$\bar{P}$ Source Note #57
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ASSYMPTOTIC DISTRIBUTIONS FOR STOCHASTIC MOMENTUM COOLING IN THE ELECTRON COOLING RING

Recent LBL cooling experiments in the Electron Cooling Ring appear to have achieved momentum cooling using the CERN notch-filter approach. It was, however, necessary to expand the linac-injected beam momentum spread (by the use of momentary rf bunching) to observe cooling. This note summarizes some calculations regarding expected assymptotic distributions using system parameters similar to those of the cooling system during the tests.

The assymptotic distribution $\Psi(E)$, can be determined from the condition that the flux, $\Phi(E)$, be constant and equal zero. That is:

$$\Phi(E) = \alpha \Psi + (\beta_1 + \beta_2 \Psi) \frac{\partial \Psi}{\partial E}$$

where

$$\alpha \equiv 2e f_o^2 Rm \sum_n Re_n G(E)$$

$$\beta_1 \equiv 2k T f_o^2 R_\kappa m \sum_n |G_n(E)|^2$$

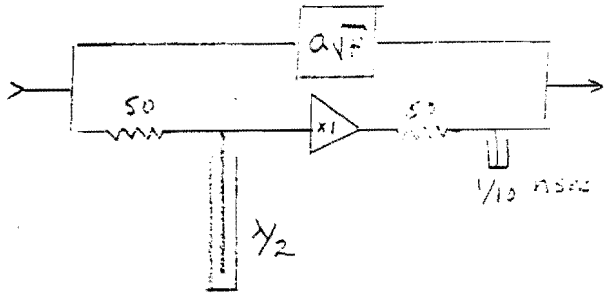
$$\beta_2 = \frac{2ef_o^2 R_\kappa m}{\eta} \sum_n \frac{|G_n(E)|^2}{n}$$

and m = effective number of pick-ups (See comment later)

$R \equiv \sqrt{R_{\rho U} R_\kappa}$ (85 Ω assumed for each)

$G_n \equiv$ Voltage transfer function of amplifer plus filter system

The model used for the filter was a shorted $\lambda/2$ stub followed by a short differentiating stub.



where

$a = -6 \times 10^{-6}$

Loss in λ_2 stb = $1.4 \times 10^{-6} \sqrt{f}$ db/m

Assumed band width = 150 MHz (50-200)

The attached figures indicate some of the results. The "tails" in the distribution are a consequence of the thermal noise in the system. Reducing the noise to zero produces no-tail solutions, just as an analytical solution (no noise, perfect filter) predicts.

The uncompensated filter case for 25-30 watts output is in reasonable agreement with observations (as I recall) during the recent experiments. Another conclusion is that passive compensation of the filter loss can produce about a factor of three less momentum spread, all other things equal, in the set-up now installed in the ECR.

I made several simplifying assumptions in the calculations. In particular, I let m be constant, 16, and used a center-of-bandwidth wavelength of 0.7 meters and a kicker length of 2 meters to estimate the power. I suspect the powers indicated on the curves may be estimated a bit too low since the kicker length is somewhat less than 2 meters. Anyway------

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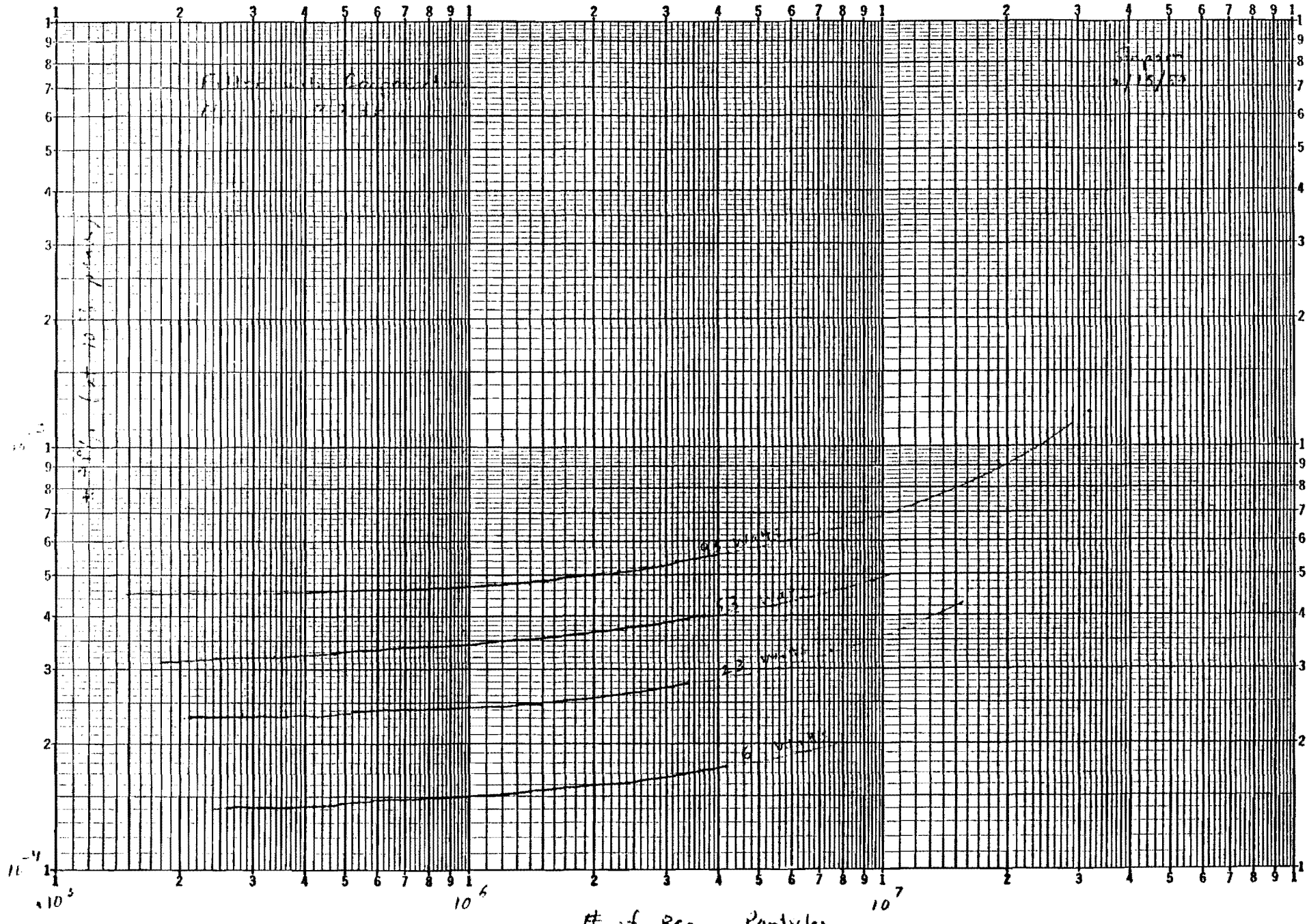

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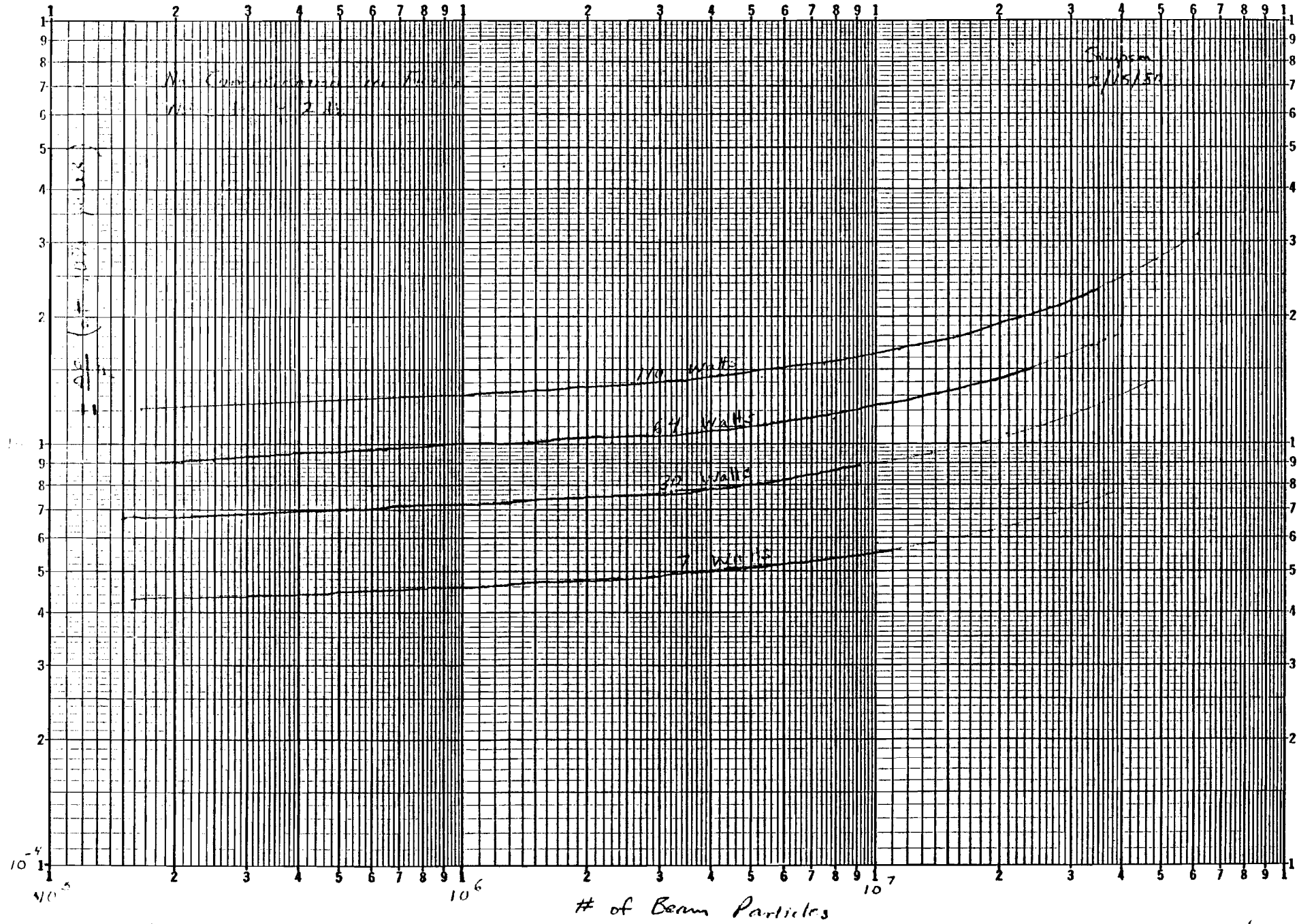

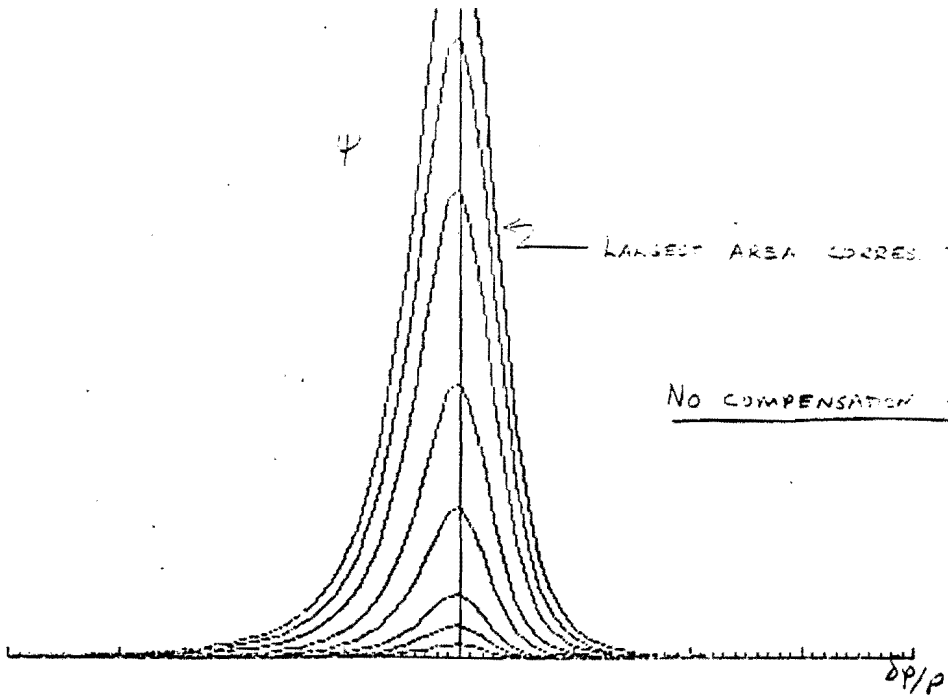

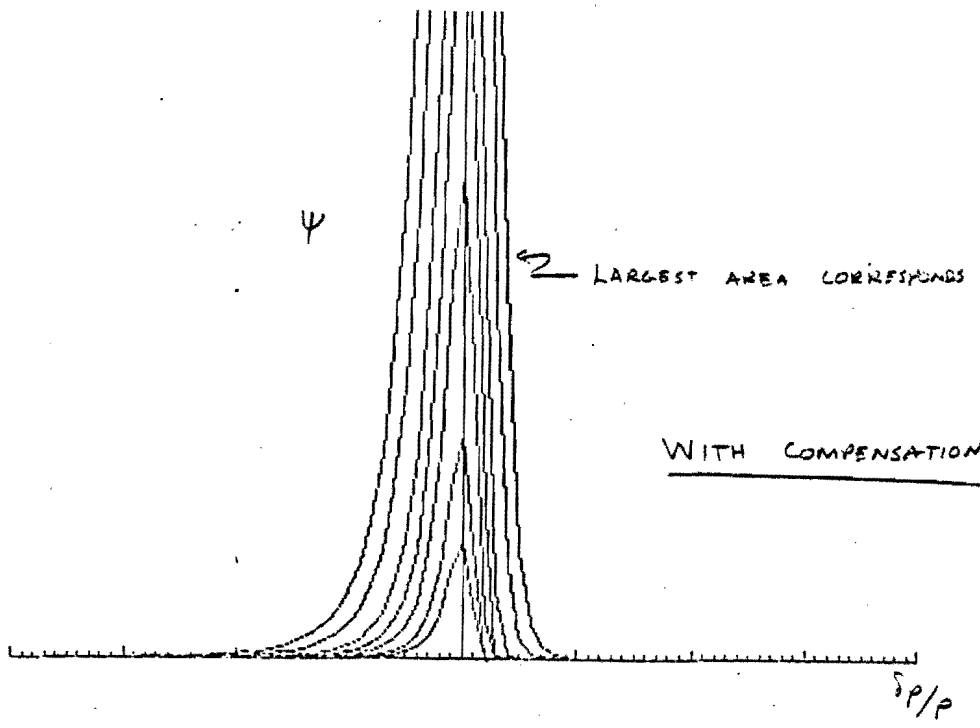

FIGURES SHOWING QUALITATIVE EFFECTS OF FILTER COMPENSATION