CALCULATIONS OF TRANSVERSE HEATING IN THE ANTIPROTON ACCUMULATOR

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1. Introduction

Transverse heating by the stacktail momentum cooling system in the Antiproton Accumulator has been identified as a problem for some time. Various fixes have been implemented to reduce this to a manageable level, but as antiproton stacking rates and stack sizes increase, transverse heating effects are likely to continually be a nuisance. In this note I will present calculations and estimates of heating effects due to fundamental physics and to mechanical and electrical misalignments. A subsequent note [1] will describe heating effects due to closed-loop feedback.

2. Kicker Physics

Assume that the kickers only perturb the beam slightly, so that the beam travels through the kickers in a straight-line path. Also assume that the kicker fields are localized, with field-free regions on either side. Then the conditions of the Panofsky-Wenzel theorem are satisfied. This theorem in its general form is [2]:

$$\frac{\partial}{\partial t} \Delta \vec{p}_{\perp} = -e \vec{\nabla}_{\perp} V \tag{1}$$

where:

V is the beam voltage gain in passing through the kicker (eV is the energy gain),

 $\vec{\nabla}_{\perp}$ is the transverse gradient $(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y})$, and

 $\Delta \vec{p}_{\perp}$ is the transverse momentum kick received.

Equation 1 may be split into x and y equations, partial derivatives may be taken with x and y, and the resultant equations may be added to yield:

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial x} \Delta p_x + \frac{\partial}{\partial y} \Delta p_y \right) = -e \nabla_{\perp}^2 V$$
⁽²⁾

From manipulations to the wave equation, it may be shown [2] that:

$$\nabla_{\perp}^{2}V + \frac{1}{\left(\beta\gamma c\right)^{2}}\frac{\partial^{2}V}{\partial t^{2}} = 0$$
(3)

Putting equations 2 and 3 together, assuming that V has a sinusoidal behavior (so that convective derivatives may be ignored), and integrating, one finds:

$$\left(\frac{\partial}{\partial x}\Delta p_{x} + \frac{\partial}{\partial y}\Delta p_{y}\right) = \frac{e}{\left(\beta\gamma c\right)^{2}}\frac{\partial V}{\partial t}$$
(4)

where an arbitrary constant of integration, corresponding to static deflections, has been ignored.

These expressions are quite general (subject to the initial assumptions, given above) and are true for any kicker or pickup.

A longitudinal kicker is designed to give a large longitudinal kick and no transverse kick to a particle on-axis. There will be an optic axis of the kicker, on which V will be non-zero and $\vec{\nabla}_{\perp}$ and \vec{p}_{\perp} will be 0. For this case, equation 4 may be Taylor-expanded about the optic axis, resulting in:

$$\left(\frac{\Delta p_x}{x} + \frac{\Delta p_y}{y}\right) = \frac{e}{\left(\beta\gamma c\right)^2} \frac{\partial V}{\partial t}$$
 5)

Hence, off-axis particles will receive a non-zero transverse kick in traversing a longitudinal kicker. Any device which changes beam energy must also impart a transverse kick to off-axis particles, and there is no way to avoid this.

The off-axis sensitivity in each plane is a function of kicker geometry, and will generally not be equal, unless the kicker is designed as an annular ring. However, for an even number of arbitrary, identical kickers with alternate rotations of 90° (as in the Fermilab Antiproton Accumulator), the kicker system will have equal sensitivities to x and y offsets, and this will be:

$$\frac{\Delta p_x}{x} = \frac{\Delta p_y}{y} = \frac{e}{2(\beta \gamma c)^2} \frac{\partial V}{\partial t}$$
(6)

3. Heating Due to Off-Center Beam in Ideal System

A beam going through a longitudinal kicker off-center will be kicked transversely due to the above effect. This will produce a beam offset 90° away in betatron phase, which will result in emittance growth as the beam is smeared out in phase space due to nonlinearities. For an offset in the x direction, the growth in beam size is simply given by [3, equation 7.24]:

$$\sigma^2 = \sigma_0^2 + \frac{1}{2}\Delta x^2 \tag{7}$$

A transverse beam kick through an angle $\Delta x'$ will give rise to emittance growth, through the relation $\Delta x = \beta \Delta x'$. It will be convenient to express $\Delta x'$ as $\Delta x' = \Delta p_x/p_x$. Since $\sigma^2 \propto \varepsilon$, the fractional emittance growth $\Delta \varepsilon/\varepsilon$ is simply $\Delta(\sigma^2)/\sigma^2$. Putting these relations together, and using Fermilab units of unnormalized emittance, $6\pi\sigma^2 = \beta\varepsilon_{FNAL}$, equation 7 may be re-expressed as:

$$\sigma^{2} = \sigma_{0}^{2} + \frac{1}{2}\beta^{2} \frac{(\Delta p_{x})^{2}}{p^{2}}$$
 8a)

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{1}{2} \frac{\beta^2}{\sigma_0^2} \frac{\left(\Delta p_x\right)^2}{p^2} = 3\pi \frac{\beta}{\varepsilon_{FNAL}} \frac{\left(\Delta p_x\right)^2}{p^2}$$
 8b)

This result is in agreement with that of Mane and Jackson [4].

Assuming a longitudinal kicker system with equal sensitivities in x and y, equation 6 may be substituted into equation 8, and the emittance growth may be expressed as:

$$\sigma^{2} = \sigma_{0}^{2} + \frac{1}{8} \frac{x^{2} \beta_{t}^{2} e^{2}}{p^{2} (\beta_{r} \gamma c)^{4}} \left(\frac{\partial V}{\partial t}\right)^{2}$$
9a)

$$\frac{\Delta\varepsilon}{\varepsilon} = 3\pi \frac{\beta_l}{\varepsilon} \frac{\left(\Delta p_x\right)^2}{p^2} = \frac{3\pi}{4} \frac{\beta_l x^2 e^2}{\varepsilon p^2 \left(\beta_r \gamma c\right)^4} \left(\frac{\partial V}{\partial t}\right)^2$$
 9b)

where the subscripts l and r have been introduced to distinguish lattice β and relativistic β .

This is the emittance growth for a single pass through the longitudinal kicker assembly. For multiple passes, additional kicks will occur before the beam has had time to smooth itself out due to nonlinearities. The kicks will be summed vectorially; subsequent kicks may either add to or subtract from the emittance growth. However, if the kicks received on subsequent turns are uncorrelated (as should be the case for a longitudinal cooling system which filters out revolution harmonics), the contributions will simply sum up in quadrature. Additionally, if the cooling system operates at high frequency and there is a non-zero slip factor, the resultant longitudinal smearing of beam will tend to uncorrelate the kicks as well. This results in an emittance growth rate for a white-noise frequency distribution which will be equal to equation 9b multiplied by the revolution frequency:

$$\frac{\partial \varepsilon_{FNAL}}{\partial t} = 3\pi f_{rev} \beta_l \frac{(\Delta p_x)^2}{p^2}$$
 10a)

$$\frac{\Delta p_x}{\sigma_x} = \frac{1}{2} \frac{e}{\left(\beta_r \gamma c\right)^2} \left(\frac{\partial V}{\partial t}\right)_{RMS}$$
 10b)

However, the frequency distribution of kicker power for the Antiproton Accumulator at Fermilab is not uniform across a Schottky band. Only power at the betatron sidebands contributes to beam heating. An additional factor K_{fd} must be added to account for the frequency distribution of kicker power. John Marriner has estimated that this effect increases heating power on the betatron sidebands by a factor of about 8 [5][†]. Since the emittance growth rates above are proportional to heating power, they will be multiplied by this factor K_{fd} of 8:

$$\frac{\partial \varepsilon_{FNAL}}{\partial t} = 3\pi K_{fd} f_{rev} \beta_l \frac{\left(\Delta p_x\right)^2}{p^2}$$
 11)

The result of an off-center beam in a longitudinal kicker tank is an increase in beam emittance which is linear in time, and which varies quadratically with the off-centering distance.

In the FNAL Antiproton Accumulator, the beam voltage gain, V, will be roughly equal to twice the maximum kicker plate voltage multiplied by a transit time factor and a geometric factor. The nominal stacktail momentum kicker power is about 1 kW, and there are 256 kicker plates (16 pairs in each of 8 tanks), each with an impedance of 100 ohms. Thus

[†]I am somewhat sceptical that the factor K_{fd} is this large. I would not expect it to be much larger than 2. Since I cannot mathematically justify reducing this factor, I will use Marriner's estimate of 8 in the following calculations, but it should be kept in mind that the true value may be smaller, which would reduce the heating effects and lengthen the calculated emittance growth times.

each plate has a power consumption of about 4 W, and has a voltage of about 20 V RMS. Assuming that the transit time factor is about 1 and the geometric factor is about 0.9 (we will calculate this later, in equation 15), and that the beam voltage gain is twice the plate voltage (true for a 1/4 wave kicker), the RMS beam voltage gain, V, is about 36 V per kicker plate pair, 580 V per kicker tank, or about 4.6 kV on a single pass through all kicker tanks. The rate of change of V is just $V\omega$, where ω is about $2\pi \times 1.5 \times 10^9$ Hz, or about 4.3×10^{13} V/s.

For the Antiproton Accumulator, the revolution frequency f_{rev} is about 6.3*10⁵ Hz, β_l is about 10 m, p is about 9 GeV/c, and $\beta\gamma$ is about 9. The nominal (unnormalized) beam emittance is about 2π mm-mrad, and K_{fd} is about 8.

Putting these values in the equations above, one estimates that for a 1mm beam offset, the RMS transverse momentum kick per turn Δp_x is about 0.9 eV/c, and the emittance growth rate is about 4.6*10⁻¹² m/sec. Thus the emittance grows by 2π mm-mrad (i.e. the nominal beam size doubles) in about 1.4*10⁶ sec, or about 16 days. This is obviously not a limiting emittance growth mechanism.

4. Natural Heating in Ideal System

A beam with finite size, in traversing a kicker, will also be heated transversely, similarly to an off-center beam. The calculations follow those for the off-center beam, except that x is replaced by σ_x . This results in an exponential growth in emittance, with a time constant given by:

$$T^{-1} = \frac{1}{2} K_{fd} f_{rev} \beta_l^2 \frac{\left(\frac{\Delta p_x}{\sigma_x}\right)^2}{p^2}$$
 12a)

$$\frac{\Delta p_x}{\sigma_x} = \frac{1}{2} \frac{e}{\left(\beta_r \gamma c\right)^2} \left(\frac{\partial V}{\partial t}\right)_{RMS}$$
 12b)

Again, putting in values pertinent to the Accumulator ring (where the nominal σ is about 1mm), one finds that $\frac{\Delta p_x}{\sigma_x}$ is about 900 eV/c per meter, and the time constant is about 4*10⁵ sec, or about 5 days.[†]

[†]These values should be taken somewhat tentatively. For a centered beam, it will be quadrupole resonances, not the dipole (betatron) resonances, which couple energy into the

This is also not a limiting emittance growth mechanism at present. It should be noted, however, that this is a fundamental heating effect which cannot be avoided. If the stacking rate increases dramatically in future incarnations of the Antiproton Accumulator, this effect may become problematic. The only way to reduce the heating without reducing the cooling rate is to make β_l smaller at the kickers.

5. Effects of Tank-to-Tank Variations

The field due to an individual kicker tank will not have equal x and y sensitivities, and will not satisfy equation 6. The off-axis effects of an individual kicker tank may be estimated by assuming that the second term of equation 3 is small, so that a two-dimensional Laplace's equation is satisfied. For a quarter-wave kicker plate, a particle traversing immediately adjacent to the plate will receive an RMS voltage gain V_{max} of twice the RMS plate voltage V_{plate} . This is only strictly true if the particle passes over the center of a plate; if it passes off-center, it will see a somewhat lower gain due to the additional wave propagation time to the edges of the kicker plate. If this "droop" in voltage gain toward the edges of the kicker is ignored, the boundary conditions for the beam voltage gain V are equal to potentials of the cross-sectional kicker geometry, but with the actual plate voltage V_{plate} doubled to V_{max} , the maximum beam voltage gain. Thus the transverse dependence of V can be calculated by solving a two-dimensional Laplace's equation with these boundary conditions, which can be done by conformal mapping.[†]

The result of these conformal mapping manipulations is the potential produced by a pair of electrostatic plates (or the voltage gain of a beam passing between parallel kicker plates) driven in either sum or difference mode:

beam to cause heating. Thus, the factor K_{fd} will likely be somewhat different from the assumed value of 8.

[†]Hint: start with a parallel plate capacitor, where potential is proportional to y. Transform z by $z' = -\coth(\frac{z}{2})$, scale this result by z'' = az', where a is real, transform this by $z''' = \ln(\frac{z''-1}{z''+1})$, and scale the result by $Z = (\frac{z'''}{\pi} - \frac{i}{2})g$, where g is the full gap of the kicker.

$$\Phi_{sum} = \frac{V_{max}}{\pi} \tan^{-1} \left[\frac{2\cos Y \tanh \frac{W}{4}}{\cosh X - \sin Y - \tanh^2 \frac{W}{4} (\cosh X + \sin Y)} \right]$$

$$\pm \frac{V_{max}}{\pi} \tan^{-1} \left[\frac{2\cos Y \tanh \frac{W}{4}}{\cosh X + \sin Y - \tanh^2 \frac{W}{4} (\cosh X - \sin Y)} \right]$$
13)

where $Y = \frac{\pi y}{g}$, $X = \frac{\pi x}{g}$, $W = \frac{\pi w}{g}$, g is the full gap of the kicker plates, w is the full width of the kicker plates, V_{max} is the maximum beam voltage gain (equal to twice the plate voltage, V_{plate}) and the arctangent takes values from 0 to π .

Expanding the sum mode potential to second order about the origin, one finds the beam voltage gain V:

$$V = \Phi_{sum} \approx V_{max} \left[\frac{2}{\pi} \tan^{-1} \left(\sinh \frac{W}{2} \right) + \frac{\pi}{g^2} \left(\operatorname{sec} h \frac{W}{2} \right) \left(\tanh \frac{W}{2} \right) \left(y^2 - x^2 \right) \right]$$
 14)

For w=1.7g, which is approximately the aspect ratio for stacktail kicker tanks in the Antiproton Accumulator, this reduces to:

$$V \approx V_{max} \left[0.91 + 0.43 \frac{(y^2 - x^2)}{g^2} \right]$$
 15)

Taking the x component of equation 2, and Taylor-expanding as in equation 5, one finds:

$$\Delta p_x \approx ix \frac{e}{\omega} \frac{\partial^2 V}{\partial x^2}$$
 16)

Substituting from equation 15, this becomes:

$$\Delta p_x \approx -i0.9x \frac{eV_{max}}{\omega g^2}$$
 17)

Substituting a maximum beam voltage gain of 580 V per tank, a frequency of 1.5 GHz, and a gap of 3 cm into equation 17, one finds that a beam with a 1 mm offset receives a momentum kick of about 120 eV/c per kicker tank. This is *much* larger than the fundamental heating kicks discussed earlier, and is due to the use of parallel plates rather than a cylindrically symmetric structure to accomplish the kicks. However, horizontally

and vertically oriented tanks will give kicks in opposite directions, which should cause this effect to cancel to zero.

If the kicker tank amplitudes and phases are not exactly matched, these momentum kicks will not be canceled exactly, and some residual momentum kick will result. The TWT's that drive these tanks can have significant variation in amplitude. If we assume that they are matched to one another to ± 1 dB RMS across the microwave band, each tank will give a momentum kick of 120 ± 15 eV/c, and the eight tanks will subtract to give a residual RMS momentum kick of about ± 42 eV/c. Putting this in equation 11, we find that the beam will grow by 2π mm mrad (double in size) in about 10 minutes for a 1 mm beam offset. Putting these values into equation 12, we find that the time constant for growth of a centered beam is about 3 minutes.

Since the TWT's used in the Antiproton Accumulator have experienced large drifts in the past due to aging effects, it is doubtful that they will be matched to ± 1 dB. Hence, these numbers are likely somewhat optimistic, and this is potentially a serious heating contribution.

However, these calculations have ignored the contribution of delta kickers. It should be possible to cancel this effect almost exactly by the use of delta kickers to impart an equal and opposite kick to the beam. With properly adjusted delta kickers and stable gains, this should not be a problem.

6. Dispersion Effects

If there is a non-zero value of the dispersion at the kickers, betatron oscillations will be excited just as in an RF accelerating cavity. This effect has been calculated by Edwards and Syphers [3], among others. Converting their equation 7.112 to unnormalized FNAL emittance and adding a constant K_{fd} , one finds:

$$\frac{\partial \varepsilon}{\partial t} = 3\pi K_{fd} f_{rev} H \frac{e^2 V_{RMS}^2}{p^2 c^2 \beta_r^4}$$
 16)

where:

$$H = \gamma D^2 + 2\alpha D D' + \beta D'^2$$
 17)

For the Antiproton Accumulator, V_{RMS} is about 4.6 kV, β_r is about 1, p is about 9 GeV/c, and f_{rev} is about 630 kHz, as before. If we assume that α is about 0, γ is about 1/ β , where

 β is about 10 meters. If we further assume a dispersion error D of about 0.1 m and no D' error, we find that H is about 1 mm. Substituting these values into equation 16, we find that the emittance growth rate is about 1.2*10⁻⁸ m/s, and the emittance grows by 2π mmmrad in about 500 sec, or about 8 minutes.

This could also be a serious contribution to emittance growth if there is significant dispersion at the kickers. However, it is thought that the dispersion at the kickers is well under 0.1 meter, so this is not likely to be the major contributor to beam heating.

7. Difference Mode Effects

The above calculations assume that the kickers are perfect, longitudinal kickers and impart no transverse kicks. In the real world, there will be some transverse kicking due to imperfections. We assume that the beam is electrically centered, or that delta kickers are used, so that on average there are no transverse kicks across the microwave band. However, it is possible that the top and bottom (or left and right) plates in a kicker will not track one another perfectly in frequency. Such mistracking could be due to electronics imperfections (in hybrids, splitters, connectors, or combiner boards) or due to physical differences in the plates themselves. A similar mistracking will occur if the kickers impart a net transverse kick which is not tracked perfectly in frequency by the delta kickers.

Assume two plates on opposite sides of the beam, each having nominally the same voltage, but with a small amount of difference mode. The difference mode voltage may be approximated as:

$$V_{d} = V_{nom} \left(\frac{\Delta A_{dB} \ln(10)}{20} + i \frac{2\pi \Delta \phi_{deg}}{360} \right)$$
 18)

where V_{nom} is the nominal (sum-mode) voltage on the plates, V_d is the voltage difference on the two plates, ΔA_{dB} is the amplitude difference in dB, and $\Delta \phi_{deg}$ is the phase difference in degrees.

We will assume, for ease of estimation, that the system response is nominally constant across the microwave band. We will also assume that the plate voltage amplitudes (or the delta kickers) are properly adjusted such that the average value of V_d across the band is zero. The quantity of interest will then be the RMS value of V_d across the microwave band, since this will contribute to heating. Conceptually, it is as if the beam has a number of independent resonant modes, one for each sideband, and V_d pumps power into these modes.

The transverse momentum kick received from a voltage difference on the plates will be roughly:

$$\Delta p_x = F_x \Delta t \cong eE_x \frac{\Delta l}{\beta_r c} \cong \frac{e}{\beta_r c} \frac{V_d \Delta l}{g}$$
⁽¹⁹⁾

where Δl is the total effective length of the electrodes, and g is the gap between the two electrodes.[†]

We will assume that each electrode has a gap about equal to its length, that there are 128 electrode pairs, and that the electrode voltage V_{nom} is about 20 V, as calculated earlier. Then, for a 0.2 dB RMS amplitude error or a 1° RMS phase error, the beam sees a voltage difference of about 0.5 V and RMS kicks of about 60 eV/c across the microwave band. Putting this in equation 11, we find that the beam will double in size in about 300 seconds, or about 10 minutes. This is also potentially a serious heating problem.

8. Kicker Tank Centering Code

Because a number of these heating mechanisms depend on beam offsets in the kickers, it is important to make sure that the beam is electrically centered in the kickers. For this reason, an application code was written to move kicker tanks so that beam is centered.

This code uses a network analyzer to drive the momentum kickers on a number of betatron lines. A kicker is driven, and the beam response is picked up by wideband Schottky pickups. The powers in each line are summed to yield the total power (subtotals are also calculated for each third of the microwave band and for each of the two sidebands). The kicker tank is then moved, and another frequency scan is taken. The results of a number of such measurements are plotted, and the user may set the kicker tank to the desired location,

[†]In this expression, it has been assumed that the electric field between the plates is V_d/g . Because of the finite width and length of the electrodes, the electric field will be somewhat less than this. For electrodes of finite width but infinite length, the problem may be solved by conformal mapping. The potential takes the difference form of equation 13. Expanding this to first order, the electric field becomes $\frac{V_d}{g} \tanh(\frac{W}{2})$. For our case, where w=1.7g, this only reduces the field by about 1%, thus is a negligible correction.

which is generally the location which minimizes total power transferred to the beam (figures 1,2).

Previously, the kicker tanks were centered by an eyeball-integration of total power on the screen of the network analyzer. This application code allows centering the kicker tanks much more quickly and more reliably. Operation of the program is fairly self-explanatory, and further documentation may be found in the extensive help features of the code itself.

This code has revealed an intriguing, systematic effect in the sideband powers. It can be seen (figure 2) that the power minimum for the upper sideband occurs at a slightly different position than that for the lower sideband. For all kicker tanks in which the plates are horizontally opposed (tanks 1,3,7, and 9), the minimum for the upper sideband is lower in x (further inside the ring). For tanks in which plates are vertically opposed (tanks 2,6,8,and 10), the upper sideband's power minimum is further outside the ring than the minimum for the lower sideband. This is presumably due to coupling between longitudinal and transverse motion, but the details of this effect are not understood at present.[†]

9. Conclusions

A number of basic, open-loop mechanisms exist which can couple power from longitudinal kickers into transverse beam motion, thereby heating the beam transversely. A number of these mechanisms are dependent upon beam offsets, thus it is very important to ensure that beam is centered between the kicker tanks, or that delta kickers are accurately adjusted to cancel transverse kicks from an off-centered beam.

Tank-to-tank variations in kicker power are another serious contribution to transverse heating. This can be addressed by carefully matching TWT powers or by accurately adjusting delta kickers. Dispersion at the kickers is another potential source of transverse heating; it must be kept well under 0.1 meter to avoid problems.

The remaining source of transverse heating which I have identified is due to mistracking of kicker plates or delta kickers across the microwave band. If the delta kickers are adjusted properly and the dispersion is eliminated, this is likely to be the dominant source of transverse heating. It can only be addressed by accurately matching the responses of kicker

[†]It will also be noted that for this scan, there is a difference in amplitude at the minima at the two sidebands. This, however, does not seem to be very significant or consistent.

plates (and delta kickers) across the microwave band, which implies the use of high quality microwave splitters and consistent electrode construction.

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LONGITUDINAL KICKER TANK CENTERING

♦Pgm Tools♦



Fig. 1 Main window of Kicker tank centering program.

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Fig. 2 Plot window of kicker tonk centering program.