TOLERANCE OF THE ALIGNMENT AND CONSTRUCTION OF THE PRE-COOLER MAGNETS

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We estimate the allowable error of the alignment and construction of the bending and quadrupole magnets for the Pre-Cooler. The results are tabulated as follows,

The most probable closed orbit as for the bending magnets,

1. gap or field error
   \[ \langle \Delta x \rangle_{\text{rms}} \approx 3.9 \frac{\sigma_B}{B} \langle \Delta \rangle_{\text{rms}} \]
2. length error
   \[ \langle \Delta x \rangle_{\text{rms}} \approx 2.5 \langle \Delta l \rangle_{\text{rms}} \]
3. longitudinal shift
   \[ \langle \Delta \rangle_{\text{rms}} \approx 1.3 \langle \Delta l \rangle_{\text{rms}} \]
4. transverse shift
   none
5. tilt
   \[ \langle \Delta y \rangle_{\text{rms}} \approx 3.6 \theta \langle \Delta \theta \rangle_{\text{rms}} \]

The most probable closed orbit and stop band width as for the quadrupole magnets,

1. transverse shift
   \[ \langle \Delta z \rangle_{\text{rms}} \approx 24 \langle \Delta z \rangle_{\text{rms}} \]
   \( z = x, y \)
2. longitudinal shift
   \[ \langle \delta V_z \rangle_{\text{rms}} \approx 0.2 \langle \Delta \delta \rangle_{\text{rms}} \]
3. length error
   \[ \langle \delta V_z \rangle_{\text{rms}} \approx 8.6 \langle \Delta l \rangle_{\text{rms}} \]
4. gradient error
   \[ \langle \delta V_z \rangle_{\text{rms}} \approx 5.2 \langle \Delta V/V \rangle_{\text{rms}} \]
5. tilt
   \[ \langle \delta V_z \rangle_{\text{rms}} \approx 2.7 \theta \langle \Delta \theta \rangle_{\text{rms}} \]
The items 2, 3 and 4 are concerned with half integer resonance and item 5 with sum resonance. In the following the accuracy requirements are made for the closed orbit \( \leq 1 \) mm, and the stop band width \( \leq 0.005 \).

### $1$ BENDING MAGNETS

The closed orbit disturbed by the bending magnet imperfections is given by the following formula, \( (\mathbf{z}=x, y) \)

\[
\begin{align*}
\frac{\dot{z}^2}{\dot{y}^2} &= \beta \frac{\dot{y}^2}{\dot{x}^2} \\
\frac{\dot{y}^2}{\dot{x}^2} &= \frac{V^2}{4 \sin^2 \pi v} \int_{\phi}^{\phi+2\pi} \int_{\phi}^{\phi+2\pi} f(z) f(x) \cos \pi (z - x) d\xi d\eta
\end{align*}
\]

The rms of the closed orbit for the random distribution is

\[
<z>_{\text{rms}} \approx \frac{V \sqrt{\beta}}{2 \sin \pi v} \sqrt{\sum (f_i \Delta \phi_i)^2}
\]

Since \( f = \beta \frac{\Delta B}{B} \), \( \Delta \phi = \frac{\Delta B}{B} \) and the kick by the error field is \( \frac{dz}{dt} = k = \frac{\Delta (B)}{B} \), we obtain the relation

\[
<z>_{\text{rms}} \approx \sqrt{\beta} \sqrt{\sum \beta_i k_i^2}
\]

### 1-1 FIELD ERROR; \( \Delta B/B \)

The kick is \( k = \frac{B \Delta B}{B} \), then the c.c. is

\[
<z>_{\text{rms}} \approx \sqrt{NB} \sqrt{\frac{\beta x}{B}} <\frac{\Delta B}{B}>_{\text{rms}}
\]

\[
\approx 3.7 <\frac{\Delta B}{B}>_{\text{rms}}
\]

\( (N_B = 128, \frac{\beta x}{\nu x} \approx 7.06, \frac{R}{\nu} = 1.56, \delta = 31.8 ) \)

To get: \( <z>_{\text{rms}} \leq 1 \) mm for this error we need to have

\[
<\frac{\Delta B}{B}>_{\text{rms}} \leq 2.6 \times 10^{-4}
\]
1.2 LENGTH ERROR; 

The kick is \( k = \frac{BA}{BG} \), then

\[
<x>_{rms} \approx \sqrt{N_B} \frac{\beta x}{a} \frac{1}{s} <\Delta l>_{rms}
\]

\[\approx 2.5 <\Delta l>_{rms}\]

For \( <\Delta l>_{rms} \leq 1 \) mm we need

\[<\Delta l>_{rms} \leq 0.4 \) mm \] \( (3) \)

1.3 TRANSVERSE SHIFT

No effect.

1.4 LONGITUDINAL SHIFT; \( \Delta s \)

Since the kick due to the shift are inverse at the entrance and exit of the bending magnet, we get

\[
<x>_{rms} \approx \sqrt{\beta x} \sqrt{\sum_i \left( \sqrt{\beta_{EN}^i} - \sqrt{\beta_{EX}^i} \right)^2 p_i^2}
\]

\[\approx \sqrt{N_B} \frac{\sqrt{\beta x}}{\sqrt{\beta_{EN}^i} - \sqrt{\beta_{EX}^i}} \frac{\epsilon_B}{s} <\Delta s>_{rms}
\]

\[\approx 1.3 <\Delta s>_{rms}\]

where \( k = \frac{BA}{BG} \) and \( |\sqrt{\beta_{EN}^i} - \sqrt{\beta_{EX}^i}| \approx 0.85 \).

Then we need for \( <\Delta s>_{rms} \leq 1 \) mm

\[<\Delta s>_{rms} \leq 0.8 \) mm \] \( (4) \)

1.5 TILT; \( \Theta \)

The tilt around the longitudinal axis gives the field

\[\Delta B_y = \frac{1}{2} B \Theta^2 \) and \( \Delta B_x = B \Theta \]

Then with \( \beta_y = 6.5 \) we have

\[
<x>_{rms} \approx \sqrt{N_B} \frac{\beta x}{2s} \frac{\epsilon_B}{s} <\Theta^2>_{rms}
\]

\[\approx 2.0 <\Theta^2>_{rms}\]
\[
\langle y \rangle_{\text{rms}} \approx \sqrt{\frac{N_F \beta_y}{N_D \beta_{yF}}} \frac{\beta_D}{S} \langle \theta \rangle_{\text{rms}} \\
\approx 3.6 \langle \theta \rangle_{\text{rms}}
\]

Therefore we need for \( \langle y \rangle_{\text{rms}} \leq 1 \text{ mm} \)

\( \langle \theta \rangle_{\text{rms}} \leq 0.3 \text{ mrad} \) \( \quad (5) \)

### 2 Qualupole Magnets

#### 2-1 Transverse Shift; \( \delta x, \delta y \)

With the shift of the magnet \( \delta x \) and \( \delta y \) the potential is

\[
\phi(x, y) = B(xy - \delta y - y\delta x)
\]

Hence the field error is

\[
\Delta B_y = -B \delta x \quad \text{and} \quad \Delta B_x = -B \delta y
\]

The kick is

\[
\begin{align*}
\delta x &= -\frac{B' \rho_0}{E_S} \delta x \\
\delta y &= -\frac{B' \rho_0}{E_S} \delta y
\end{align*}
\]

Therefore

\[
\langle x \rangle_{\text{rms}} \approx \sqrt{\frac{\beta_x}{\sqrt{N_F \beta_{xF} + N_D \beta_{xD}}} \frac{\beta_0 \rho_0}{E_S} \langle \delta x \rangle_{\text{rms}}} \\
\approx 24 \langle \delta x \rangle_{\text{rms}}
\]

\[
\langle y \rangle_{\text{rms}} \approx 24 \langle \delta y \rangle_{\text{rms}}
\]

( \( N_F = 48, N_D = 52, \beta_{xF} = 15, \beta_{xD} = 2.7, \beta' = 15, l_Q = 0.61, B_S = 29.6, \beta_y = 16, \beta_{yF} = 2.7 \). For \( \langle z \rangle_{\text{rms}} \leq 1 \text{ mm} \) we need

\( \langle \delta z \rangle_{\text{rms}} \leq 0.04 \text{ mm} \) \( \quad (6) \)

This is the most severe requirement.

#### 2-2 Gradient Error; \( \Delta \beta / \beta' \)

The gradient error introduces the tune shift
\[ \Delta \nu = -\frac{1}{4\pi} \oint \beta \Delta k \, ds \]

For the random distribution of the gradient error the most probable tune shift is

\[ \langle \Delta \nu \rangle_{\text{rms}} \approx \frac{1}{4\pi} \sqrt{\sum \beta_i^2 \Delta k_i^2 \Delta \nu_i^2} \]  

Then for \( \Delta k = \frac{\Delta B}{B} \) we have

\[ \langle \Delta \nu_x \rangle_{\text{rms}} \approx \frac{k_{l,a}}{4\pi} \sqrt{N_F \beta_x^2 + N_D \beta_x^2} < \frac{\Delta B}{B} >_{\text{rms}} \]
\[ \approx 2.6 < \frac{\Delta B}{B} >_{\text{rms}} \]

\[ \langle \Delta \nu_y \rangle_{\text{rms}} \approx \frac{k_{l,a}}{4\pi} \sqrt{N_F \beta_y^2 + N_D \beta_y^2} < \frac{\Delta B}{B} >_{\text{rms}} \]
\[ \approx 2.6 < \frac{\Delta B}{B} >_{\text{rms}} \]

The tune shift is very small and not harmful, but half integer resonance might be harmful. The stop band width of the resonance is twice the tune shift. For the width limit \( \langle \Delta \nu \rangle_{\text{rms}} \leq 0.005 \) we need

\[ \langle \frac{\Delta B}{B} \rangle_{\text{rms}} \leq 10^{-3} \]  

2-3 LENGTH ERROR: \( \Delta l \)

This gives the gradient \( k = \frac{1}{B} \frac{\Delta B}{\Delta \lambda} \) with the length \( \Delta l \), then the band width is

\[ \langle \Delta \nu_x \rangle_{\text{rms}} \approx \frac{2}{4\pi} \frac{B}{B_S} \sqrt{N_F \beta_x^2 + N_D \beta_x^2} < \Delta l >_{\text{rms}} \]
\[ \approx 8.6 < \Delta l >_{\text{rms}} \]

For \( \langle \Delta \nu \rangle_{\text{rms}} \leq 0.005 \) we need

\[ < \Delta l >_{\text{rms}} \leq 0.6 \text{ mm} \]  

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5
2-4 LONGITUDINAL SHIFT: $\Delta \lambda$

This gives the field $k = \frac{B'}{B_3}$ and the length $\Delta \lambda$ with inverse sign in the both ends of the magnet. Then the width is

$$<\delta \nu^2_{y,\text{rms}} > \approx \frac{2}{4\pi} \frac{B'}{B_3} \sqrt{N_F(\Delta \beta y^2)+N_B(\Delta \beta^2 y^2)} <\Delta \lambda> \text{rms}$$

$$\approx 0.2 <\Delta \lambda> \text{rms}$$

where $\Delta \beta$ is the difference of $\beta$ between the two ends. Then we need

$$<\Delta \lambda> \text{rms} \leq 25 \text{ mm} \quad (10)$$

2-5 TILT: $\theta$

Since the potential is mixed with the skew component $X$

$$\phi(x,y) = B'xy + \theta B'(x^2-y^2)$$

we have an xy coupling. The width of the sum resonance is given by

$$|\delta \nu| = \frac{C_n}{\sqrt{2 \nu_x \nu_y}}$$

For uncorrelated tilt of the magnets, we have

$$<C_n> \text{rms} \approx \frac{2F}{\sqrt{N_\theta}} \frac{R^2}{B_3} \beta <\theta> \text{rms}$$

Hence

$$|\delta \nu| \approx 2.7 <\theta> \text{rms}$$

where $F (= 6.9 \times 10^{-2})$ is the filling factor of the quadrupole magnets in the circumference. For $|\delta \nu| \leq 0.005$ we need

$$<\theta> \text{rms} \leq 2 \text{ mrad} \quad (11)$$

REFERENCE

1. L.C. Tang; FN-183