Fermilab

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NOTES ON BEAM PROFILE MONITOR

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Magnesium Jet

The design goals of the magnesium jet were to 1) reach an ion count rate of greater than 10,000 cps, 2) keep the detector system from being contaminated with magnesium, and 3) have a long operating time (> 1000 hrs) without magnesium replacement.

Assumed Beam and Vacuum Characteristics

10^7 protons circulating at 1 Mhz

$\Rightarrow N_i$ (numbers of protons passing given point) $= 10^{13}$ protons/sec

σ_m (ionization cross section for protons on magnesium) $= 2 \times 10^{-18}$ cm^2

background pressure in beam pipe $= 5 \times 10^{-10}$ T

background counts ~ 500 cps

Oven Characteristics

(Oven design based on RADAX II specifications.)

effective crucible capacity = 8 cm^3

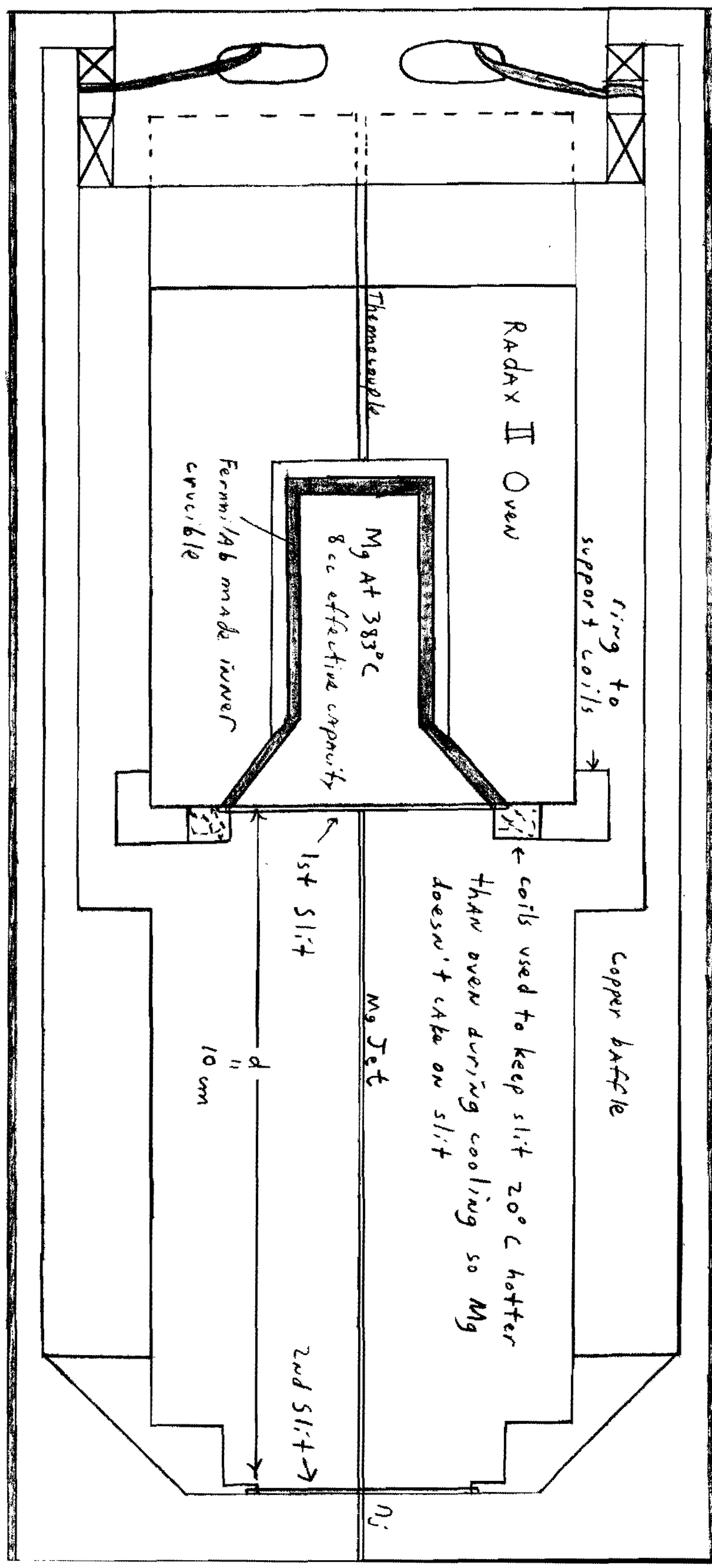
operating temp. = 383° C T = 656°K

n_0 (density of magnesium leaving oven) = 1.47×10^{13} $\frac{\text{part}}{\text{cm}^3}$

Fig 1 : Oven Assembly

Slit 2: 3 cm x 1mm Area $A_2 = 3 \times 10^{-1}\ cm^2$

n_j = Jet density



oven slit dimensions:

slit 1   60 .2 mm diameter holes,
.5 mm center-to-center

$$V = \text{velocity of magnesium} = 14{,}551 \sqrt{\frac{T(K^\circ)}{m\,(g/\text{mole})}}\ \text{cm/s}$$

$$m = 24.3\ g/\text{mole} \qquad \Rightarrow \qquad V = 7.56 \times 10^4\ \frac{\text{cm}}{\text{s}}$$

Mg Depletion

$F_0 \equiv$ oven flow $= n_0 V A_1$ $A_1 \equiv$ area of slit #1
$= 60\pi r^2 = 1.885 \times 10^{-2}\ \text{cm}^2$

$$\therefore\ F_0 = \left(1.47 \times 10^{13}\ \frac{\text{part.}}{\text{cm}^3}\right)\left(7.56 \times 10^4\ \frac{\text{cm}}{\text{s}}\right)\left(1.885 \times 10^{-2}\ \text{cm}^2\right)$$

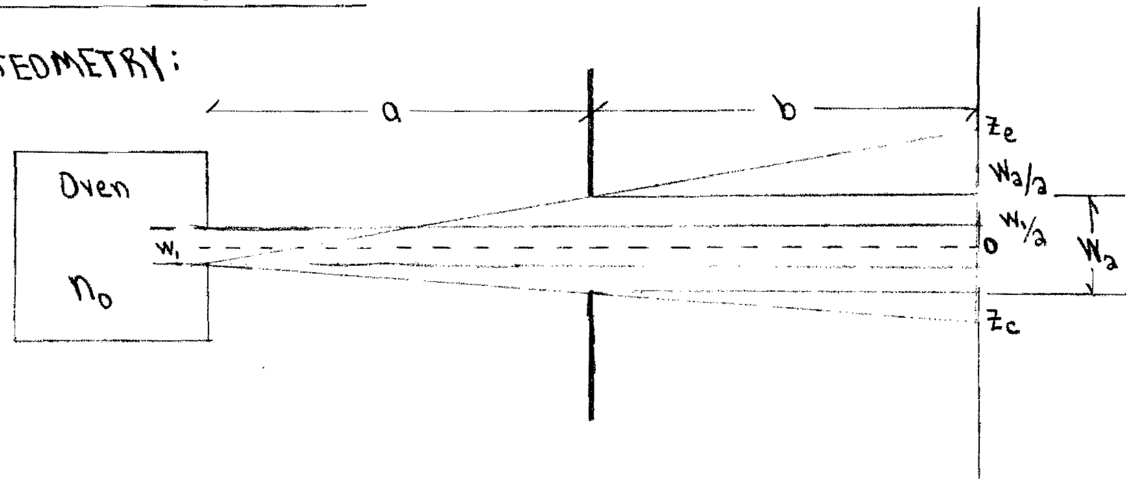
$$= 2.1 \times 10^{16}\ \frac{\text{part.}}{\text{sec}}$$

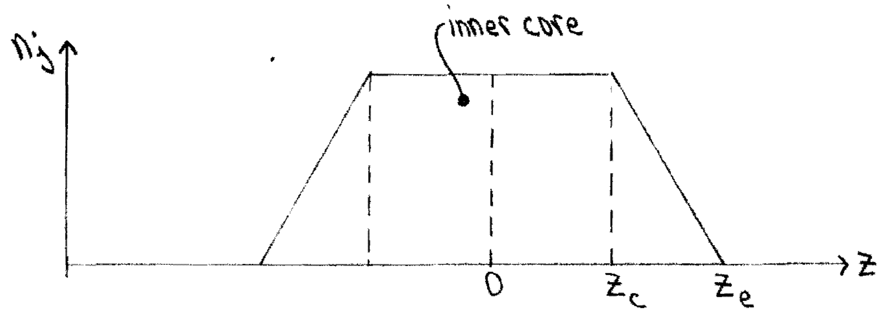
$$\frac{F_0}{N_A} = \frac{2.1 \times 10^{16}\ \frac{\text{part}}{\text{sec}}}{6.02 \times 10^{23}\ \frac{\text{part}}{\text{mole}}} \times 24.3\ \frac{g}{\text{mole}} \times 3600\ \frac{\text{sec}}{\text{hr}} \times \frac{1\ \text{cm}^3\ \text{of Mg}}{1.692\ g}$$

$$= 1.8 \times 10^{-3}\ \frac{\text{cm}^3}{\text{hr}} \qquad \Rightarrow \qquad \frac{8\ \text{cm}^3}{1.8 \times 10^{-3}\ \frac{\text{cm}^3}{\text{hr}}} = 4446\ \text{hrs of operation}$$

JET Characteristics

GEOMETRY:





Flat distribution cutoff edge z_c

$$\frac{z - \frac{W_1}{2}}{a+b} = \frac{z - \frac{W_2}{2}}{b} \Rightarrow z\left(\frac{1}{b} - \frac{1}{a+b}\right) = \frac{W_2}{2b} - \frac{W_1}{2(a+b)}$$

$$z \cdot \frac{a}{(b+a)\,b} = \frac{W_2}{2b} - \frac{W_1}{2(a+b)}$$

or $$z_c = \frac{1}{2a}\left[(b+a)\,w_2 - b\,w_1\right]$$

Edge Z_e

$$\frac{\frac{W_2}{2} + \frac{W_1}{2}}{a} = \frac{Z_e + W_1/2}{a+b}$$

$$Z_e = \frac{a+b}{2a}\left(w_2 + w_1\right) - \frac{w_1}{2}$$

CORE Jet Density

$$n_j = n_0 \Omega = n_0 \frac{A_2}{2\pi d^2} = \frac{\left(1.47\times10^{13}\ \frac{\text{part}}{\text{cm}^3}\right)\left(3\times10^{-1}\ \text{cm}^2\right)}{2\pi\ (10\ \text{cm})^2}$$

$$= 7.02\times10^{9}\ \frac{\text{part.}}{\text{cm}^3}$$

Ion Count (to be detected by microchannel plates)

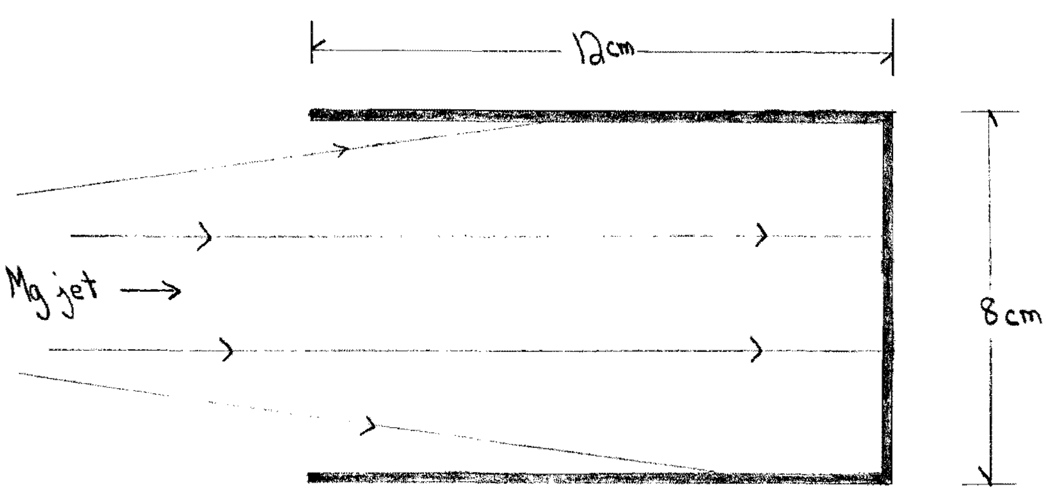
$$N_c\ (\text{counts/sec}) = N_i\ \sigma_m\ t\ (\text{thickness of Mg ribbon})\ n_j$$

$$= \left(10^{13}\ \frac{\text{protons}}{\text{sec}}\right)\left(2\times10^{-18}\ \text{cm}^2\right)\left(.1\ \text{cm}\right)\left(7.02\times10^{9}\ \frac{\text{part.}}{\text{cm}^3}\right)$$

$$= 14{,}040 \text{ counts per second}$$

COLLECTOR Characteristics

(To ensure the MCP's and beam pipe are not contaminated.)



Copper cylinder at 300°K

FOR Mg impacting on Cu at room temperature, the thermal accomodation coefficient $\alpha_T = 1$ (i.e. none of the Mg bounces for a jet of 380-400 C°). This implies that all the Mg that gets away from the collector does so due to desorption.

$1-\alpha_s$ ($\alpha_s \equiv$ sticking coefficient) $= \alpha_d$, the amount of Mg which gets away relative to the amount impinging. Assuming a worst case calculation – magnesium spread evenly over collector, maximum evaporation rate (all Mg gets away) – then $N_e \equiv$ # of particles that get away per second, and is equal to uA_3.

$u \equiv$ evaporation rate $\quad A_3 \equiv$ collector cross section $\simeq 74\ cm^2$

$$u = 3.513 \times 10^{22} \frac{P_t}{(MT)^{1/2}}\ sec^{-1}\ cm^{-2} = 7.127 \times 10^{21} \frac{P_t}{T^{1/2}}$$

$P_t \equiv$ vapor pressure of Mg in Torr at temperature T.

where $\log P_t = -\frac{7620}{T} + 2.5 \log T + 1.73736$ (for Mg)

$\therefore$ at 300 °K $\quad P_t \simeq 3.4 \times 10^{-18}$ Torr

and $u = 1.39 \times 10^3\ part./cm^2 \cdot sec$

therefore, $N_e = \left(1.39 \times 10^3 \frac{part}{cm^2 \cdot sec}\right) \times \left(74\ cm^2\right) = 1.0286 \times 10^5\ part/sec$

$$N_w \equiv \text{weight of Mg escaping} = \left(1.0286 \times 10^5 \frac{part}{sec}\right) \times \left(\frac{1\ mole}{6.02 \times 10^{23}\ part}\right)$$
$$\times \left(24.3 \frac{g}{mole}\right) \times \left(86{,}400 \frac{sec}{day}\right) \times \left(365 \frac{days}{yr}\right) \cong 1.31 \times 10^{-10}\ g/yr$$

Lifetime of MCP

It can be assumed that a MCP is ruined by chemical contamination if a monolayer of Mg forms over it. Then assume that during and after baking any Mg in the chamber deposits evenly over everything in ~ 1 meter radius area. Using a spherical surface as a convenience: $S = 4\pi r^2 \sim 12\ m^2 \sim 1.2\times10^5\ cm^2$, D = average particle spacing ~ 3Å. So, in a monolayer each particle takes up ≈ a surface area $\delta S = 7\times10^{-16}\ cm^2$.

$$N \equiv \#\text{ of atoms needed} = \frac{S}{\delta S} \simeq 1.8\times10^{20}\text{ atoms} \times 4.04\times10^{-23}\ \frac{g}{\text{part. of Mg}}$$

$$= 7.2\times10^{-3}\ g \text{ of Mg needed to ruin the plates.}$$

$$\therefore \text{ detector lifetime} \equiv T = \frac{7.2\times10^{-3}\ g}{1.3\times10^{-10}\ g/yr} = 5.55\times10^{7}\ yrs.$$

Thus, as long as the collector and oven array are separated from the system during baking (see figure 2) there should be no difficulty with plate contamination.

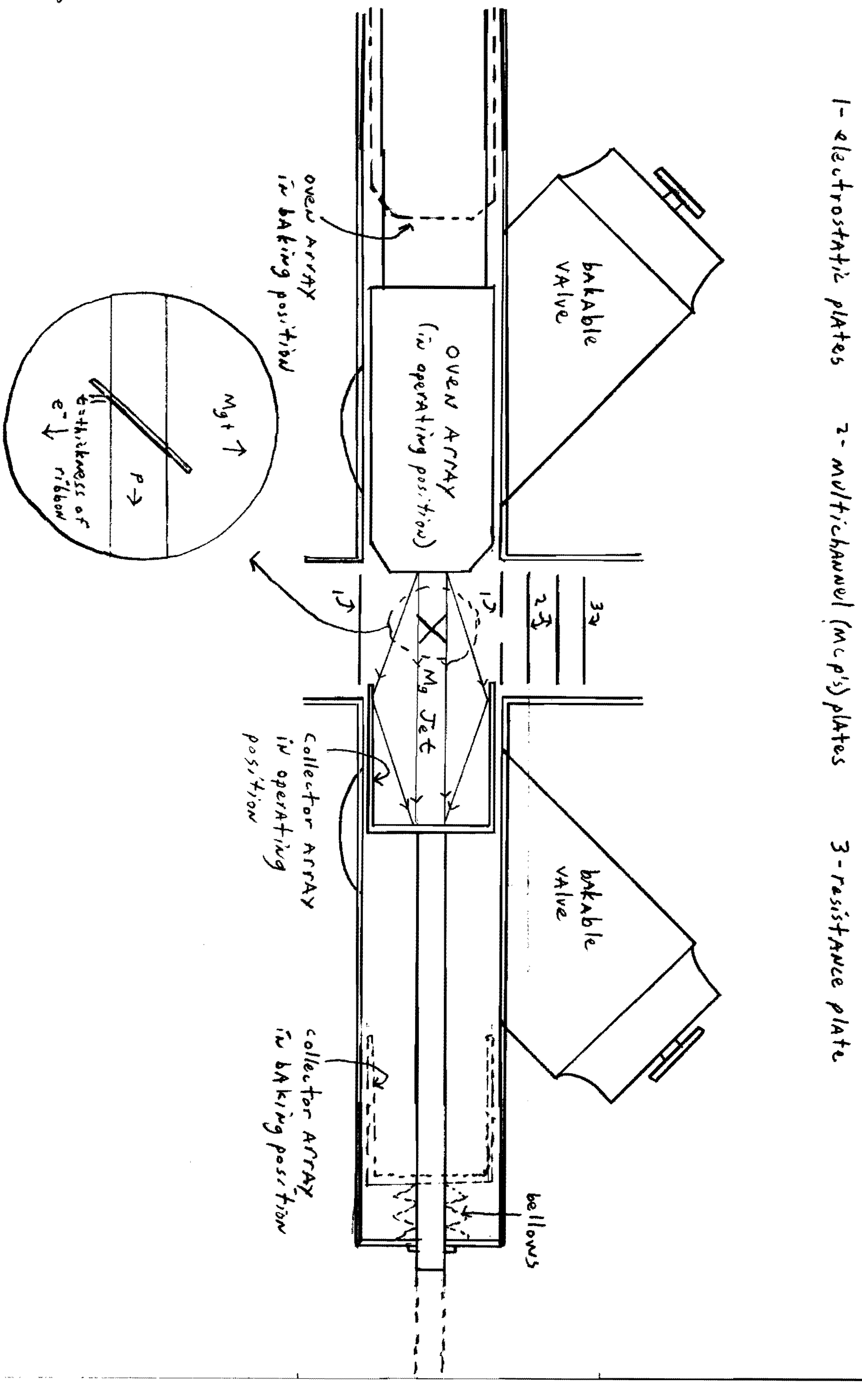
Sticking Coefficient

$$r \equiv \text{rate of Mg deposition on collector} = \left(7.02\times10^{9}\ \frac{part}{cm^3}\right) \times \left(7.56\times10^{4}\ \frac{cm}{s}\right)\times\left(.3\ cm^2\right)_{\text{area of 2nd slit}} = 1.59\times10^{14}\ \frac{part}{sec}$$

$$\left(1.59\times10^{14}\ \frac{part}{sec}\right)\times\left(1.273\times10^{-15}\ \frac{g\cdot sec}{part\cdot yr}\right) = .203\ g/yr$$

and $\alpha_d = \frac{N_w}{r} = \frac{1.31\times10^{-10}\ g/yr}{.203\ g/yr} \cong 6.5\times10^{-10}$ $\therefore\ \alpha_s = 1-\alpha_d \cong 1$

Figure 2: Complete Setup for the Magnesium Jet



Hydrogen Jet

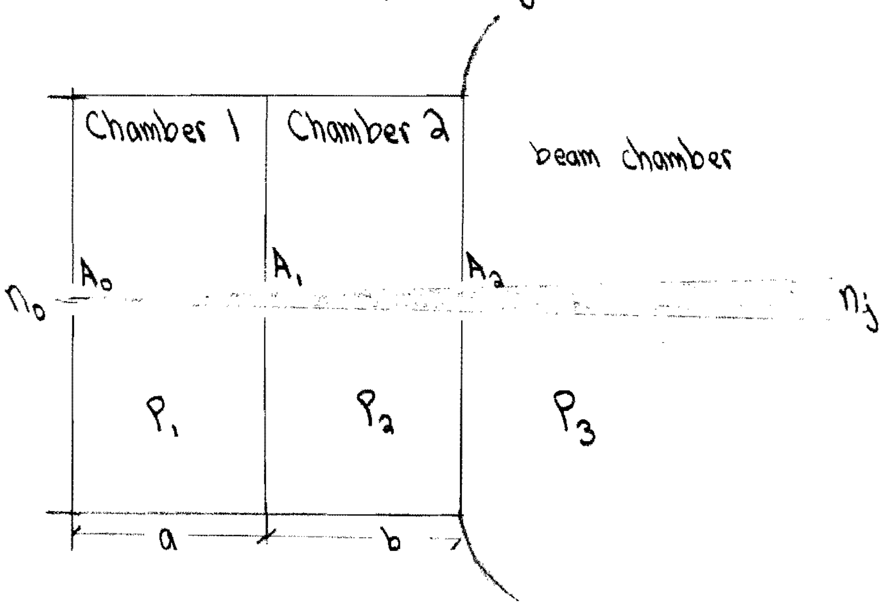
GOALS

* $1 \times 10^4 \frac{\text{ions}}{\text{sec}}$ striking MCP's due to jet

* high signal to noise ratio (low background)

METHOD

Utilize titanium pumping and scrapers to form the jet.



Rough dimensions for calculation purposes:

$a = 5$ cm
$b = 5$ cm

$A_0 = 10^{-2}\ \text{cm}^2$

$A_1 = 5\ \text{cm} \times .1\ \text{cm}$

$A_2 = 5\ \text{cm} \times .1\ \text{cm}$

CALCULATIONS

counts wanted from jet $\sim 1\times10^{4}\ \frac{\text{ions}}{\text{sec}}$

so, $1\times10^{4}\ \frac{\text{ions}}{\text{sec}} = (\text{Hit probability})\left(1\times10^{7}\ \frac{\text{protons}}{\text{bunch}}\right)\left(10^{6}\ \frac{\text{bunch}}{\text{sec}}\right)$

$$= 1\times10^{-9} = n_j\,\sigma\,t$$

$$= n_j\,(10^{-18}\text{cm}^2)(.10\text{ cm})$$

$$\therefore\ n_j = 1\times10^{10}\ \frac{\text{Protons}}{\text{cm}^3}$$

$$n_j = n_0\,\frac{d\Omega}{2\pi}$$

so, $n_0 = n_j\,\frac{2\pi}{d\Omega}$ $\qquad d\Omega = \frac{A}{r^2} = \frac{(5\text{cm})(.1\text{cm})}{100\text{ cm}^2} = 5\times10^{-3}$

$$\therefore\ n_0 = (1\times10^{10})\left(\frac{2\pi}{5\times10^{-3}}\right) = 1.3\times10^{13}\ \frac{p}{\text{cm}^3}$$

since $P = \frac{n}{V}RT$ $\qquad \frac{n}{V} = \left(1.3\times10^{13}\frac{p}{\text{cm}^3}\right)\left(\frac{\text{mole}}{6.02\times10^{23}\text{ part}}\right) = 2.6\times10^{-11}\ \frac{\text{mole}}{\text{cm}^3}$

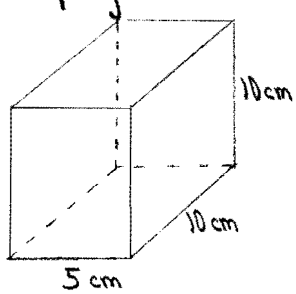
$$\therefore\ P_0 = \left(2.6\times10^{-11}\ \frac{\text{mole}}{\text{cm}^3}\right)\left(6.236\times10^{4}\ \frac{\text{cm}^3\cdot T}{°K\cdot\text{mole}}\right)(300\ °K)$$

$$= 4\times10^{-4}\ \text{Torr} = P_0$$

Titanium Pumping

$10\ \frac{l}{s\cdot\text{cm}^2}$ maximum for fresh titanium

pumping in Chambers 1 and 2:



$= 400\text{ cm}^2$ $\qquad$ more realistic pumping speed

$$(400\text{cm}^2)\left(1\ \frac{l}{s\cdot\text{cm}^2}\right) = 400\ \frac{l}{s} = 4\times10^{5}\ \frac{\text{cm}^3}{s}$$

Ultimate pressure in chamber due to gas load: $P = \frac{Q}{S}$

$Q \equiv$ throughput

$S \equiv$ pumping speed

$$Q_0 = P_0 \overset{\text{conductance}}{C_0} = P_0 \frac{\bar{v} A_0}{4} \quad 300°K$$

$$= (4 \times 10^{-4}\ T)(1.8 \times 10^5 \tfrac{cm}{s})(1 \times 10^{-2}\ cm^2)\tfrac{1}{4}$$

$$Q_0 = 1.8 \times 10^{-1}\ \frac{T\ cm^3}{s} = 1.8 \times 10^{-4}\ \frac{T \cdot l}{s}$$

$$P_1 = \frac{Q_0}{S} = \frac{1.8 \times 10^{-1}\ \frac{T\ cm^3}{s}}{4 \times 10^5\ \frac{cm^3}{s}} = .45 \times 10^{-6}\ T = 4.5 \times 10^{-7}\ T$$

$$Q_1 = P_1 \frac{\bar{v} A_1}{4} = (4.5 \times 10^{-7}\ T)(1.8 \times 10^5 \tfrac{cm}{s})(.5\ cm^2)\tfrac{1}{4}$$

$$= .010\ \frac{cm^3 \cdot T}{s}$$

$$P_2 = \frac{Q_1}{S} = \frac{.010\ \frac{cm^3 \cdot T}{s}}{4 \times 10^5\ \frac{cm^3}{s}} = 2.5 \times 10^{-8}\ T$$

$$Q_2 = P_2 \frac{\bar{v}}{4} A_2 = (2.5 \times 10^{-8}\ T)\left(\frac{1.8 \times 10^5\ \frac{cm}{s}}{4}\right)(.5\ cm^2)$$

$$= 5.6 \times 10^{-4}\ \frac{cm^3 \cdot T}{s}$$

$$P_3 = \frac{Q_2}{S} = \frac{5.6 \times 10^{-4}\ \frac{cm^3 \cdot T}{s}}{4 \times 10^5\ \frac{cm^3}{s}} = 1.4 \times 10^{-9}\ T$$

jet counts: 10,000 cps

background:

$$n_b \sigma t \left(1\times10^{13} \frac{\text{protons}}{\text{sec}}\right) \qquad t = 3.5\ \text{cm} \ \left(\text{region in beam pipe}\right)$$

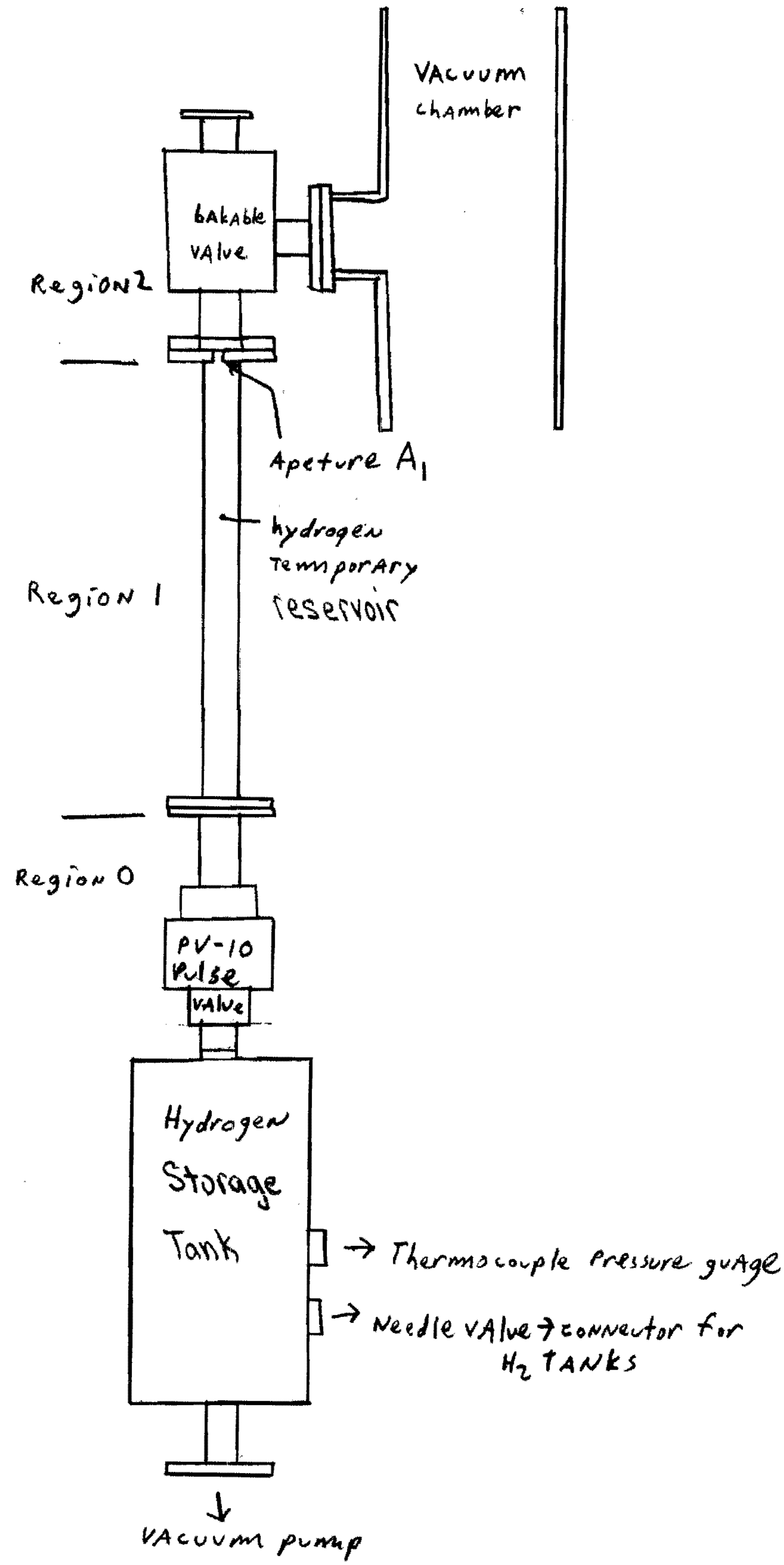
$$n_b = \frac{P(T)}{\left(3.12\times10^{-17}\ \frac{T\,cm^3}{Part}\right)} \quad \text{at } 300\ ^\circ K$$

$$n_b = \frac{1.4\times10^{-9}\ T}{3.12\times10^{-17}} = 4.49\times10^{7}\ P/cm^3$$

$$\left(4.49\times10^{7} \frac{P}{cm^3}\right)\left(1\times10^{-18}\ cm^2\right)\left(3.5\ cm\right)\left(1\times10^{13}\ P/s\right) = 1,571\ \text{cps}$$

$$S/N \sim 6$$

Figure III: The Hydrogen bump



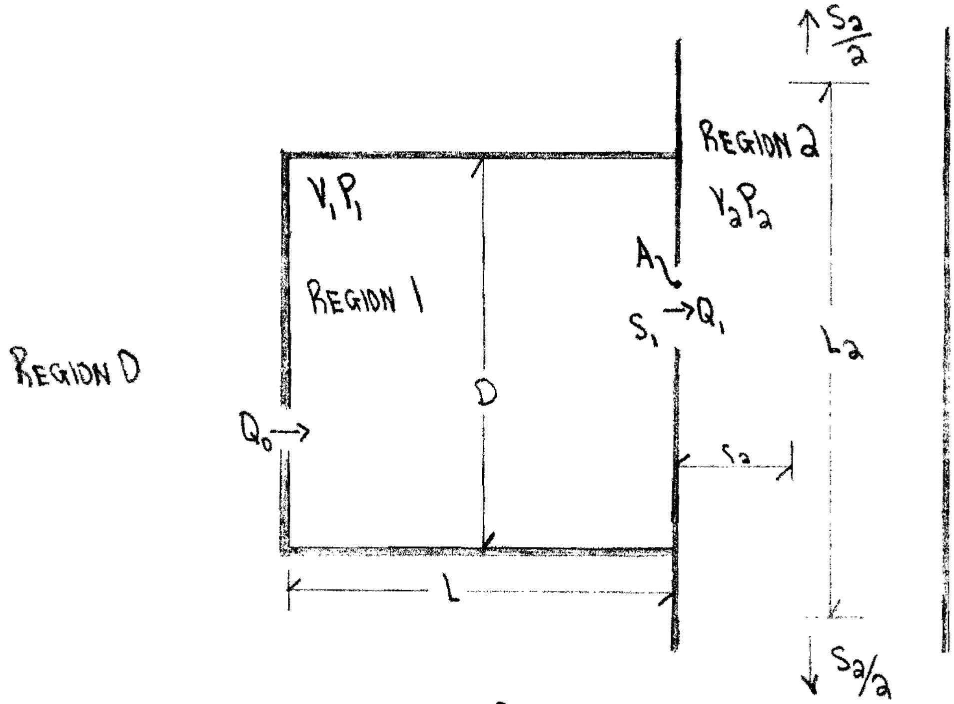
Hydrogen Bump

Beam Characteristics

10^7 protons at 1 MHz $\Rightarrow N_p = 10^{13}$ P/sec

$\sigma_i \equiv$ cross section for hydrogen ionization $\sim 10^{-18}$ cm^2

Bump Geometry and Characteristics



$A \equiv$ aperture area $= \pi \frac{d_A^2}{4}$

$V_2 \equiv \pi r_2^2 L_2 = \pi (5\text{ cm})^2 (100\text{ cm}) = 7.854\ \ell$

$V_1 = \pi \frac{D^2}{4} L$

$P_i \equiv$ Pressure in region i; in TORR

$Q_i \equiv$ throughput from region i to region $i+1$, in $\frac{T\cdot\ell}{sec}$

$t \equiv$ time in seconds

$S_i \equiv$ pumping speed; in ℓ/s $\qquad S_2 = 600\ \ell/s$

$T \equiv 300\,°K$

$m \equiv 2.016$ g/mole (H_2)

$c_i \equiv$ conductance of aperture i

Derivations

$t < 5$ ms (time region 1)

assumptions:
$$Q_0 > 0$$
$$P_1 \gg P_2$$
$$Q_0 \gg P_1 S_1$$
$$d_A \ll D$$

$$S_1 = C_A\left[1 - \frac{P_2}{P_1}\right] \cong C_A \qquad Q_0 - P_1 S_1 = V_1 \frac{\partial P_1}{\partial t} \approx Q_0$$

$$\Rightarrow P_1 = \frac{Q_0 t}{V_1}$$

$$Q_1 = P_1 S_1 \sim P_1 C_A$$

$$Q_1 - P_2 S_2 = V_2 \frac{\partial P_2}{\partial t} \Rightarrow \frac{Q_0 C_A}{V_1 V_2} t - \frac{P_2 S_2}{V_2} = \frac{\partial P_2}{\partial t}$$

Let $P_2 = a + bt + ce^{dt}$

then,

$$\frac{Q_0 C_A}{V_1 V_2} t - \frac{(a + bt + ce^{dt}) S_2}{V_2} = b + cd\, e^{dt}$$

implying that: $d = \frac{-S_2}{V_2}$ $\quad a = \frac{-V_2}{S_2} b$ $\quad b = \frac{Q_0 C_A}{V_1 S_2}$

$$a = \frac{-Q_0 C_A V_2}{V_1 S_2^2}$$

so, $$P_2 = \frac{-Q_0 C_A V_2}{V_1 S_2^2} + \frac{Q_0 C_A}{V_1 S_2} t + C e^{-S_2 t/V_2}$$

let $t=0$, then $$P_2 = P_{2i} = \frac{-Q_0 C_A V_2}{V_1 S_2^2} + C$$

or $$C = P_{2i} + \frac{Q_0 C_A V_2}{V_1 S_2^2}$$

finally:
$$P_2 = \frac{-Q_0 C_A V_2}{V_1 S_2^2} + \frac{Q_0 C_A}{V_1 S_2} t + \left(P_{2i} + \frac{Q_0 C_A V_2}{V_1 S_2^2}\right) e^{-S_2 t/V_2}$$

at Δt (5 msec):

$$P_1 \equiv P_{11} = \frac{Q_0}{V_1} \Delta t$$

$$P_2 \equiv P_{22} = \frac{-Q_0 C_A V_2}{V_1 S_2^2} + \frac{Q_0 C_A}{V_1 S_2} \Delta t + \left(P_{2i} + \frac{Q_0 C_A V_2}{V_1 S_2^2}\right) e^{\frac{-S_2 \Delta t}{V_2}}$$

Time Region 2

$t > 5$ msec

assumptions:

$$Q_0 = 0$$
$$P_1 \gg P_2$$
$$S_2 V_1 \neq C_A V_2$$

$S_1 \approx C_A$

$$-P_1 S_1 = V_1 \frac{\partial P_1}{\partial t} \qquad P_1 = P_{11} e^{\left(\frac{-S_1 t}{V_1}\right)} = P_{11} e^{-\frac{C_A t}{V_1}}$$

$$Q_1 = P_1 S_1 \approx P_1 C_A \approx P_{11} C_A \exp\left[\frac{-C_A t}{V_1}\right]$$

$$Q_1 - P_2 S_2 = V_2 \frac{\partial P_2}{\partial t} \quad \Rightarrow \quad \frac{P_{11} C_A}{V_2} \exp\left[\frac{-C_A t}{V_1}\right] - \frac{P_2 S_2}{V_2} = \frac{\partial P_2}{\partial t}$$

H_2 Depletion in Storage Tank

Assuming that the tank is refilled when its pressure has dropped to $.90\ P_0$ (i.e. $.1\ P_0 V_T$ units of gas utilized), and the amount of H_2 used per shot is $Q_0 \Delta t$, the number of shots obtained before refilling is:

$$N = \frac{P_0 V_T}{10\ Q_0 \Delta t} .$$

Pressure in Beam Pipe if Valve Fails Open

Here the pressure in P_1 will increase until

$$Q_1 = Q_0 \quad \Rightarrow \quad P_2 = \frac{Q_0}{S_2}$$

Number of Counts Obtained

H_2 pressure: $$P(\text{Torr}) = \frac{n_{H_2}\left(\frac{\text{part}}{\text{cm}^3}\right) \times R_0 \times T\,(K^\circ)}{N_A} = \frac{n R_0 T}{N_A}$$

using $R_0 = 6.263 \times 10^4 \ \frac{\text{Torr}\cdot\text{cm}^3}{\text{mole}\cdot K^\circ}$

$T = 300^\circ K$

$$\Rightarrow \quad n_{H_2} = (3.2 \times 10^{16})\ P(\text{Torr})$$

$$N_i \equiv \#\text{ of counts/sec} = n_{H_2}\, \sigma_i\, L_1\, N_p$$

$L_1 \equiv$ effective MCP length $= 10$ cm

and $$N_i = (3.2 \times 10^{16})(10^{-18}\,\text{cm}^2)(10\,\text{cm})\left(10^{13}\,\frac{\text{part}}{\text{sec}}\right) P(\text{Torr})$$

$$= (3.2 \times 10^{12})\ P(\text{Torr}) \ \frac{\text{counts}}{\text{sec}}$$

implying that 3.125×10^{-13} T·s of gas yields 1 count

$$1 \text{ count} \equiv 3.125 \times 10^{-13} \text{ T·s}$$

Applied Design

$L = 30$ cm

$D = 1.27$ cm

$D_A = .15$ cm

$C_A = 2.86 \left(\frac{T}{M}\right)^{1/2} D_A^2 \ \ell/\text{sec} = .785 \frac{\ell}{\text{sec}}$

$T = 300\,°K$

$V_2 = 7.84\ \ell$ $\qquad S_2 = 600\ \ell/\text{sec}$

$V_T \sim 1\ \ell$

$\Delta t = 5 \times 10^{-3}$ sec

$V_1 = .038\ \ell$

$\frac{C_A}{V_1} = 20.66 \qquad \frac{S_2}{V_2} = 76.53 \qquad P_{2i} = 5 \times 10^{-10}$ Torr

$S_2 V_1 - C_A V_2 = 16.646\ \ell^2/\text{sec} \qquad C_A V_1 = 2.98 \times 10^{-2}\ \ell^2/\text{sec}$

$\frac{C_A V_1}{S_2 V_1 - C_A V_2} = 1.79 \times 10^{-3} \qquad \frac{C_A V_2}{V_1 S_2^2} = 4.5 \times 10^{-4}$

$\frac{C_A}{V_1 S_2} = 3.443 \times 10^{-2}$

For the PV-10 pulse valve, $Q_0 = C_1 P_0$, where C_1 varies depending on the applied signal. We use an average C_1 value of $2.6 \times 10^{-3} \frac{\ell}{\text{sec}}$.

Let $P_2 = a e^{\frac{-S_2 t}{V_2}} + b e^{\frac{-C_A t}{V_1}}$

then,

$$\frac{P_{11} C_A}{V_2} e^{-\frac{C_A t}{V_1}} - \cancel{\frac{S_2}{V_2} a e^{\frac{-S_2 t}{V_2}}} - \frac{S_2}{V_2} b e^{\frac{-C_A t}{V_1}}$$

$$= \cancel{\frac{-a S_2}{V_2} e^{\frac{-S_2 t}{V_2}}} - \frac{b C_A}{V_1} e^{\frac{-C_A t}{V_1}}$$

so, $\frac{P_{11} C_A}{V_2} - \frac{S_2}{V_2} b + \frac{C_A}{V_1} b = 0$

$$\Rightarrow b\left(\frac{C_A}{V_1} - \frac{S_2}{V_2}\right) = -\frac{P_{11} C_A}{V_2}$$

$$\Rightarrow b = \frac{-P_{11} C_A V_1}{C_A V_2 - S_2 V_1} = \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2}$$

and $P_2 = a e^{\left(\frac{-S_2 t}{V_2}\right)} + \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2} e^{-\frac{C_A t}{V_1}}$

at $t = 0$ $\quad P_2 = P_{22} = a + \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2}$

or, $a = P_{22} - \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2}$

finally,

$$P_2 = \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2} e^{-\frac{C_A t}{V_1}} + \left(P_{22} - \frac{P_{11} C_A V_1}{S_2 V_1 - C_A V_2}\right) e^{-\frac{S_2}{V_2} t}$$

time region 1

$$P_1 = (6.92 \times 10^{-2}) P_0 t$$

$$P_2 = (-1.183 \times 10^{-6}) P_0 + (9.06 \times 10^{-5}) P_0 t + (5 \times 10^{-10} + 1.183 \times 10^{-6} P_0) e^{-76.5t}$$

$$P_{11} = 3.46 \times 10^{-4} P_0$$

$$P_{22} = (-7.3 \times 10^{-7}) P_0 + (1.682)(5 \times 10^{-10} + 1.183 \times 10^{-6} P_0)$$

time region 2

$$P_1 = P_{11} e^{-20.66t}$$

$$P_2 = P_{11} (1.792 \times 10^{-3}) e^{-20.66t} + (P_{22} - P_{11}(1.792 \times 10^{-3})) e^{-76.53t}$$

depletion in storage tank

$$N = \frac{\cancel{P_0'} (1 \ell)}{10 \, (2.63 \times 10^{-3} \, \frac{\ell}{sec}) \, \cancel{P_0'} (5 \times 10^{-3} \, sec)} \cong 7605 \text{ shots}$$

valve failure pressure

$$P = \frac{2.63 \times 10^{-3} P_0}{600 \, \ell/sec} = 4.38 \times 10^{-6} P_0$$

Results

Chosing $P_0 = 2 \times 10^{-2}$ Torr

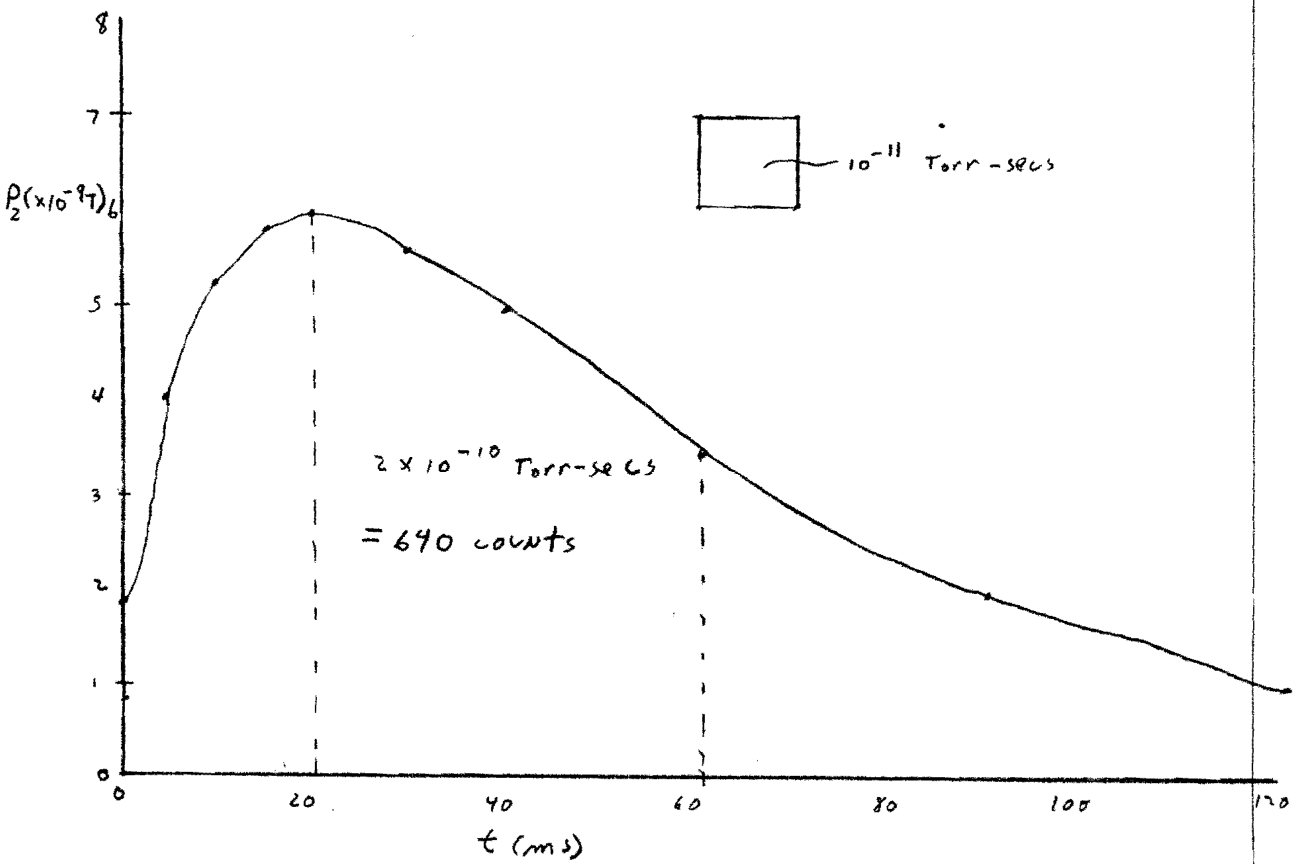
then, $P_{11} = 6.92 \times 10^{-6}$ T

$P_{22} = 1.88 \times 10^{-9}$ T

$P_2 = (1.24 \times 10^{-8}) e^{-20.66t} - (1.052 \times 10^{-8}) e^{-76.53t}$

$P_{fail} = 8.76 \times 10^{-8}$ T

t (beyond initial 5 ms)	real time (ms)	P_2 ($\times 10^{-9}$ T)
0 ms	5	1.88
5	10	4.01
10	15	5.19
15	20	5.76
20	25	5.93
30	35	5.61
40	45	4.93
60	65	3.48
90	95	1.92
120	125	1.04



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