism?

- KRD, E. Dudas, & T. Gherghetta, hep-ph/9811428.

Yes!

- Consider $D = 5$ for concreteness:
  $\Longrightarrow$ bulk field $\Psi \equiv (\psi_1, \bar{\psi}_2)^T$ in Weyl rep.

- Assume lepton-number conserved on brane:
  $\Longrightarrow$ no primordial $\nu_L \nu_L$ mass term
  (just as in usual seesaw mechanism)

- Allow lepton-number breaking in bulk:
  $\Longrightarrow$ "bare" Majorana mass term $\frac{1}{2} M_0 \bar{\Psi}^c \Psi$.

- Allow general Dirac couplings (m_1, m_2) of brane field ν_L to bulk fields (ψ_1, ψ_2).

- Expand $\psi_{1,2}$ in KK modes:

$$\begin{aligned}
\psi_1(x,y) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_1^{(n)}(x) \cos(ny/R) \\
\psi_2(x,y) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \psi_2^{(n)}(x) \sin(ny/R) .
\end{aligned}$$

$\Longrightarrow$ no zero-mode for ψ_2.

- For $n \geq 1$, define

$$N^{(n)} \equiv (\psi_1^{(n)} + \psi_2^{(n)})/\sqrt{2}$$
$$M^{(n)} \equiv (\psi_1^{(n)} - \psi_2^{(n)})/\sqrt{2}.$$

Then ν_L mixes with entire spectrum of KK states!

Basis: $(\nu_L, \psi_1^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, ...)$

Mass mixing matrix takes the form :

$$\begin{pmatrix} 0 & m & m_N & m_M & m_N & m_M & \cdots \\ m & M_0 & 0 & 0 & 0 & 0 & \cdots \\ m_N & 0 & M_0 + \frac{1}{R} & 0 & 0 & 0 & \cdots \\ m_M & 0 & 0 & M_0 - \frac{1}{R} & 0 & 0 & \cdots \\ m_N & 0 & 0 & 0 & M_0 + \frac{2}{R} & 0 & \cdots \\ m_M & 0 & 0 & 0 & 0 & M_0 - \frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

This leads to many unexpected higher-dimensional features depending on parameters $m, m_N, m_M, M_0,$ *and* R*...*

Simplest case ("orbifold case"):

- Assume SM brane located at orbifold fixed point $y = 0$

  $\Longrightarrow$ then no coupling $\nu_L \psi_2$, since $\psi_2 = 0$ on brane

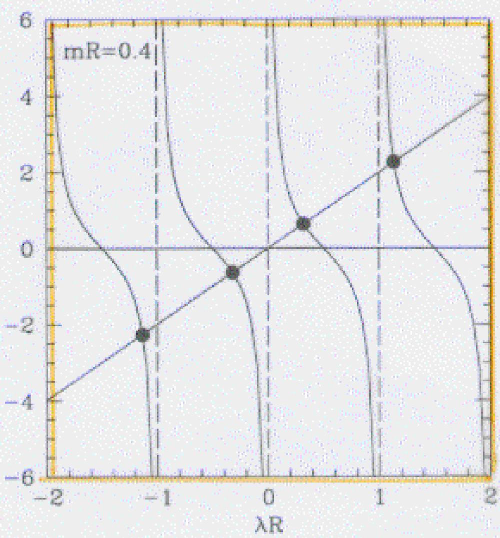
  $\Longrightarrow$ $m_M = m_N = m$.

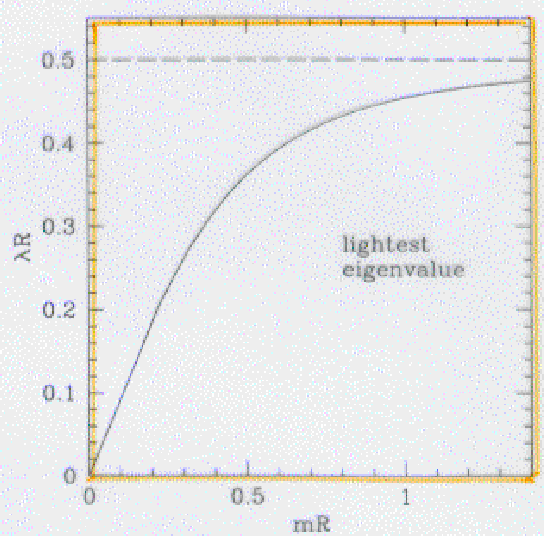
- Assume $M_0 = 0$ (lepton-number conserved in bulk).

Then mass matrix takes the form

$$\begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & 0 & 0 & 0 & 0 & 0 & \dots \\ m & 0 & 1/R & 0 & 0 & 0 & \dots \\ m & 0 & 0 & -1/R & 0 & 0 & \dots \\ m & 0 & 0 & 0 & 2/R & 0 & \dots \\ m & 0 & 0 & 0 & 0 & -2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} .$$

Eigenvalues are solutions to $\lambda R = \pi (mR)^2 \cot(\pi \lambda R)$.





All eigenvalues paired (Dirac masses only)

- for $mR \ll 1$, find $m_\nu \approx m$ (no seesaw behavior)
- for $mR \gg 1$, find $m_\nu \approx (2R)^{-1}$
  - — independent of Yukawa coupling m!
  - — neutrino mass bounded by the radius!

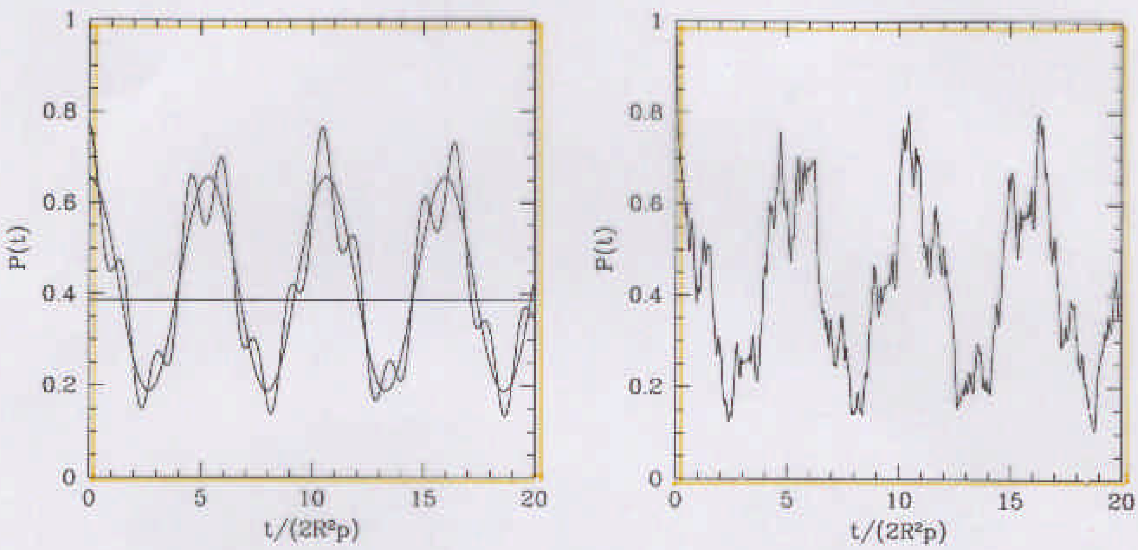
But is the actual value of m_ν important for oscillations?

Consider oscillations between ν_L and KK states
(analogue of usual neutrino/anti-neutrino oscillations).

If U diagonalizes the mass matrix, then

$$P_{\nu_L \to \nu_L}(t) = \left| \sum_k |U_{\nu k}|^2 \exp\left(\frac{i\lambda_k^2 t}{2p}\right)\right|^2 .$$

We then find...



- Neutrino oscillations still effectively periodic.
- Deficits and regenerations are *never total*!
- Oscillation length set by first eigenvalue interval $\approx R^{-1}$ rather than by m_ν itself!

$$\delta m^2 \approx 10^{-4}\ \text{eV}^2 \quad \Longrightarrow \quad R \approx 10^{-5}\ \text{meters !}$$

Bare Majorana case:

Consider effects of bare Majorana mass M_0 in bulk.
Note: we expect $M_0 \approx M_s \gg R^{-1}$, hence $m_\nu \sim \mathcal{O}(m^2/M_0)$.

$$\begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & M_0 & 0 & 0 & 0 & 0 & \dots \\ m & 0 & M_0+\frac{1}{R} & 0 & 0 & 0 & \dots \\ m & 0 & 0 & M_0-\frac{1}{R} & 0 & 0 & \dots \\ m & 0 & 0 & 0 & M_0+\frac{2}{R} & 0 & \dots \\ m & 0 & 0 & 0 & 0 & M_0-\frac{2}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} .$$

But now define $\epsilon \equiv M_0 \pmod{R^{-1}}$ (smallest diagonal entry)
$\Longrightarrow$ all other diagonal entries are $\epsilon \pm k'/R$...

$$\begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & \epsilon & 0 & 0 & 0 & 0 & \dots \\ m & 0 & \epsilon+\frac{1}{R} & 0 & 0 & 0 & \dots \\ m & 0 & 0 & \epsilon-\frac{1}{R} & 0 & 0 & \dots \\ m & 0 & 0 & 0 & \epsilon+\frac{2}{R} & 0 & \dots \\ m & 0 & 0 & 0 & 0 & \epsilon-\frac{2}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} .$$

Heavy scale M_0 *decouples* from the physics!

$\Longrightarrow$ only scale remaining is R^{-1}

$\Longrightarrow$ resulting (Majorana) neutrino mass is $m_\nu \sim \mathcal{O}(m^2 R)$.

The KK seesaws have replaced M_0 with R^{-1} !

Scherk-Schwarz special case:

- One possible way to generate $M_0 \neq 0$ is to break lepton-number in the bulk via the Scherk-Schwarz mechanism:

$$\psi_{1,2}(2\pi R) \;=\; -\psi_{1,2}(0)\;.$$

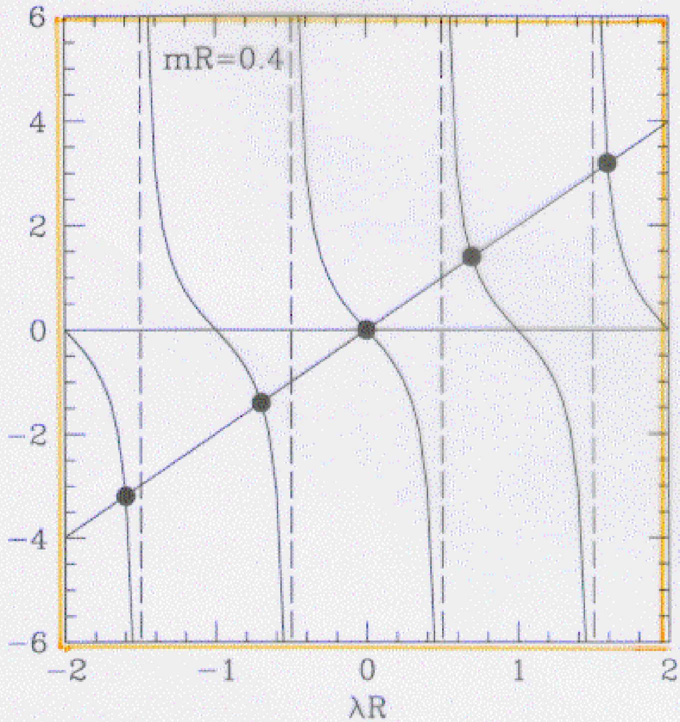
- This is a global (non-local) breaking of lepton-number.
- In such cases, we then find

$$\boxed{M_0 \;=\; \tfrac{1}{2}R^{-1}}$$

This precise value is fixed *topologically*.

$$\begin{pmatrix} 0 & m & m & m & m & \cdots \\ m & 1/(2R) & 0 & 0 & 0 & \cdots \\ m & 0 & -1/(2R) & 0 & 0 & \cdots \\ m & 0 & 0 & 3/(2R) & 0 & \cdots \\ m & 0 & 0 & 0 & -3/(2R) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Eigenvalues are solutions to $\lambda R = -\pi(mR)^2 \tan(\pi\lambda R)$.



- Tangent curves have been shifted by half-period
  - $\Longrightarrow$ neutrino is exactly massless!
  - $\Longrightarrow$ result holds for all values for mR!
  - $\Longrightarrow$ all other KK states have *Dirac* masses!

- Neutrino mass eigenstate is exactly given by

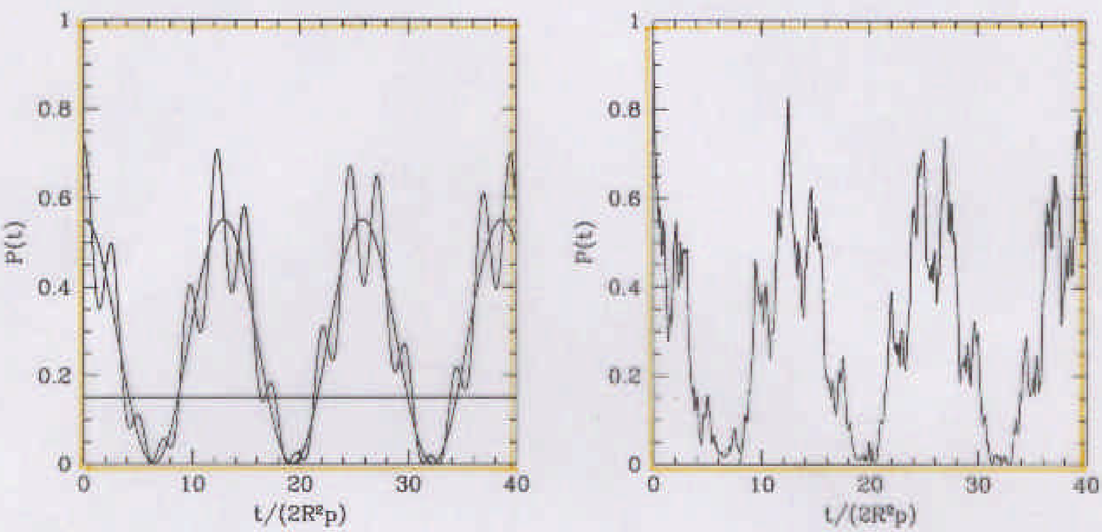
$$|\tilde{\nu}_L\rangle \;\sim\; |\nu_L\rangle - mR\sum_{k=1}^{\infty}\frac{1}{k-1/2}\left\{|N^{(k-1)}\rangle - |M^{(k)}\rangle\right\}$$

  - $\Longrightarrow$ mostly $|\nu_L\rangle$ for $mR \ll 1$, as required!
  - $\Longrightarrow$ this combination cancels neutrino mass exactly!

But what about neutrino oscillations?

$$P_{\nu_L \to \nu_L}(t) = \left| \sum_k |U_{\nu k}|^2 \exp\left(\frac{i\lambda_k^2 t}{2p}\right) \right|^2 .$$

Note: $\mathcal{M}$-matrix non-diagonal $\Longrightarrow$ U-matrix non-diagonal.
Thus, still have oscillations!



- Neutrino oscillations without neutrino masses in $D \geq 5$!
- Oscillations still effectively periodic.
- Regenerations never total, but deficits total.
- Oscillation length set by first eigenvalue interval $\approx R^{-1}$.
- Easy to generalize to flavor oscillations, even with $(\nu_e, \nu_\mu, \nu_\tau)$ massless! Oscillations *indirect* via KK states...

Thus, in such scenarios, neutrino oscillations are evidence not for neutrino masses, but for extra spacetime dimensions!

- KRD, E. Dudas, & T. Gherghetta, hep-ph/9811428.

Finally, in Type I string theory, there also exists a new way to break lepton number...

- Thus far, assumed SM brane at $y = 0$ (fixed point).
- But in Type I string theory, can shift branes away from fixed points to arbitrary location $y^* \neq 0$.
  Brane/bulk coupling then takes the form

$$m\,\bar{\nu}_L\,(\Psi + \Psi^c)\Big|_{y=y*} + \text{h.c.} \qquad \text{where} \quad \Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix}$$

$\Longrightarrow$ generates a coupling between ν_L and ψ_2!
$\Longrightarrow$ establishes unequal Dirac couplings $m_N \neq m_M$ in seesaw matrix:

$$\begin{aligned} m_N^{(n)} &\equiv m\,[\cos(ny^*/R) + \sin(ny^*/R)] \\ m_M^{(n)} &\equiv m\,[\cos(ny^*/R) - \sin(ny^*/R)]\ . \end{aligned}$$

- This splits the Dirac masses into unequal Majorana masses:

$$\lambda_\pm \;=\; \tfrac{1}{2}\left[\mu \pm \sqrt{\mu^2 + 4m^2}\right]\ , \quad \text{where}$$

$$\mu \;=\; -m^2 R \sum_{k=1}^{n} \frac{1}{k}\,\sin\left(\frac{2ky^*}{R}\right)\ .$$

Thus, brane-shifting is a uniquely *stringy* way of breaking lepton-number, establishing a seesaw, and generating a Majorana neutrino mass.

Extension to flavor ...

$\Rightarrow$ three LH ν_i on brane

Bulk ?

- three corresponding bulk neutrinos
- 3×3 mixing bmatrices
  for brane/bulk couplings

- Mohapatra & Perez-Lorenzana
- Barbieri, Creminelli, Strumia

⋮

LOTS of parameters!

ANOTHER ALTERNATIVE ?

A "compact" model

KRD & I. Sarcevic
(in progress)

- three left-handed ν_i on brane (flavors)
  - "bare" Majorana masses m_i
    BUT no mixing!

- only one bulk neutrino

- universal brane/bulk coupling

$$\left(\begin{array}{ccc|ccccc}
m_1 & & 0 & m & m & m & m & \cdots \\
 & m_2 & & m & m & m & m & \cdots \\
0 & & m_3 & m & m & m & m & \cdots \\
\hline
m & m & m & 1/R & & 0 & & \\
m & m & m & & -1/R & & & \\
m & m & m & & 0 & 2/R & & \\
m & m & m & & & & -2/R & \\
\vdots & & & & & & & \ddots
\end{array}\right)$$

eigenvalues: $$\boxed{\tan \lambda \pi R = \pi m^2 R \sum \frac{1}{\lambda - m_i}}$$

$\Rightarrow$ still yields flavor oscillations $\nu_i \leftrightarrow \nu_j$ indirectly through KK states!

- Thus, flavor oscillations $\nu_i \leftrightarrow \nu_j$ are possible even if
  - $\rightarrow$ brane theory (SM) is flavor-diagonal
  - $\rightarrow$ bulk theory is flavor-neutral
  - $\rightarrow$ brane/bulk coupling is flavor-blind!
- ONLY five parameters $\Rightarrow$ all masses & "mixings"!

Thus, with extra large dimensions, it is possible to...

- generate light neutrinos via power-law Yukawa running
- suppress neutrino masses via higher-dim. volume factors
- establish seesaw mechanism via towers of KK states
- break lepton-number via Type I brane shifting
- have neutrino oscillation wavelengths set by radii
- have neutrino oscillations without neutrino masses!

Moreover, these scenarios can easily be generalized to

- multiple extra spacetime dimensions with different radii
- different Ψ fields in different directions
- indirect *flavor* oscillations via flavor/KK non-diagonality.

Rich neutrino phenomenology is possible in extra dimensions!

Of course, these scenarios are at best only qualitative.

- Still must perform detailed experimental comparisons
  - — atmospheric neutrinos *vs.* solar neutrinos, ...
  - — effects of KK admixtures (*e.g.*, $\pi \to \mu\bar{\nu}_\mu$).
- Theoretical issues:
  - — incorporate mechanisms within realistic string models
  - — brane dynamics? (branes not truly rigid)
  - — other methods of protecting/breaking lepton-number.

Many ideas, but further study needed!