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Correlator Filters for Stochastic Cooling

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I. Introduction

The shorted stub filter although simple in concept has the disadvantage of being subject to a large notch dispersion. Although this dispersion can be reduced by various compensation techniques, the problems of imperfections and impedance missmatches makes this filter quite difficult to work with in practice.

This note will describe the correlator type filter which can provide a notch filter with all the necessary properties for realistic cooling systems. Since the entire filter uses 50Ω matched components, the filter is less sensitive to imperfections and the filter is easier to produce in practice.

This note will deal only with non-superconducting cable, because the system is easier to model and to produce and because it maybe more reliable to operate. Many of the ideas presented are directly applicable to a superconducting filter. A future note will discuss the use of low loss transmission cable, which could provide the power handling capabilities for post-TWT filters, a property previously considered impossible without superconducting cable.

II. Basic Correlator Filter

Fig. (1a) shows the basic correlator filter which uses the frequency correlation between a long low attenuation cable (γ_1, ℓ_1) and a short high attenuation cable (γ_2, ℓ_2) to yield a repetitive notch spacing, f_o. The voltage transfer function is

$$T(f) = \frac{V_o(f)}{V_i(f)} = \frac{1}{2}\left(e^{-\gamma_2\ell_2} - (+)\ \xi\ e^{-\gamma_1\ell_1}\right) \qquad (1)$$

$$= \frac{1}{2}\, e^{-\gamma_2\ell_2}\left(1 - (+)\ \xi\ e^{-(\gamma_1\ell_1 - \gamma_2\ell_2)}\right)$$

where

$$\gamma = \alpha + i\beta$$
$$\alpha = \alpha_c + \alpha_d$$
$$\beta = \beta_o + \alpha_c$$
$$\beta_o = \frac{2\pi f}{v_o}$$

for

α_c = conductor loss

α_d = dielectric loss

v_o = phase velocity of idea cable.

(see $\vec{p}$ Note 233 for definitions of these quantities). Fig. (2) shows the phasor diagrams of this filter, which indicates that notches will occur whenever V_1 and V_2 have a 0° or 180° relative phase, depending on the sign of the combiner. The transfer function can be written as

$$T(f) = \frac{1}{2}\, e^{-\gamma_2\ell_2}\left[1 - (+)\ \xi\ A e^{\frac{-2\pi i f}{f_o}} D\right] \qquad (2)$$

where

$$A = \mathrm{EXP}\left[\alpha_2\ell_2 - \alpha_1\ell_1\right]$$

$$D = \mathrm{EXP}\left[i\left(\alpha_{c2}\ell_2 - \alpha_{c1}\ell_1\right)\right]$$

and

$$f_o = \frac{1}{\left(\frac{\ell_1}{v_{o1}} - \frac{\ell_2}{v_{o2}}\right)} = \text{notch frequency spacing.}$$

Assumming $\alpha_{c1}\ell_1 = \alpha_{c2}\ell_2$ (i.e. $D = 1$), then for the 180° power combiner (negative sign) the notches (minimum of T(f)) occur at $\bar{f} = nf_o$ for $n = 0,1,2 \ldots$ and peaks occur at $\hat{f} = (n + \frac{1}{2})\, f_o$. For a 0° combiner the notches and peaks are interchanged. If $\alpha_{c1}\ell_1 \neq \alpha_{c2}\ell_2$ then the notch spacing will vary with frequency. The notch depth will be determined by the parameters

ξ and A. ξ can be adjusted to yield exact cancellation ($-\infty$ db deep notches) at any frequency. For the basic correlator filter the two most obvious properties which need to be satisfied are:

$$1)\quad \alpha_{c1}\ell_1 = \alpha_{c2}\ell_2 \quad \text{(zero notch dispersion)} \tag{3a}$$

$$2)\quad f_o = \left(\frac{\ell_1}{v_{01}} - \frac{\ell_2}{v_{02}}\right) \quad \text{(notch spacing)} \tag{3b}$$

Fig. (3) shows the expected notches for the ANLator(*) filter in the 1-2 GHZ bandwidth. Fig. (4) shows the expected notch depth

$$\text{Notch depth} = \frac{T(\bar{f}_n)}{T(\hat{f}_n)} \approx \frac{(1 - \xi A)}{(1 + \xi A)} \tag{4}$$

The calculated notch depth varies from 35 db to -6 db over the 1-2 GHZ bandwidth and agrees quite well with the measured values. Fig. (5) shows the measured notch dispersion for the ANLator filter, which had an RMS $\delta_n < \pm 3 \times 10^{-6}$ from 1-2 GHZ. This dispersion was dominated by combiner/splitter phase variations and not by cable properties.

The basic correlator has three major deficiencies

1) notch depth variation
2) phase variation at the notch
3) large and variable attenuation at the peak transfer function.

The compensation for these problems will be discussed in the following section.

(*)The ANLator filter is the prototype correlator filter built by Argonne in July 1982. It used ≈400 m of the 7/8" foam cable RG-333 and ≈15 m of RG-178 cable. Combiners used limited the filter to the 1-2 GHz bandwidth.

III. Compensated Correlator

A. Notch depth variation

The source of the notch depth variation is the factor A. Fig. (6) shows the variation of the attenuation versus frequency for the cables used in the

ANLator. The low loss (RG-333) cable shows the effect of the dielectric loss ($\propto f$) on the attenuation. However the high loss cable (RG-178) has such high skin loss ($\propto \sqrt{f}$) that the dielectric loss has negligable effect on the total attenuation dependence. The variation in A by only a few db results in a large variation of notch depth, as shown in Fig. (7).

The solution to this problem is to tune the dielectric of the high loss line such that

$$\alpha_1 \ell_1 = \alpha_2 \ell_2$$

Since the zero dispersion condition, Eq. (3a), requires $\alpha_{c1}\ell_1 = \alpha_{c2}\ell_2$ this implies

$$\alpha_{d2} = \alpha_{d1} \frac{\ell_1}{\ell_2} \tag{5}$$

or

$$\frac{\tan \delta_{e2}}{v_{o2}} = \frac{\tan \delta_{e1}}{v_{o1}} \frac{\ell_1}{\ell_2} \tag{6}$$

Optimizing Eqs. (3) and (6) for a particular dielectric material and conductors is not straight forward. However, after picking appropriate materials, an adequate solution can usually be obtained. Figs. (8&9) show one solution using RG-333 and a commercially available high loss cable LA50141, but substituting a 35% nylon and 65% TFE dielectric filling for the 100% TFE. The length of LA50141 corresponds to a 37 nsec signal delay, but this delay can be reduced to 18 nsec by going to a smaller diameter cable LA50085-SS with a stainless steel outer conductor instead of copper (i.e. special order cable).

B. Phase Variation

Fig. (9) shows the variation of the phase of $T(\bar{f})$ at the notch. This variation arises from the phase dispersion in the $e^{-\gamma_2 \ell_2}$ factor of Eq. (2). Although not necessarily a serious problem, this does reduce the forcing term for stochastic cooling. This dispersion can be countered using an opposite dispersion cable from the filter to the kicker, as shown in Fig. (1b).

Using the notation in $\bar{p}$ Note No. 233 the phase of the propagation constant is given by

$$\beta \ell = \beta_o \ell (1 + d_m - \delta_n)$$

where

$$\beta_o = \frac{2\pi f}{v_o}$$

$$d_m = \frac{L_{sm}}{2L_o}$$

and

$$\delta_n = 0 \text{ at } f = f_m = mf_o.$$

Setting the timing of the signal in Fig. (1b) equal to the beam timing $\left(t_B = \frac{K}{f_o} + \Delta t(E)\right)$ at the midband frequency f_m gives

$$2\pi f_m \, t_B = \beta_2(f_m)\, \ell_2 + \beta_3(f_m)\, \ell_3 + \beta_4(f_m)\, \ell_4$$

or

$$\ell_3 = \frac{v_{03}}{(1 + d_{m3})}\left[t_B - \frac{\ell_2(1 + d_{m2})}{v_{o2}} - \frac{\ell_4\ (1 + {}_{m4})}{v_{o4}} \right] \tag{7}$$

Setting the timing equal at the upper frequency of the bandwidth (f_2) gives

$$\ell_3 = -\frac{v_{o3}}{\delta_{n3}} \left(\frac{\ell_2 \delta_{n2}}{v_{o2}} + \frac{\ell_4 \delta_{n4}}{v_{o4}}\right)\Bigg|_{f = f_2} \tag{8}$$

Eq. (7) + (8) can then be solved for ℓ_3 and ℓ_4. Since $\ell_3 > 0$ and $\delta_{n2} > 0$ then for $\delta_{n3} < 0$, we must have δ_{n4} the same sign as δ_{n2} or

$$\left|\frac{\ell_4\ \delta_{n4}}{v_{o4}}\right| < \frac{\ell_2 \delta_{n2}}{v_{o2}} .$$

If we try to match the dispersion with $\ell_4 = 0$, then the constraint on δ_{n3} at $f = f_2$ is

$$\delta_{n3} = -\frac{\ell_2 \delta_{n2}}{v_{o2}} \left[t_\beta - \frac{\ell_2\ (1 + d_{m2})}{v_{o2}} \right]^{-1} (1 + d_{m3})$$

For the previous correlator in Figs. (8&9), $t_B = 0.3/f_o$ and $\ell_4 = 0$

$$\delta_{n3} = -7.6 \times 10^{-5} \text{ at 2 GHZ.}$$

Since this can't be provided by readily available cables, we must use two different types of cables as shown in Fig. (1b).

Figures (10) and (11) show a correlator notch filter with phase compensation. The cables used were:

1) 7/8" foam Heliax: 425 meters
2) LA50141-SS with 26% Nylon: 5.5 meters (30 nsec)
3) 1-5/8" airline cable: 77.7 meters
4) 7/8" foam Heliax: 47.8 meters

No beam dispersion has been used and no time delay or phase dispersion for other system components has been included in this calculation.

The phase variation for the correlator has been reduced from ±30° to ±8° over the 1-2 GHZ band.

C. Variation of the Peak Attenuation

The variation in the peak atenuation comes from the exponent $(\alpha_2 \ell_2 + \alpha_3 \ell_3 + \alpha_4 \ell_4)$ changing with frequency. This variation, although not necessarily undesirable, can be reduced by using the gain variation of a stripline coupler. For a peak transfer function $a(f) = |T(\hat{f})|^2$ and an octive bandwidth coupler with gain variation

$$G^2 = G_0{}^2 \left| \sin \frac{\pi f}{2f_c} \right|^2 ,$$

we can try to match the peak variation over the octive bandwidth to yield as flat as response as possible. An amplifier is also required if we want to keep $a(f) = 0\text{db}$. However, we find that for an octive bandwidth the coupler only yields a maximum of 6 db gain variation since

$$\frac{G^2(f_2)}{G^2(f_1)} = \left|\frac{\sin \frac{\pi f_2}{2f_c}}{\sin \frac{\pi f_1}{2f_c}}\right|^2 = 4\cos^2\left(\frac{\pi f_1}{2f_c}\right) < 4 \text{ (or 6db)}$$

To overcome this we can add several couplers each contributing about 6 db of gain variation. Fig (12a) shows the result of adding two 6 db couplers (4-8 GHz) to the phase compensated correlator presented in Figs (10) and (11). The two curves are the variation with frequency of the peak transfer function a(f) and the notch or minimum transfer function $|T(\bar{f})|^2$. Fig. (12b) shows the same curves for the correlator presented in Figs. (10) and (11), without gain compensation. In order to maintain a(f) ≈ 0db an amplifier gain of 60 db was also required. The net gain variation of the peak of the transfer function has been reduced from 16 db to about 4 db for the 1-2 GHZ bandwidth.

IV. Summary of Correlator Transfer Function

$$T(f) = G(f)\, F(f)\, \left(1-\xi A\, D\, e^{-2\pi i f/f_o}\right)$$

where

$A = \mathrm{EXP}\,(\alpha_2\ell_2 - \alpha_1\ell_1)$ attenuation balance

$D = \mathrm{EXP}\,(i\alpha_{c2}\ell_2 - i\alpha_{c1}\ell_1)$ dispersion balance

$f_o = \left(\frac{\ell_1}{v_{o1}} - \frac{\ell_2}{v_{o2}}\right)^{-1}$ notch frequency

$$F(f) = \mathrm{EXP}\left[-\left(\gamma_2\ell_2 + \gamma_3\ell_3 + \gamma_4\ell_4\right) + 2\pi i f t_B\right]$$

$$G(f) = 0.5\, G_0^2\left[10^{\frac{a}{20}}\, e^{\frac{i\pi}{2}}\, \sin\left(\frac{\pi f}{2f_c}\right)\right]^n$$

t_B = beam time from pickup to kicker

f_c = frequency of maximum coupling

a = coupler maximum coupling in db

n = the number of couplers used

G_o = amplifier gain

ξ = variable attenuator for notch depth tuning.

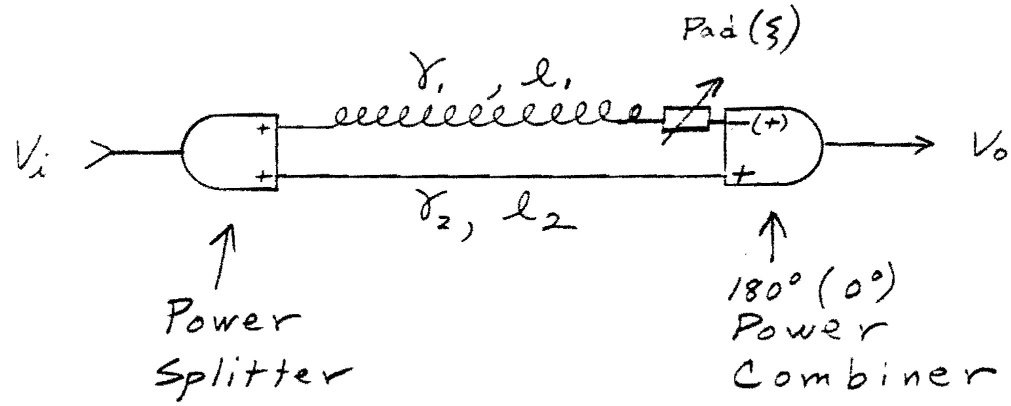


Fig. (1a) Schematic of Correlator Filter

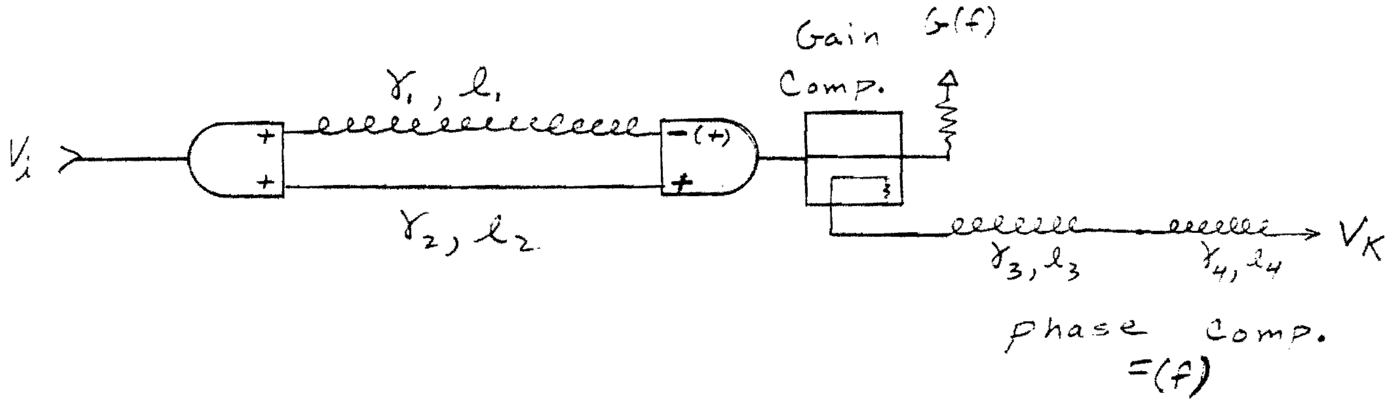


Fig. (1b) Schematic of Compensated Correlator

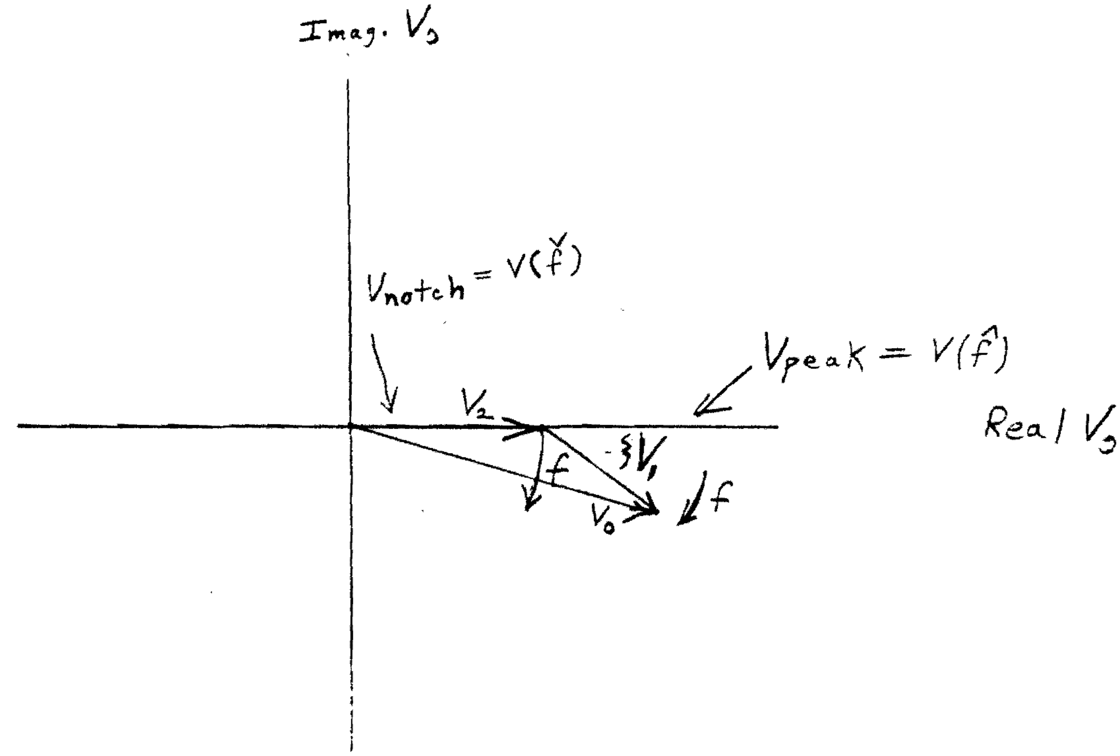


Fig. 2 Phasor Diagram of Correlator Filter

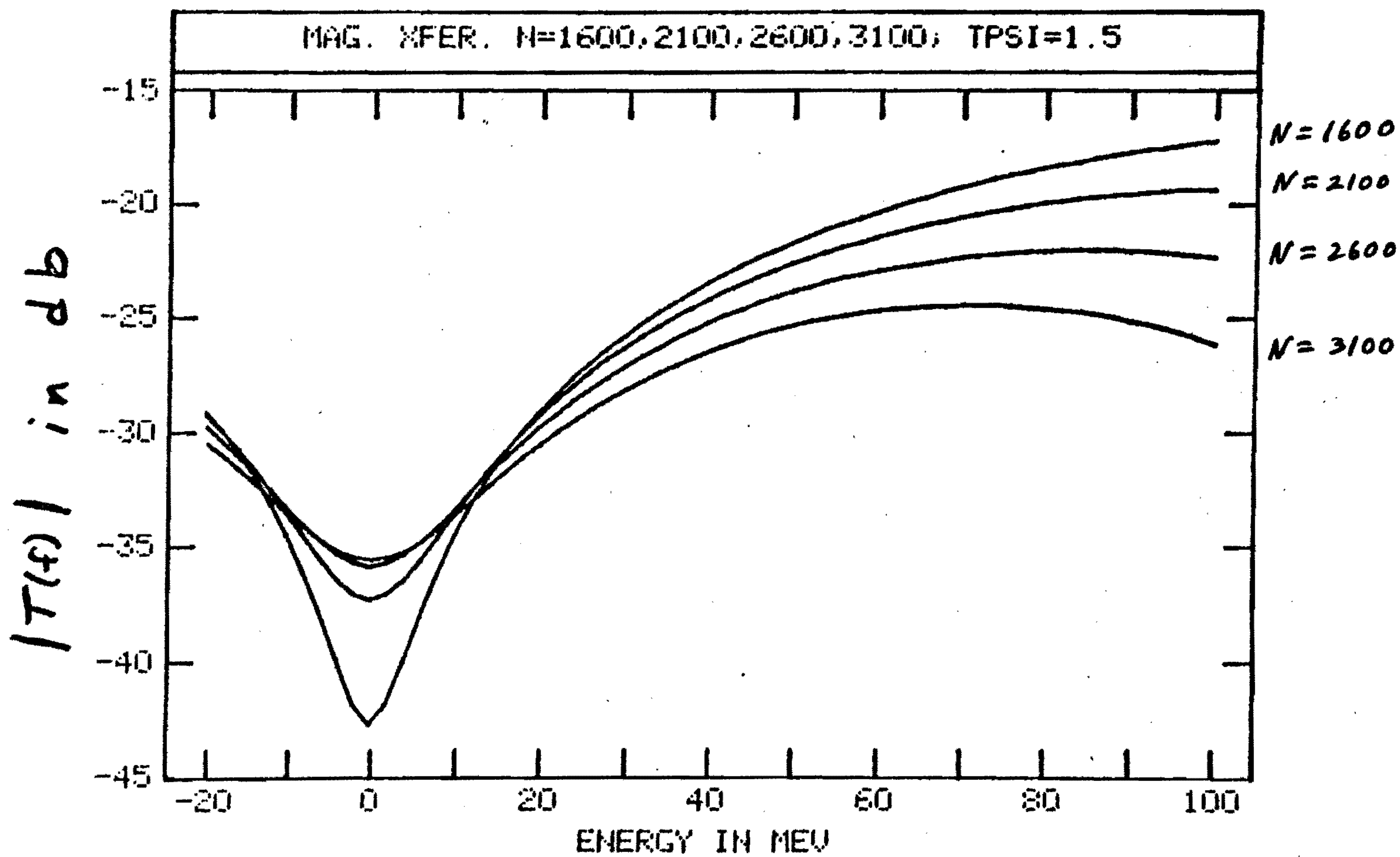


Fig.(3a) $|T(f)|$ for notch frequency $f_n = n f_o$
(f_o = 0.63 MHZ)

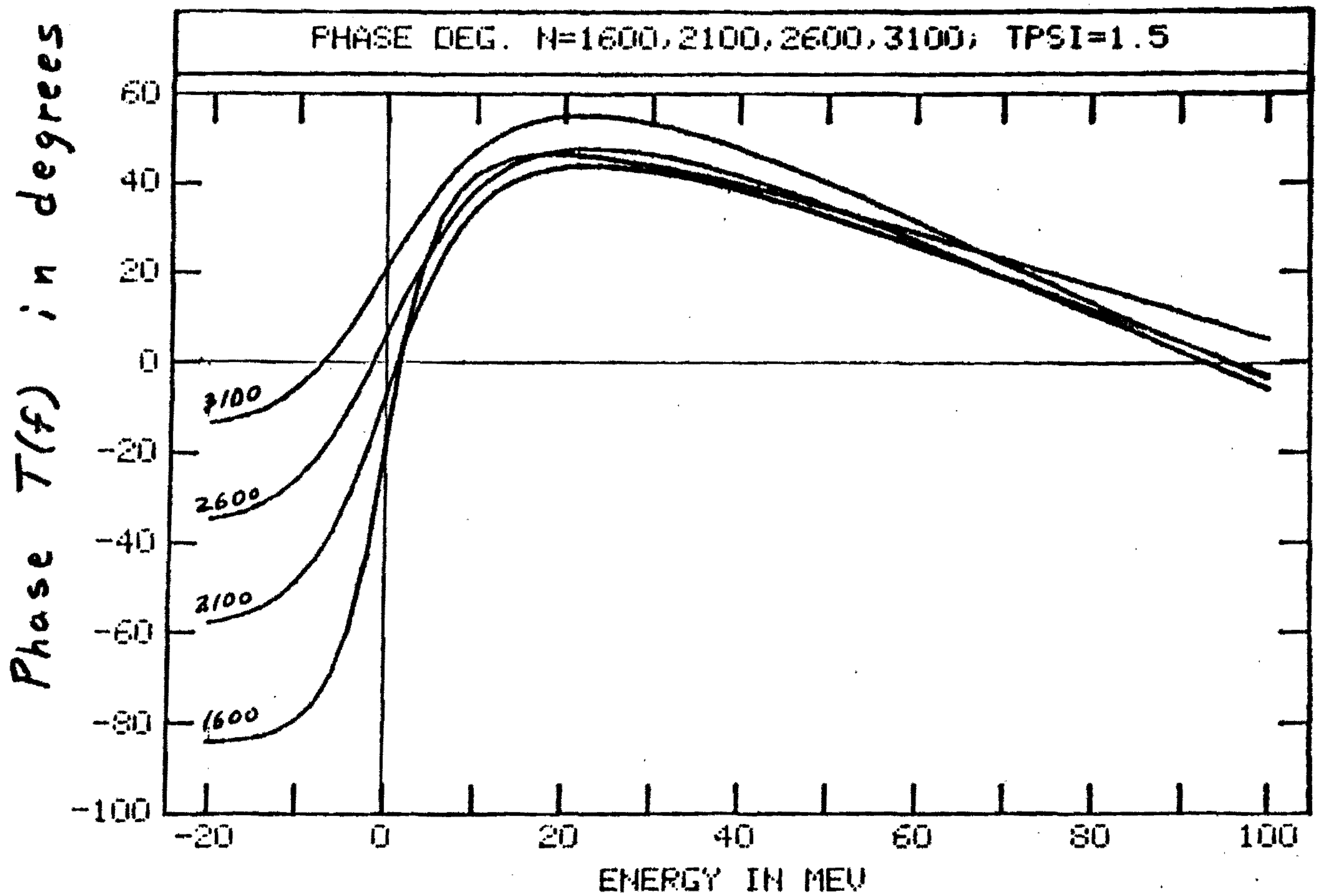


Fig. (3b) Phase $T(f)$ for notch frequencies $f_n = n f_0$
($f_0 = 0.63$ MHZ)

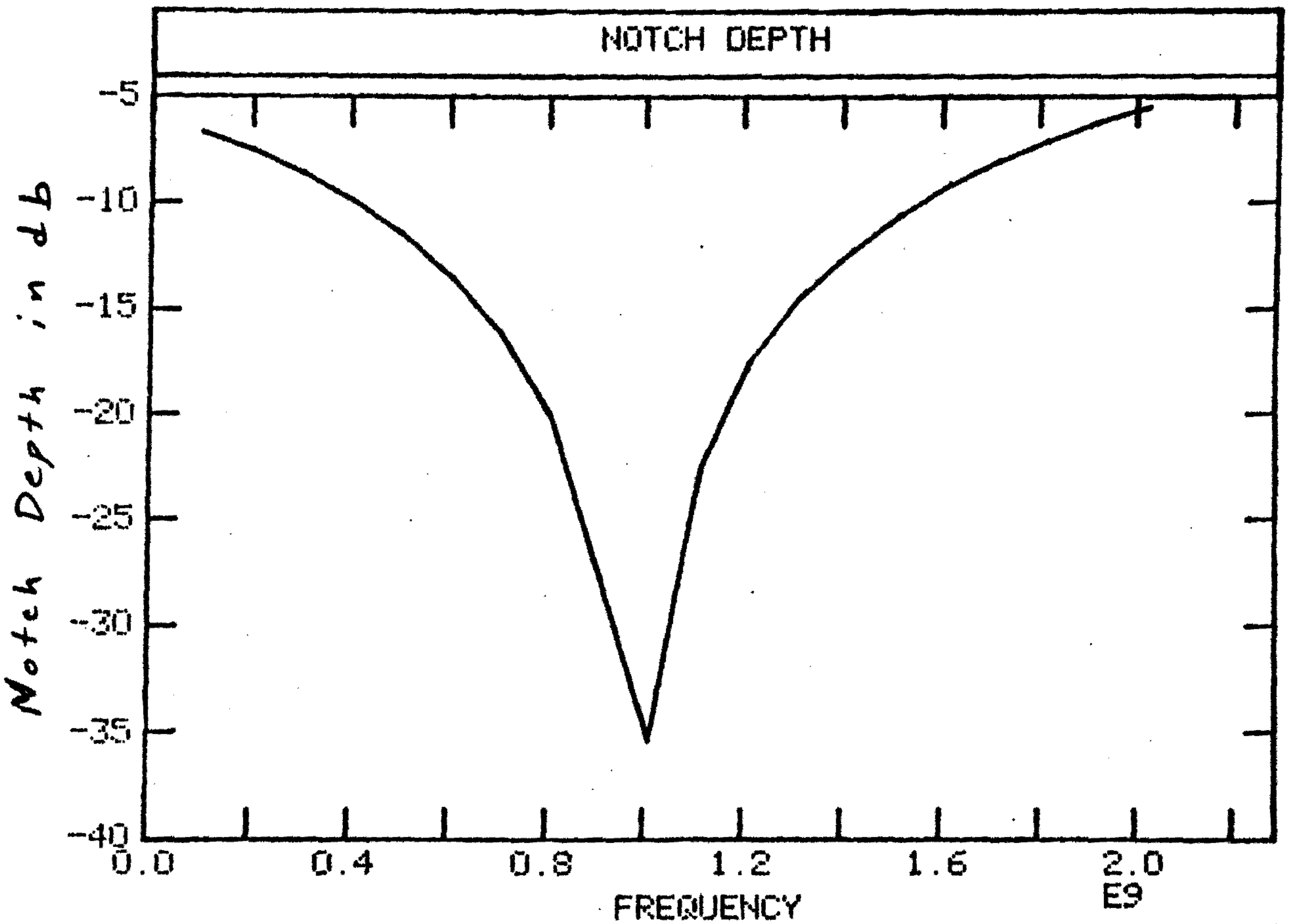
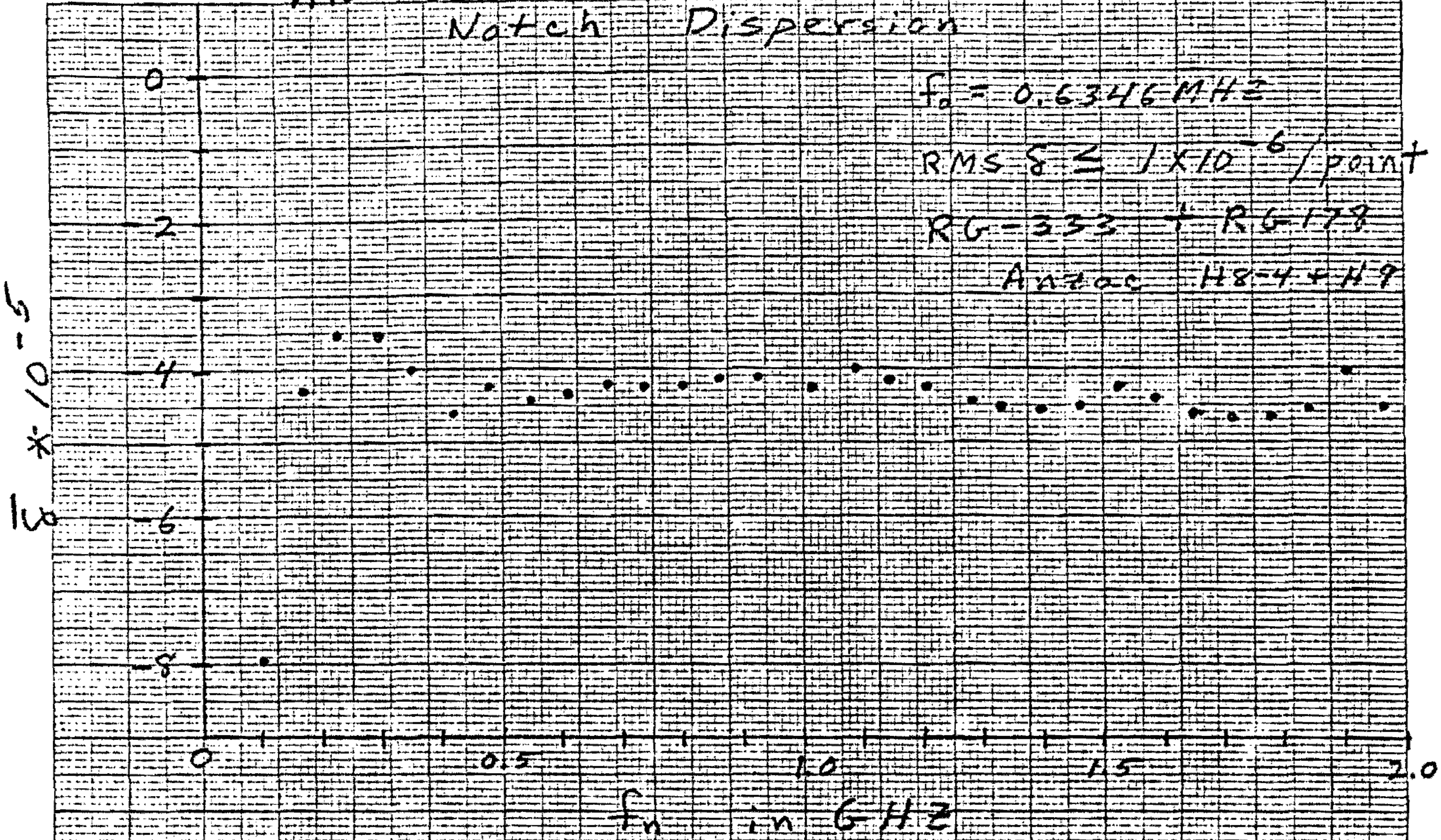


Fig.(4) Notch Depth as function of frequency

S.L. Kramer
6/25/82

ANLator Filter



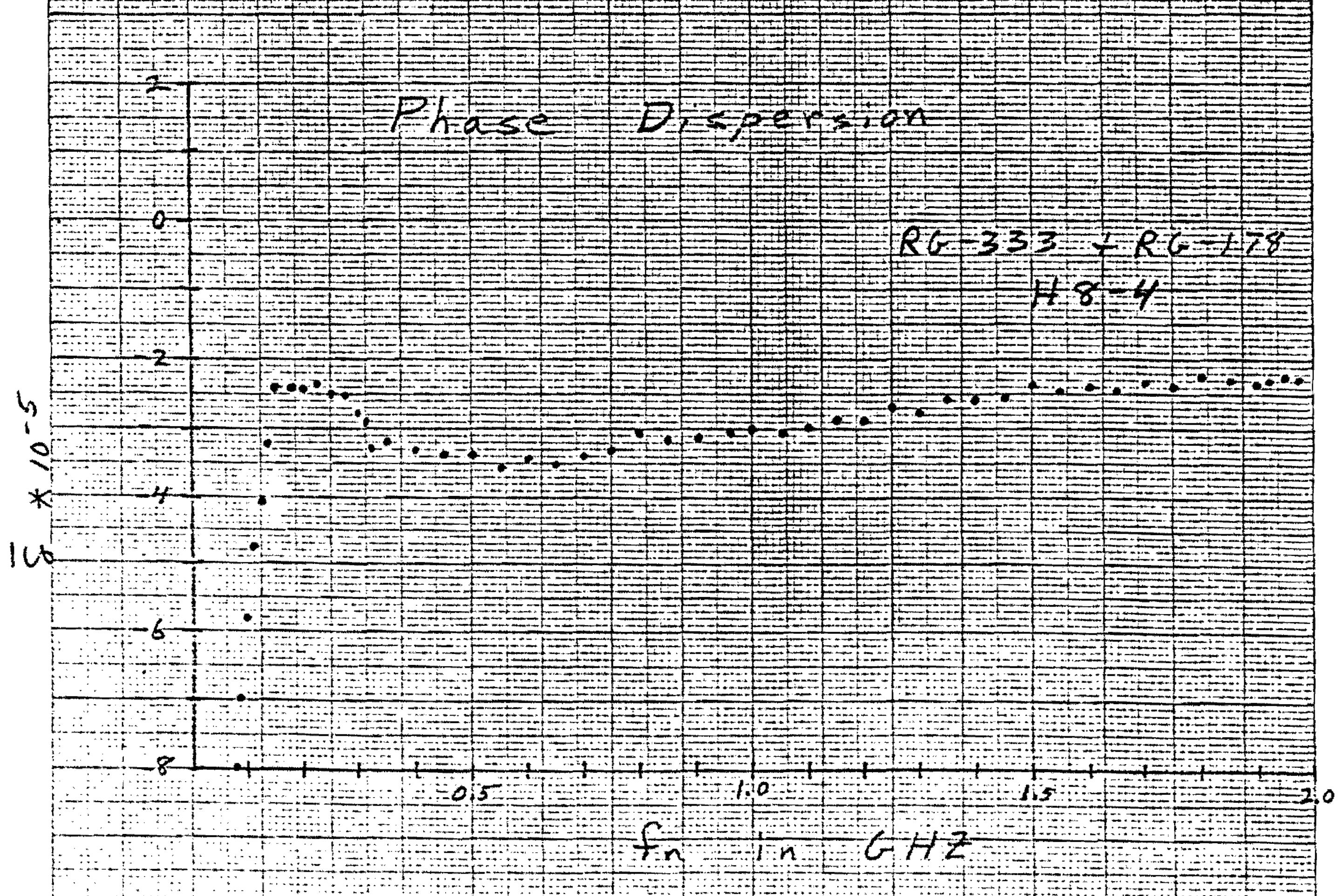


Fig 5 Notch Dispersion of The ANLator Filter

Attenuation for ANLator Filters

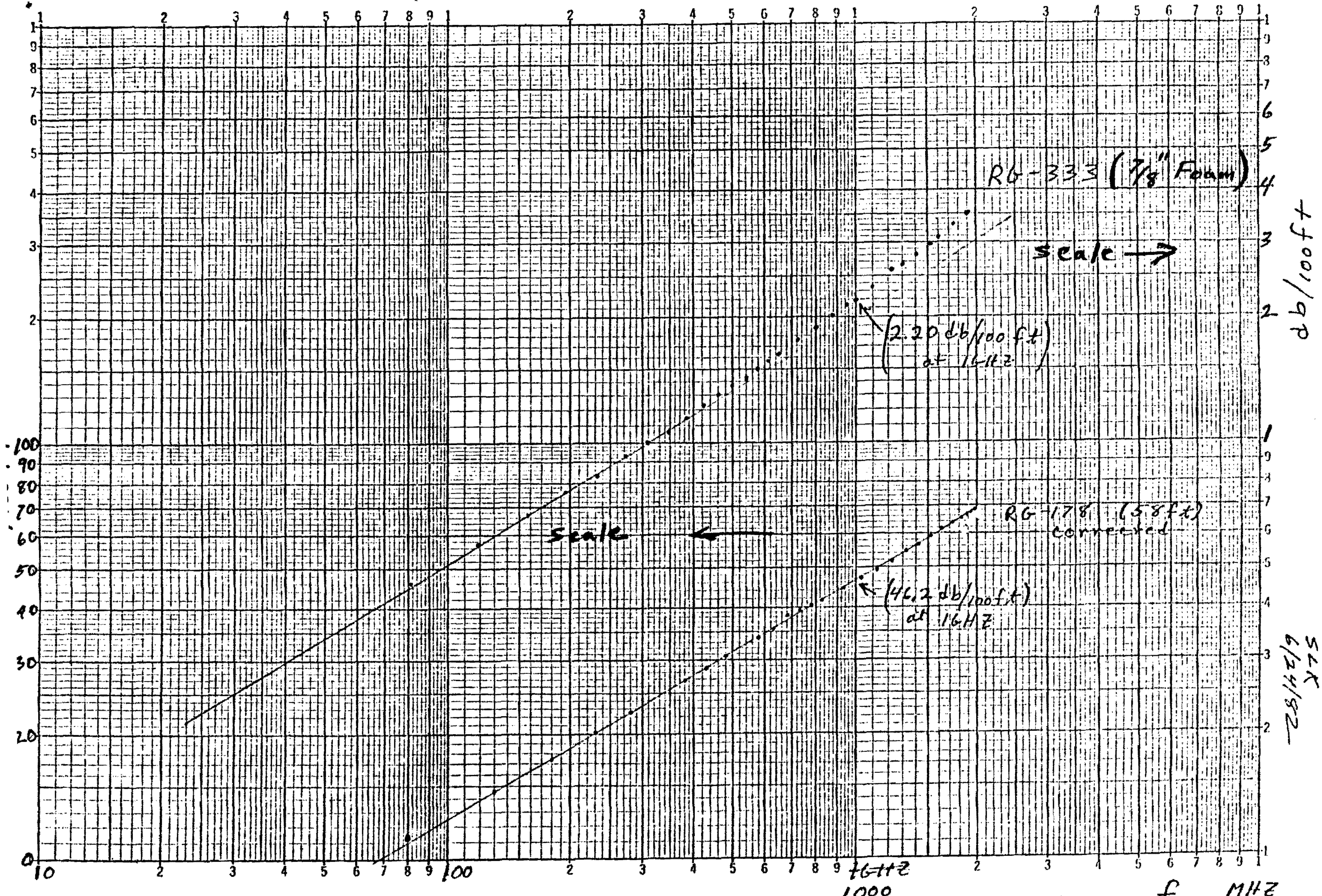
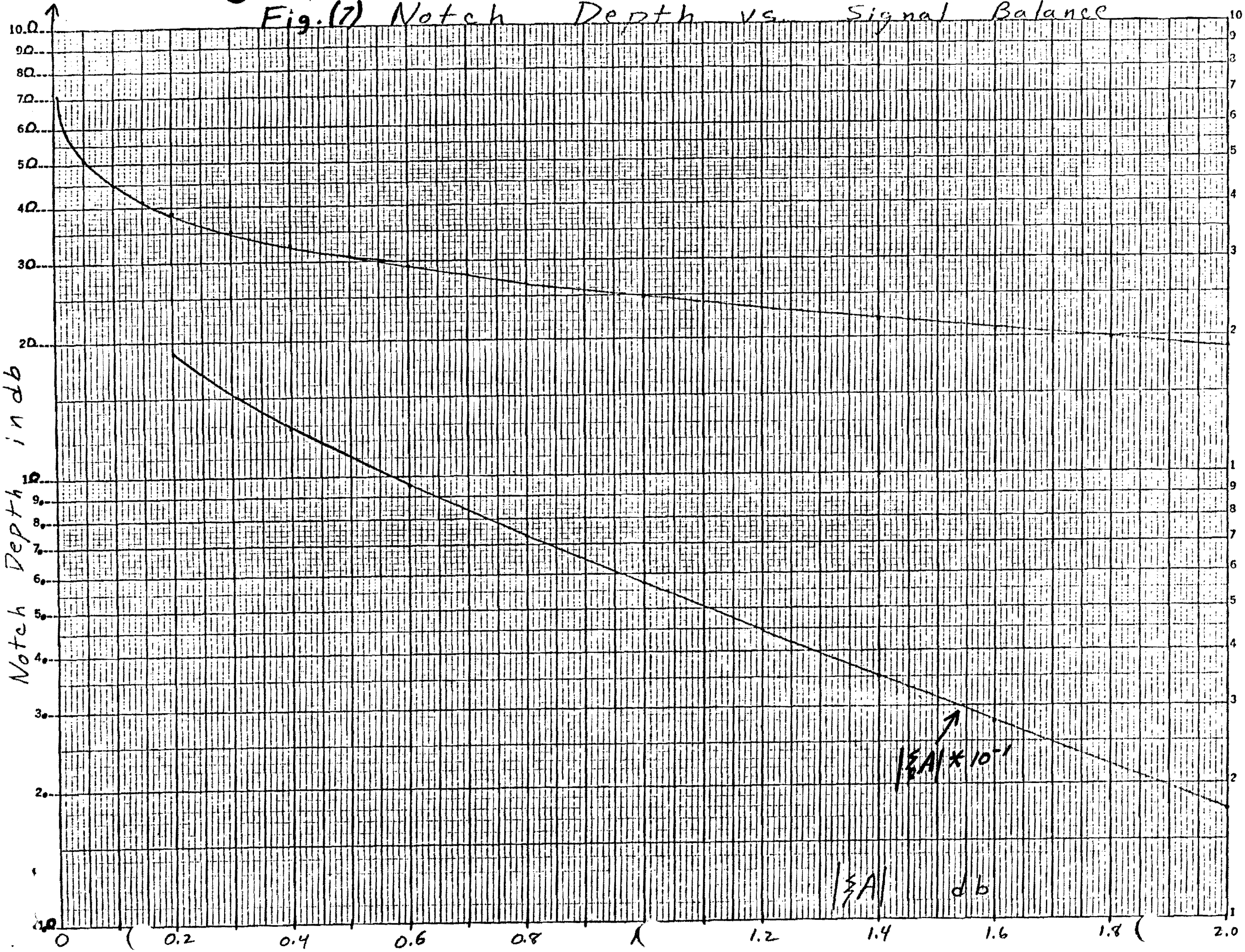


Fig. (6) Attenuation of the Cables used in the ANLator

Fig. (7) Notch Depth vs. Signal Balance



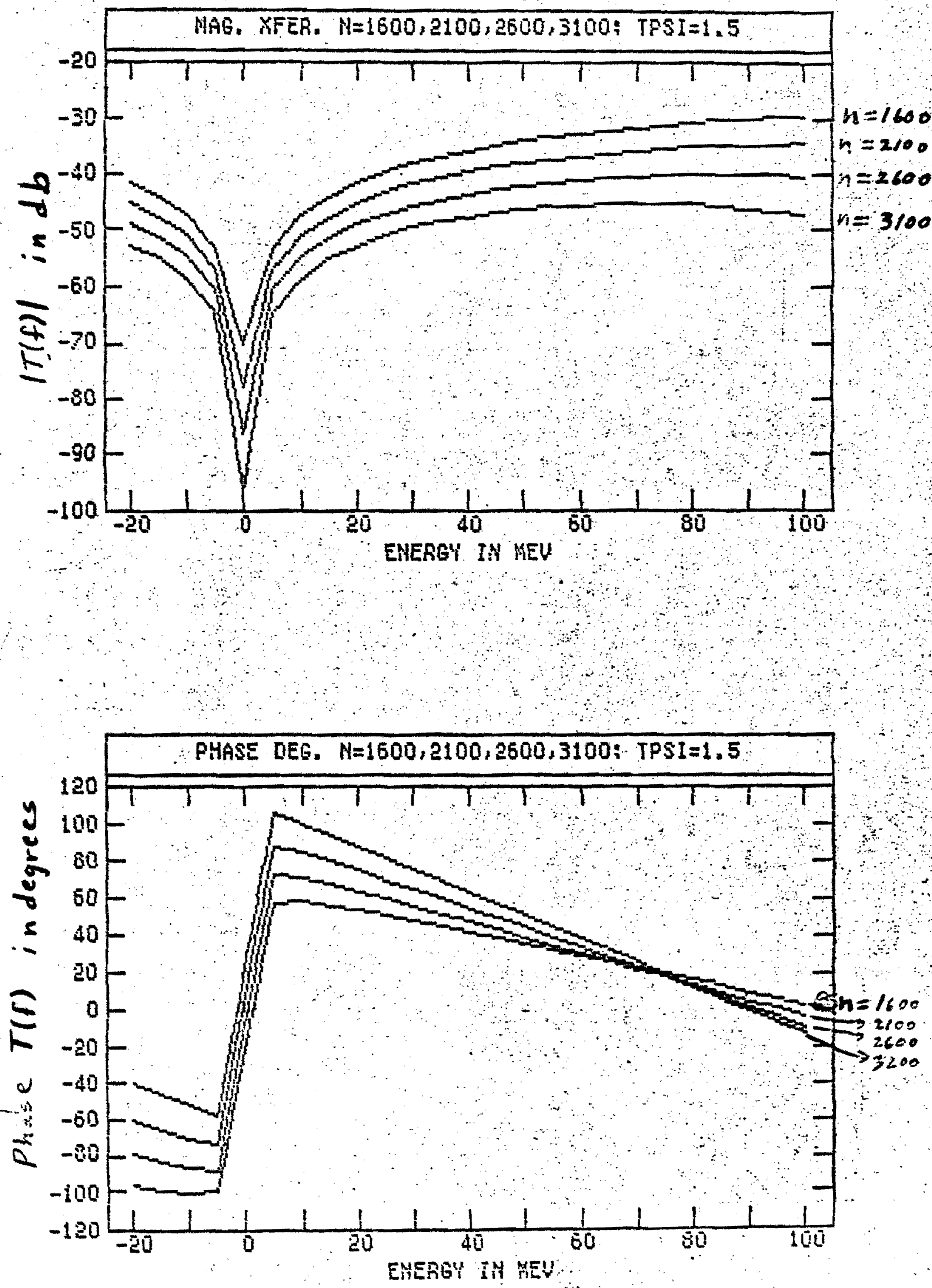
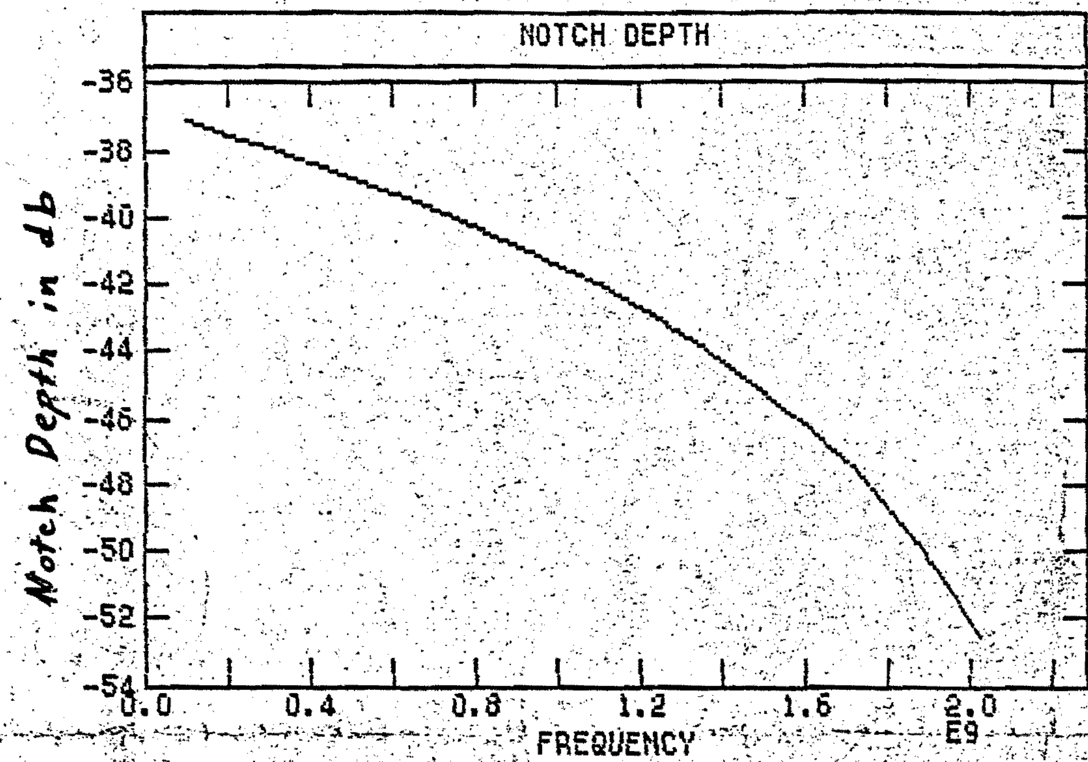


Fig. (8) T(f) for Tuned Correlator Filter
RG-333 and LA50141 (35% Nylon)

F = 2.0245E9 ATT2 = 772.18 DEPTH = -52.543 ATF = -98.874 ANG =
27.281 DEAN = .21047 ATTEN1 = 11.882 ATTEN2 = 775.71



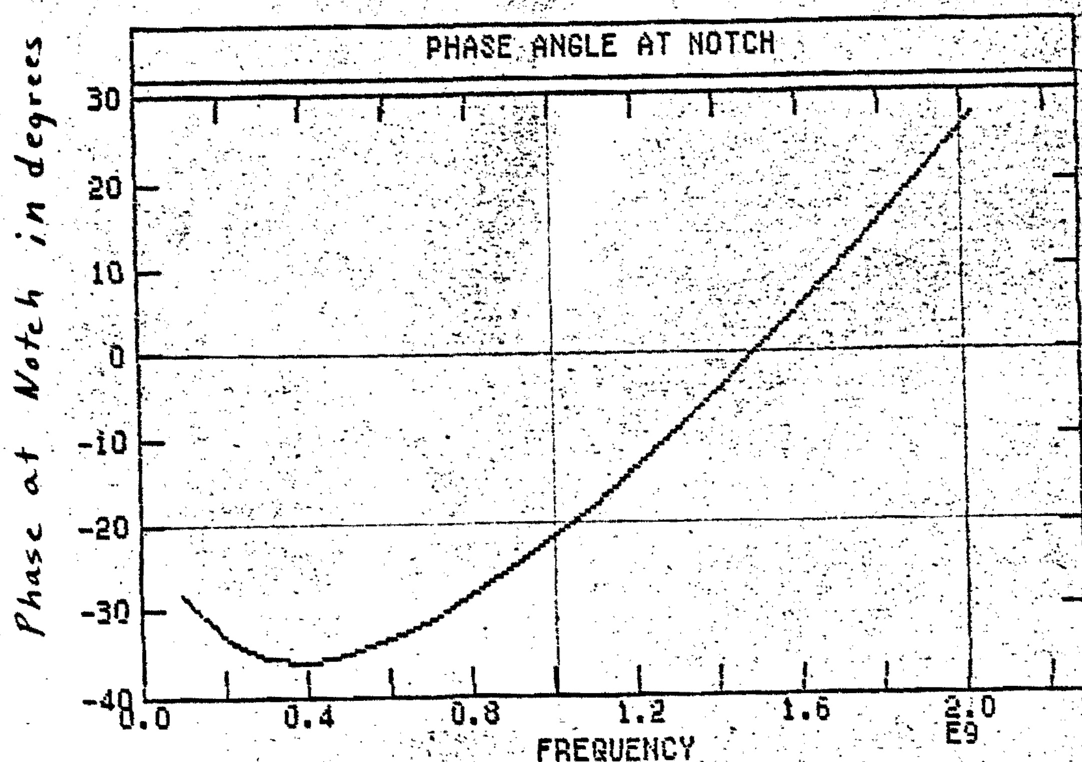


Fig. (9) Notch Depth and Phase for tuned Correlato
RG-333 and LA50141 (35% Nylon)

CABLE ATTENUATION MATCH

Fig.(10a) Phase Compensated Correlator

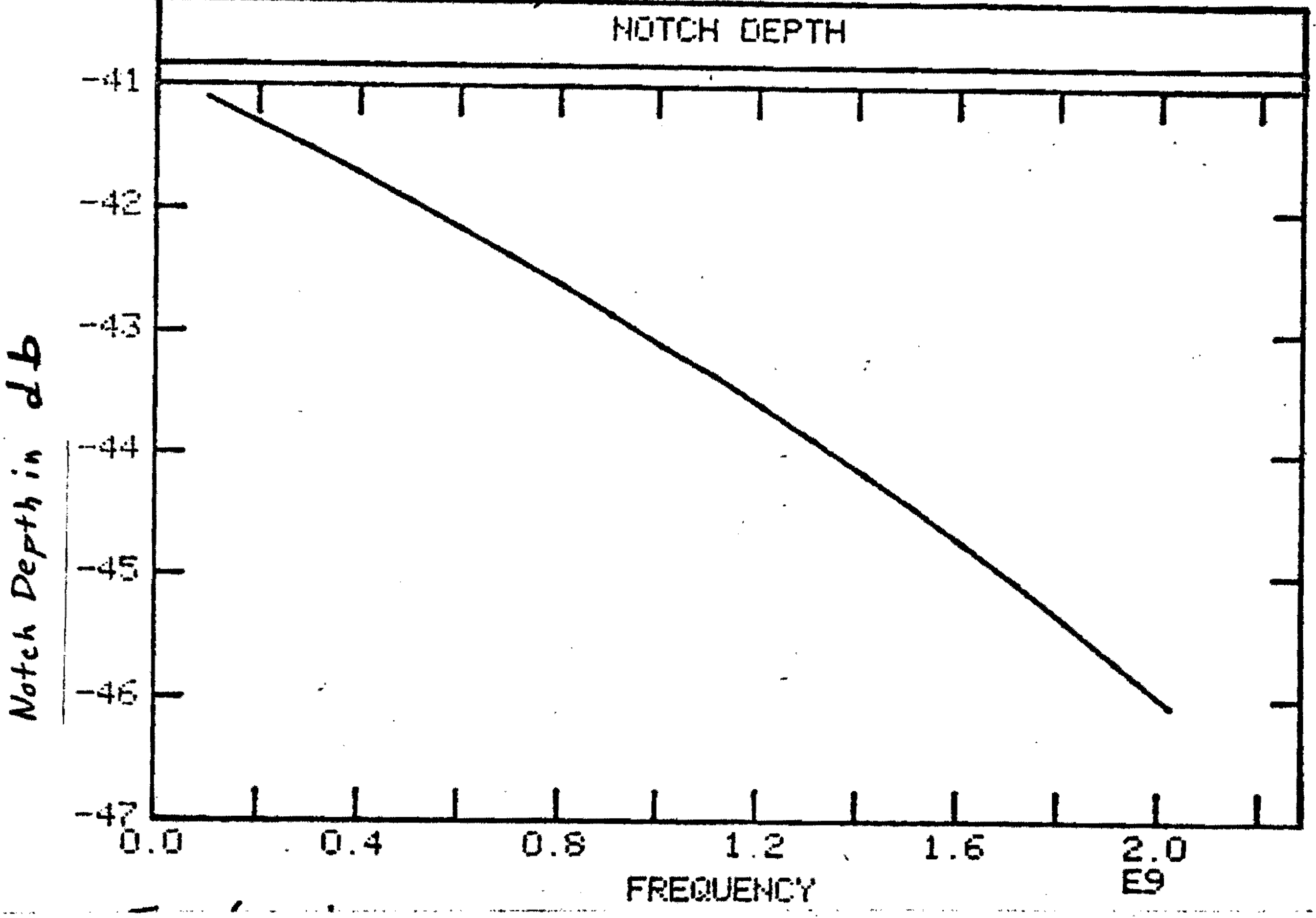


Fig.(10b) Phase Compensated Correlator

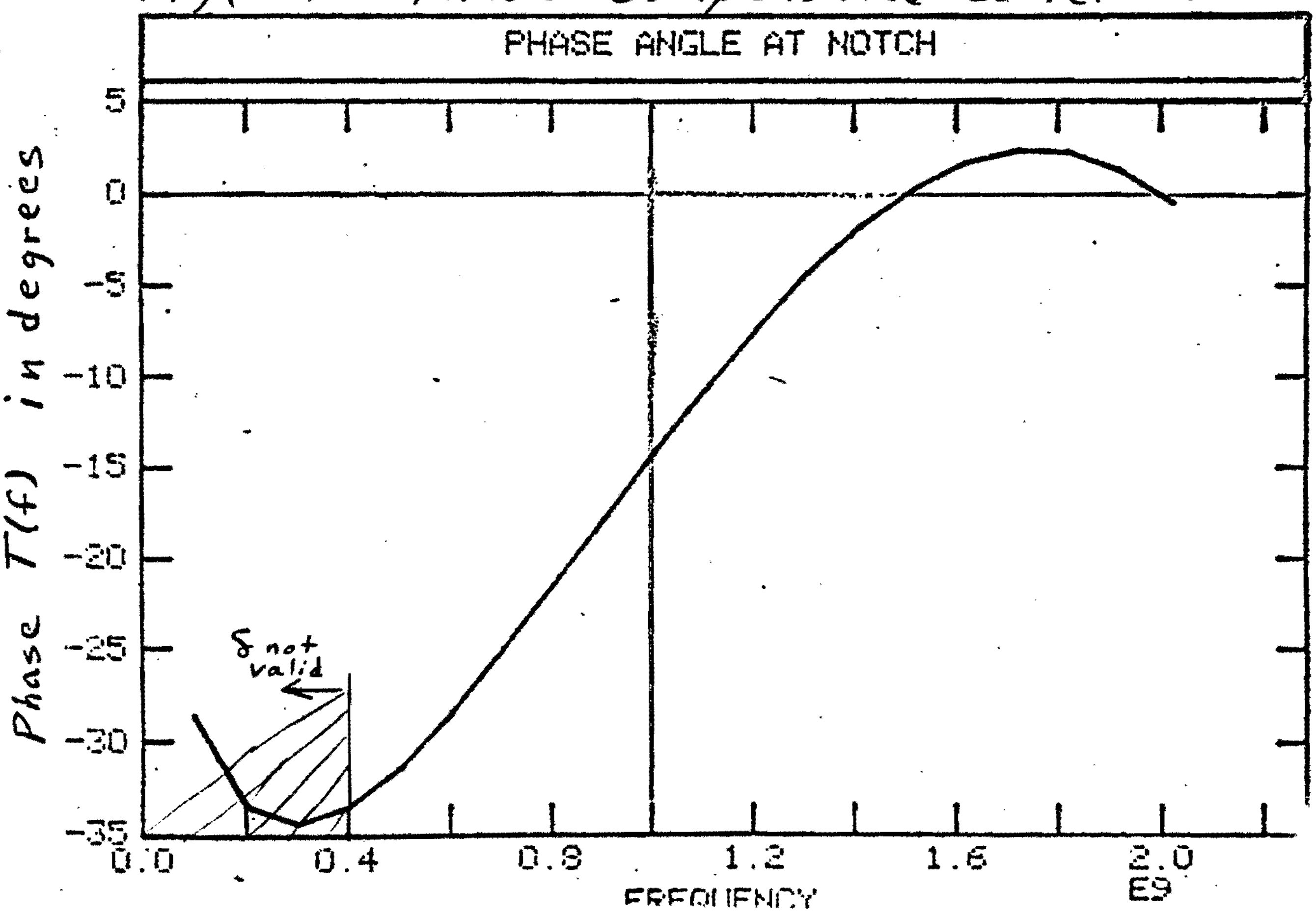


Fig. (11a) Phase Compensated Correlator

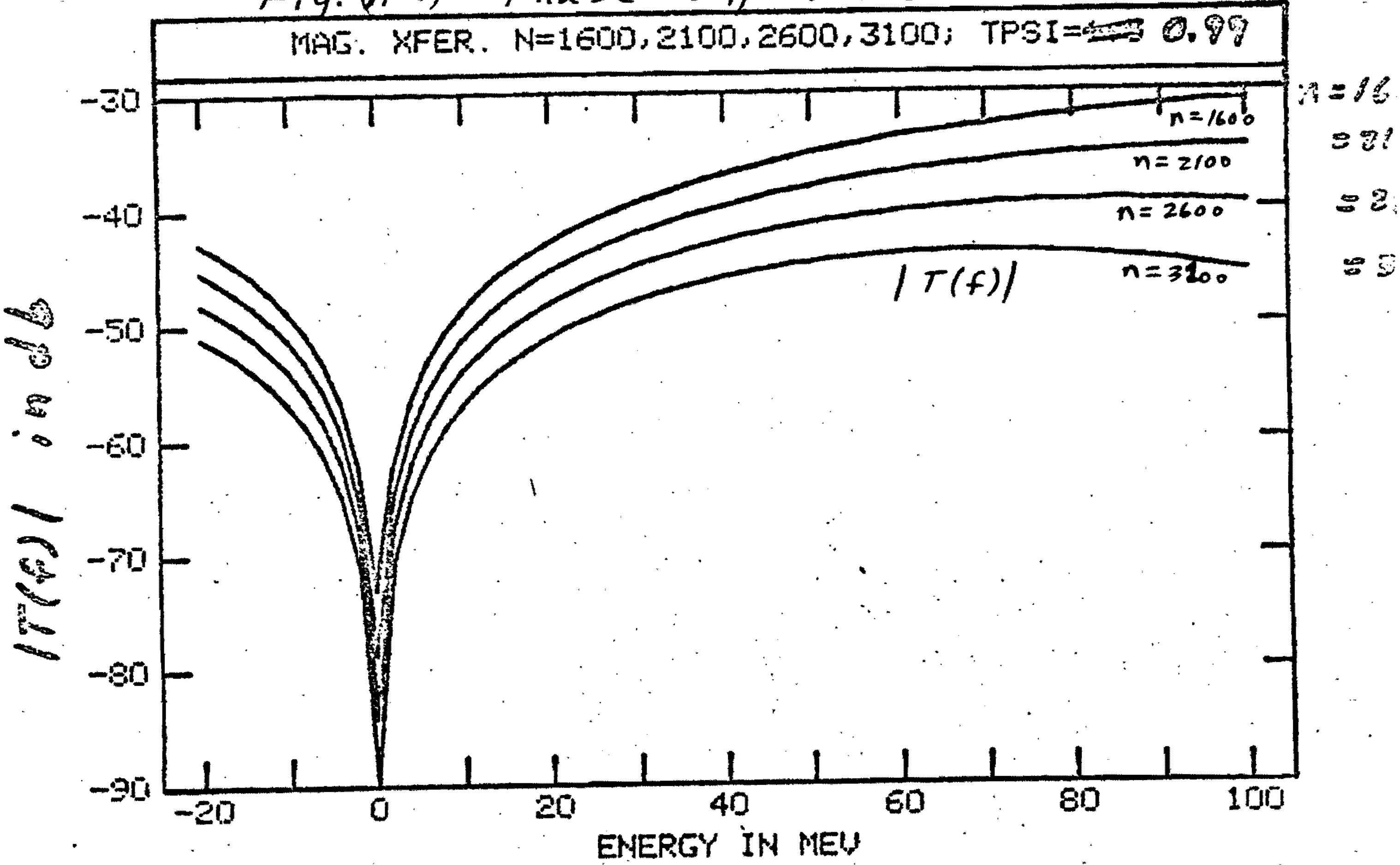


Fig. (11b) Phase Compensated Correlator

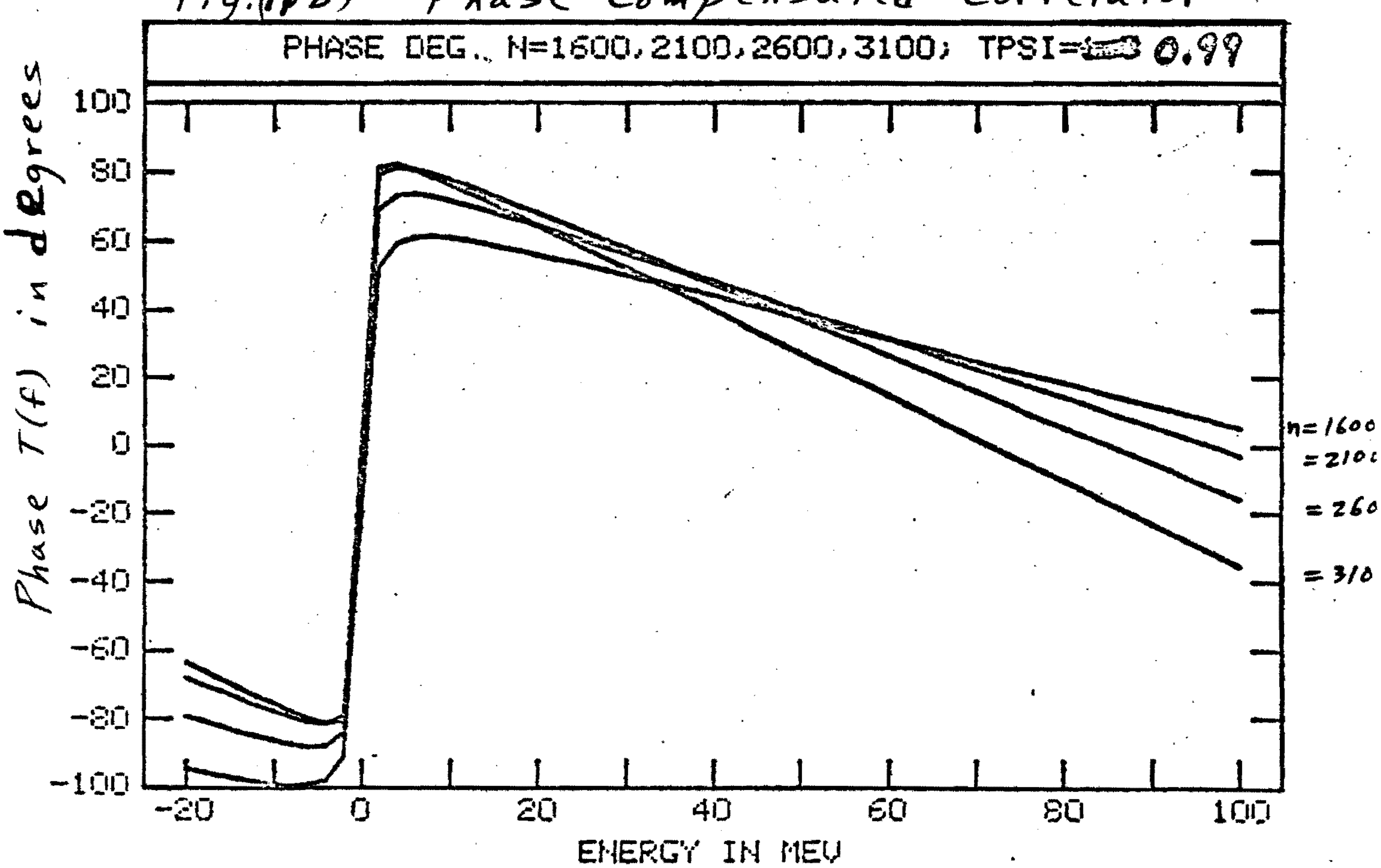


Fig. (12a) Gain + Phase Compensated Correlator

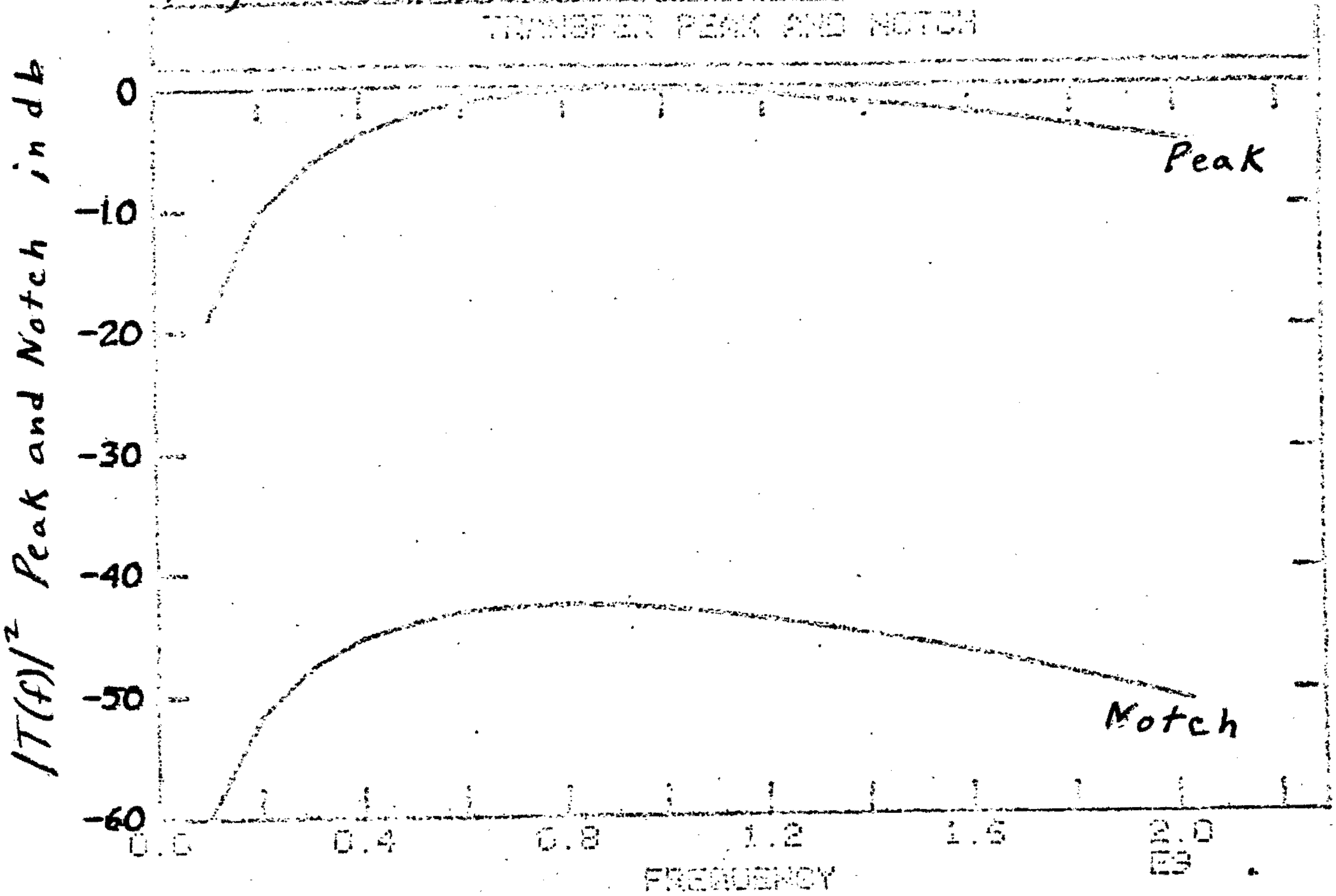


Fig. (12b) Phase Compensated Correlator (Figs. 10+11)